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Entropy-based Serviceability Assessment of Water Distribution Networks, Subjected to Natural and Man-made Hazards

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ABSTRACT

In this study a modified entropy-based measure is presented for evaluating the serviceability level of water distribution networks in which the hydraulic uncertainties, such as flow rates in pipes, as well as the uncertainties due to mechanical parameters, like failure probabilities of links, are considered simultaneously. In the proposed entropy calculation method, the connectivity order of the network demand nodes is incorporated in the entropy calculations by defining a factor based on the ratio of the nodal demand to the total flow rates of all links of the network. The failure probability of the network links has been incorporated using a penalty function based on their failure probability in any specified hazard scenario. Then, this penalty function is inserted satisfactorily in the existing hydraulic entropy function (defined by previous researchers) of the network. In this way, the effect of mechanical behavior of links is also taken into account in the hydraulic entropy function of the network, while keeping its simplicity and applicability. By calculating the entropy values of some sample networks, it has been shown that the proposed entropy-based index is an efficient tool to find the optimum hydraulic layout for designing a new system, or to make decision on the best mitigation plan for an existing network subjected to different natural and man-made hazards.

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1. INTRODUCTION

A water distribution system is a network of source nodes, pipes, demand nodes and other hydraulic components such as pumps, valves and tanks. The objective of the water distribution system is to supply water at a sufficient pressure and quantity to all its users, including water for the purpose of fire-fighting. Quantification of water distribution networks' reliability, as a lifeline system whose failure causes serious social, economical and environment consequences, has been considered as one of the most important research topics in risk management in the past decades [1, 2] as well as the water system maintenance [3]. Reliability of a water distribution network can be defined as the probability that the given demand nodes in the system receive sufficient supply with satisfactory pressure head [4]. There are several measures of reliability for water distribution networks, proposed by various researchers [5-9]. But, one of the reasons that reliability has not become a common phase in design practice yet is its complexity [10].

Redundancy, on the other hand, in a water distribution network implies the reserve capacity of the network, and that the demand nodes have alternative supply paths in the event if links fail in presenting the desired service [11, 12]. Redundancy, which is related to reliability, is an aspect of the overall system performance that is often neglected. A redundant network is inherently very pleased and reliable. Seismic performance of lifeline networks during the past earthquakes have turned out that a single redundancy can provide a remarkable increase in the system reliability. In other words, networks with some amount of redundancy have much higher capacity to respond to partial failure in the network [13]. Thus, redundancy can be considered as a surrogate measure for the reliability of water distribution networks. Tanyimboh and Templeman [14, 15] improved the preliminary idea expressed by Awumah and his colleagues [11, 12] and proposed a better definition of the entropy function for water distribution networks. To find the maximum-entropy flow distribution, Tanyimboh and Templeman developed a non-iterative algorithm for single-source

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networks. The only known data are assumed to be the network topology, the flow directions in each pipe, and the supplies and demands at each node. As the parameters such as pipe length, diameter and roughness are not known, there will be an almost infinite number of possible flow distributions unless the network is of the branching-tree type. They formulated their non-iterative algorithm using the path entropy concept and Laplace's principle of insufficient reason. They also tried to extend the single-source algorithm to cover multiple-source networks using the super-source idea. But this extended version of the single-source algorithm for multiple-source networks was shown to be inconsistent in a discussion presented by Walters [16]. Based on the single-source algorithm, Yassin-Kassab et al. [17] presented a non-iterative algorithm for calculating the maximum-entropy flow distribution in multiple source networks. The more delicate definition of the entropy function, based on Tanyimboh and Templeman, is presented in the next section of the paper with a discussion on its interpretation.

Later the relationship between the entropy and reliability of water distribution network was investigated by Tanyimboh and Templeman [18]. That study supports the hypothesis that water distribution networks, designed to carry the maximum entropy flows, will be more reliable. Further studies by Tanyimboh and Sheahan [19] explored the possibility of optimizing the layout of water distribution systems by using a minimum-cost/maximum-entropy design concept. In that research they tried on advancing the entropy flows in water distribution network to the stage where applications are possible, but the actual interpretation of the meaning of network entropy has never been fully elucidated.

Ang and Jowitt [20, 21] investigated the concept of network entropy using a simple water distribution network. Their investigation was concentrated on the relationship between the total power dissipated by the water distribution network and the numerical value of the network's entropy. In another article by them [22], an alternative method to calculate the network entropy of water distribution systems was presented, which gives new insights into the concept of network entropy. That alternative method, termed the Path Entropy Method (PEM), offers a simpler explanation to the entropy of branching-tree networks and the maximum entropy of water distribution networks. The formulation of the PEM was based on the fact that the entropy of the water distribution network arises because of the different paths available to a water molecule to move from a super-source to a super-sink. More explanation about PEM is given in the next section of the paper.

It should be noted that for a water distribution system to be serviceable, only being connected to the source is not sufficient, but it is also necessary to ensure that a given node can fully supply its corresponding demand. That is why hydraulic calculation has to be included in determination of mechanical-hydraulic reliability. Previously defined redundancy indices for water distribution networks in the literature are generally based on only one of hydraulic or mechanical characteristics of the network and they do not consider both characteristics simultaneously in their calculations. This is while the network risk is highly affected by both of these characteristics. In this regards, the aim of this paper is to explore deficiencies of previous definitions for the entropy of water distribution networks and to present a new weighted entropy-based measure for assessing serviceability of water distribution networks considering both aforementioned characteristics of the system.

2. ENTROPY FUNCTION FOR WATER DISTRIBUTION NETWORKS

One of the most appropriate entropy functions for water distribution networks was defined by Tanyimboh and Templeman [14]. Their formulation of the entropy function mainly relied on Shannon's measure of uncertainty, which is the underlying principle of information theory. They assumed that the available information on the water distribution network were the topological layout, the supply and demand at all nodes, and the flow direction in each pipe segment. Flow direction in each pipe is critical, as there will be a maximum-entropy flow distribution for each set of flow directions. It is worth mentioning that length, diameter, and roughness of pipes are not used directly in their formulation. Unless the network has a branching structure, there will be a very large number of feasible flow patterns. They define network entropy function as:

$$\frac{S}{K} = S = S_0 + \sum_{n=1}^{N} P_n S_n \tag{1}$$

where S is the entropy defined by Shannon, N is the total number of nodes, and K is the Boltzman constant which is usually set to unity (it will be shown in this paper that this can be true only in special cases). The entropy of the external inflows, S_0, is represented by:

$$S_0 = -\sum_{i \in I} P_{0,i} \ln P_{0,i} \tag{2}$$

where I is the set of all source nodes,

$$P_{0,i} = \frac{q_{0,i}}{T_0} \tag{3}$$

where $q_{0,i}$ is the external inflow at source node i and T_0 is the total supply or demand. The second term in the entropy function consists of the outflow entropy at each node, S_n, weighted by the ratio, P_n, of the total inflow of each node to the total inflow of the whole network.

$$P_n = \frac{T_n}{T_0} \tag{4}$$

where T_n is the total inflow at node n. An important point in the definition of outflow is that it is inclusive of any demand at the node. In Equation (1) the outflow entropy at each node S_n is given by:

$$S_n = -\sum_{j \in ND_n} P_{n,j}\, ln\, P_{n,j} \tag{5}$$

where ND_n is the set of all outflows from node n, and

$$P_{n,j} = \frac{q_{n,j}}{T_n} \tag{6}$$

where $q_{n,j}$ is the flow from node n to node j.

Entropy function, given by Equation (1) shows that the entropy of a water distribution network has two components. The first part is the amount of entropy in the external inflows and the second part consists of the weighted entropy values at every demand node. Informational entropy measures the amount of uncertainty in a situation or system. For a water distribution network, the uncertainty can be imagined from the viewpoint of a water molecule. Now it is tried to illustrate the concept of entropy function using the simple example of water distribution network, shown in Figure 1. The water distribution network, shown in Figure 1, has a single source node and three demand nodes and the flows are assigned to the links in such a way that the entropy value of the network becomes maximum. The entropy of the external inflows S_0 is in fact the uncertainty faced by a water molecule moving from the super-source to the individual supply nodes. For all nodes, the entropy would be non-zero only if there are two or more paths for the water molecule to take at each node. However, the entropy S_n, calculated for each node n, is the probability of entering each of the pipes connected to that node for the water molecule arriving at that node, which is expressed by the P_n term in Equation (1). Details of entropy calculation of the sample network with its tree diagram are shown in Figure 2. From the above-mentioned definition, it is clear that the entropy of a water distribution network can be represented by the number of paths available for a water molecule moving from the super-source to the super-sink (see Figure 3). Based on this observation, an alternative way of calculating network entropy is the path entropy method (PEM) [22]. The diagram of applying PEM to the sample network and its entropy calculation are shown in Figure 3. In Figure 3, the number of paths from the super-source to the super-sink and the amount of flow in each path are shown. Development of the PEM diagram includes two main steps. The first step is to establish the number of paths from the source nodes to each demand node and drawing the PEM diagram with all nodes and links. The second step involves determining the flow carried by each link, which is performed by an inspection of the flow rates in all of the network links. Once the PEM diagram is developed, the calculation of the network entropy is relatively straightforward, as compared to the network entropy equations by Tanyimboh and Templeman [14].

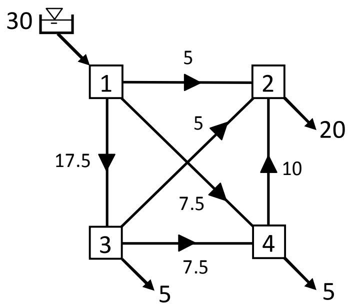
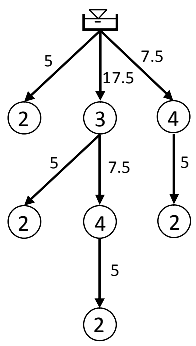
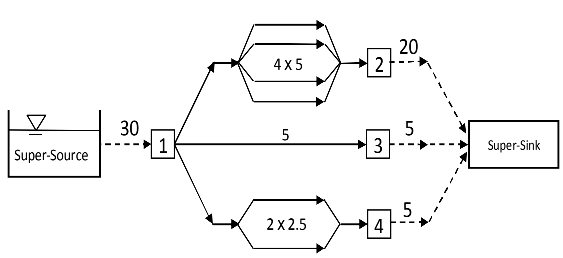


Figure 1. Schematic diagram of the physical system sample, fully–connected network with maximum network entropy based on Equation (1).



$$\begin{aligned} S = &-\frac{30}{30}\times\left[\frac{5}{30}ln\left(\frac{5}{30}\right)+\frac{17.5}{30}ln\left(\frac{17.5}{30}\right)+\frac{7.5}{30}ln\left(\frac{7.5}{30}\right)\right] \\ &-\frac{17.5}{30}\times\left[2\times\frac{5}{17.5}ln\left(\frac{5}{17.5}\right)+\frac{7.5}{30}ln\left(\frac{7.5}{17.5}\right)\right] \\ &-\frac{15}{30}\times\left[\frac{5}{15}ln\left(\frac{5}{15}\right)+\frac{10}{30}ln\left(\frac{10}{15}\right)\right]=1.9073 \end{aligned}$$

Figure 2. Tree diagram of sample network, shown in Figure 1, with entropy calculation.



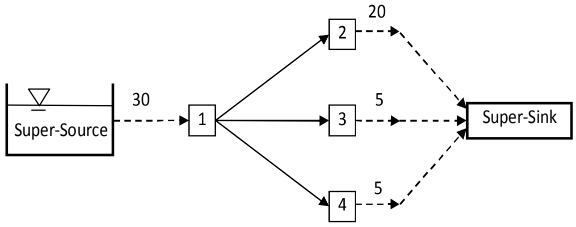
$$S = -4\times\frac{5}{30}\ln\left(\frac{5}{30}\right)-1\times\frac{5}{30}\ln\left(\frac{5}{30}\right)-2\times\frac{2.5}{30}\ln\left(\frac{2.5}{30}\right)=1.9073$$

Figure 3. PEM diagram of sample network with entropy calculation.

However, it must be noted that the less complicated entropy calculation is a result of the efforts spent in organizing the data into a PEM diagram. The true strength of the PEM lies in its ability to give new insights into the meaning of the network entropy, such as the entropy of branching-tree networks and the maximum-entropy flows of a single-source network with given flow directions, which will be discussed in the next section.

3. DISCUSSION ON TANYIMBOH AND TEMPLEMAN'S DEFENITION OF ENTROPY FUNCTION

In a discussion presented by Walters [16], it has been stated that all of the trees, each of which connects all of the demand nodes to the source in a network, have the same minimum entropy value. Afterwards, Ang and Jowitt [22] showed this fact by path entropy method. Figure 4 shows all different layouts of branching-tree networks, related to the sample network, shown in Figure 1. As it is seen in Figure 4, in all of the shown layouts there is only one path from the source node to each demand node, and therefore, from the informational point of view, all of them have essentially the same entropy. The PEM diagram for the branching-tree sample network is shown in Figure 5, which can be used for representing any of the different layouts.



$$S = -\frac{20}{30}\ln\left(\frac{20}{30}\right) - 2\times\frac{5}{30}\ln\left(\frac{5}{30}\right) = 0.8676$$

Figure 5. PEM diagram of the tree-branching networks

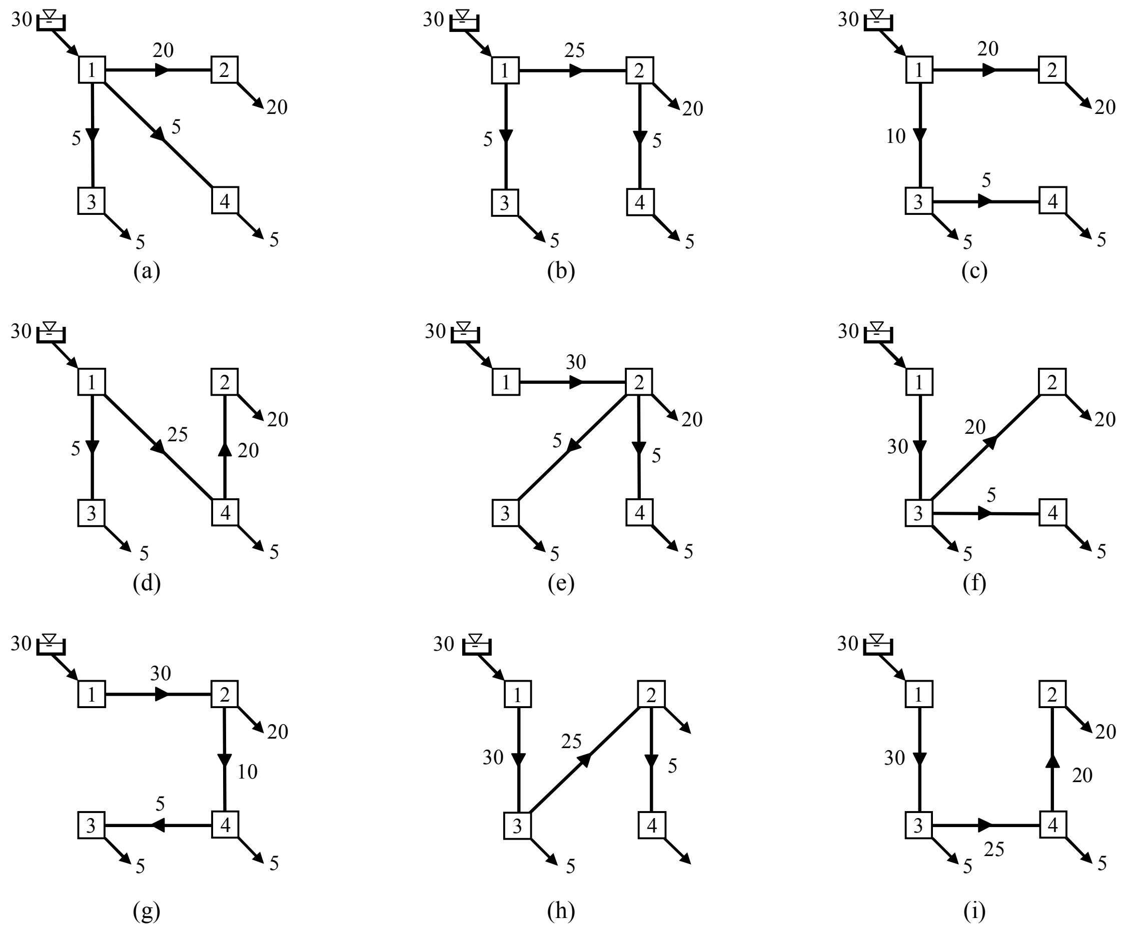


Figure 4. Various branching-tree networks of the sample fully-connected network shown in Figure 1

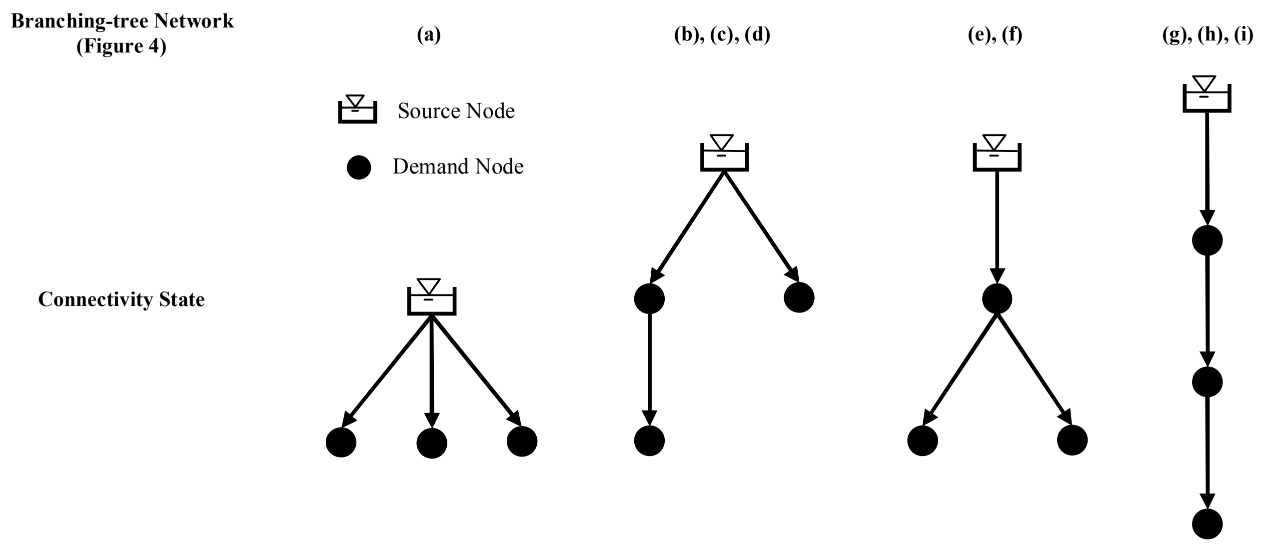


Figure 6. Different supply-demand connectivity states of the branching-tree networks shown in Figure 4.

In all of these networks, a water molecule moving from the super-source to the super-sink is only uncertain about the demand node it would arrive. However, when there is just one arriving link to a demand node, which is the case for a branching-tree network, the uncertainty disappears. Thus, Tanyimboh and Templeman's definition of entropy function cannot consider any difference between branching-tree networks with different layouts, but same number of supply and demand nodes (like networks shown in Figure 4), which all have the same PEM diagram as shown in Figure 5. However, with a cursory look at these networks, it can be easily seen that some of them are more sensitive than the others which damages their links.

For instance, if the link 1-3 in networks (f), (c) and (d) in Figure 4 gets damaged due to any hazards like earthquake, the amount of loss would be different, that is 30, 10 and 5 l/s, respectively. Therefore, the amount of loss of service in a network depends not only on its main configuration as parallel or series, but also on the connectivity order of different demand nodes to the supply node. Consideration of this factor can improve evaluation of networks' reliability. For better recognition of the main difference between the sensitivity states of various branching-tree networks, the networks shown in Figure 4 are looked at more deeply. These networks can be divided into four major categories based on their connectivity state as shown in Figure 6.

As it is seen in Figure 4, in network (a) each of the demand nodes is connected to the supply node by a direct or independent link, while in networks (b), (c) and (d) two demand nodes are directly connected to the supply node, and the third one is connected indirectly, via two links in series. In (e) and (f) networks just one demand node is directly connected to the supply node, and the other two nodes are connected indirectly via two links.

Finally, in networks (g), (h), (i) one demand node is directly connected to the supply node, the second one is connected via two links, and the third one is connected via three links. It is obvious that the first connectivity state has the greatest redundancy, or the least sensitivity, because of independent paths between each demand node to the supply node, which means that failure of any link, affects only the serviceability of one demand node. Conversely, the fourth connectivity state is the most vulnerable state, with no redundancy, since the serviceability levels of its various demand nodes are highly dependent.

Furthermore, it should be noted that although all networks falling in each of the connectivity state categories shown in Figure 4 have the same connectivity dependency, their reliability levels, are not the same because of the different connectivity orders of their demand nodes to the supply node. For example, refereeing to Figure 4, if the link between demand nodes 2 and 4 in network (b) is cut, the network loss of service will be only 5 l/s, while the same cut in network (d), which has the same sensitivity state as network (b), will result in 20 l/s loss of service.

The last point which should be taken into consideration for serviceability evaluation of water supply networks subjected to any kind of natural or man-made hazard is the vulnerability of its components subjected to that hazard. This has not been addressed in the previous studies on the serviceability evaluation of networks based on the entropy concept. In the next sections of the paper some modifications are proposed for resolving the mentioned shortcomings.

4. INCORPORATING THE CONNECTIVITY STATE AND ORDER IN THE NETWORK SERVICEABILITY EVALUATION

To add the effect of connectivity state and order which in fact determines the sensitivity state of a network, the authors have defined a penalty factor (Tp) for each link as the total amount of loss, if that link is cut. The value of this factor for each link shows, in some way, the sensitivity of the network to the cut of that link. Based on that definition a new weighting factor ($P_n^{'}$) has been introduced as:

$$P_n^{'} = \frac{T_n}{T_{p0}} \tag{7}$$

where T_{p0} is the summation of penalty factors for all links in the network. This weighting factor has been used instead of the previous one, P_n in Equation (1), in the calculation of networks' entropy values. These penalty factors could be modified by some other factors like the importance of demand nodes (for example, the existence of a fire fighting valve).

The mentioned modification can make a distinction between the networks with different patterns but the same entropy values, however, the network's links failure probabilities are not included yet in the entropy calculation, and consequently, the network reliability evaluation. For example, two different networks with the same patterns (like pattern (b) in Figure 6) but different link failure probability have identical entropy values, while they do not have the same reliabilities. The entropy values, calculated by the modified formula, based on Equation (7), for all of the branching-tree networks, shown in Figure 4, are given in Table 1.

As it can be seen in Table 1, only network (a) has a similar value with the former definition of entropy and it is because in this network each of the demand nodes has an independent link to the supply node. The entropy value decreases by reduction in path independency for different demand nodes while connection of the nodes with higher demand to the supply node by more intermediate links can decrease entropy drastically (like networks (e) and (f)). The other interesting point here is that network (g) has higher entropy value than networks (f) and (d), in spite of the fact that demand nodes of network (g) are completely in series, while in networks (f) and (d) the demand nodes are partially in series. These results show that the network vulnerability depends not only on the network pattern as series or parallel, but also on the connectivity order of its demand nodes.

It can be shown, mathematically (see Appendix 1), that the suggested weighting factor, given by Equation (7), behaves like a new Boltzman's constant for a single supply network which is obtained from the following equation:

TABLE 1. Entropy values for branching-tree networks shown in Figure 4 using new weighting factor (Equation (7))

Network	**a**	**b**	**c**
Entropy value	0.8676	0.7436	0.7436
Network	**d**	**e**	**f**
Entropy value	0.5206	0.6507	0.4732
Network	**g**	**h**	**i**
Entropy value	0.5784	0.4338	0.3470

$$K = \frac{T_o}{T_{p0}} \tag{8}$$

For example, entropy of network (c) using pass entropy method is obtained as follows:

$$S = -\frac{30}{35} \times \left[\frac{20}{30} In\left(\frac{20}{30}\right) + 2 \times \frac{5}{30} In\left(\frac{5}{30}\right) \right] = 0.7436$$

The Boltzman's constant, given by Equation (8), is equal to unity only when the total supply (T_0) is equal to sum of penalty numbers (T_{p0}). It means that the unit Boltzman's constant can be used only when all demand nodes are connected to the supply node independently, and this constant decreases with growth of penalty factors. The new Boltzman's constant gives a better illustration for serviceability of the network.

5. INCORPORATING THE PROBABILITY OF FAILURE IN THE NETWORK SERVICEABILITY EVALUATION

As mentioned in the preceding section, although Tanyimboh and Templeman's entropy function for water distribution networks has its benefits and simplicity, but it cannot identify different patterns as well as link-failure probability. This is while the network's risk is highly affected by both hydraulic and mechanical characteristics of a system. Thus, a new weighted entropy function is presented here which can consider both aforementioned characteristics of the system in its formulation, while keeping simplicity of the pervious definition. For this purpose, a failure penalty function is defined for different links of the network based on their probability of failure in the specified hazard scenario. This failure penalty function is incorporated in the hydraulic entropy function of the network in an appropriate manner so that the effect of mechanical behaviour of links is considered in the network's entropy. In this manner the amount of supply loss due to absence of each link in the network is taken into account by corresponding failure penalty function. The modified entropy function for water distribution network is defined as:

$$\frac{S_N}{K} = S_N = S_0 + \sum_{n=1}^{N} P_n^{'} S_n - ln(\varepsilon) \quad (9)$$

where S_N is the new entropy value, and the other parameters are same as in Equation (1), except S_n which is calculated by the following equation:

$$S_n = -\sum_{j \in ND_n} P_{n,j}\, ln\left(P_{n,j}/\left(1 - Pf_{n,j}\right)\right) \quad (10)$$

where $P_{n,j}$ is obtained from Equation (6). $Pf_{n,j}$ is the failure probability of the link between node n and node j, which can be obtained using analytical failure estimation of a specified scenario or using expert judgement. The term $-ln(\varepsilon)$ in Equation (10) has been considered to prevent creation of negative entropy values. In fact, the failure probability of links cannot be considered equal to 1 in Equation (10) since the denominator will be zero in that case. Therefore, it is assumed that the failure probability of a definitely damaged link is equal to 1-ε instead of 1; ε being a small value between zero and one, such as 0.01. It should be noted that adding this biased value to the proposed entropy function does not affect its concept, because the entropy function is a comparative index. It can be shown that the minimum value of S_n will be equal to zero (see Appendix 2). Moreover, in the proposed entropy function it is assumed, like the previous studies that when a link is in failure state, it is completely nonoperational and no water molecule can reach the demand node from that link. In the other words, leakage state is not considered here.

In order to investigate the behaviour of the proposed entropy function, a simple network is considered with one supply and one demand node and two parallel links in which failure probability of links are P_{f1} and P_{f2}. In this network a water molecule has only two choices, P_1 is probability of selecting the first link and P_2 is probability of selecting the second one. This network is shown schematically in Figure 7a and its Venn's diagram is shown in Figure 7b.

The behavior of the proposed entropy function of this network in shown in Figure 8 in two different states: a) when all links of the network have the same failure probability, and b) when the network links have different failure probabilities.

As it is seen in Figure 8a, behaviour of the proposed entropy function is quite similar to Tanyimboh and Templeman's function (the dashed curve in the figure), when failure probability of all links is maximum (0.99). If the network's links have the same failure probability, but less than maximum, the proposed entropy function behaves as before, and its maximum point does not change, but its values are shifted up in over its whole range by some value which depends on the failure probability of its links. However, if the network's links have different failure probabilities the proposed entropy function will not behave as before, and its maximum will occur when the link with lower failure probability have more flow than the link with higher failure probability. Mathematically, the probability of link i being operational which is defined as follow:

$$Po_i = (1 - Pf_i) \quad (11)$$

the maximum entropy of the network with two parallel links and one demand node will be obtained when the ratio of flow in link 1 to the total network inflow is:

$$x_{max} = \frac{Po_1}{Po_1 + Po_2} \quad (12)$$

Equation (12) shows that when the failure probabilities of the two links of the network are the same, the maximum entropy will be achieved when both links carry the same amount of flow, but unequal failure probabilities of links will result in other flow ratios. For example, Figure 8b compares the state of equal failure probabilities of links (the solid curve) with two other states of failure probabilities, which results in unequal flow ratios in the network's links (dashed and dotted curves) for achieving the maximum entropy.

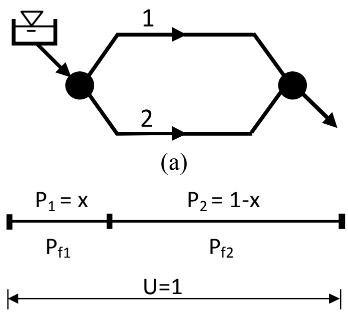
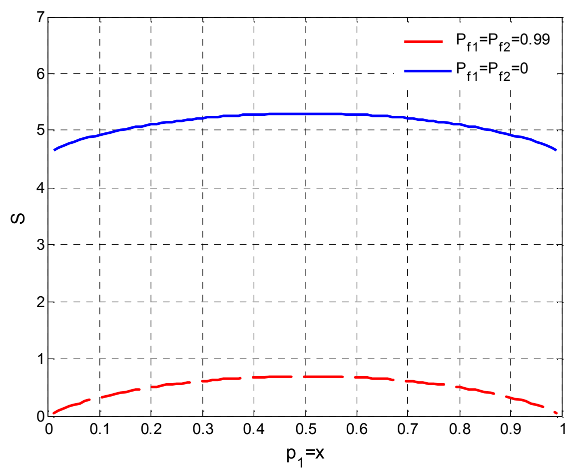
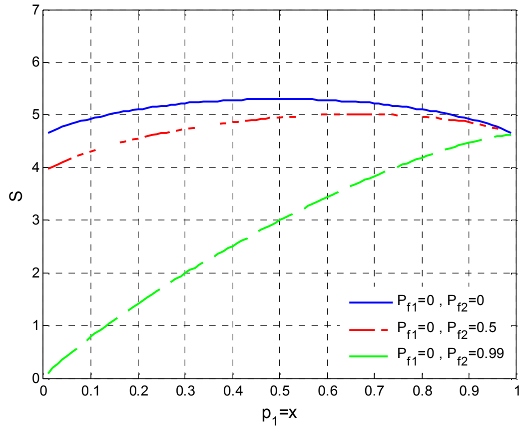


Figure 7. The 2-link sample network and its Venn's diagram

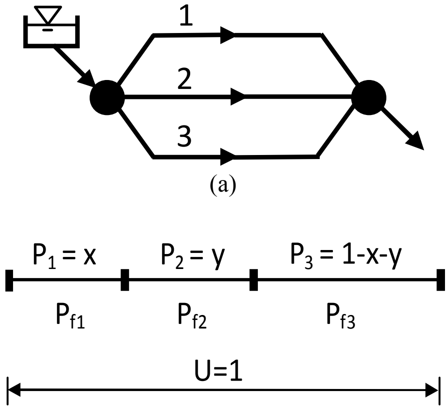


(a)



(b)

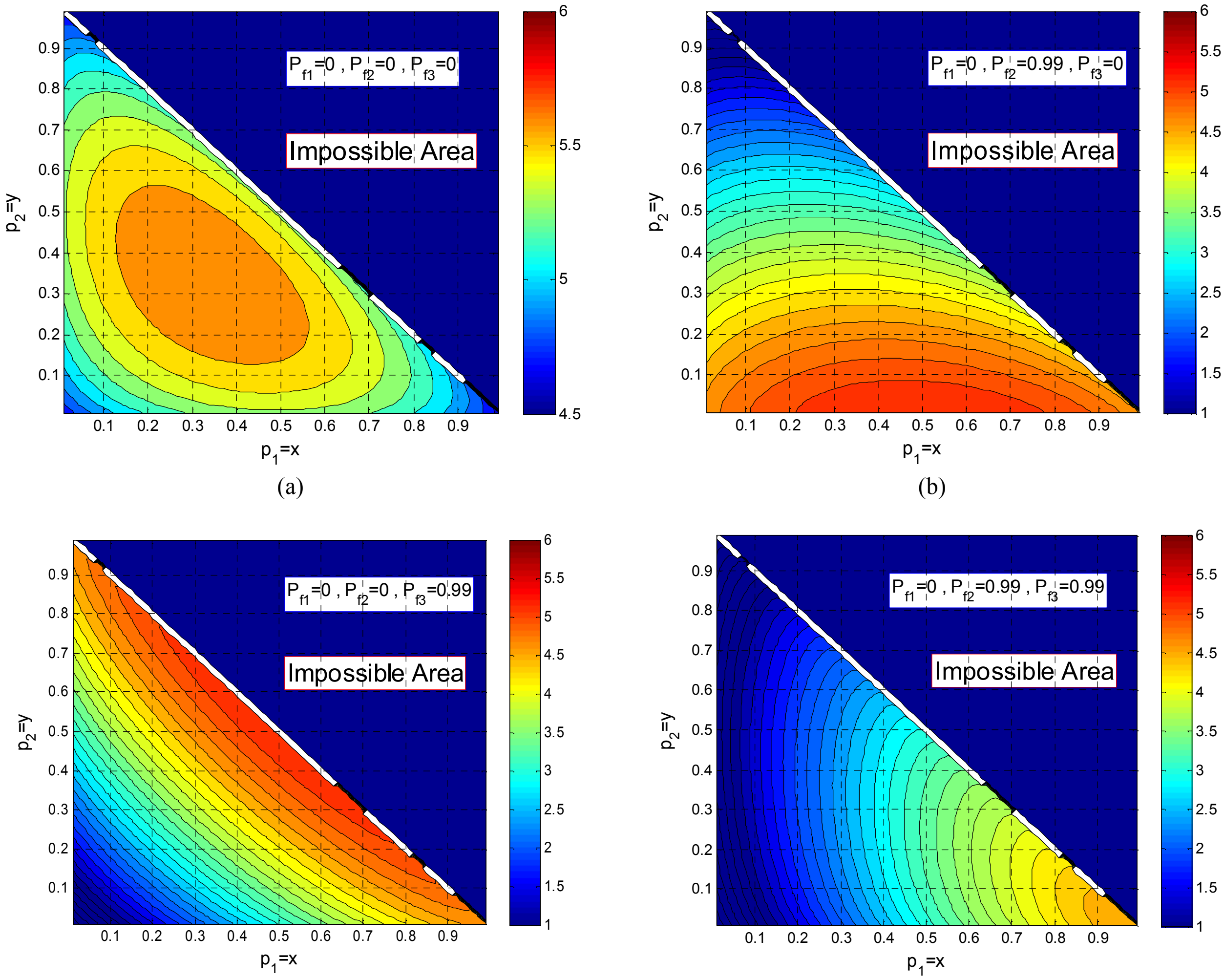
Figure 8. The proposed entropy function diagrams for the sample network with 2 links in two states of (a) the same failure probability of links, and (b) different failure probabilities of links (ε=0.01)



(a)

(b)

Figure 9. The 3-link sample network and its Venn's diagram



(a) (b)

(c) (d)

Figure 10. Variation of the proposed entropy function for the sample network with three parallel links, shown in Figure 9 (ε=0.01)

As another sample, Figure 9 shows a network with one supply and one demand node and three parallel links in which failure probability of links are P_{f1}, P_{f2} and P_{f3}. In this network a water molecule has only three choices, P_1 is probability of selecting the link 1, P_2 is probability of selecting link 2, and obviously, probability of selecting link 3 will be 1-P_1-P_2. This network is shown schematically in Figure 9a and its Venn's diagram in Figure 9b.

Figure 10 shows the results of entropy calculations for the sample network shown in Figure 9, in which values of P_1 and P_2 are shown in the two horizontal and vertical axes, respectively, and variation of the entropy values are shown using contours.

As it is observed in Figure 10a, when all links have the same failure probability the flow ratio in all of them is the same as 0.33. But, as shown in Figure 10 b and c, when two links have the same failure probability of zero, and the third one has the failure probability of 0.99 (breakage for ε=0.01) the flow ratio for the two intact links is 0.5, and that of the fully damaged link is zero. Finally, Figure 10d shows the state in which two links have the maximum failure probability of 0.99, and the third one is intact. Therefore, the maximum entropy is achieved when the flow ratio for the first two links is zero and that of the intact link is 1.

It should be notified that in the above examples due to the parallel configuration of the networks T_{p0}=T_p (see Equation (4) and Equation (7)) use of either P or P' in Equation (9) does not affect the entropy values. However, if the network has some links in series, then $T_{p0}\neq T_p$, and therefore, use of P in Equation (9) leads to different results from those obtained by P' in that equation, as illustrated in the next example. In this example, there are two links in series (see Figure 11), and two states of connectivity order have been considered for it based on the demand ratio in its nodes.

Since this network is of branching-tree type, its entropy calculated by Tanyimboh and Templeman's formula (Equation (1)) is single value of 0.6365. This is while using the proposed formula, in which the total flow in all branches of the network as well as the failure probability of its links are taken into account (Equation (9)), the entropy values are obtained as given in Table 2.

As it is observed in Table 2, with increase in the failure probability of links the entropy value decreases. Also when the node with higher demand is closer to the supply node, the entropy value is higher. It is reminded that the entropy values obtained by Equation (9) are much larger than those obtained by Equation (1) due to the existence of term $ln(\varepsilon)$. However, as the absolute values of entropy are not important in each case, and are considered comparatively, this difference does not matter.

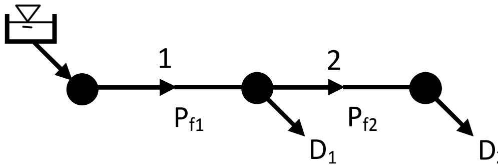


Figure 11. The sample network with two links in series

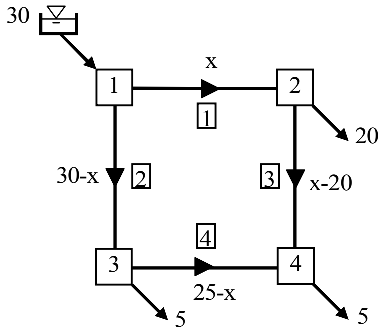


Figure 12. Sample water distribution network with one source and three demand

TABLE 2. Entropy value (S_n) of the sample network shown in Figure 11 (ε=0.01)

	D_1/D_2=2	D_1/D_2=0.5
P_{f1}= P_{f2}=0	5.0826	4.9871
P_{f1}= P_{f2}=0.50	4.3894	4.2939
P_{f1}= P_{f2}=0.75	3.6963	3.6008
P_{f1}=0; P_{f2}=0.50	4.9093	4.7098

6. APPLICATION TO A SAMPLE LOOP NETWORK

To show how the proposed modifications affect the entropy values, they have been applied to the water distribution network example presented by Ang and Jowitt [20], and the results have been compared with those obtained by the conventional Tanyimboh and Templeman's method. The aforementioned example network has one supply and three demand nodes, as shown in Figure 12, in which one of the nodes has greater demand than the other two ones.

It is supposed that there is no limitation on the flow rate and flow direction in the network's links, so that all possible patterns of the network could be considered. However, there are a total of three different sets of flow directions for the topological layout of the network (see Figure 12). If flow rate of the first link is assumed to be "x", the flow rate in the remaining links can be easily determined from the flow equilibrium at each node as shown in Figure 12. Thus, regarding the summation of all nodal demands and all possible flow directions from the supply node, flow rate of the first and the second link vary from zero to 30, the third link from -20 to 10, and that of the forth link from -5 to 25 (negative flow rate means that the flow direction is inverse to the assumed flow direction) (see Table 3).

Based on the entropy function proposed by Tanyimboh and Templeman (Equation (1)), entropy variation of the sample network versus x has been shown in Figure 13.

As it can be easily seen, the graph has four similar minimal values each of which belongs to one of the branching-tree sub-systems; and also there are three relative maximal values each of which belongs to a specific flow direction pattern. As expected, all the branching-tree networks have similar minimal entropy and all patterns have their relative maximal values, while absolute maximum occurs in the pattern in which the greater nodal demand has more flow paths to the supply node. This example also shows that although the flow entropy function proposed by Tanyimboh and Templeman is simple and efficient for investigating the serviceability of water distribution networks in normal conditions, but it cannot make distinction between various states of the network in which there are different failure probabilities of links, and also the sensitivity of the network to its link functionality. Entropy of the sample network based on the proposed equation has been shown in Figure 14.

TABLE 3. Possible flow rates for the links of the sample network shown in Figure 12

Link number	Start node	End node	Flow rate (Q_i)
1	1	2	$0 \leq x \leq 30$
2	1	3	$0 \leq 30\text{-}x \leq 30$
3	2	4	$-20 \leq x\text{-}20 \leq 10$
4	3	4	$-5 \leq 25\text{-}x \leq 25$

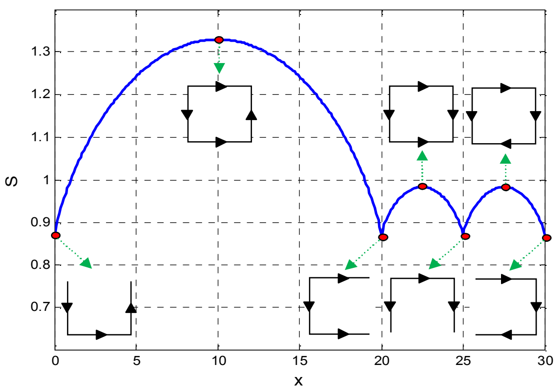
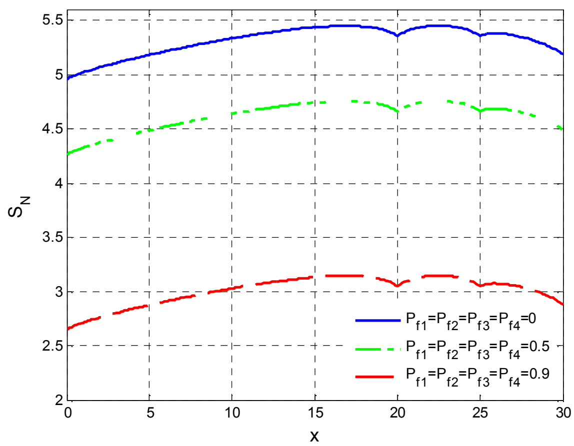
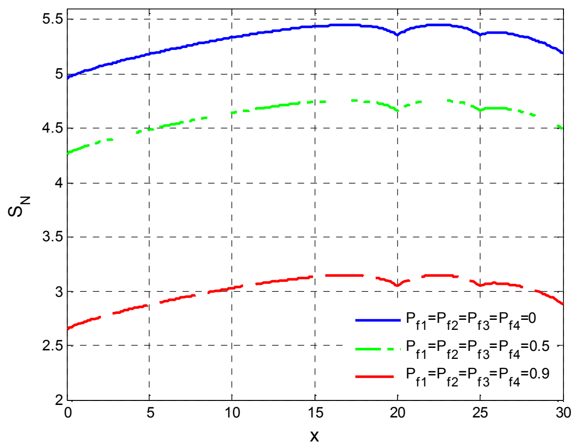


Figure 13. PEM diagram of sample network with entropy calculation. Entropy of the sample network versus flow rate at first link (x) based on the entropy function proposed by Tanyimboh and Templeman [14]

In Figure 14, part (a) shows the case in which all links have the same failure probability, and part (b) shows the case in which the failure probability of one link is 0.99 (almost cut for ε=0.01). As it is seen in Figure 14a, when the failure probability of all links is zero, the proposed function has a trend similar to that of Tanyimboh and Templeman's function, with some minor differences due to use of *P'* instead of *P* in Equation (1) (note that ln(ε) has been added to all values). For example, by using Tanyimboh and Templeman's formula, the entropy values for case of x=5 and x=15 are the same (see Figure 14), while by inclusion of connectivity order and failure probability of links (using Equation (9)) the entropy in case of x=5 is much lower that the case of x=15. Furthermore, the branching-tree network with x=0 (absence of link 1) has much lower entropy value than the branching-tree network with x=20.



(a)



(b)

Figure 14. Entropy values of the sample network based on the proposed entropy function; (a) all links have similar failure probability, (b) failure probability of one of the links is (1- ε), with ε=0.01

Figure 14a also shows that by using the modified formula (Equation (9)), it is possible that some branching-tree networks could be more reliable than loop networks. For example, consider the case in which x=20 (which means the absence of link 3, resulting in a branching-tree network with three links) has higher entropy value than the case in which x=5 (which means having a loop network with four links).

Sensitivity of the network's entropy to the damage of one of its links has been shown in Figure 14b. Since by elimination of a link, the network is converted to a branching-tree network, and also it is desired that all demand nodes be serviceable, the optimum pattern in this case will be a branching-tree pattern. In fact, the branching-tree pattern which has not the desired vulnerable link is not sensitive to the elimination of that link and by distancing from this pattern, network's entropy decrease more. This figure shows the importance of connectivity order of demand nodes in any given pattern. For example, links 2, 4, 3 and 1 are, respectively, the most important links in the pattern with x=5, while for pattern with x=15 this order will change to links 1 and 2 together, then 4 and finally 3. It is notable that both of these two patterns have the same T&T entropy value. Also, because of less decrease of network's entropy with elimination of one of links in pattern with x=15, this pattern is less vulnerable than pattern with x=5 to elimination of one link. In some patterns elimination of one link decreases the entropy value more considerably than the other links (like x=22.5 and 27.5), this represents that these networks have high risk to sabotage.

If instead of elimination of one link (i.e. the case in which the failure probability of that link is 0.99) the links are partially damaged (which is the case with failure probabilities less than 0.99) the variation of entropy values of the sample network will be as shown in Figure 15.

In cases shown in Figure 15, it is assumed that one of the links has a failure probability of 0.5. It is obvious that in these cases the maximum entropy is resulted by a situation which differs from the fully damaged situation.

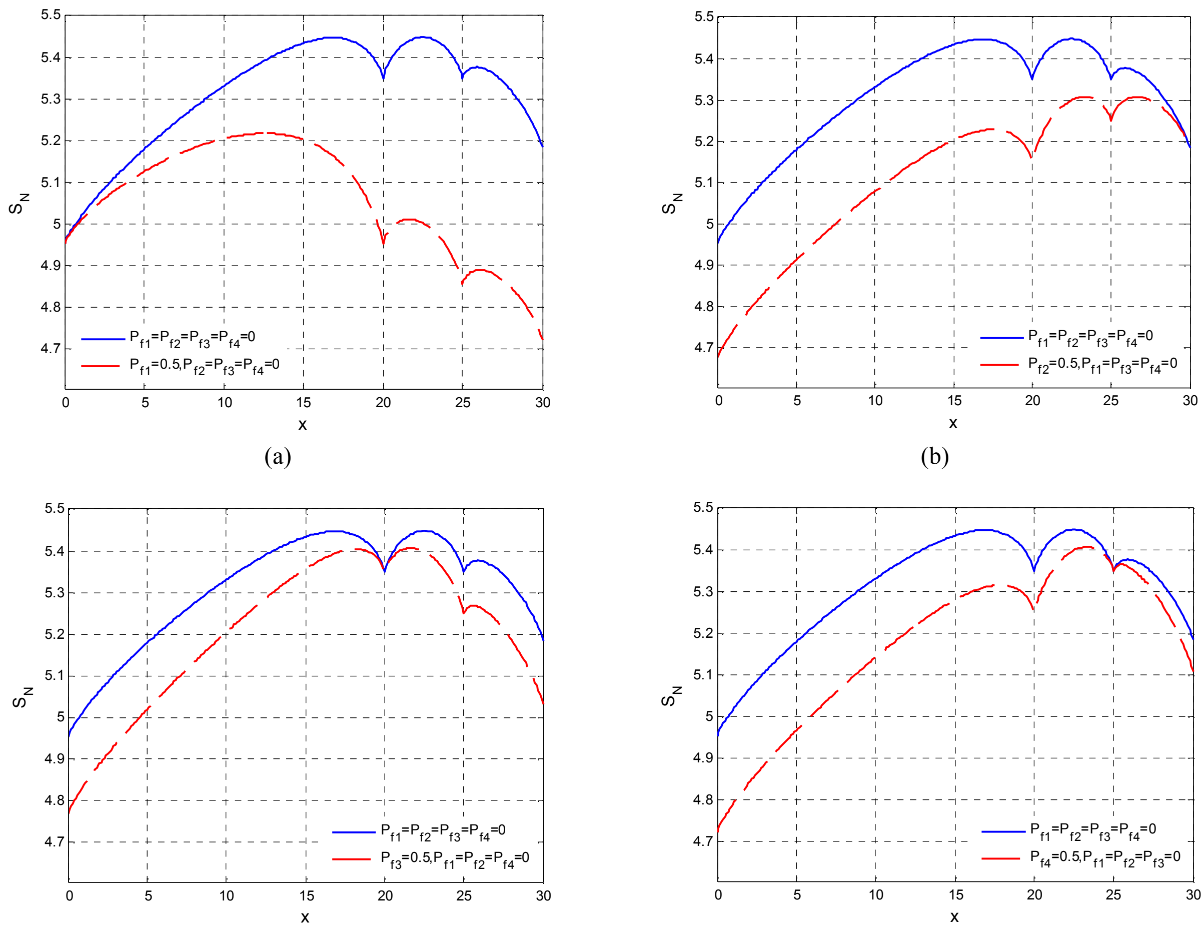


Figure 15. Entropy value of the sample network based on the proposed entropy function when only one of the network's link has partial failure probability; (a) only 1st link has partial failure probability, (b) only 2nd link has partial failure probability, (c) only 3rd link has partial failure probability and (d) only 4th link has partial failure probability

7. CONCLUSIONS

In this study after reviewing the existing method proposed by other researchers for calculating the entropy of water distribution networks, and using it as a serviceability index, two modifications have been proposed for improving the entropy-based serviceability evaluation of these networks. Based on the discussions and numerical calculations presented in the paper, it can be concluded that:

- The Tanyimboh and Templeman's entropy formulation is only based on hydraulic parameters of the network, and therefore, cannot take into account the effect of mechanical damages, due to either natural or man-made hazards.
- Incorporating the connectivity order in the entropy calculations shows that, as expected, if the demand nodes with larger demands are located closer to the supply node the serviceability of the network will be more reliable. This fact was not considered in the previous entropy calculations.
- Incorporating the failure probability of the network links in the entropy calculations again shows that, as expected, for achieving the larger serviceability level, the links with higher failure probabilities should have lower flow rate. This fact was not also considered in the previous entropy calculations.
- The major advantage of the previous entropy definition for water distribution networks is its simplicity. The proposed modified formula in this study, in addition to that advantage, has the capability of incorporating connectivity order of the network demand nodes as well as the failure probability of its links in the entropy calculations and serviceability evaluation. Therefore, it is recommended that the proposed entropy function is used for identifying the most important links of the network in different hazard scenarios, and selecting the optimum mitigation plan on that basis.

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APPENDIX 1

Mathematical proof that the suggested weighting factor, given by Equation (7), behaves like a new Boltzman's constant for a single supply network

$$\frac{S_N}{K} = S_0 + \sum_{n=1}^{N} P'_n S_n$$

$$= -\sum_{i\in I} P_{0,i} \ln P_{0,i} - \sum_{n=1}^{N} P'_n \sum_{j\in ND_n} P_{n,j} \ln P_{n,j}$$

$$= -\sum_{i\in I} \frac{q_{0,i}}{T_0} \ln\frac{q_{0,i}}{T_0} - \sum_{n=1}^{N} \frac{T_n}{T_{p0}} \sum_{j\in ND_n} \frac{q_{n,j}}{T_n} \ln\frac{q_{n,j}}{T_n}$$

$$= -\sum_{i\in I} \frac{q_{0,i}}{T_0} \ln\frac{q_{0,i}}{T_0} - \sum_{n=1}^{N} \frac{T_n}{T_{p0}} \times \frac{T_0}{T_0} \sum_{j\in ND_n} \frac{q_{n,j}}{T_n} \ln\frac{q_{n,j}}{T_n}$$

$$= -\sum_{i\in I} \frac{q_{0,i}}{T_0} \ln\frac{q_{0,i}}{T_0} - \frac{T_0}{T_{p0}} \sum_{n=1}^{N} \frac{T_n}{T_0} \sum_{j\in ND_n} \frac{q_{n,j}}{T_n} \ln\frac{q_{n,j}}{T_n}$$

$$= S_0 + \frac{T_0}{T_{p0}} \sum_{n=1}^{N} P_n S_n$$

for a network with single supply node:

$$S_0 = o \Rightarrow \frac{S_N}{K} = \frac{T_0}{T_{p0}} S \Rightarrow K = \frac{T_0}{T_{p0}}$$

APPENDIX 2

Mathematical proof that minimum value of S_N is equal to zero

$$S_N = S_0 + \sum_{n=1}^{N} P'_n S_n - \ln(\varepsilon)$$

$$= S_0 - \sum_{n=1}^{N} P'_n \sum_{j\in ND_n} P_{n,j} \ln\left(P_{n,j} / \left(1 - Pf_{n,j}\right)\right) - \ln(\varepsilon)$$

$$= S_0 - \sum_{n=1}^{N} P'_n \left[\sum_{j\in ND_n} P_{n,j} \ln P_{n,j} - \sum_{j\in ND_n} P_{n,j} \ln\left(1 - Pf_{n,j}\right) \right] - \ln(\varepsilon)$$

$$= S_0 - \sum_{n=1}^{N} \frac{T_n}{T_{p0}} \left[\sum_{j\in ND_n} \frac{q_{n,j}}{T_n} \ln\frac{q_{n,j}}{T_n} - \sum_{j\in ND_n} \frac{q_{n,j}}{T_n} \ln\left(1 - Pf_{n,j}\right) \right] - \ln(\varepsilon)$$

$$= S_0 - \sum_{n=1}^{N} P'_n \sum_{j\in ND_n} P_{n,j} \ln P_{n,j} + \sum_{n=1}^{N} \sum_{j\in ND_n} \frac{q_{n,j}}{T_{p0}} \ln\left(1 - Pf_{n,j}\right) - \ln(\varepsilon)$$

$$= S + \sum_{n=1}^{N} \sum_{j\in ND_n} \frac{q_{n,j}}{T_{p0}} \ln\left(1 - Pf_{n,j}\right) - \ln(\varepsilon) \qquad (I)$$

$$if\ Pf_{n,j} = (1 - \varepsilon)$$

$$\Rightarrow \sum_{n=1}^{N} \sum_{j\in ND_n} \frac{q_{n,j}}{T_{p0}} \ln\left(1 - Pf_{n,j}\right) = \ln(\varepsilon) \sum_{n=1}^{N} \sum_{j\in ND_n} \frac{q_{n,j}}{T_{p0}}$$

$$= \ln(\varepsilon) \sum_{n=1}^{N} \frac{q_n}{T_{p0}} = \frac{\ln(\varepsilon)}{T_{p0}} \sum_{n=1}^{N} q_n \le \ln(\varepsilon) \qquad (II)$$

$$Min(S) = 0 \qquad (III)$$

$$I, II, II \Rightarrow The\ minimum\ value\ of\ S_N = 0$$

Entropy-based Serviceability Assessment of Water Distribution Networks, Subjected to Natural and Man-made Hazards

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چکیده

در این مطالعه یک معیار اصلاح شده بر اساس مفهوم آنتروپی اطلاعات برای ارزیابی سطح خدمت‌رسانی شبکه‌های توزیع آب ارائه می‌گردد؛ به نحوی که در آن به طور همزمان اثر عدم قطعیت‌های هیدرولیکی (مانند میزان جریان در لوله‌ها) و عدم قطعیت‌های مکانیکی (مانند احتمال گسیختگی خطوط جریان) لحاظ شده است. در روش پیشنهاد شده برای محاسبه آنتروپی شبکه، توالی اتصال گره‌های مصرف شبکه، با تعریف ضریبی در رابطه به صورت نسبت نیاز گره مصرف به نرخ جریان تمام لینک‌های شبکه دیده شده است. احتمال گسیختگی لینک‌ها نیز با تعریف یک تابع جریمه بر اساس احتمال گسیختگی هر یک از لینک‌ها تحت یک سناریوی خطر مشخص، دیده شده و به شکل مناسب در تابع آنتروپی هیدرولیکی موجود (ارائه شده توسط پژوهشگران پیشین) اعمال گردیده‌است. بدین ترتیب ضمن در نظر گرفتن اثر رفتار مکانیکی لینک‌ها در محاسبه آنتروپی هیدرولیکی شبکه‌ها سادگی روش نیز حفظ شده است. با محاسبه مقادیر آنتروپی برای تعدادی شبکه آب نمونه، کارایی شاخص پیشنهادی جهت دستیابی به پیکره‌بندی بهینه هیدرولیکی برای یک شبکه جدید و نیز انتخاب بهترین برنامه‌ریزی برای کاهش خسارت در یک شبکه موجود در برابر خطرات مختلف طبیعی یا ساخته دست انسان، نشان داده شده‌است.

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