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A Biomechanical Approach for the Study of Deformation of Red Cells in Narrow Capillaries

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ABSTRACT

This model focuses on the behavior of capillary-tissue fluid exchange system when the diameter of the capillary is less than that of red cell. In vivo and in vitro observations indicate that the width of the gap between the red cell and the vessel wall is generally small compared to the radius of the capillary for a single file flow of red cell in narrow vessel, particularly if the vessel diameter is less than about 6 μ m through squeezing flow of plasma in between the gap between a cell moving through a capillary of smaller diameter than that of the cell. The study reveals the results for the resistance to flow for different values of deformed red cell shapes, cell velocities and permeability. The analysis concludes that the resistance to flow decreases as permeability decreases because of these changes, tissues behaves like an impermeable surface. It has been shown that the resistance to flow in gap decreases as cell velocity increases. The significance of the present model over the existing models has been pointed out by comparing the results with other theories both analytically and numerically. This information of blood could be useful in the development of new diagnosis tools for many diseases.

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1. INTRODUCTION

Blood is a concentrated suspension, containing 40-45% by volume of red blood cells (erythrocytes) suspended in plasma. The mechanical properties of human RBCs have been studied extensively [1, 2]. The unstressed shape of a normal human RBC is a biconcave disc with a diameter of 8 μm and a thickness of 2 μm. The interior of the cell behaves as a viscous incompressible fluid. The cell membrane consists of a lipid bilayer and a cytoskeleton which consists of a network of protein molecules. The membrane strongly resists area changes, and its elastic modulus of isotropic dilation is ~500 dyn/cm, whereas its modulus of shear deformation is about 0.006 dyn/cm [3, 4]. The lipid molecules that comprise the lipid bilayer can slide past each other relatively easily, but resist being pulled apart. The cell membrane has a relatively small bending modulus, about 1.8 × 10−12 dyn·cm [5]. The membrane also possesses a viscous resistance to transient in-plane shear deformations. The viscoelastic behaviour of the membrane in shear can be represented by a Kelvin solid model, in which the total shear stress is represented as the sum of viscous and elastic contributions [6, 7, and 8]. The viscous component arises from the fluid-like behaviour of the lipid bilayer, and the elastic component arises from the stretching of the cytoskeleton.

Blood flow is responsible for nutrient and waste transport within the closed-loop, cardiovascular network [9, 10, 11]. Typically flow is laminar in healthy arteries, but the presence of abnormal flow conditions can promote the development of cardiovascular disease [12]. Blood flow in capillaries of diameters smaller than that of a cell with a single file movement of a cell is of great interest to physiologists involved in microvascular research.

Fluid mechanical and biochemical processes occurring during the movement of an erythrocyte through the capillary are very complex [13, 14, 15]. The red cells are typically deformed from their resting biconcave disk shape to parachute, slipper or bullet like shape depending on the pressure differences across the cell membrane, the tube diameter and the hematocrit [16, 17]. The stress exerted on the cell by the surrounded plasma deform into the shapes of close fitting within the tube, and there exist a layer of plasma between the red cell and the tube wall due to the

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pressure developed. This layer is potentially important in reference to both mass transfer, resistance to flow and residence times of red cells and plasma in the capillary.

Therefore, in this paper a mathematical model for the single file of red cells in a capillary of radius less than 6 μm surrounded by tissue has been described. In tissue, fluid flow is governed by Darcy's law and plasma by a Newtonian fluid. Unreformed cell shape near the wall is assumed to be parabolic and the deformation of the cell is depending on local pressure.

2. FORMULATION OF THE PROBLEM

The red blood cell is modeled as an axisymmtric containing an incompressible fluid. Single file flow of red blood cells is considered and cell to cell interactions are neglected. 'H' is the thickness of the porous matrix. u_0 is the velocity of the cell at $y'=h'$, $h'=(\alpha\pm\beta)'(P'-P_0)+(x'^2/4a')$ is the fluid film thickness. 'a' is the focal length of the initially assumed shape of parabola and $(\alpha\pm\beta)'(p'-p_0)$ is the further deformation due to increased pressure in the wedge formed in between the parabola and the capillary. The flow region is divided into two regions and the governing equations are written separately in two regions as given below.

2. 1. Region-I:

Capillary region-Equation of motion in capillary region is given by:

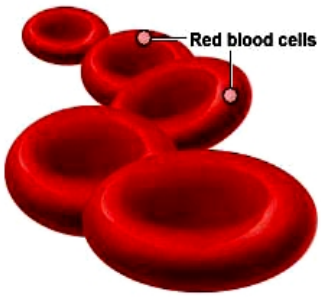

Figure 1. (a): Red Cells

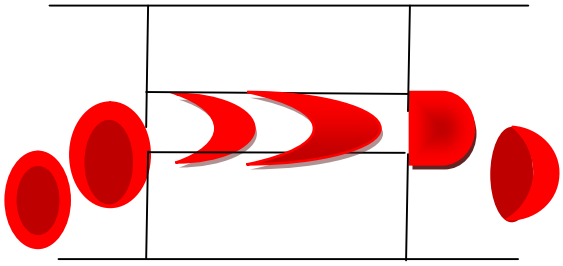

Figure 1. (b) Deformation of Red Cells

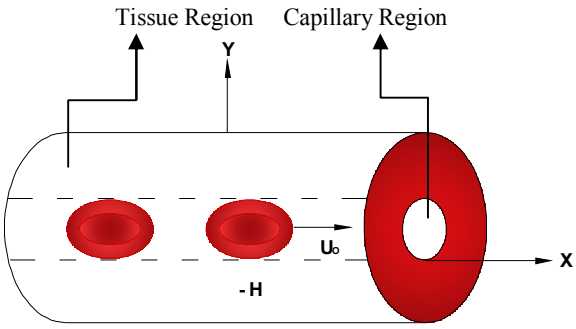

Figure 1. (c) Blood Vessel

$$\left(-\frac{\partial p'}{\partial x'}\right)+\mu\frac{\partial^2 u'}{\partial y'^2}=0 \tag{1}$$

where, μ is the viscosity of the fluid and p' is the pressure in the fluid film region.
Equation of continuity is given by

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0 \tag{2}$$

where, u' and v' are the axial and transverse velocities of the fluid in the capillary.

2. 2. Region-II:

Porous region- Using Darcy's law, the velocity components in porous matrix can be written as:

$$\vec{u}'=-\frac{K}{\mu}\frac{\partial \bar{p}'}{\partial x'} \tag{3}$$

$$\vec{v}'=-\frac{K}{\mu}\frac{\partial \bar{p}'}{\partial y'} \tag{4}$$

where $\vec{u}'$ and $\vec{v}'$ are the axial and transverse velocities of the fluid in porous matrix, K is the matrix permeability $\bar{p}'(x,y)$ is the pressure distribution in the porous matrix introducing the equation of continuity. The governing equation for pressure distribution in the porous matrix is

$$\frac{\partial^2 \bar{p}'}{\partial x'^2}+\frac{\partial^2 \bar{p}'}{\partial y'^2}=0 \tag{5}$$

2. 3. BOUNDARY AND MATCHING CONDITIONS

Following matching and boundary conditions are introduced to solve the above equations

$$\left.\begin{aligned}
u' &= \left(-\sigma\frac{\partial u'}{\partial x'}\right) && \text{at} && y'=0\\
u' &= \left(-\frac{k}{\mu}\frac{\partial \bar{p}}{\partial x'}\right) && \text{at} && y'=0,\\
v' &= 0 && \text{at} && y'=h'\\
v' &= \left(-\frac{k}{\mu}\frac{\partial \bar{p}'}{\partial y'}\right) && \text{at} && y'=0,\\
\frac{\partial \bar{p}'}{\partial x'} &= 0 && \text{at} && x'=0\\
\bar{p}' &= 0 && \text{at} && x'=1',
\end{aligned}\right\} \tag{6}$$

$$\left.\begin{array}{lll} \frac{\partial \bar{p}'}{\partial y'}=0 & \text{at} & x'=0 \\ p'=P_0 & \text{at} & x'=0, \end{array}\right\}$$

where σ' is the slip parameter, P_0 is the reference pressure, α and β are the radial compliances of the capillary and cell which are not separately significant, but appear only in the linear combination $(\alpha+\beta)'$, 2l is the effective length of the capillary and the initial gap at the nip of the parabola much smaller and is assumed to be than H the thickness of the porous layer and is taken to be zero.

2. 4. NON-DIMENSIONAL SCHEME Following non-dimensional scheme is introduced to solve the above equations**:**

$$\left.\begin{array}{ll} x=x'/H'; & y=y'/H'; \\ p=p'/P_0; & u=u'/u_0; \\ e=\rho u_0 H/\mu; & v=v'/u_0; \\ (\alpha+\beta)=(\alpha+\beta)'/\left(H'^3/\rho u_0^2\right); & \\ p=p'/P_0; & \sigma=\sigma'/H'; \\ \varepsilon=H'/4a'; & h=\eta(p-1)+\varepsilon x^2 \end{array}\right\} \quad (7)$$

2. 5. BOUNDARY AND MATCHING CONDITIONS Following boundary conditions and matching conditions are introduced to solve the above equations:

$$\left.\begin{array}{lll} u=1 & \text{at} & y=h, \\ u=-\sigma\frac{\partial \omega}{\partial y} & \text{at} & y=0 \\ v=0 & \text{at} & y=h, \\ v=-\frac{KP_0}{\mu U_0 H}\frac{\partial \bar{p}}{\partial y} & \text{at} & y=0 \\ \frac{\partial \bar{P}}{\partial x}=0 & \text{at} & x=0, \\ \bar{p}=0 & \text{at} & x=\frac{l'}{H'}=l \\ \frac{\partial \bar{P}}{\partial y}=0 & \text{at} & y=-1, \\ p=1 & \text{at} & x=\frac{l'}{H'}=l \\ p=\bar{p}(x,0) & & \end{array}\right\} \quad (8)$$

3. SOLUTION OF THE PROBLEM

Solving the equation of motion, equation of continuity and boundary and matching conditions in capillary region, the expression for the velocity distribution is obtained as

$$u=\frac{P_0 H}{U_0 \mu}\frac{\partial p}{\partial x}\left(y^2-\frac{(y-\sigma)}{(h-\sigma)}h^2\right)+\frac{y-\sigma}{h-\sigma} \quad (9)$$

On Solving the Laplace equation, the pressure distribution in the porous region is obtained as:

$$\bar{p}=\sum_{n=0}^{\infty} 2En\cosh\{\alpha_n(H+y)\}\cos(\alpha_n x) \quad (10)$$

where $\alpha_n=(2n+1)\pi/21)$ the value of En is obtained by Fourier cosine series expansion, as:

$$En=\frac{1}{\alpha_n}\operatorname{sech}(\alpha_n H)\cos(n\pi) \quad (11)$$

Using velocity profile in the equation of continuity, the pressure distribution in capillary region and the corresponding matching velocities at the common boundary (y=0), we get

$$P=[Z_n(P_0A_1x^2+A_2x^4+A_3x^2)]-[Z_n(\alpha_n^2/2)(P_0x^4(A_1/6)+x^6(2A_2/5)+x^4(A_3/6))]+1-[Z_n(P_0A_1+A_2+A_3)]+[Z_n(\alpha_n^2/2)(P_0(A_1/6)+(2A_2/5)+(A_3/6))] \quad (12)$$

The Flow Resistance is obtained as:

$$R^*=(1/Q)[Zn\left(P_0A_1x^2+A_2x^4+A_3x^2\right)]-[Zn(\alpha_n^2/2)(P_0x^4(A_1/6)+x^6(2A_2/5)+x^4(A_3/6))] \quad (13)$$

where

$$Zn=[(3K/2H^2\sigma)\sum_{n=0}^{\infty}En\alpha_n^3\sinh\alpha_n(H+y)] \quad (14)$$

$$A_1=((-3\sigma/2\eta^2)+(1/8\sigma)-(3/4\eta)),$$

$$A_2=(-(3\varepsilon'\sigma/12\eta^3)+(\varepsilon'/48\sigma\eta)-(3\varepsilon'/24\eta^2))$$

$$A_3=(-(\sigma/2\eta^3)+(1/8\sigma\eta)-(3/8\eta^2)) \quad (15)$$

$$\eta=(H^2P_0(\alpha+\beta)/\rho U_0^2)$$

4. RESULTS AND DISCUSSION

In order to have estimate of the quantitative effects of various parameters involved in the analysis computer codes were developed and to evaluate the analytical results obtained for resistance to blood flow for normal condition have been determine.

The results are shown in Figures 2-5 by using the values of parameter based on experimental data in capillary. The results of the analysis are presented through the graphs in figures numbered from Figures 2–5. Figure 2 represents the variation of resistance to flow for different values of ε.

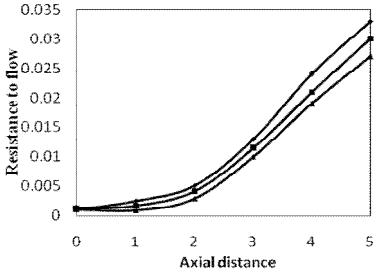

Figure 2. Variation of resistance to flow for different shapes of cell

As the value of ε increases, resistance to flow decreases. Due to large variation of the pressure developed in the wedge, the corresponding deformations in the cell and the capillary wall occur near the narrowest part in the gap region between the cell and the tube wall affect the resistance to flow. The increases in the pressure will naturally deform the cell and this deformation will produce corresponding effect on the resistance to flow.

Figure 3 presents the variation of resistance to flow for different cell velocities. As cell velocity increases the resistance to flow in the gap decreases.

According to the model, the changes in the cell width with increasing cell velocity are accompanied by decreasing flow resistance. This is due to the fact that at low velocities the cells bulge outward almost filling a 6μmcapillary and become more elongated and streamlined with increasing velocity [16]. At high velocity the increasing width of plasma layer between the cell and the wall leads to reduction in flow resistance.

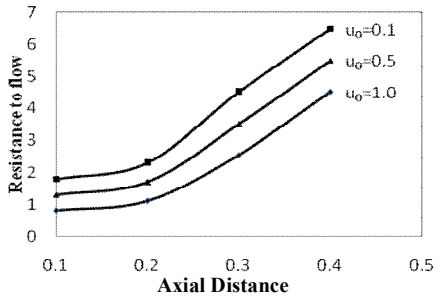

Figure 3. Variation of resistance to flow with axial distance

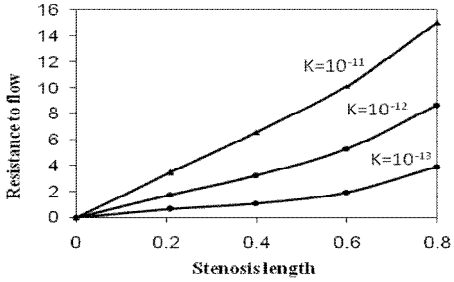

Figure 4. Variation of resistance to flow different permeability

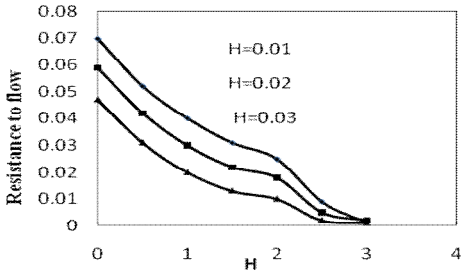

Figure 5. Variation of resistance to flow for different H

Figure 4 depicts the variation of flow resistance for different values of permeability. As value of permeability decreases the resistance to flow in the gap between the cell and capillary wall decreases. In this case, tissue behaves like an impermeable surface with respect to fluid flow into the tissue and a lubrication layer is maintained between cell and the wall necessary to maintain the flow of red blood cell as discussed by [4]. Figure 5 presents the variation of resistance to flow for different values of H. As the values of H increases the resistance to flow in the gap decreases. These results are similar to the results of [12].

5. CONCLUSION

In this paper a model is used to focus the behavior of capillary-tissue fluid exchange system when the diameter of the capillary is less than that of red cell. The study reveals the results for the resistance to flow for different values of deformed red cell shapes, cell velocities and permeability. The analysis concludes that the resistance to flow decreases as permeability decreases because of these changes, tissues behaves like an impermeable surface. It has been shown that the resistance to flow in gap decreases as cell velocity increases. The significance of the present model over the existing models has been pointed out by comparing the results with other theories both analytically and numerically. This model is able to predict the main characteristics of the physiological flows and would be helpful for the people working in the field of biomedical science as well as to the medical practitioners.

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چکیده

این مدل رفتار تغییرات سیال را در بافت مویرگ زمانیکه قطر مویرگ کمتر از سلول قرمز است را مورد بررسی قرار داده است. مشاهدات درون و برون بافتی نشان می دهند که فاصله بین سلول قرمز و دیواره رگ به طور کلی در مقایسه با شعاع مویرگ برای یک تک جریان سیال سلول قرمز در رگ، کوچک باشد به ویژه اگر قطر رگ در میان فاصله بین سلول متحرک درون مویرگی با قطر کوچکتر از سلول، تحت فشار جریان پلاسما حدوداً کمتر از $6\mu m$ باشد. این مطالعه نتایج مقاومت جریان برای مقادیر متفاوت از تغییر شکل سلول قرمز، سرعتها و قابلیت نفوذ سلول را اشکار می سازد. نتایج انالیز نشان می دهد که مقاومت جریان بر اساس کاهش نفوذپذیری، کاهش می یابد و به علت این تغییرات ، رفتار بافت ها همانند سطوح غیرقابل نفوذ می گردد. این مطلب نشان می دهد که مقاومت جریان در شکاف با کاهش سرعت سلول کاهش می یابد. اهمیت این مدل در مقایسه با مدلهای موجود در این است که نتایج را به صورت عددی و تحلیلی مورد بررسی قرار می دهد. این یافتهای خونی، در توسعه ابزار جدید تشخیص بسیاری از بیماری ها می تواند مفید واقع شود.

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