PERFORMANCE MODEL AND ANALYSIS OF BLOOD FLOW IN SMALL VESSELS WITH MAGNETIC EFFECTS

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ABSTRACT In this investigation, a two-fluid model consisting of a core region of suspension of all the erythrocytes (particles) in plasma (fluid) assumed to be a particle-fluid mixture and a peripheral layer of cell-free plasma (Newtonian fluid), has been proposed to represent blood flow in small diameter tubes with magnetic effects . The analytical results obtained in the proposed model for effective viscosity, velocity profiles and flow rate have been evaluated numerically for various values of the parameters. Quantitative comparison depicted that present model represents blood flow at hematocrit (≤ 40%) and in vessels up to 70μm in diameter. Using experimental values of the parameters, the flow rate for normal and diseased blood has been computed and compared with corresponding values obtained from a well known experimentally tested model in the literature. The effects of a magnetic field have been used to control the flow, which may be useful in certain hypertension cases, etc.

Keywords Blood flow; plasma; erythrocyte; hematocrit; hartmann number; hypertension.

چکیده در این تحقیق، یک مدل دو سیالی شامل یک ناحیه‌ی از هسته با ذرات معلق گلبول‌های قرمز (ذرات) در پلاسما (مایع)، که یک مخلوط ذرات- مایع فرض می شود، و یک لایه محیطی از پلاسما عاری از سلول (سیال نیوتنی) ، برای نشان دادن جریان خون در لوله های با قطر کوچک با اثرات مغناطیسی پیشنهاد شده است. نتایج تحلیلی به دست آمده در مدل پیشنهادی برای گرانروی موثر، پروفیل سرعت و سرعت جریان به صورت عددی برای مقادیر مختلف پارامترها ارزیابی شده است. مقایسه کمی که مناسب بودن مدل مورد مطالعه را تایید می‌نماید،نشان می‌دهد جریان خون در هماتوکریت (≤٤٠٪) و در عروق تا قطر ۷۰μm می‌باشد. با استفاده از مقادیرتجربی پارامترها، سرعت جریان برای خون سالم و بیمار محاسبه و با مقادیرمعادل به دست آمده از یک مدل معتبرو آزمایش شده تجربی در لیتریچر مقایسه شده است.از اثرات میدان مغناطیسی برای کنترل جریان، که ممکن است در بعضی از موارد فشار خون بالا، و غیره مفید باشد استفاده شده است

1. INTRODUCTION

Physiological fluid dynamics is a relatively new area that deals with the fluid dynamics of MHD biological fluids. During the last few decades, extensive literature has become available on MHD flows of biological fluids.

Such flows have numerous applications in bioengineering and medical sciences. The Blood vessels are part of circulatory system, which easily pass nutrients, blood, hormones, and other important substances to and fro from body cells which maintain homeostasis. Furthermore, the blood vessels are responsible for transmitting blood throughout the body. The three major types of blood vessels are: arteries, veins, and capillaries. The blood flow in the small vessels of human circulatory system has become a matter of great significance for scientific research for many years. Mathematical approach of the problem has gone through prolonged changes and modifications resultingfrom new evidence uncovered through improved experimental measurements. New blood vessels provide nutrients to proliferating cancer cells, which is in favor of tumor growth. Tumor cells need an adequate blood supply in order to perform vital cellular functions. Accordingly, the disturbance of blood flow is good predictor of the

causes of disease, hence regional blood flow measured which permits earlier cancer detection. Nowadays approach to cancer treatment is multidisciplinary, one that involves varying combination of surgery, radiation therapy, chemotherapy, and targeted therapies. Small blood vessel disease is a chronic medical condition, which adversely affects coronary arterial and jeopardizes heart health. Consistently, and in close association with the onset of atherosclerosis, small blood vessel disease impacts arteries ability to expand in order to accommodate proper blood flow.

One of the leading causes of deaths in the world is heart related diseases. The heart diseases mainly occur due to temporary deficiency of oxygen or blood supply to the heart. This deficiency may be due to a constriction or obstruction in the blood supply to that part; the constriction involves the deposition of some fatty substances like cholesterol, cellular waste product, calcium, etc. The atherosclerosis may cause heart attack.

This study brings out many interesting fluid mechanical phenomena due to the magnetic field and presence of the peripheral layer. Blood has been modeled as two –fluid model with the core region consisting of suspension of all the erythrocytes, and plasma in the peripheral region as a newtonian fluid. It is noted that the velocity and flow rate decreases, while the effective viscosity increases with magnetic field and hematocrit . The experimental studies and the theoretical treatments of blood flow phenomena are very useful for the diagnosis of a number of cardiovascular diseases and development of pathological patterns in human or animal physiology, and for other clinical purposes and practical applications.

Many researchers have studied blood flow in artery by considering blood as either Newtonian or non-Newtonian fluids. The study of magnetic field is very important both from theoretical and practical point of view; because most of the natural flow problems are connected with magnetic field. Experimental investigation of Cokelet [1] and theoretical observations of Haynes [2] indicate that blood can no longer be treated as a single-phase homogenous viscous fluid in small size vessels (of diameter $\leq$ 1000μm). It is surprising to note that the individuality of the red cells (of diameter 8μm) is important even in such large vessels (with diameter up to 100 cells diameter). Accurate description of flow in capillary vessels, whose diameters (4-10μm) are equal or smaller than that of a red blood cell, requires consideration of red cells as discrete particles. Also, certain observed phenomena in blood flow including Fahraeus-Lindqvist effect (decrease of apparent viscosity with decreasing diameter of blood vessels), non-Newtonian behavior, and so on cannot be explained fully by considering blood as a single-phase homogenous fluid [3, 4].

Consider two-phase theoretical model to address pulsatile blood flow in the entrance region of an artery [5]. Srivastava et al. [6] studied the theory to study the effects of external body acceleration on blood flow through small diameter tubes while Srivastava [7, 8] dealt with the problem of blood flow through stenotic vessels representing blood by an erythrocytes-plasma suspension. And most recently, Jung et al. [9, 10] discussed steady and pulsatile flow of particulate buildup on the inside curvature of coronary artery using multiphase of dense suspension hemodynamic. Haynes [2] presented a two-fluid theoretical model for blood flow consisting of a core region of suspension of all the erythrocytes as a homogeneous Newtonian viscous fluid and a cell-free plasma layer as a Newtonian fluid of constant viscosity (equal to the viscosity of water).

Bugliarello and Sevilla [11] presented blood in small diameter tubes by a two-layered model assuming peripheral and core fluids as Newtonian fluids of different viscosities. Following the theoretical study of Haynes [2] and experimentally tested model of Bugliorello and Sevilla [11], two-fluid modeling of blood flow has been discussed and used by a good number of researchers. Chaturani and Upadhya [12, 13]] addressed the flow of blood in small diameter tubes using the two-layered model of micro-polar and couple stress fluids, respectively. Pralhad and Schultz model used a two-fluid model of polar fluid to analyze the flow of blood through stenotic arteries [14]. Two-fluid model analyses have been carried out by Srivastava [8, 15, 16] to observe the effects of a non-symmetrical stenosis on blood flow characteristics. A mathematical analysis of blood flow through arteries using finite element has been presented previously [17]. Galerkin approaches and studied a MHD flow in stenosis artery using finite difference technique [18]. However, the

suction phenomenon of tissue in blood vessels has not been given any consideration in these studies. For a realistic approach this factor influences the blood flow in vessels.

Sharan and Popel [19] suggested a modification on the models of Haynes [2] and Bugliarello and Sevilla [11] assuming the viscosity in the peripheral layer to be higher than that of plasma due to additional dissipation of energy caused by the red cells motion near the cell-free layer. Wang and Bassingthwaighte [20] applied the two-layered models of Haynes [2] and Sharan and Popel [19] to discuss the flow of blood in narrow curved tubes, etc. Kumar et al. [21] founded a performance of blood flow in stenosed arterial model. Kenjeres [22] developed a comprehensive mathematical model for simulations of blood flow under the presence of strong non-uniform magnetic fields. Kumar and Saket [23] investigated reliability of convective diffusion process in stenosis blood vessels. Gupta [24] investigated finite element Galerkin's scheme for flow in blood vessels with magnetic effects. Rathod and Tanveer [25] studied the plusatile flow of blood through a porous medium under the influence of periodic body acceleration by considering blood as a couple stress, incompressible, electrical conducting fluid in presence of magnetic field.

Kumar [26] proposed a computational model of blood flow in the presence of atherosclerosis. Gupta [27] developed a performance and analysis of blood flow through carotid artery. Kumar [28] made a performance modeling and analysis of blood flow and cross-sectional area of an artery with magnetic effects. Gupta et al. [29] investigated steady blood flow in an artery with mild stenosis, by considering the blood as power-law fluid under the influence of linear and quadratic radial variation of viscosity. Some researchers profound simulation of variable viscosity and Jeffrey fluid model for blood flow through a tapered artery with a stenosis and geometrical shapes [30-32].

The purpose of present paper is to investigate the flow of blood in small vessels involving a two-fluid model with magnetic effects. The mathematical model considers a two-layered model of blood, consisting of a core region of suspension of all the erythrocytes (small spherical non-flexible particles), assumed to be a particle fluid suspension (i.e., a suspension of red cells in plasma) and a peripheral layer of plasma (Newtonian fluid). This study which presents a theoretical model for blood, seems to be the only one of its kind which enables one to observe the simultaneous effects of hematocrit and the peripheral layer on the flow characteristics while flowing through small vessels.

The organization of paper is as follows: The mathematical analysis of the problem along with requisite assumptions and notations has been provided in section 2. Section 3 presents the numerical results. Conclusion is given in section 4.

2. MATHEMATICAL ANALYSIS

Consider the axi-symmetric flow of blood in a uniform circular tube of radius R. The induced magnetic field is neglected. Blood is represented by a two-fluid model consisting of a core region (central layer) of suspension of all the erythrocytes assumed to be a particle-fluid mixture (i.e., a suspension of red cells in plasma) of radius R_1 and a peripheral layer of plasma (Newtonian fluid) of thickness (R-R_1) as depicts in Figure 1. The application of magneto-hydrodynamics in physiological problems is of growing interest. The flow of blood can be controlled by applying magnetic field of sufficient strength.

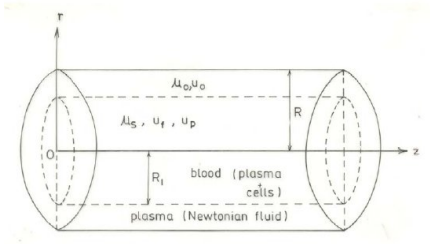

Figure 1. Flow Geometry of blood in vessels with magnetic effects

Under the simplified assumptions stated in [5], the governing equations of the flow are [7, 33-36] as given below:

$$\frac{dp}{dz} = \frac{\mu_0}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_0, \qquad R_1 < r < R \tag{1}$$

$$(1-C)\frac{dp}{dz} = (1-C)\frac{\mu_s C}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_f + CS(u_p - u_f) - Mu_f, \qquad 0 \le r \le R_1 \tag{2}$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_p}{\partial r} = 0 \ at \ r = 0, \tag{3}$$

where (r, z) are (radial, axial) coordinates, (u_f, u_p) are the axial velocity of (fluid, particle) in core region ($0 \le r \le R_1$), (μ_0 ,u_0) are the (viscosity , fluid velocity) in the peripheral region ($R_1 \le r \le R$), $\mu_s \cong \mu_s$ (C) is the suspension viscosity in the core region, C denotes the constant volume fraction density of the particles (called hematocrit) and S is the drag coefficient of interaction between the two phases (fluid and particle). The expression for the drag coefficient of interaction S and empirical relation for the viscosity of suspension μ_s as given below[7, 37-39]:

$$S = \frac{9}{2}\frac{\mu_0}{a_0^{\,2}}\frac{\left[4 + 3(8C - 3C^2)^{1/2} + 3C + 3M\right]}{(2-3C)^2} \tag{4}$$

$$\mu_s = \frac{\mu_0}{1 - qC}; \; q = 0.07\exp[2.49C + \frac{1107^0 K}{T}\exp(-1.69C], \tag{5}$$

where a_o is the radius of a particle and T is measured in absolute temperature.

The boundary conditions are the standard no-slip conditions of velocities and shear stresses at the tube wall and the interface, and are given by:

$$u_0 = 0, \; at \; r = R \tag{6}$$

$$u_0 = u_f \; and \; \tau_0 = \tau_f \; at \, r = R_1 \tag{7}$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_p}{\partial r} = 0 \; at \;\; r = 0 \tag{8}$$

where $\tau_f = (1-C)\mu_s \frac{\partial u_f}{\partial r}$ and $\tau_f = (1-C)\mu_s \frac{\partial u_f}{\partial r}$ as the shear stresses of the peripheral and central layers, respectively.

The expressions for the velocities u_0, u_f and u_p obtained as the solutions of Equations (1) – (3), using the boundary conditions (6) – (8), are given as:

$$u_0 = -\frac{R^2}{4\mu_0}\frac{dp}{dz}\left(1 - \frac{r^2}{R^2}\right), \qquad R_1 \le r \le R \tag{9}$$

$$u_f = -\frac{R^2}{4(1-C)\mu_0}\frac{dp}{dz}\mu'\left(\frac{R_1^2}{R^2} - \frac{r^2}{R^2}\right) + (1-C-M) \quad 0 \le r \le R_1 \tag{10}$$

$$u_p = -\frac{R^2}{4(1-C)\mu_0}\{\frac{dp}{dz}[\mu'\left(\frac{R_1^2}{R^2} - \frac{r^2}{R^2}\right)\} + (1-C-M)$$
$$\{1 - \left(\frac{R_1^2}{R^2} + \frac{4(1-C-M)\mu_0}{SR^2}\right)\} \; 0 \le r \le R_1 \tag{11}$$

where $\mu' = \frac{\mu_0}{\mu_s}$.

The flow flux (volumetric flow rate) is given by

$$Q = Q_0 + Q_f + Q_p \tag{12}$$

where

$$Q_0 = 2\pi \int_{R_1} r\, u_0 dr, Q_f = 2\pi(1 - C - M)\int_{R_1} r\, u_f dr$$

and

$$Q_p = 2\pi(C - M)\int_{R_1} r\, u_p dr$$

Substituting Equations (9) – (11) into Equation (12), the expression for flow flux as given by:

$$u_p = -\frac{\pi R^4}{8(1-C)\mu_0}\frac{dp}{dz}[(1-C-M)(1-(\frac{R_1}{R})^4 + \mu'(\frac{R_1}{R})^4 + \eta^2(\frac{R_1}{R})^2] \tag{13}$$

where $\eta^2 = 8C(1-C-M)\frac{\mu_0}{SR^2}$, a non-dimensional suspension parameter. The use of the fact that total flux is equal to the sum of the fluxes across the two regions (peripheral and core) determines the relations [2, 40-42].

$$R_1 = \alpha R \tag{14}$$

Substituting Equations (14) into Equation (13), yields the following expression for the effective (apparent) viscosity as given below:

$$\mu_e = \frac{(1-C-M)\mu_0}{(1-C-M)(1-\alpha^4) + \mu'\alpha^4 + \eta^2\alpha^2} \tag{15}$$

where $R_1 = R$ (i.e., in the absence of the peripheral layer), above results reduces to the case of a single layered model of a particle-fluid suspension as given below:

$$\mu_{es} = \frac{(1 - C - M)\mu_0}{\mu' + \eta^2} \tag{16}$$

It is worth mentioning that in the absence of the particles (i.e., $C = 0$), the core mixture changes to the same fluid as in the peripheral region and thus the role of the peripheral layer automatically

disappears. In addition, when core mixture behaves as a single-phase fluid of constant viscosity (i.e., $\mu_s = \mu_1 \neq \mu_0$), one obtains the same expression for effective viscosity derived from steady Newtonian fluid model of Bugliarello and Sevilla [11] as

$$\mu_{eb} = \frac{\mu_0}{1 - \alpha^4 + \mu' \alpha^4} \tag{17}$$

Equation (17) recovers the result obtained in Haynes [2] when μ_o = 1cp.

3. NUMERICAL RESULTS

In the present section, numerical results have been provided to explore the effects of various parameters on the velocity profiles, flow rate, etc. For this purpose, we develop a program coded in MATLAB software. The results have been numerically worked out for various combinations of the parameters involved in the solution. The corresponding results obtained from the theoretical model of Haynes [2], considering a two-phase fluid in the core region and experimentally tested steady flow model of the literature [2, 6, 11, 43] using a single- phase fluid (blood) of constant viscosity for a given hematocrit have been evaluated for the experimental values of the parameters available from the published literature of [6, 11, 43, 44] at 25.5 ^{0}C. The value of α is calculated from the relation: α = 1- ε/R, in which ε ≅ ε (C) denotes the peripheral layer thickness for a given hematocrit [2].

Owing to the significance of viscosity, the effective viscosity at 20% and 40% hematocrit (red cell concentration) has been computed for different size blood vessels. The results obtained are arranged in Table 1 and compared with the corresponding theoretical values of Haynes [2] and Chaturani and Upadhya [13], and experimental values of Bugliarello and Sevilla [11]. For numerical computation of the results for effective viscosity given in Equation (15), the mixture viscosity μs has been computed using empirical relation (5) for two values of the pressure gradient, – dp/dz =67.5 dyne/mm^3 and 76.0 dyne /mm^3 The effective viscosity deviates from experimental value with increasing hematocrit and also with the vessel size.

TABLE 1. Experimental results compared with literature

Vessel Diameter (μm)	α	Effective Viscosity (cp) Present	Haynes [2]	Bugliarello & Sevilla [11]	Chaturani & Upadhya [13]
20% Hematocrit, ε=4.67 μm					
20	0.382	1.211	1.247	1.245	1.261
30	0.588	1.240	1.340	1.334	1.389
40	0.766	1.265	1.429	1.420	1.511
70	0.866	1.315	1.753	1.607	1.679
100	0.906	1.340	1.753	1.723	1.869
40% Hematocrit, ε=3.12 μm					
20	0.688	1.243	1.423	1.391	1.445
30	0.792	1.282	1.656	1.581	1.709
40	0.844	1.390	1.979	1.741	2.080
70	0.910	1.355	2.307	2.075	2.440
100	0.937	1.377	2.624	2.272	2.667

However, one notices that present model exhibits Fahraeus – Lindqvist effect (i.e., apparent viscosity of blood decreases with decreasing diameter of blood vessel). The axial velocity profiles (u_f, u_p and u_o) computed from the present theory (Equations (9) – (11), the corresponding model derived (using erythrocytes-plasma suspension to represent blood in the core region similar to the present proposed model) from Haynes [2] and the steady flow model of Bugliarello and Sevilla [11] at 20% and 40% hematocrit are displayed graphically in Figures 2 and 3 respectively. To evaluate the results obtained for velocity profiles in Haynes [2] and Bugliarello and Sevilla [11], the mixture viscosity (or blood viscosity) has been taken to be 2.18 cp and 3.10 cp for 20% and 40% hematocrit, respectively from published literature [6, 11, 43, 45].

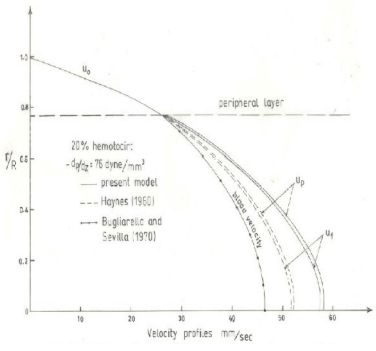

Figure 2. Velocity Profiles at 20 % Hematocrit in the blood vessels diameter 40 ml micron and Hartmann number (M) =1.0.

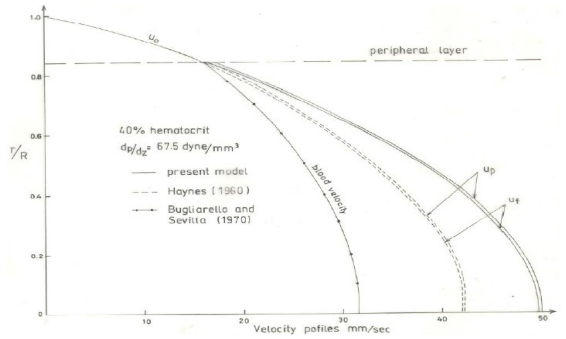

Figure 3. Velocity Profiles at 20 % Hematocrit in the blood vessels diameter 40 ml micron Hartmann number (M) =1.0.

The volumetric flow rate(*Q)* vs pressure gradient (*–dp/dz)* computed from the proposed model (Equation (13)) and the experimentally tested models [11, 46, 47] at 20% and 40% hematocrits have been plotted in Figure 4. It may be noted that the magnitudes of the flow rate (Q) obtained in the proposed theory are in reasonable agreement with the corresponding value obtained in the literature [11, 48, 49], particularly for low pressure gradients. To emphasize further on the study presented above, volumetric flow rate *Q* vs pressure gradient *–dp/dz* for normal and diseased blood (Hb SS, plasma cell dycrasias, hypertension (controlled), hypertension (uncontrolled) and polycythemia) in a 70μm diameter vessel using the present theoretical approach and the experimental data available from published literature [11, 45, 50] has been shown in Figure 5 and compared with the corresponding values obtained in Bugliarello and Sevilla [11]. Various values of the parameters used for the purpose are arranged in Table 2.

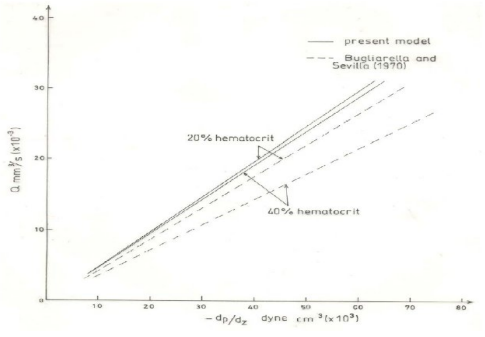

Figure 4. Pressure flow rate relationship in a 40 mili micron diameter blood vessels for various hematocrit and Hartmann Number (M) =1.08

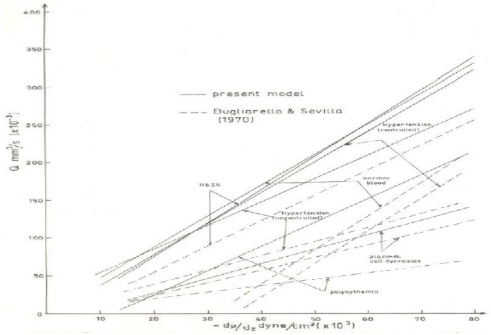

Figure 5. Flow rate for normal and diseased blood in 70 milimicron blood vessel diameter

TABLE 2. Experimental data for diseased and normal blood in 70 μm diameter vessels [50]

Disease	Hematocrit (%)	μ_0 (cp)	μ_s	α
Hb SS(Sickle cell)	24.81	1.31	5.15	0.795
Plasma cell dycrasias	28.00	3.08	5.44	0.817
Normal Blood	42.61	1.25	4.04	0.921
Hypertension (Uncontrolled)	43.29	1.53	5.16	0.925
Hypertension (Uncontrolled)	43.31	1.29	4.87	0.929
Polycythemia	63.20	1.51	7.70	0.991

The flow rates obtained from the study deviate from those reported by Bugliarello and Sevilla [11], with increasing values of the pressure gradient and also with increasing hematocrit.

4. CONCLUSION

The present algorithm is economica; and efficient, having a sharp convergence. The magnetic field reduces the stress. A two-layered model consisting of a core region of suspension of all the erythrocytes in plasma (i.e., particle- fluid mixture) and a peripheral layer of plasma (Newtonian fluid) has been proposed to describe blood flow in small diameter vessels with magnetic effects. It is however felt that a considerable amount of further research is essential to make the model useful for higher parameter values (hematocrit and vessel size) and to overcome the discrepancies of some other approximations used in the formulation. It is hoped that our investigation may be helpful for the medical practitioners and other persons in the area of bio fluid dynamics to understand the flow of

blood in the presence of magnetic effects. The results derived may be useful for hypertension patients through magnetic therapy. It is clear from the above discussion that magnetic field affects largely on the flow velocities of blood and viscosity. So,by taking appropriate values of magnetic field we may regulate the velocities and volumetric flow rate. The present work studied magnetic effect of blood flow through small vessel, which is of great interest for the purpose of medical sciences. Magnetic field applied is affecting the flow of blood through small vessel, which is useful for the problem like blood pressure hypertension etc. Applications of magnetic effects of blood flow through small vessel is showing the variations in temperature of the object which is helpful for the purpose of thermal therapy in the treatment of tumor, glands etc. The treatment for small vessel disease involves medications to control the narrowing of small blood vessels that could lead to a heart attack.

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6. REFERENCES

1. Cokelet, G.R., "The rheology of human blood: In Biomechanics (Ed. Y.C. Fung et al.)", Prentice Hall, Englewood Cliffs, New Jersey, (1972), 145-150.
2. Haynes, R.H., "Physical basis of the dependence of blood viscosity on tube radius", ***American Journal of Physiology,*** Vol. 198, (1960), 1193–1205.
3. Skalak, R., "Mehanics of microcirculation: In Biomechanics, Its foundation and Objectives" (Ed. Y.C. Fun), Prentice- Hall Publ. Co., Englewood cliffs, (1972), 63–103.
4. Casson, N.A., "Flow equation of pigment oil suspension of printing ink type: In Rheology of Disperse Systems (Ed. C. C. Mill)", Pergamon Press, London, (1959), 84–120.
5. Srivastava, L.M., and Srivastava., V.P., "On two-phase model pf pulsatile blood flow with entrance effects", ***Biorheology,*** Vol. 20, (1983), 761–777.
6. Srivastava, L.M., Edemeka, U.E. and Srivastava., V.P., "Particulate suspension blood flow under external body acceleration", ***International Journal of Biomedical Computing,*** Vol. 37, (1994), 113–129.
7. Srivastava, V.P., "Particle – fluid suspension model of blood flow through stenotic vessels", ***International Journal of Biomedical Computing,*** 38, (1995), 141–154.
8. Srivastava, V.P., "Particulate suspension blood flow through stenotic arteries: effect of hematocrit and stenosis shape", ***Indian Journal of Pure and Applied Mathematics,*** Vol. 33, (2002), 1353-1360.
9. Jung, J., Hassanein, A. and Lyczkowshi, R.W., "Hemodynamic computation using multiphase flow dynamics in right coronary artery", ***Annals of Biomedical Engineering,*** Vol. 34, (2006a), 393-402.
10. Jung, J., Lyczkowshi, R.W. Panchal, C.B. and Hassanein, A., "Hemodynamic simulation of pulsatile flow in a coronary artery", ***Journal of Biomechanics***, Vol. 39, (2006b), 2064-2073.
11. Bugliarello, G., and Sevilla, J., "Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes", ***Biorheology,*** Vol. 7, (1970), 85–107.
12. Chaturani, P., and Upadhya V.S., "On miropolar fluid model for blood flow through narrow tubes", ***Biorheology,*** Vol. 16, (1979), 419-428.
13. Chaturani, P., and Upadhya V.S., "A two-fluid model for blood flow through small diameter tubes with non-zero couple stress boundary condition at the interface", ***Biorheology,*** Vol. 18, (1981), 245-253.
14. Pralhad, R.N., and Schultz, T.J., "Two-layered blood flow in stenosed tubes for different diseases", ***Biorheology,*** Vol. (1988), 25, 715-726.
15. Srivastava, V. P., "Blood flow through stenosed vessels with a peripheral plasma layer and applications", ***Automedica,*** Vol.18, (2000), 271-300.
16. Srivastava, V.P., "Flow of a couple stress fluids representing blood flow through stenotic vessels with a peripheral layer", ***Indian Journal of Pure and Applied Mathematics,*** Vol. 34, (2003), 1727-1740.
17. Sharma, G.C. Jain, M. and Kumar, A., "Finite element Galerkin's approach for a computational study of arterial flow", ***International Journal Applied Mathematics and Mechanics,*** Vol.22 No. 9, (2001), 1012-1018.
18. Sharma, G. C., Jain, M., and Kumar, A., "MHD flow in stenosed artery", ***Proceedings in Int. Conf. on Mathematical modelling Jan. 29-31 Rookee,*** (2001), 683-687.
19. Sharan, M., and Popel, A.S., "A two-phase model for flow of blood in narrow tubes with increased effective viscosity near the wall", ***Biorheology,*** Vol. 38, (2001), 415-428.
20. Wang, C.Y., and Bassingthwaighte, J. B., "Blood flow in small curved tubes", ***Journal of Biomechanical Engineering,*** Vol. 125, (2003), 910-913.
21. Kumar, A., Varshney, C.L. and Singh, V.P., "Analytical

study of blood flow with periodic body acceleration in the presence of magnetic field", ***Society for Mathematical Biology Conference July 30 - August 2, 2008, Centre for Mathematical Medicine, Fields Institute Toronto, Canada,*** (2008).
22. Kenjeres, S., "Numerical analysis of blood flow in realistic arteries subjected to strong non-uniform magnetic fields", ***International Journal of heat and fluid flow***, Vol. 29, No. 3, (2008),752-764.
23. Kumar A., and Saket R.K., "Reliability of Convective Diffusion Process in Stenosis Blood Vessels", ***Chemical product and process modeling,*** Vol. 3 No. 1, (2008), 1-17.
24. Gupta A.K., "Finite Element Galerkin's scheme for flow in blood vessels with magnetic effects, International Journal of Applied Systemic Studies", ***Inderscience publishing,*** Vol. 2, No. 3, (2009), 284-293.
25. Rathod, V.P., and Tanveer, S., "Pulsatile of couple stress fluid through a porous medium with a periodic body acceleration and magnetic field", ***Bull. Malay. Math. Sci. Soc.,*** Vol. 32, No. 2, (2009), 245-259.
26. Kumar, A., "Computational model of blood flow in the presence of atherosclerosis", ***International Federation for Medical and Biological Engineering (IFMBE) Conference, 6th World Congress on Biomechanics (WCB 2010) Edited C.T. Lim and J.C.H. Goh (Eds.): WCB 2010, IFMBE Proceedings,*** Vol. 31, (2010) 1591–1594.
27. Gupta A.K., "Performance and analysis of blood flow through carotid artery", ***International Journal of Engineering and Business Management,*** Vol. 3, No. 4, (2011), 1-6.
28. Kumar A., "Performance modeling and Analysis of blood flow and cross-sectional area of an artery with magnetic effects", ***3rd International conference on Chemical engineering and Advanced Materials CEAM 2011 –VF,*** November, (2011), 14-28, accepted. .
29. Gupta, S., Gupta, M. and Singh, S.P., "Effect of radial viscosity variation on non-Newtonian flow of blood in a stenosed artery", ***International Journal of Applied Mathematics and Mechancs,*** Vol. 8, No. 2, (2012), 51-61.
30. Akbar, N.S., and Nadeem, S., "Simulation of variable viscosity and Jeffrey fuid model for blood flow through a tapered artery with a stenosis", ***Communications in Theoretical Physics***, Vol. 57, (2012), 133–140.
31. Fung, Y.C., "Biomechanics, its scope, history and some problems of continuum mechanics in physiology", ***Applied Mechanics Review,*** Vol. (1964), 21, 1-20.
32. Gupta, B.B., Enigma, K.M. and Jaffrey, M.Y., "A three layer semi – empirical model for blood flow and particulate suspension through narrow tubes", ***ASME Journal of Biomechanical Engineering,*** Vol. 104, (1982) 129–135.
33. Drew, D.A., "Low concentration of two-phase flow near a stagnant point", ***Physics Fluids,*** Vol. 17, (1974), 1688-1691.
34. Lew, H.S., and Fung, Y.C., "Entry flow into blood vessels at arbitrary Reynolds number", ***Journal of Biomechanics,*** Vol. 3, (1970), 23–30.
35. Misra, J.C., and Pandey, S.K., "Peristaltic flow of a multilayered power-law fluid through a cylindrical tube", ***International Journal of Engineering Science,*** Vol. 39, (2001), 387-402.
36. Muller, V.A., "Die mehrschichtige der fortpflonzungesges chwindigkeit and der damfung der druckwellen in dehnba ren rohren von deren wellenlange", ***Helv. Physiology Pharmacology Acta***, Vol. 9, (1951), 162-176.
37. Muller, V.A., "Die mehrschichtige rohrwand als model fur die aorta", ***Helv. Physiology Pharmacology Acta***, Vol. 17, (1959), 131–145.
38. Merril, E.W., Cokelet, G.R. Britten, A. and Wells, R.E., "Non-Newtonian rheology of human blood – effect of fibrinogen deduced by subtraction", ***Circulation Research,*** Vol. 13, (1963), 48–56.
39. Womersley, J.R., "Oscillatory flow in arteries. I. The constrained elastic tube as a model of arterial flow and pulse transmission. II. The reflexion of pulse wave at junctions and rigid inserts in the arterial system", ***Journal of Medicine and Biology,*** Vol. 2, (1958),178–187,
40. Haynes, R.H., and Burton, A.C., "Role of non-Newtonian behavior of blood in homodynamic", ***American Journal of Physiology***, Vol. 197, (1959), 943–952.
41. Hershey, D., Byrnes, R.E. Deadens, R.L. and Roam A.M., "Blood theology: temperature dependent of the power law model" ***Presented at the A.I.CH.E. Meeting, Boston,*** (1964).
42. Hershey, D. and Chow S.J., "Blood flow in rigid tubes: thickness and slip velocity of plasma film at the wall", ***Journal of Applied Physiology,*** Vol. 21, (1966), 27–36.
43. Sud, V.K. and Sekhon, G.S., "Arterial flow under periodic body acceleration", ***Bulletin of Mathematical* Biology,** Vol. 47, (1985), 35– 52.
44. Charm, S.E., and Kurland G.S., "Blood Flow and Microcirculation", Wiley, New York, (1974).
45. Womersley, J.R., "Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known", ***Journal of Physiology,*** Vol.127, (1954), 38–51.
46. Merrill, E.W., Cokelet, G.R., Britten, A. and Wells, R.E., "Rheology of human blood and red cells plasma membrane", ***Biophysiology Anat,*** Vol. 4, (1964), 51–63.
47. Huckaback, C.E., and Hahn A.W., "A generalized approach to the modeling of arterial blood flow", ***Journal of Applied Physiology,*** Vol. 27, (1968), 27–34.
48. Womersley, J.R., "Oscillatory motion of a viscous liquid in a thin walled elastic tube, I. The linear approximation for long waves", ***Philosophical Magzine***, Vol. 46, (1955), 199– 221.
49. Womersley, J.R., "An elastic theory of pulse transmission and oscillatory flow in mammalian arteries". ***WADC Report TR – 56-614,*** (1957).
50. Shu, C., "Hemorheology in clinical medicine", ***Clinical Hemo-rheology,*** Vol. 2, (1982), 137–142.