MULTI-OBJECTIVE OPTIMIZATION OF SOLAR THERMAL ENERGY STORAGE USING HYBRID OF PARTICLE SWARM OPTIMIZATION, MULTIPLE CROSSOVER AND MUTATION OPERATOR

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Abstract Increasing of net energy storage (Q net) and discharge time of phase change material (t PCM) simultaneously, are important purpose in the design of solar systems. In the present paper, multi-objective (MO) based on hybrid of Particle Swarm Optimization (PSO) and multiple crossover and mutation operator is used for Pareto based optimization of solar systems. The conflicting objectives are Q net and t PCM and design variables are some geometrical parameters of solar system. The Pareto results of MO hybrid of PSO and multiple crossover and mutation operator methods are compared with that of multi-objective genetic algorithms (NSGA II). It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of solar systems can be discovered.

Keywords Particle Swarm Optimization, Multi-Objective Optimization, Multiple Crossover and Mutation Operator, Solar System, PCM.

چکیده افزایش میزان ذخیره انرژی و زمان دشارژ مواد پی سی ام به طور همزمان از اهداف مهم در طراحی سیستم های خورشیدی می باشند.در این مقاله بهینه سازی چند منظوره بر اساس بهینه سازی دسته ذرات و تقاطع چندگانه و عملگر جهشی برای بهینه سازی سیستم خورشیدی پارتویی استفاده شده است. اجزادی دارای تعارض گرمای خالص و زمان دشارژ پی سی ام بوده و متغیر های طراحی شامل پارامتر های جغرافیایی سیستم خورشیدی می باشند. نتایج پارتوی بهینه سازی چند گانه ترکیبی از بهینه سازی دسته ذرات، تقاطع چندگانه و روش های عملگر جهشی با آلگوریتم ژنتیک چند منظوره مقایسه شده اند. نشان داده شده است که که روابط جالب و مهمی به عنوان اصول طراحی بهینه در ارتباط عملکرد سیستم خورشیدی بدست می آید.

1. INTRODUCTION

Using thermal storage systems to gain hot water is becoming very popular as a result of limited fossil fuel sources and the requirement of protecting environment related to not polluting characteristics of this kind of energy. Many studies have been done to calculate transferred heat within Phase Change Material (PCM) and net heat gain by means of whole system. Stritih [1] embarked on an experimental study and achieved temperature distribution along the PCM. Canbazoglu et al. [2] developed an experimentally investigation for calculating the midpoint tank temperature with

Nomenclature

Q_{net}	Net energy stored
t	Discharge time
PCM	Phase Change Material
PSO	Particle Swarm Optimization
MO	Multi Objective
NSGA II	Non Dominated Sorting Genetic Algorithms
F_R	Correction factor
U_t	Total heat transfer coefficient
h_f	Heat transfer coefficient
$\dot{Q}_w$	Total stored heat
φ	Latitude
Q_s	Sensible heat
Q_L	Latent heat
A_c	Collector area, m^2
C_p	Specific heat, J/kgK
C_n	Cash flow, $
D	Diameter, m
F'	Collector efficiency factor
F_R	Collector heat removal factor
H	Instantaneous solar radiation, W/m^2
H_b	Beam component of solar energy incident on the plane of measurement, W/m^2
H_d	Diffuse component of solar energy incident on the plane of measurement, W/m^2
h	Convection heat transfer coefficient, W/m^2K
h_{SL}	Specific melting latent heat of PCM during the phase change, kj/kgK
Q	Energy, kJ
R	Ratio of total radiation on tilted surface to that on plane of measurement
R_b	Ratio of beam radiation on tilted surface to that on plane of measurement
r	Internal rate of return (IRR)
S	Absorbed solar energy per unit area W/m^2
s	Slope of plane from horizontal
T	Temperature, K
ΔT_1	The difference between the maximum temperature of hot water in the heat storage tank and the melting temperature of PCM, K
ΔT_2	The difference between the temperature of hot water taken from the heat storage tank and the melting temperature of PCM, K
U	Total heat transfer coefficient, W/m^2K
W	Pipe distance in the collector, m
w	Clock angle

sodium thiosulfate pentahydrate PCM. Aghbalou et al. [3] dealt with the exergetic optimization of a solar thermal energy system which consisted of a solar collector and a rectangular water storage tank that contains a PCM distributed in an assembly of slabs. Mettawee and Assassa [4] used paraffin wax as PCM in an experimental study to investigate the performance of a compact PCM solar collector based on latent heat storage. Koca et al. [5] developed an experimental system with analysis of energy and exergy for a latent heat storage system with PCM. Varol et al. [6] made a series of predictions by using three different soft computing techniques to introduce an efficient method for calculating useful energy.

Optimization in engineering design has always been of great importance and interest particularly in solving complex real-world design problems. Basically, the optimization process is defined as to find a set of values for a vector of design variables so that it leads to an optimum value of an objective or cost function. There are many calculus-based methods including gradient approaches to single objective optimization and are well documented in Arora [7] and Rao [8]. However, some basic difficulties in the gradient methods, such as their strong dependence on the initial guess, cause them to find local optima rather than global ones. Consequently, some other heuristic optimization methods like Particle Swarm Optimization (PSO) have been used extensively during the last decade. PSO, first introduced by Kennedy and Eberhart [9], is one of the modern heuristic algorithms. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems [9, 10]. The PSO technique can generate a high-quality solution within short calculation time and stable convergence characteristic than other stochastic methods [11, 12]. In the present paper we deal with a multi-objective optimization problem. In these problems, there are several objective or cost functions (a vector of objectives) to be optimized (minimized or maximized) simultaneously. These objectives often conflict with each other so that improving one of them will deteriorate another. Therefore, there is no single optimal solution as the best with respect to all the objective functions. Instead, there is a set of optimal solutions, known as Pareto optimal solutions or Pareto front [13- 16] for multi-objective optimization problems. The concept of Pareto front or set of optimal solutions in the space of objective functions in multi-objective optimization problems (MOPs) stands for a set of solutions that are non-dominated to each other but are superior to the rest of solutions in the search space. This means that it is not possible to find a single solution to be superior to all other solutions with respect to all objectives so that changing the vector of design variables in such a Pareto front consisting of these non-dominated solutions could not lead to the improvement of all objectives simultaneously. Consequently, such a change will lead to deteriorating of at least one objective. Thus, each solution of the Pareto set includes at least one objective inferior to that of another solution in that Pareto set, although both are superior to others in the rest of search space.

In this paper, multi-objective PSO method is used for Pareto approach optimization of solar systems. The conflicting objective functions are net energy stored (Q_{net}) and discharge time of PCM (t_{PCM}) and design variables are some geometrical parameters of solar system. It is shown that some interesting and important relationships as useful optimal design principles involved in the performance of solar network can be discovered. Such important optimal principles would not have been obtained without the use of Pareto optimization approach.

2. MATHEMATICAL MODELING OF SOLAR SYSTEM

The schematic of thermal storage unit under analysis is shown in Figure (1). It consists of a flat plate solar collector, a storage water tank with disodium hydrogen phosphate-dodecahydrate as the PCM content tank inside, a pipeline and several valves. Water temperature rises due to flowing through the collector when Sun is shining. Then, it goes into the water tank including the PCM tank. Heat energy transfers from warm water to the solid PCM, which its melting point is 29 centigrade degrees, and changes its phase into liquid. After that, water is ready to use. When the Sun descends, network water comes to the storage tank from bypass pipeline and receives thermal energy stored in fluid PCM during the day. In this process, water temperature increases while the PCM phase is changing. After that, water goes to pipeline to be consumed.

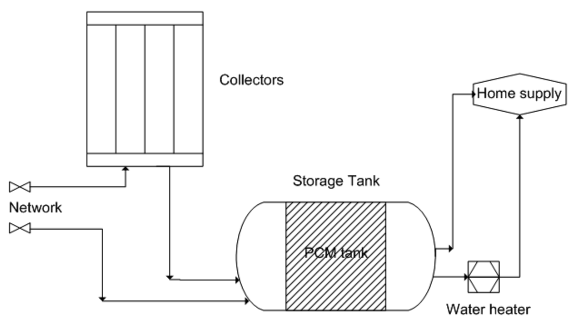

Figure 1. Solar system set up

Energy analysis was carried out to evaluate the amount of heat energy stored by solar collector with the PCM. For a solar collector shown schematically in Figure (1), the useful energy that increases water temperature during the flowing inside the collector before the water and PCM tank is represented in Equation (1);

$$Q_u = A_c F_R \left[S - U(T_i - T_a) \right] \tag{1}$$

where F_R can be calculated using Equation (2);

$$F_R = \frac{\dot{m} C_p}{A_c U} \left[1 - e^{-\left[A_c U F' / \dot{m} C_p \right]} \right] \tag{2}$$

Collector efficiency factor, F' could be achieved from Equation (3)

$$F' = \frac{U_t^{-1}}{W\left[U_L \left(D + (W - D) F \right) \right] + \frac{1}{\pi D_i \; h_f}} \tag{3}$$

Total heat transfer coefficient is calculated from the expression as below

$$U_t = \left[\frac{N}{\left(\frac{344}{T_p} \right) \left[\frac{T_p - T_a}{N + f} \right]^{0/31}} + \frac{1}{h_w} \right]^{-1} + \frac{\sigma (T_a + T_p)(T_a^2 + T_p^2)}{\left[\epsilon_p + 0/0425 N (1 - \epsilon_p) \right]^{-1} + \left[\frac{2N + f - 1}{\epsilon_g} \right] - N} \tag{4}$$

Net thermal energy gained with the absorber of collector may be obtained by multiplying instantaneous solar radiation by heat gain coefficient and effective absorptivity of solar collector which is shown in Equation (5) as below;

$$S \quad HR(\quad) \tag{5}$$

Effective absorptivity of solar collector can be defined with Equation (6)

$$(\tau \alpha) = \rho \times \sum_{n=0}^{\infty} \left[(1 - \alpha) \rho d \right]^n = \frac{\rho \alpha}{1 - (1 - \alpha) \rho d} \tag{6}$$

Heat transfer resistance, R which is ratio of total radiation on tilted surface to that on plant of measurement can be calculated as bellow [17]

$$R = \frac{H_b}{H} R_b + \frac{H_d}{H} \tag{7}$$

R_b is the ratio of beam radiation on tilted surface to that on plant of measurement may be obtained by Equation (8)

$$R_b = \frac{\cos(\varphi - s)\cos\delta \cos w + \sin(\varphi - s)\sin\delta}{\cos\varphi \cos\delta \cos w + \sin\varphi \sin\delta} \tag{8}$$

When water flows into the storage tank, we should apply energy balance equations including both PCM and water. Sensible and latent heat are represented in Equations (9) and (10)

$$Q_S = m_w c_{p,w} \Delta T + m_{PCM} (c_{p,PCM-L} \Delta T_1 + c_{p,PCM-s} \Delta T_2) \tag{9}$$

$$Q_L = m_{PCM} h_{SL} \tag{10}$$

The total heat for the storage tank may be obtained by summation of sensible and latent heat as shown in Equation (11)

$$Q_{PCM} = Q_S + Q_L \tag{11}$$

where Q_{PCM} is the total energy stored in the PCM. After this, the discharge time of PCM which is the period that hot water is available after sunset can be calculated

$$t_{PCM} = \frac{Q_{PCM}}{\dot{Q}_W} \tag{12}$$

In Equation (12) $\dot{Q}_w$ is the total heat stored per unit time in the conventional storage tank. And finally,

$$Q_{net} = m_w C_{p,w} (T_{out} - T_{in}) - \dot{Q}_{PCM} \tag{13}$$

It is obvious that in a solar system the net energy stored (Q_{net}) and discharge time of PCM (t_{PCM})

should be maximized. Design variables in present study are: inner diameter of pipes (D), area of collectors (A_C), water mass flow rate($\dot{m}$), latitude (φ) and the mass of PCM (M_{PCM}). The range of variations of design variables are shown in table (1) moreover the constants in the mathematical modeling are shown in table (2).

TABLE 1: Design variables and their range of variations

Design Variable	From	To
D (m)	0.008	0.02
A_C (m^2)	0. 25	1
$\dot{m}$ (kg/s)	0.0015	0.005
φ (deg)	10	45
M_{PCM} (kg)	5	20

TABLE 2. The constants used in mathematical modeling of solar system

Parameter	Value
Melting point of PCM (C)	35
Melting latent heat of PCM (kJ/kg)	278.84
Density of PCM (kg/m^3)	1522
Ambient temperature (C)	25
Inlet water temperature (C)	15
Specific heat of solid (kJ/kg k)	1.55
Specific heat of Liquid (kJ/kg k)	2.51

3. PARTICLE SWARM OPTIMIZATION HYBRID WITH MULTIPLE CROSSOVER AND MUTATION OPERATOR

In this section, a novel PSO is proposed which is improved by utilizing multiple crossover and mutation operator to update the particle positions. In the follow, basic concepts of PSO, multiple crossover and mutation operator are introduced and in the next section, hybrid of these operators is described.

3.1. Basic Concept of Particle Swarm Optimization James Kennedy and Russell C. Eberhart [9] originally proposed the PSO algorithm for optimization. PSO is a population-based search algorithm based on the simulation of the social behavior of birds within a flock. Although originally adopted for balancing weights in neural networks [18], PSO soon became a very popular global optimizer, mainly in problems in which the decision variables are real numbers ([18], [19]).

In PSO, particles are "flown" through hyper-dimensional search space. Changes to the position of the particles within the search space are based on the social-psychological tendency of individuals to emulate the success of other individuals. The position of each particle is changed according to its own experience and that of its neighbors. Let $\vec{x}_i(t)$ denote the position of particle p_i, at time step t. The position of p_i is then changed by adding a velocity $\vec{v}_i(t)$ to the current position, i.e.:

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \tag{14}$$

The velocity vector reflects the socially exchanged information and, in general, is defined in the following way:

$$\vec{v}_i(t+1) = W\,\vec{v}_i(t) + Cr(\vec{x}_{global\ best} - \vec{x}_i(t)) \tag{15}$$

where $r \in [0,1]$ is random value, C is the social learning factor and represents the attraction that a particle has toward the success of its neighbors, W is the inertia weight which is employed to control the impact of the previous history of velocities on the current velocity of a given particle. $\vec{x}_{global\ best}$ is the position of the best particle of the entire swarm.

3.2. Basic Concept of Multiple Crossover and Mutation Operator

3.2.1. Multiple Crossovers Unlike the traditional crossover by using only two chromosomes, a novel crossover formula that contains three parent chromosomes is proposed in this study. We assume that chromosome $\vec{x}_i(t)$ selected from the population randomly. Also, let $\rho \in [0,1]$ be a random number. If $\rho \ge \rho_{Crossover}$, then the following multiple-crossover is performed to generate new chromosome

$$\vec{x}_i(t) = \vec{x}_i(t) + \sigma(2\vec{x}_i(t) - \vec{x}_{i-1}(t) - \vec{x}_{i-2}(t)) \tag{16}$$

where $\sigma \in [0,1]$ is a random value. If $\rho \leq \rho_{Crossover}$, no crossover operation is performed.

3.2.2. Mutation Operator The mutation operator provides a possible mutation on some chosen chromosome $\vec{x}_i(t)$. Also, let $\vartheta \in [0,1]$ be a random number. If $\theta \geq p_{Mutation}$, then the following mutation operator is performed to generate new chromosome

$$\vec{x}_i(t) = \vec{x}_i(t) + \xi \times \kappa \tag{17}$$

where $\xi \in [0,1]$ is a random value and κ is a positive constant. If $\theta \leq p_{Mutation}$ no mutation operation is performed.

3.3. Hybrid of PSO and Multiple Crossover and Mutation Operator The flow chart of the Hybrid of PSO and multiple crossover and mutation operator is shown in Figure (2).

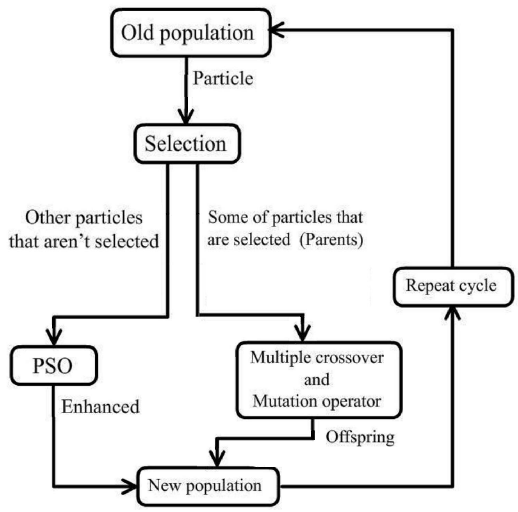

Figure 2. The flow chart of the hybrid of PSO and multiple crossover and mutation operator

3.4. Hybrid of PSO and Multiple Crossover and Mutation Operator for Multi Objective Problems Optimization problems that have more than one objective function are rather common in every field or area of knowledge. In such problems, the objectives to be optimized are normally in conflict with respect to each other, which means that there is no single solution for these problems. Instead, we aim to find good "trade-off" solutions that represent the best possible compromises among the objectives. PSO is a heuristic search technique [9] that simulates the movements of a flock of birds which aim to find food. The relative simplicity of PSO and the fact that is a population-based technique have made it a natural candidate to be extended for multi objective optimization. Moore and Chapman [20] proposed the first extension of the PSO strategy for solving multi objective problems in an unpublished manuscript in 1999. After this early attempt, a great interest to extend PSO arose among researchers, but interestingly, the next proposal was not published until 2002. Nevertheless, there are currently different proposals of multi objective PSOs reported in the specialized literature.

We are interested in solving problems of the type:

$$\text{Minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), ..., f_k(\vec{x})] \tag{18}$$

Subject to:

$$g_i(\vec{x}) \leq 0 \quad i = 1,2,...,m \tag{19}$$

$$h_i(\vec{x}) = 0 \quad i = 1,2,..., p \tag{20}$$

where $\vec{x} = [x_1, x_2, ..., x_n]^T$ is the vector of decision variables, $f_i : R^n \to R, i = 1,...,k$ are the objective functions and $g_i, h_j : R^n \to R, i = 1,...,m, j = 1,..., p$ are the constraint functions of the problem. To describe the concept of optimality in which we are interested, we will introduce next a few definitions.

3.4.1. Dominance Given two vectors $\vec{x}, \vec{y} \in R^k$ we say that $\vec{x} \leq \vec{y}$ if $\vec{x}_i \leq \vec{y}_i$ for $i = 1,...,k$ and that $\vec{x}$ dominates $\vec{y}$ (denoted by $\vec{x} \prec \vec{y}$) if $\vec{x} \leq \vec{y}$ and $\vec{x} \neq \vec{y}$.

3.4.2. Non-Dominance We say that a vector of decision variables $\vec{x} \in \chi \subset R^n$ is non-dominated with respect to χ, if there does not exist another $\vec{x}' \in \chi$ such that $\vec{f}(\vec{x}') \prec \vec{f}(\vec{x})$.

3.4.3. Pareto-optimal We say that a vector of decision variables $\vec{x}^* \in F \subset R^n$ (*F* is the feasible region) is Pareto-optimal if it is non-dominated with respect to *F*.

3.4.4. Pareto Optimal Set The Pareto Optimal Set p^* is defined by: $p^* = \{\vec{x} \in F \mid \vec{x}\ is\ pareto-optimal\}$

3.4.5. Pareto Front The Pareto Front pF^* is defined by: $pF^* = \{\vec{f}(\vec{x}) \in R^k \mid \vec{x} \in p^*\}$

We thus wish to determine the Pareto optimal set from the set *F* of all the decision variable vectors that satisfy Equations (19) and (20). Note however that in practice, not all the Pareto optimal set is normally desirable (e.g., it may not be desirable to have different solutions that map to the same values in objective function space) or achievable. In order to apply the PSO strategy for solving multi objective optimization problems, it is obvious that the original scheme has to be modified. The solution set of a problem with multiple objectives does not consist of a single solution (as in global optimization). Instead, in multi objective optimization, we aim to find a set of different solutions (the so-called Pareto optimal set).

When solving single-objective optimization problems, $\vec{x}_{global\,best}$ is used as a leader to update particles position. However, in the case of multi objective optimization problems, each particle might have a set of different leaders from which just one can be selected in order to update its position. Such set of leaders is usually stored in a different place from the swarm, that we will call "external archive": This is a repository in which the non-dominated solutions found so far are stored. However, if all of non-dominate solutions are retained in the archive then the size of the archive increases very quickly. This is an important issue because the archive has to update at each generation. Thus, this update may become very expensive, computationally speaking, if the size of the archive grows too much. Therefore, archive tends to be bounded, which makes necessary the use of an additional criterion to decide which non-dominated solutions to retain.

In this paper, it is adopted ε-elimination technique to prune the archive. In this approach the entire particles in the archive have a radius of neighborhood equal to ε and if a particle has the distance less than ε to another particle will be eliminated in the objective function space. Here, the following equation is used to determined ε:

$$\varepsilon = \frac{t}{\sigma \times \max imum\ \ generation} \tag{21}$$

where *t* is time step and σ is a positive constant. The contents of the external archive are also reported as the final output of the algorithm.

In this paper, the leader selection technique is based on density measures. For this propose, a neighborhood radius for each particle in archive is defined. Then, number of neighborhoods of these particles is calculated in the objective function space. Particles whose fewer number of neighborhood are preferred as leader.

4. MULTI-OBJECTIVE OPTIMIZATION OF SOLAR SYSTEM USING PSO HYBRID WITH MULTIPLE CROSSOVER AND MUTATION OPERATOR

In order to investigate the optimal performance of the solar thermal energy storage in different conditions, PSO method is now employed in a multi-objective optimization procedure. Two conflicting objectives in this study are the net energy (Q_{net}) and discharge time of PCM (t_{PCM}) that are to be simultaneously optimized with respect to the design variables *D*, A_C, $\dot{m}$, φ and M_{PCM}.

The parameters of PSO are cited in Table (3). Furthermore, over iteration, the inertia weight W is linearly decreased with W_1=0.9 and W_2=0.4 and C is linearly increased with C_{2i}=0.5 and C_{2i}=2.5. The term $v_i(t)$ is limited to the range $[-v_{ave}, +v_{ave}]$, which $v_{ave} = \frac{x_{\max} - x_{\min}}{2}$. While the velocity violates this range, it will be multiplied by a random number between [0,1]. To provide the similar conditions, the constant values of the archive pruning are set as 0.01 for both of algorithms $\sigma = 0.01$ and $\varepsilon - dominance = 0.01$.

TABLE 3. Multi-objective optimization algorithms are used in the comparison and their parameter configurations

Algorithm	Population size	Pruning of archive	Crossover probability	Mutation probability
Proposed MOPSO	256	$\sigma = 0.01$	0.4	0.1
NSGAII	256	$\varepsilon = 0.8$	0.8	0.1

Figure 3 depicts the obtained non-dominated optimum design points as a Pareto front of those two objective functions. There are four optimum design points, namely, *A, B, C* and *D* whose corresponding designs variables and objective functions is shown in Table (4). These points clearly demonstrate tradeoffs in objective functions Q_{net} and t_{PCM} from which an appropriate design can be compromisingly chosen. It is clear from figure 3 that all the optimum design points in the Pareto front are non-dominated and could be chosen by a designer as optimum solar system. Evidently, choosing a better value for any objective function in the Pareto front would cause a worse value for another objective. The corresponding decision variables of the Pareto front shown in figure 3 are the best possible design points so that if any other set of decision variables is chosen, the corresponding values of the pair of objectives will locate a point inferior to this Pareto front. Such inferior area in the space of the two objectives is in fact bottom/ left side of Figure 3.

In Figure 3, the design points *A* and *D* stand for the best Q_{net} and the best t_{PCM} respectively. Moreover, the design point, *B* exhibit important optimal design concepts. In fact, optimum design point *B* obtained in this paper exhibits a decrease in Q_{net} (about 4.2%) in comparison with that of point *A,* whilst its t_{PCM} improves about 22.4% in comparison with that of point *A*.

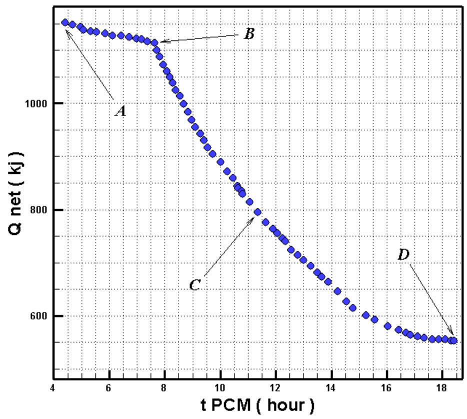

Figure 3. Multi-objective Pareto result for solar system

It is now desired to find a trade-off optimum design points compromising both objective functions. This can be achieved by the method employed in this paper, namely, the mapping method. In this method, the values of objective functions of all non-dominated points are mapped into interval 0 and 1.Using the sum of these values for each non-dominated point, the trade-off point simply is one having the minimum sum of those values. Consequently, optimum design point *C* is the trade-off points which have been obtained from the mapping method.

TABLE 4: The values of objective functions and their associated design variables of the optimum points

Point	Design Variables					Objective Functions	
	D (m)	A_C (m^2)	$\dot{m}$ (kg/s)	φ (deg)	M_{PCM} (kg)	t_{PCM} (hour)	Q_{net} (kj)
A	0.008	0.668	0.0034	29.54	5.000	4.251	1150.908
B	0.008	0.668	0.0034	29.54	10.587	7.513	1120.162
C	0.008	0.668	0.0022	29.54	13.371	11.231	800.912
D	0.008	0.668	0.0015	29.54	13.371	18.207	550.001

The Pareto front obtained from the propose method (Figure 3) has been superimposed with the Pareto front of multi-objective genetic algorithms [21] in Figure 4. It can be clearly seen from this figure that the propose Pareto front achieves better objective functions than NSGA II for present case study, which demonstrate the effectiveness of this paper in obtaining the Pareto front.

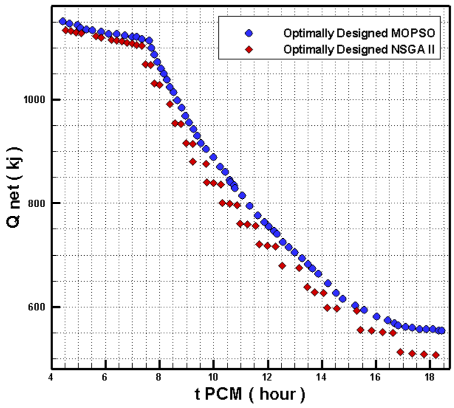

Figure 4. Overlay graph of the obtained Pareto front of MOPSO and NSGA II methods

There are some interesting design facts which can be used in the design of such solar systems. Figure 5 demonstrates the optimal behaviors of t_{PCM} with respect to mass flow rate between points *B* and *D*.

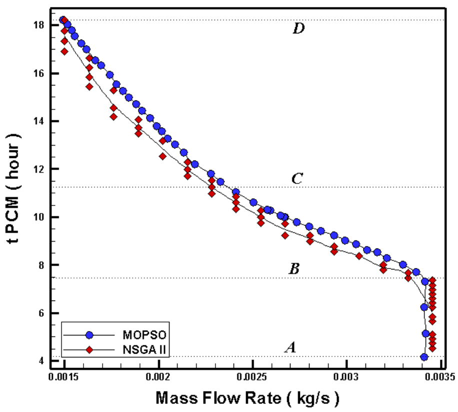

Figure 5. Optimal variations of discharge time with respect to mass flow rate

Figure 6 represents the optimal relationship of net energy and mass flow rate in the form of

$$Q_{net} \propto \dot{m} \quad (23)$$

These useful relationships that are indefeasible between the optimum design variables of a solar system cannot be discovered without the use of multi-objective Pareto optimization process presented in this paper.

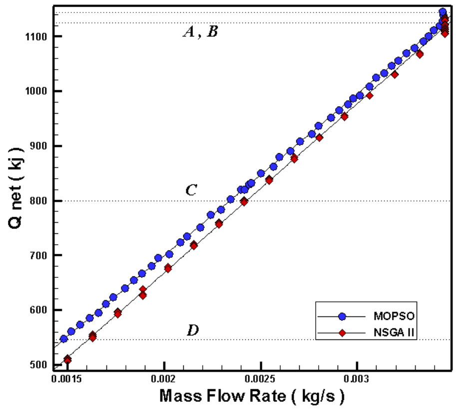

Figure 6. Optimal variations of net energy storage with respect to mass flow rate

5. CONCLUSION

Multi-objective Pareto based on optimization of solar system has been successfully investigated using PSO method. Current Pareto optimal solutions display tradeoff information between maximization of net energy stored and discharge time of PCM. Such tradeoff information is very helpful to a higher-level decision-maker in selecting a design with other considerations. The Pareto front of MO hybrid of PSO, multiple crossover and mutation operator and NSGA II methods have been compared and showed that the MOPSO method achieves better objective functions than NSGA II for present case study.

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