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RESEARCH NOTE

Boundary Element Analysis of a Lined Tunnel Problem

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ABSTRACT

The boundary element technique is a practical problem-solving tool and useful in comparison to other currently popular alternatives. In this paper, a circular tunnel with a diameter of 3.4 m is to be excavated through a clayey soil and surrounded by concrete lining of a thickness 0.2 m, so the clear internal diameter is 3.0 m. A plane strain analysis is adopted for such problem and the soil is assumed to be homogeneous, isotropic and linearly elastic medium. The program MRBEM (multi-region BEM) is adopted to analyze a 2D circular lined tunnel in a soil medium of an infinite extent. The existence of a lined tunnel, as in the case of any structure, is affected by the soil medium and affects the surrounding soil. This may be understood throughout the study of many parameters that influence the tunnel's behavior. The parametric study for the lined tunnel covers the influence of the following factors: depth of the tunnel and undrained shear strength of soil. It was found that ground surface settlement, due to the effect of surcharge load, shows its maximum value at both sides of the lined tunnel and not above the center of the tunnel. The maximum positive stresses at the lining interface occur at the crown and invert regions. These stresses tend to decrease gradually until they reach their minimum negative values at the springline region.

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1. INTRODUCTION

With the modern development of experimental research, theoretical analysis becomes very powerful and soundly based. A successful theoretical analysis is capable of producing accurate and reliable results without the risk or expense required to build a test rig and carry out an actual experimental investigation [1].

Boundary element models are widely used in geomechanics problems for computing stresses and displacements around underground excavations. Most of these models assume the rock mass to be a homogeneous, isotropic, linearly elastic solid, although heterogeneity and anisotropy can also be analyzed by the boundary element method [2- 4]. An important development of the boundary element approach is that half-plane and bounded half-plane problems can be solved without making additional numerical approximations. Problems involving layers of finite thickness are more difficult to treat as closed form fundamental solutions are difficult to derive. However, the fundamental solution for a multi-layered medium has been derived using the principle of superposition [5-6].

2. LINED TUNNEL PROBLEM

A problem of a practical importance in the soil-structure interaction field is the lined tunnel. Generally, a tunnel must be supported by the ground strength, and the lining (either concrete or steel) only plays a role of an assistant to maintain the ground strength.

The lining of a tunnel is never loaded by the stress which is initially prevailed in the ground. Luckily, the initial (or primary) stress is reduced by deformation of the ground that occurs during excavation but also after installation of the lining (here 'lining' is understood as the shell of shotcrete, which is placed as soon as possible after excavation). Deformation of the ground (soil or rock) implies a reduction of the primary stress. This is a manifestation of *arching*. Since the deformation of the ground is connected with the deformation of the lining, it follows that the load acting upon the lining depends on its deformation. This is always the case with soil-structure and constitutes an

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inherent difficulty for design as the load is not an independent variable. Thus, the question is not “which is the pressure acting upon lining”, but rather “which is the relation between pressure and deformation” [7].

Methods of efficiently analyzing the three-dimensional state of stress and displacement in a rock or soil mass due to tunneling were presented by Beer et al. [8]. The boundary element method (BEM) and the coupled BEM-finite element method were used. Special infinite boundary elements were used to model the portion of the tunnel which is very long and can be assumed to extend to infinity. The methods are particularly efficient and accurate for determining the state of stress at the tunnel face and near the tunnel intersections. A test example and a practical application in tunneling are shown.

A numerical method of viscoelastic boundary element analysis using the time marching method was presented by Shinokawa and Mitsui [9] and was combined with the finite element method. Some numerical examples in the viscoelastic analysis were represented to confirm the applicability of the boundary element method (BEM) to geotechnical analysis. As an application of BEM to practical geotechnical engineering, the stability of a pillar (an area between two tunnels when they cross obliquely) is examined using the elastic boundary element analysis. It was concluded that BEM can be efficiently applied in practical geotechnical analysis.

Shou [10] stated that a weak zone can be considered as a layered system composed of formations with different material properties. In order to understand the potential mechanism for material instability in relation to the material contrast, it is essential to consider the problem as a multi-layered system. For the analysis of multi-layered elastic media, a linear variation displacement discontinuity method was developed based on a scheme superposing two sets of bonded half-planes and subtracting one infinite plane. The model was verified before being applied to the study of the fracture zone behaviour of tunneling through a weak zone. The results showed that the influence of the material contrast on the stress distribution might be insignificant. However, the fracture zone behaviour, especially the fracture patterns, can be quite different for different material contrasts. The material contrast and the existence of interface(s) play important roles on the behaviour of the rock mass near the interface approached by the tunneling face. The results showed that the influence of the material contrast on the stress distribution might not be so significant except at the interfaces.

This paper is directed to analyze a circular lined tunnel in a soil medium of an infinite extent. The stresses and displacements around the tunnel are investigated.

3. THE PROGRAM MRBEM

The program MRBEM (Multi-Region Boundary Element Method) is a general purpose BEM program for solving elasticity and potential problems with multiple regions. This program is suitable for solving soil-structure interaction problems in which different material models are prescribed. For the solution of heterogeneous domains, the place where cannot obtain a fundamental solution, the concept of multiple regions is adopted where the domain is subdivided into subregions, much in the same way as with the finite element method (FEM). Since at the interfaces between the regions, both tractions and displacements are not known, the number of unknowns is increased and additional equations are required to be able to solve the problem. These equations can be obtained from the conditions of equilibrium and compatibility at the region interfaces.

There are two approaches which can be taken into the consideration in the implementation of the method. In the first, the assembly procedure is modified so that larger systems of equations are obtained including the additional unknowns at the interfaces. The second method is similar to the approach taken by the FEM. Here, a stiffness matrix K is constructed for each region, the coefficients of which are the fluxes or tractions due to unit temperatures/ displacements. The matrices K for all regions are then assembled in the same way as with the FEM. The second method is more efficient and more amenable [11].

4. GENERAL DESCRIPTION OF THE PROBLEM

The program MRBEM (multi-region BEM) is adopted to analyze a 2D circular lined tunnel in a soil medium of an infinite extent. Considering the cross section of the tunnel shown in Figure 1, various locations are denoted by the indicated names.

A circular tunnel with a diameter of 3.4 m is to be excavated through a clayey soil and surrounded by concrete lining of a thickness 0.2 m, so the clear internal diameter is 3.0 m. A plane strain analysis is adopted for such problem and the soil is assumed to be homogeneous, isotropic and linearly elastic medium.

5. BOUNDARY ELEMENT DISCRETIZATION

Figure 2 shows the boundary discretization scheme of the two regions (soil and concrete) and the boundary conditions to be used in the problem. The boundaries of the two regions are discretized to numbers of linear elements with a further concentration of elements at the curved surface so as to be closer to the smoothness.

The program has the facility of symmetry about an axis therefore; half the problem is taken in order to reduce the computational effort. As only the surface of the continuum needs to be discretized in the BEM, problem extending to infinite can be described by a very small number of elements or unknowns. In addition, the boundary conditions of the infinite domain are properly defined.

The soil region is subjected to a surcharge load of 20 kN/m^2 distributed uniformly along its surface and its effects is transported to the nodes of each linear element. The water table level is assumed to be at the natural ground surface level (G.S.L), so the soil is considered as a fully saturated soil.

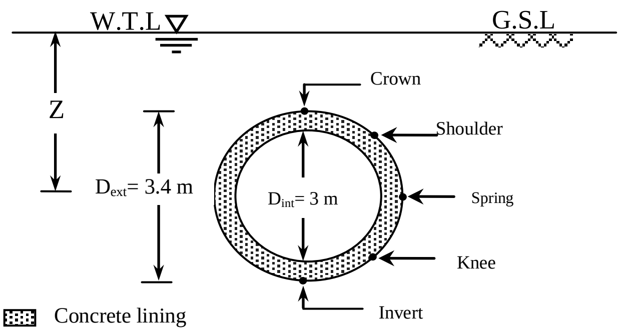


Figure 1. The problem of a tunnel with concrete lining.

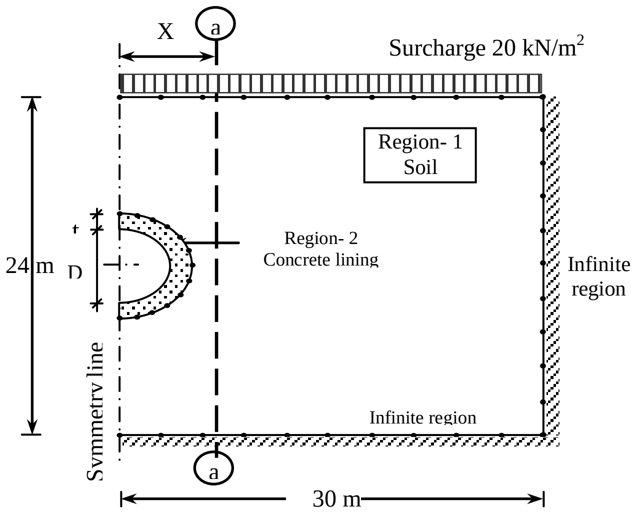


Figure 2. Boundary element discretization for the lined tunnel problem.

6. PARAMETRIC STUDY

The existence of a lined tunnel, as in the case of any structure, through the soil medium is affected by the surrounding soil. The tunnel itself also affects the surrounding soil. This may be understood throughout the study of many parameters that influence the tunnel's behavior. The parametric study for the lined tunnel covers the influence of the following factors:

- Depth of the tunnel
- Undrained shear strength of soil

Table 1 gives a brief description for the parametric study adopted in the tunnel problem.

TABLE 1. Parametric study for the lined tunnel

Tunnel with concrete lining	
Soil type	Z/D
Soft clay	2
	3
	4
Medium clay	2
	3
	4
Stiff clay	2
	3
	4

7. MATERIAL PROPERTIES

In order to analyze the problem of the lined tunnel embedded in soil, it is important to define the properties of the materials which are concerned with such problems. The program MRBEM requests two kinds of properties; those are the modulus of elasticity E and the Poisson's ratio ν.

Needless to say that many empirical equations have been published which correlate the undrained shear strength C_u of the soil with its modulus of elasticity E_s. Since the outline of this study is focusing on the application of the BEM, thus it is decided to use one of these correlations so as not to be involved in these complexities. This correlation is:

For normally consolidated clays [12]:

$$E_s = 250\ c_u \text{ to } 500\ c_u \tag{1}$$

Table 2 shows the material properties used in the lined tunnel problem.

TABLE 2. Material properties for the lined tunnel problem.

Soil				Concrete	
Soil type	c_u (kPa)	E_s (kPa)	ν	E_c (kPa)	ν
Soft	25	12500	0.49	21×10^6	0.15
Medium	60	30000			
Stiff	100	50000			

8. RESULTS AND DISCUSSION

Figure 3 shows the ground surface settlement due to the effect of the surcharge load. It is evident that the soft clay shows the larger displacement followed by medium and stiff clay. One can see that the curves are concaved upward above the region of the tunnel's centerline, especially for the soft clay, which shows a large amount of deformation at both sides of the tunnel. These results are in good agreement with those obtained by Salim [13] who used the CRISP finite element program to analyze tunnel problems. It is also clear from this figure that increasing the tunnel's depth has no significant effect on the surface settlement except that the concavity is decreased gradually and the curves come closer to straightness.

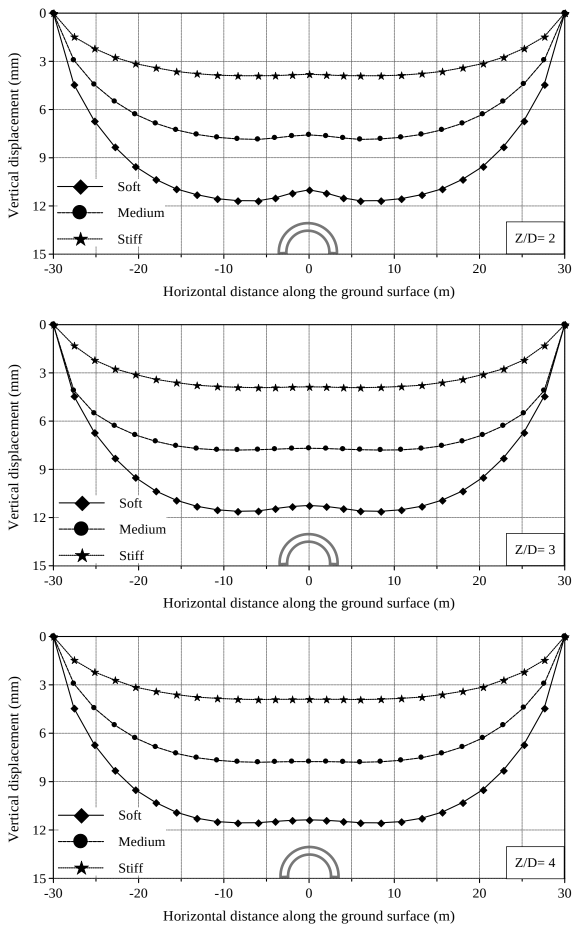


Figure 3. Distribution of vertical displacement at the ground surface for the tunnel problem.

Figure 4 shows the vertical stress at the ground surface level for different depths of tunnel. It is easy to see that the type of soil and the depth of tunnel have no significant effects on the vertical stress distribution.

Figure 5 shows the nodal displacements along section a-a (shown in Figure 2) in the soil layer which extent a distance of (X= 6 m) away form the center of the tunnel.

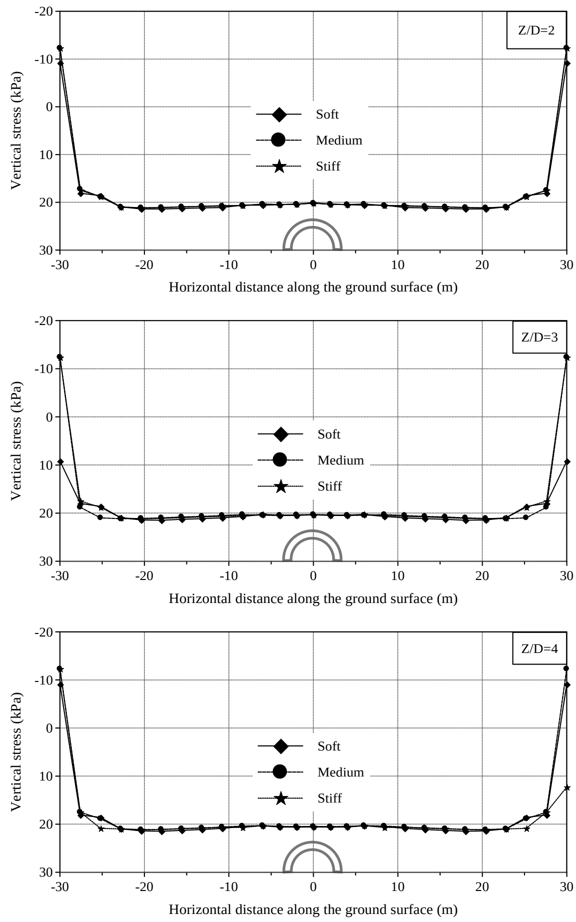
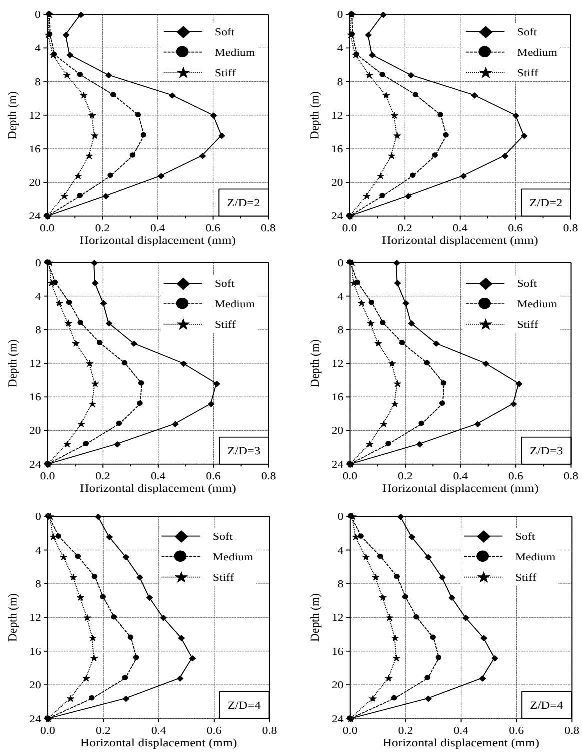


Figure 4. Distribution of vertical stresses at the ground surface for the tunnel problem.

It is apparent that the existence of the tunnel affects the variation of the horizontal displacements more than the vertical displacements. This figure illustrates that the maximum displacement, even it is very small, is not lying on the level of the tunnel's center but within the bottom half of the depth. These results are different from those found by Abdul-Raheem [14] who used the SAGE-CRISP finite element program and showed the same trends of the curves but the maximum horizontal displacement is lying at the center of tunnel. It is believed that the tunnel with its concrete lining tends to compress the bottom portion of the soil below the invert region, so this axial load is transferred laterally at that portion of the soil. It can be seen also that the lateral displacement curves are redistributed more uniformly as the tunnel sink down into the soil. The soft soil has the larger amount of lateral displacement due to its lower shear strength.

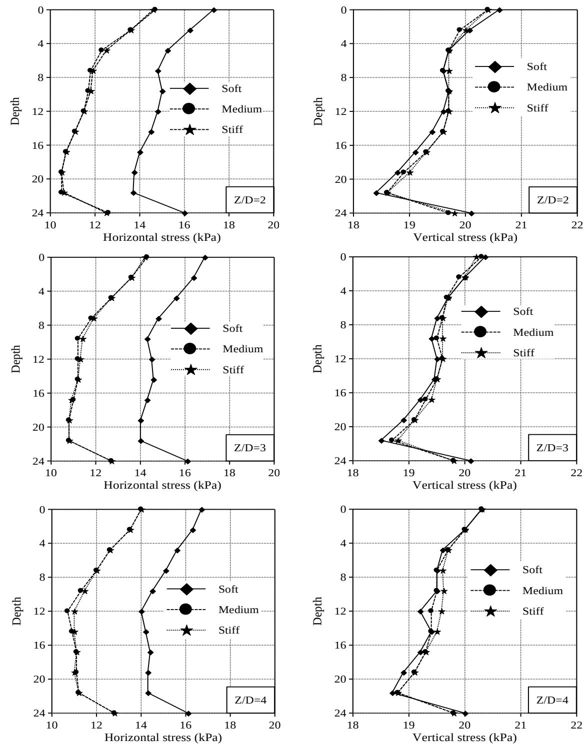
On the other side of this figure, the vertical displacement is plotted against the depth at the same section of the soil. The maximum settlement value is found at the surface layer which is in contact with the applied load, and then it decreases linearly with depth where the influence of the transferred load begins to decrease. Contrary to the horizontal displacement, the vertical displacement does not show a considerable effect throughout the increase of the tunnel's depth, and the soft clay is still having the larger amount of displacement.



a. Horizontal displacement b. Vertical displacement

Figure 5. Distribution of displacements at a distance of 6 m away from the tunnel's centerline.

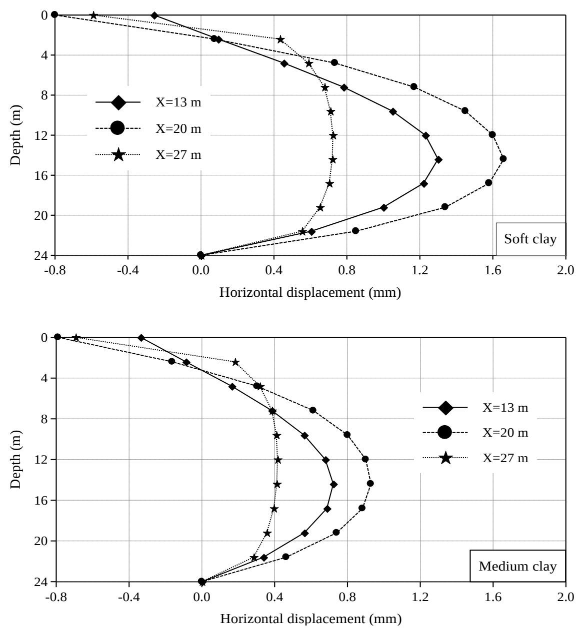
Figure 6 shows the stress distribution along a section in the soil layer which extends to a distance of (X=6 m) away form the center of the tunnel. It is obvious that the horizontal stress reaches its maximum value at the soil surface and begins to decrease with depth. If a connection is to be made with the horizontal displacement shown in Figure 5, one can notice that the minimum horizontal stress appears within a portion of depth at which the maximum horizontal displacement is calculated. This may be reinforced by the previous comment on Figure 5 which mentions that the maximum lateral displacement is a result of load which is transferred from the invert of lining to the lower soil. The vertical stress distribution along the same section is also shown in this figure. It is obvious that the different types of soil have the same behavior with a slight difference in their results.



a. Horizontal displacement b. Vertical displacement

Figure 6. Distribution of stresses at a distance of 6 m away from the centerline of tunnel.

Figure 7 shows the variation of horizontal displacement along the section a-a as it extends away from the tunnel for different distances (X).



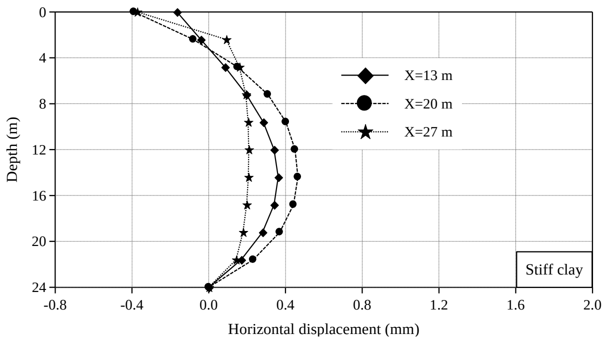
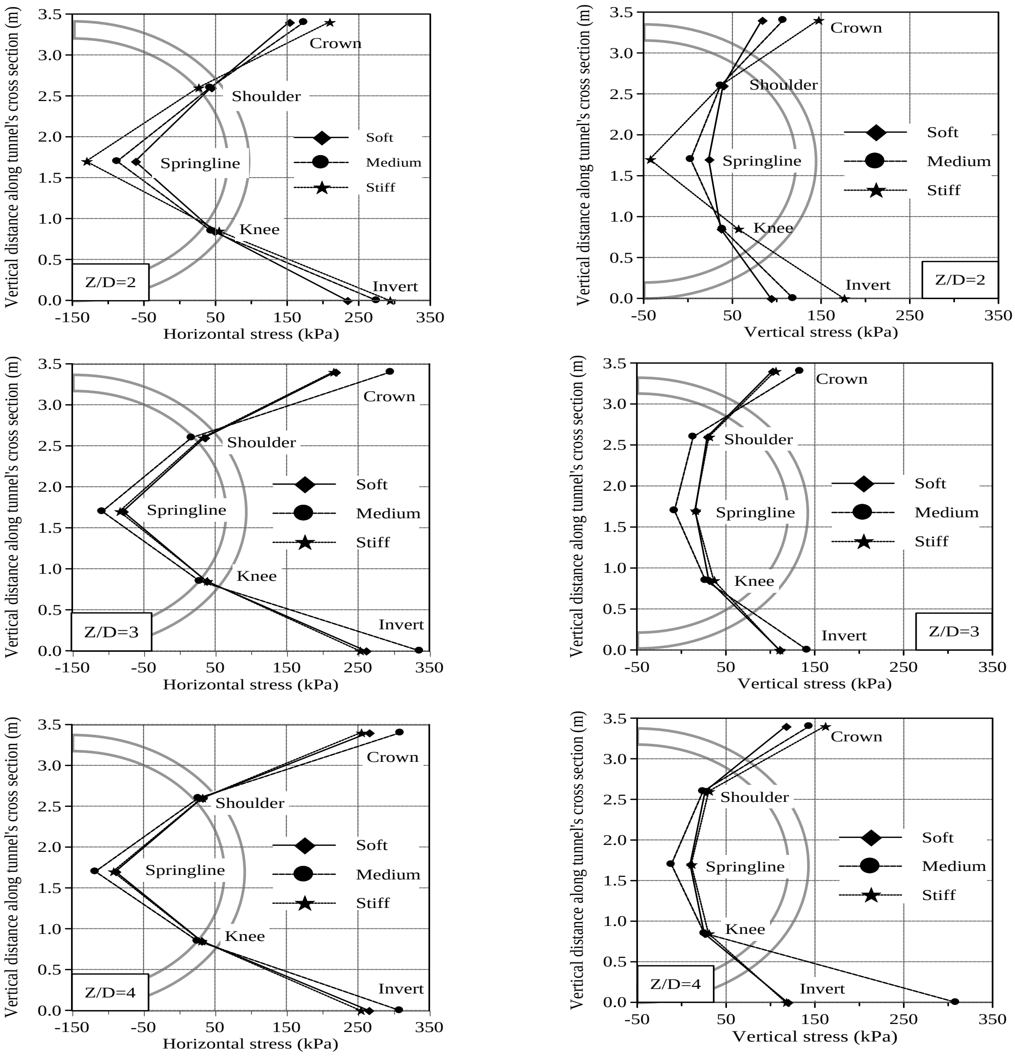


Figure 7. Distribution of horizontal displacement at different distances away from the tunnel centerline for Z/D =5.

It is obvious that the displacements are increased laterally with distance until they reach their maximum values at a distance (X=20 m) which represents two third of the horizontal distance. After that, the displacements are decreased gradually until they completely diminished at the boundary where it is assumed that the nodal displacements are equal to zero.

Figure 8 shows the distribution of stresses at the lining interface represented by the crown, shoulder, springline, knee and invert regions. It can be seen that the crown of the lining and its invert take the maximum stresses. Since these two regions are lying on the axis of symmetry, thus the assumed boundary condition is that nodes are restrained against any lateral movement which gives the highest value for the horizontal stresses.



a. Horizontal displacement

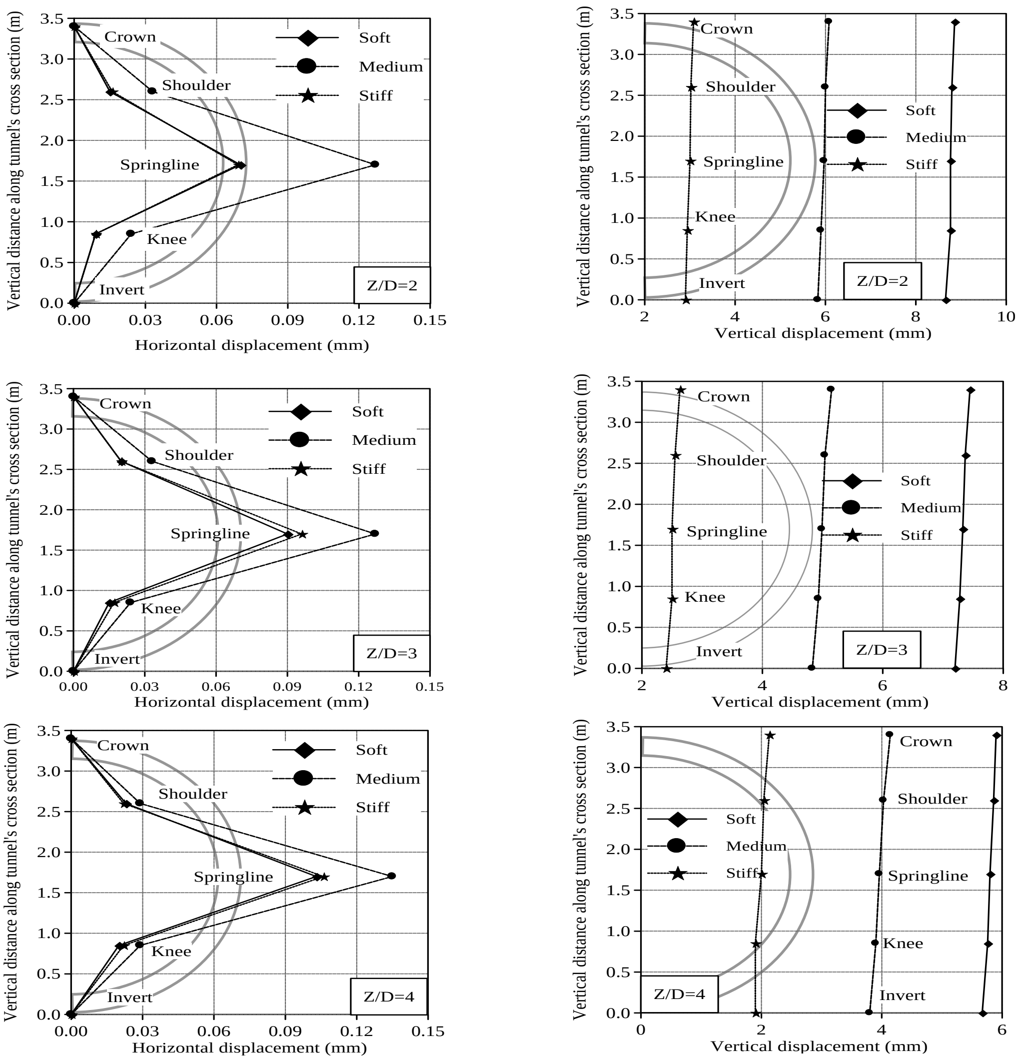
b. Vertical displacement

Figure 8. Distribution of stresses at certain points along the lining perimeter

The positive values refer to the compressive stress, so the crown is compressed under the action of the overburden pressure and also the invert under the action of the bearing soil below the tunnel. Under this pressure, the diameter of the tunnel tends to decrease axially and increase laterally. Based on this variation in the diameter, one can notice that the stresses at the shoulder begin to decrease continuously until they reach the minimum negative value at the springline region. These negative values are due to horizontal elongation in the diameter which leads the interface of the lining to be under tension at that region. These results are fairly compatible with those obtained by Al-Sadoon [15] who used the FEM to analyze the soil-structure interaction in tunnels.

In order to get a complete picture for the stress-strain relationship, the displacements at the crown, shoulder, springline, knee and invert are shown in Figure 9. One can see that the maximum horizontal displacement appears at the springline and increases with depth (Z/D) due to the thrust occurring at the crown. The shoulder and the knee represent inflection points at which the stresses and displacements are transformed from the inward movement at the crown region to the outward movement at the springline region. Therefore, it is evident that these two regions have almost constant values throughout the variation of depth.

The vertical displacements are also shown in this figure in which it is apparent that these kinds of displacements are increased with depth due to the overburden pressure.



a. Horizontal displacement

b. Vertical displacement

Figure 9. Distribution of displacements at certain points along the lining perimeter.

On the other hand, if the magnitude of the displacement is taken to be as the difference between two opposite points, then the evaluation process takes another approach as it is shown in Figure 10. This figure plots the axial and lateral strain in the concrete lining against the non-dimensional depth (Z/D), in the other words, the shortening and elongation which appears in the lining. One can notice that even if the maximum vertical displacement in the concrete lining is shown in soft clay, but the maximum non-uniform displacement (axial strain or shortening) appears when the surrounding soil is medium clay. This shortening causes a lateral outward movement at the springline region which leads the diameter to be enlarged.

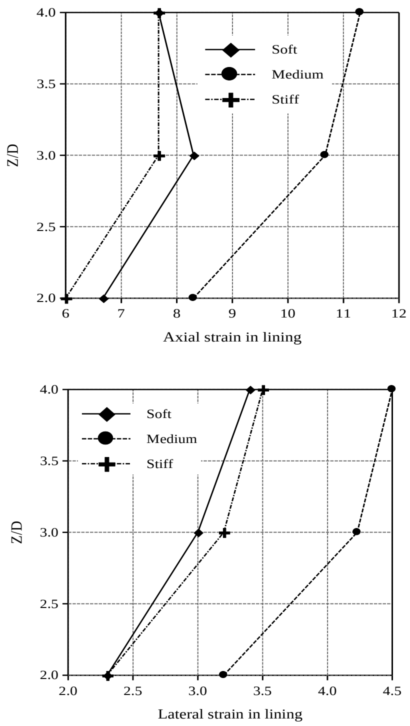


Figure 10. Axial and lateral strain in the concrete lining.

9. CONCLUSIONS

1. Ground surface settlement, due to the effect of surcharge load, shows its maximum value at both sides of the lined tunnel and not above the center of the tunnel.
2. The maximum positive stresses at the lining interface occur at the crown and invert regions. These stresses tend to decrease gradually until they reach their minimum negative values at the springline region.
3. The horizontal displacement at the lining interface shows its maximum value at the springline region. The shoulder and the knee represent inflection points at which the stresses and displacements are transformed from the inward movement at the crown to the outward movement at the springline region.
4. Although the maximum vertical displacement in the concrete lining is significant with stiff clay, the maximum difference of the vertical displacement between the crown and the invert regions (the shortening in the diameter) appears when the surrounding soil is medium clay.

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RESEARCH NOTE

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چکیده

روش عنصر مرزی یک متد عملی برای حل مسئله می‌باشد و در مقایسه با دیگر روش‌های متداول و معروف مناسب است. در این مقاله، یک تونل دایره‌ای به قطر ۳,٤ مترکه در خاک رس احاطه شده با لایه بتن به ضخامت ۰,۲ متر حفاری شده است، قطر داخلی مشخص ۳,۰ متر است.تجزیه و تحلیل کرنش سطحی برای چنین مسائلی اتخاذ می‌گردد و خاک محیطی همگن، ایزوتروپیک و الاستیک خطی فرض شده است. برنامه MRBEM (BEM چند منطقه‌ای) برای تجزیه و تحلیل یک تونل رشته‌ای مدوردوبعدی در محیط خاک در ابعاد بی نهایت استفاده شده است. وجود تونل رشته‌ای، در مورد هر ساختار، تحت تاثیر محیط خاک بوده و خاک اطراف آن را تحت تاثیر قرار می‌دهد. این مسئله در طول مطالعه بسیاری از پارامترهای مؤثر بر رفتار تونل قابل درک است.مطالعه پارامتری تونل رشته‌ای تاثیر عوامل زیر را پوشش می‌دهد: عمق تونل مقاومت برشی تخلیه نشده خاک. فرونشست سطح خاک، با توجه به اثر اضافه بار، حداکثر مقدار خود را در هر دو طرف تونل رشته‌ای و نه بالاتر از مرکز تونل نشان می‌دهد. حداکثرمقدار مثبت تنش در مناطق تاجی و قوسدار اتفاق می-افتد. این تنش ها به تدریج تمایل به کاهش یافتن دارند تا زمانی که به حداقل مقادیر منفی خود در منطقه springline برسند.

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