VIBRATION MECHANISM OF 13TH CENTURY HISTORICAL *MENAR-JONBAN* MONUMENT IN IRAN

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Abstract Historical monument of *Menar-Jonban* (shaking tower) is located in the famous city of Isfahan in central Iran. Initial construction of this interesting and unique masonry monument belongs to 700 years ago. This monument has two vibrating circular towers of 7.5 m height. These towers are separated from each other by a distance of 9.2 m and constructed on top of an ancient tomb of 10 m height. When one of the towers is shaken by the human force, the other one immediately starts to vibrate without transmitting any significant vibration to the other parts of the structure. This unique dynamic behavior has become a puzzle to architects and structural engineers for many years. Visitors from all over the world, climbing to the top of one of the towers and by shaking one, cause automatic shaking of the other tower. In this paper, the description of the structure, free and forced vibration tests setup, test results and findings on this unique structure is presented. Moreover, to identify the dynamic characteristics and behavior of this monument, analytical studies have been performed and the results of the various possible mathematical models were compared with measured response for system identification purposes.

Keywords Historical monument, Isfahan, Vibration Mechanism, masonry monument, vibration tests

چکیده بنای تاریخی منارجنبان در شهر معروف اصفهان در مرکز ایران قرار گرفته است. ساخت اولیه این بنای تاریخی جالب و منحصر به فرد به ۷۰۰ سال پیش مربوط می شود. این بنا دارای دو مناره گرد لرزنده به ارتفاع ۷.۵ متر می‌باشد. این مناره ها به فاصله ۹.۲ متر از یکدیگر و در بالای یک مقبره به ارتفاع ۱۰ متر قرار گرفته اند. هنگامی که یکی از مناره‌ها توسط شخصی به ارتعاش در می‌آید، مناره بعدی بلافاصله شروع به ارتعاش می‌کند، بدون اینکه ارتعاش قابل توجهی به دیگر قسمت‌های سازه منتقل شود. این عملکرد منحصر به فرد در طول سالیان متمادی به یک معما برای مهندسان معماری و سازه تبدیل شده است. بازدید کنندگان از تمام نقاط دنیا با رفتن به بالای یکی از مناره‌ها و مرتعش کردن آن سبب ارتعاش منار بعدی به صورت اتوماتیک می‌شوند. در این مقاله شرح سازه، برنامه آزمایش ارتعاش آزاد و اجباری، نتایج آزمایش، و یافته‌های این سازه منحصر به فرد ارائه می‌گردد. بهعلاوه، برای تعیین مشخصات دینامیکی و رفتار این بنای تاریخی مطالعات تحلیلی انجام گرفته است و به منظور تعیین هویت سیستم، نتایج مدلهای ریاضی متعدد با نتایج پاسخ‌های اندازه گیری شده مقایسه شده است.

1. INTRODUCTION

The *Menar-Jonban* (shaking tower) is one of the well-known historical monuments (tomb) located in the famous city of Isfahan. This beautiful historical city was the capital city of Iran for a very long time and contains several attractive monuments. Each of these monuments (mosques, tombs, bridges, palaces, etc.) has special characteristics from the structural and architectural viewpoints. *Menar-Jonban* (Menar in Persian means tower and Jonban means shaking) was constructed 700 years ago using masonry materials and then completed by the famous Iranian architect and constructor named "Sheikh Bahaie"[1].

This paper will try to explore the dynamic behavior of this architectural wonder of the 13th century and the phenomenon or at least to augment the insights of the observed behavior, using the forced and free vibration tests and system identification analysis. From the structural point of view, *Menar-Jonban* has three special and unique characteristics that have attracted the attention of scientists and engineers for many years:

1. Controlled dynamic of Structures: When one of the towers is put into motion by human force, the other one will start to vibrate automatically with a short time delay. Transmission of the vibration from one tower to another is a special and unique characteristic of this monument. There is a strong belief that the designer of the monument had a good understanding of structural dynamic concepts and vibration control.
2. Isolation Theory: A joint is provided between each tower-column and the connecting arch, which acts as vibration isolator from towers to the other parts of the structure. Scientists believe that two added joints reduce the stiffness of the towers and therefore the vibration of them under human simulations.
3. Material behavior: It seems that the type of the mortar used for the masonry construction of the monument and specially the towers adds special flexibility to the tower. For this reason, no crack has appeared in the towers during years of shaking.

2. ARCHITECTURAL AND STRUCTURAL CHARACTERISTICS

A three dimensional view and plan dimensions of the monument is shown in Figure 1. The entrance view of the monument is shown in Figure 2. It consists of indoor hall with the dimensions10mx13m in plane and 10m in height with arch roof, and two shaking towers located at the roof-top of the entrance hall on both sides. The monument was constructed by brick masonry units. Masonry walls around the hall are the resisting system for vertical dead loads and lateral seismic forces. Peripheral walls with the thickness of 0.8m are strengthened by using brick piers with 1.8x1.8m in cross section. One of the design characteristic of the walls is the compatibility with architectural views in the monument. The designer of the monument has used brick piers to increase load resisting and lateral rigidity of the walls; and also, created a good architectural environment inside the monument. It should be mentioned that this compatibility between architectural and structural design exists in many Iranian historical monuments. From both architectural and structural point of view, the most important characteristic of the historical monument of *Menar-Jonban* is a complete symmetry about Y axis (see Figure 1).

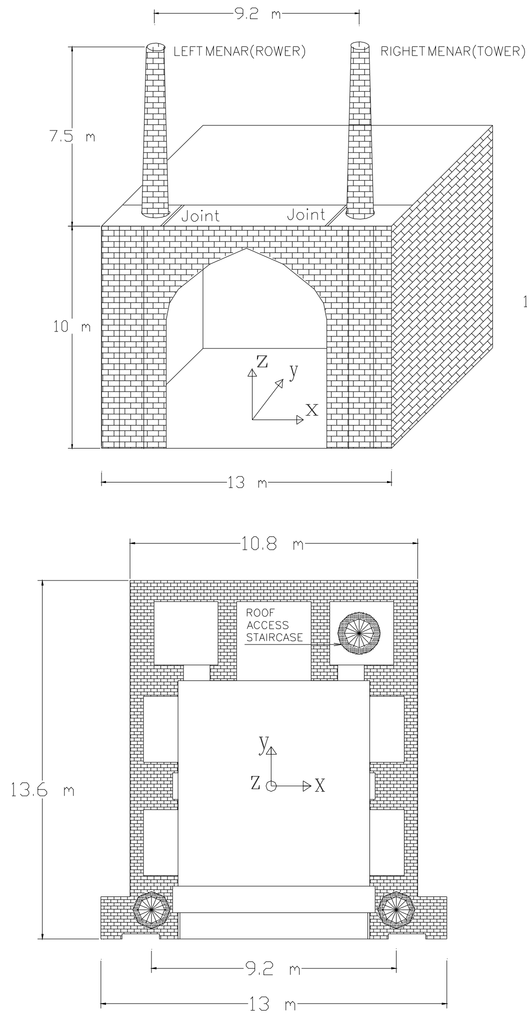


Figure 1. General view and plan of historical monument of *Menar-Jonban*

Figure 3 shows one of the towers and its entrance. Construction of two similar cylindrical brick towers on top of the roof is an interesting part of this monument. The outside diameter of these towers is 1.4m at the bottom and 0.9m at the top and the brick wall thickness is 0.2m. There is a helical staircase inside each tower for access to the top of them. A person can pass through this staircase and vibrate the tower at the top simply by shaking it. Two horizontal wooden belts are located at top and bottom levels of towers. It seems that a wooden frame is used in design of the towers to provide more flexibility.

A considerable point in designof the towers is the two joints or spaces with about 5cm in width located at the top level of the roof between each tower and the connected roof (see Figure 1). There is a strong belief that these joints have essential role in vibration mechanism of the towers. It seems that the designer considered these joints to prevent local damages in the connection regions during vibration of the towers or the whole structure.

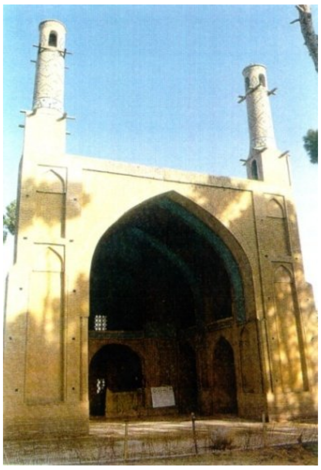


Figure 2. Front view of *Menar-Jonban*

Figure 3. View of left Menar (tower) at roof level

3. VIBRATION MECHANISM OF TOWERS

The vibration mechanism of two similar towers can be simply simulated via two similar pendulums having low damping and their resonance phenomenon as shown in Figure 4. Assume that two similar pendulums (with equal mass and length) are symmetrically tied together by a horizontal wire (Figure 4a). In this condition, by shaking one of them, the second one will start to vibrate automatically. The amplitude of the vibration and phase angle of two pendulums will be similar after some oscillations. Such a phenomenon will not occur in the case of different pendulums for example with different lengths or masses (Figure 4b).

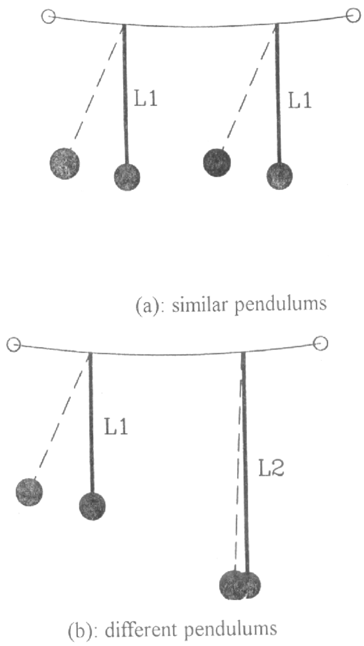


Figure 4. Resonance phenomena in vibration of similar pendulums

Generally, vibration of each pendulum can be assumed as a Single Degree of Freedom (SDOF) damped system with the following differential equation [2, 3].

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{1}$$

where m, k, and c are mass, stiffness, and damping coefficient of SDOF system, and u , $\dot{u}$, and $\ddot{u}$ are displacement, velocity, and acceleration responses, respectively. The solution of equation (1) subjected to the initial condition of $u(0)$ and $\dot{u}(0)$ is:

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\omega_D t + \left(\frac{\dot{u}(0)+\zeta\omega_n u(0)}{\omega_D}\right)\sin\omega_D t\right] \quad (2)$$

where

$$\zeta = \frac{c}{2m\omega_n} \quad (3)$$

$$\omega_D = \omega_n\sqrt{1-\zeta^2} \quad (4)$$

In the above equations ω_n, ω_D and ζ are the undamped natural frequency, damped natural frequency and the damping ratio, respectively. Natural periods in terms of natural frequencies can be explained as $T_n = 2\pi/\omega_n$ and $T_D = 2\pi/\omega_D$. The displacement amplitude of an undamped system is constant in all vibration cycles. However, in the case of damped system, the displacement amplitude decreases in each vibration cycle as shown in Figure 5. The ratio of two successive peaks of damped free vibrations can be determined as:

$$\frac{u(t)}{u(t+T_D)} = \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right) \quad (5)$$

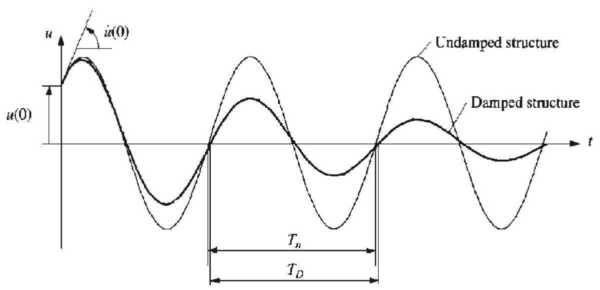


Figure 5. Free vibration of damped SDOF system [2]

In the case of a harmonic vibration, a harmonic force $p(t) = p_\circ \sin\omega t$ is considered in the right hand side of Equation 1, where $p_\circ$ and ω are the maximum amplitude of the force and its corresponding frequency, respectively. Therefore:

$$m\ddot{u} + c\dot{u} + ku = p_\circ \sin\omega t \quad (6)$$

The Steady state response of this system can be written as:

$$u(t) = u_\circ \sin(\omega t - \phi) = \frac{p_\circ}{k} R_d \sin(\omega t - \phi) \quad (7)$$

where $R_d = \frac{u_\circ}{(u_{st})_\circ}$ and ϕ are deformation response factor and phase angle, respectively as shown in Figure 6. With regard to this figure, the value of R_d is many times larger than 1 in the case of resonance frequency ($\omega/\omega_n = 1$). This implies that the deformation amplitude is much larger than the static deformation. For an undamped system, the value of R_d is unbounded at the resonance frequency. Also, in this case the phase angle is $\phi = 90^\circ$ for all values of ζ . However, in the case of damped system, the value of $u_\circ$ is equal to $u_\circ = \frac{(u_{st})_\circ}{2\zeta} = \frac{p_\circ}{c\omega_n}$. This equation implies that the response is controlled by the damping of the system.

In the case of a simple pendulum system (Figure 4), which consists of a point mass m suspended by a light string of length L, the differential equation governing for undamped small amplitude oscillations (θ) can be written as:

$$\ddot{\theta} + \frac{g}{L}\theta = 0 \quad (8)$$

Therefore, the natural period of vibration is determined as:

$$T_n = 2\pi\sqrt{\frac{L}{g}} \quad (9)$$

The above equation indicates that the natural period of a simple pendulum system only depends on the length L (g is the gravity acceleration). This equation indicates that the amplitude of two similar pendulums will be the same in the case of undamped linear oscillations.

Equations 1 to 9 are the basis for free and harmonic forced vibration of a dynamic linear system. These equations can be used to vibration study of the *Menar-Jonban* towers in the linear ranges. Based on the similarity and symmetry of twin towers, one can conclude that by shaking one of the towers, the resonance phenomenon will occur between them through the base medium. This phenomenon was clearly observed during the dynamic tests in the field. In addition, a medium such as the base roof is necessary to transmit the vibration from one tower to another.

It is important to note that the old idea of similar towers in *Menar-Jonban* as tuned vibration system has been used as Tuned Mass Dampers (TMD) for vibration control of modern structures such as high rise buildings and suspension bridges [7]. Therefore, it can be considered as a concept for seismic design of different dynamic systems.

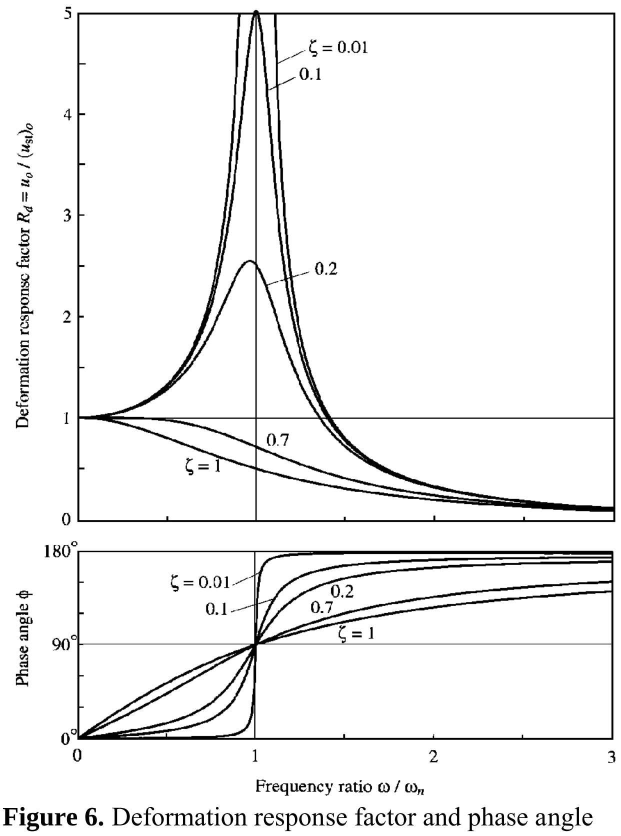
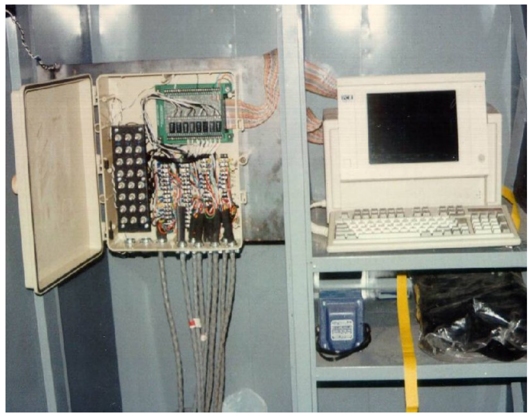


Figure 6. Deformation response factor and phase angle for a damped system excited by harmonic force [2]

4. VIBRATION TEST SETUP AND TEST PROGRAM

Forced vibration tests were performed for system identification of the *Menar-Jonban* structure. For this purpose, the structure was instrumented by 8 low noise force balance accelerometers. Produced vibrations were measured by a P.C. based digital data acquisition system. Recording system of OASYS and one component forced balance accelerometers of FBA-11 have been used in this vibration test program as shown in Figure 7.



a) OASYS Vibration recording system

b) FBA-11 accelerometers

Figure 7. a) OASYS Vibration recording system, and b) FBA-11 accelerometers installed next to the joint on the roof level.

Four different vibration test series of Test 1 to Test 4 have been used in the vibration test program. Figure 8 shows the arrangement of the accelerometers (measuring points 1 to 8) and vibration directions in each test series. Vibrations simply produced by manpower by producing harmonic push and pull on the top of the one tower. The vibration directions of two towers (A, B, C, and D or E, F, G, and H) were considered in all test series. A summery description of each test setup is as follow:

Test 1: In this test, 8 accelerometers were installed on the left tower in both X and Y directions as shown in Figure 8a. Measuring points 7 and 8 are situated at the bottom of the tower entrance on the backside. Points 1 and 2 are kept as reference points for the next test series. The horizontal accelerations along the tower-column were measured for identification of its frequencies and mode shapes. Only left tower was excited in this test.

Test 2: In this test as shown in Figure 8b, accelerometers 1 and 2 remained unchanged as reference. The Accelerometers 3 to 8 were arranged in two equal distances at the roof level. This test is used to study the vibration transmission from the left tower to the roof and also determining the roof vibrations itself. Only left tower was excited in this test.

Test 3: In this test as shown in Figure 8c, accelerometers were installed on the top and base level of both towers. All accelerometers situated at the base of the towers are arranged in Y direction and located next to the joints. The effect of joint in the vibration mechanism of the towers can be found from this test. Both left and right towers are excited in this test.

Test 4: Arrangement of accelerometers in this test as shown in Figure 8d was similar to that of Test 3, except that all accelerometers situated at the base of the towers are arranged in X direction. The symmetry of the structural system about X-axes can be studied using the results of Test 3 and Test 4. Only right tower was excited in this test.

In all of the above tests, the total duration of recording was 35 seconds comprising of 20 seconds of forced vibration measurement and 15 seconds of free vibration measurement. All of the recorded accelerations were corrected for both baseline and frequency errors [4] using Butterworth band pass filter, [5]. Damping ratios were determined from the free vibration parts of the measurements using logarithmic decay method.

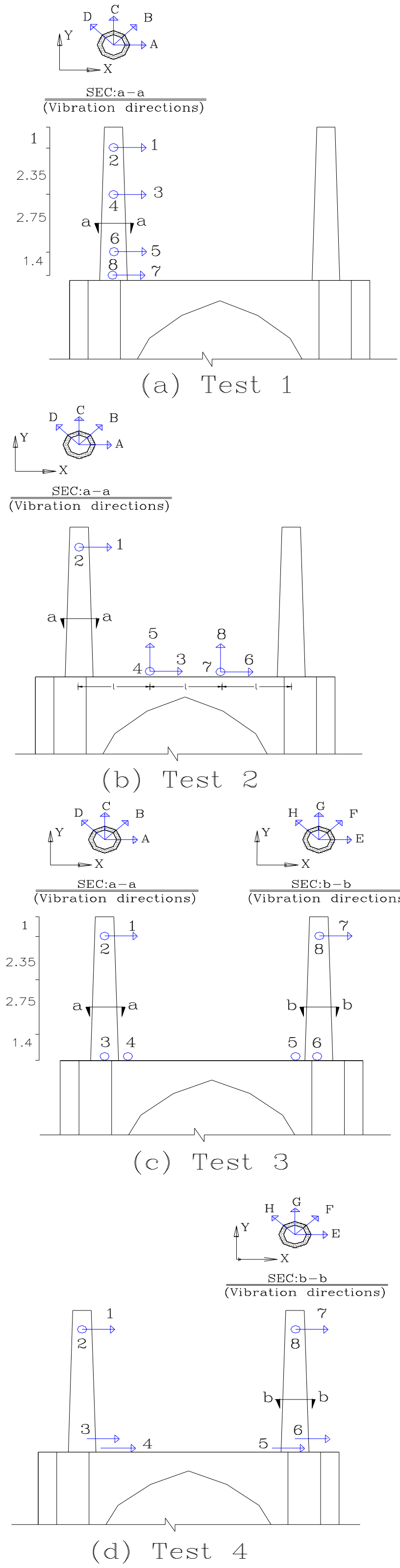


Figure 8. Arrangement of the accelerometers and vibration directions in different test series

5. VIBRATION TEST RESULTS

Dynamic characteristics of *Menar-Jonban* including frequencies, mode shapes and damping ratios have been determined by processing the recorded vibrations from actual structures. Modal frequencies and related mode shapes have been determined using Fast Furrier Transformation (FFT). An example of the corrected acceleration time histories and related FFT results obtained from Test 3 are shown in Figure 9 and Figure 10. Results presented in these figures are related to the vibration directions of A, B, C and D (TST3A, TST3B, TST3C, and TST3D). Similar results for free vibrations are shown in Figure 11 and Figure 12. Numbers 1 to 8 at the right side of these figures indicates the measuring points.

The main dynamic characteristics of the whole monument and both towers including frequencies, damping ratios, and the dominant vibration mode shapes have been identified from different vibration test series. Summary of the important test results obtained from the forced and free vibration tests are as follows:

1. The results of Test 1 show that the forced vibration frequency of the left tower is between 1.7 Hz to 2.2 Hz and its free vibration frequency is between 1.8 Hz to 2.4Hz in different directions A, B, C, and D. These values indicate that the free vibration frequencies are about 5 to 10 percent greater than the forced vibration frequencies. The reason for this enhancement can be related to the large deformation of towers (P-Δ effects) and nonlinear behavior of the masonry materials during forced vibration tests.
2. The frequency content of the left tower in different directions of Test 1 indicates that the higher frequency belongs to the X-direction. Therefore, the stiffness of this tower in the X-direction is greater than that in the Y-direction.
3. The first mode shape of the left tower obtained from Test 1 is presented in Figure 13. As indicated in this figure, the main vibration mode of the left tower in the X-direction is a bending mode shape, whereas in the Y-direction it is a linear mode shape.
4. The results of the Test 2 show that the forced vibration frequency of the left tower is between 1.8 Hz to 2.2 Hz and its free vibration frequency is between 2.0 Hz to 2.3Hz in different directions A, B, C, and D. These results are similar to the results of Test 1. In addition, the vibration amplitude at the base of the left tower (excited tower) is about 3% of the top value.
5. The amplitudes of vibration at the monument roof obtained from Test 2 indicate that the roof is rigid for in plane deformations. However, considerable displacements on the top of the towers occur due to the out of plane bending deformations.
6. The results of the Test 3 indicate two main vibration modes for the total monument system as shown in Figure 14. The first dominant vibration mode is a lateral-bending mode with frequencies of 2Hz and 2.2Hz in the forced and free vibration conditions, respectively. The second mode is the lateral-torsional-bending mode with frequencies of 5.5Hz and 5.7Hz, similarly. Also, an approximate damping ratio of 5% can be obtained from these dominant vibration modes. Similar results have been obtained by exciting of the left or right towers.
7. Based on the results of the Test 3, the maximum top displacement of the left tower (excited tower) is about 10cm ($\pm$5cm). In this condition, the maximum top displacement of the right tower (free standing tower) is about 35% of the left one and vise versa. Generally, vibration transmission from right to left tower is slightly better than left to right tower.
8. The results of Test 4 are generally similar to those of Test 3. The results obtained from this test indicate similarity and symmetry of the whole monument structural system with respect to Y axes.
9. The results of both Test 3 and Test 4 indicate that the dominant free vibration frequency of the left tower is about 4% greater than the right tower. This could be effective in preventing vibration transmission from one tower to the other one. This behavior indicates two different pendulum vibration phenomena (Figure 4b).
10. The results of both Test 3 and Test 4 also indicate that two existing joints between towers and the roof will result in a relative displacement in the X and Y directions. The displacement produced at the base of the excited tower is approximately 2 to 3 times greater than the displacement of the adjacent roof beyond the joint. This relative displacement in the Y-

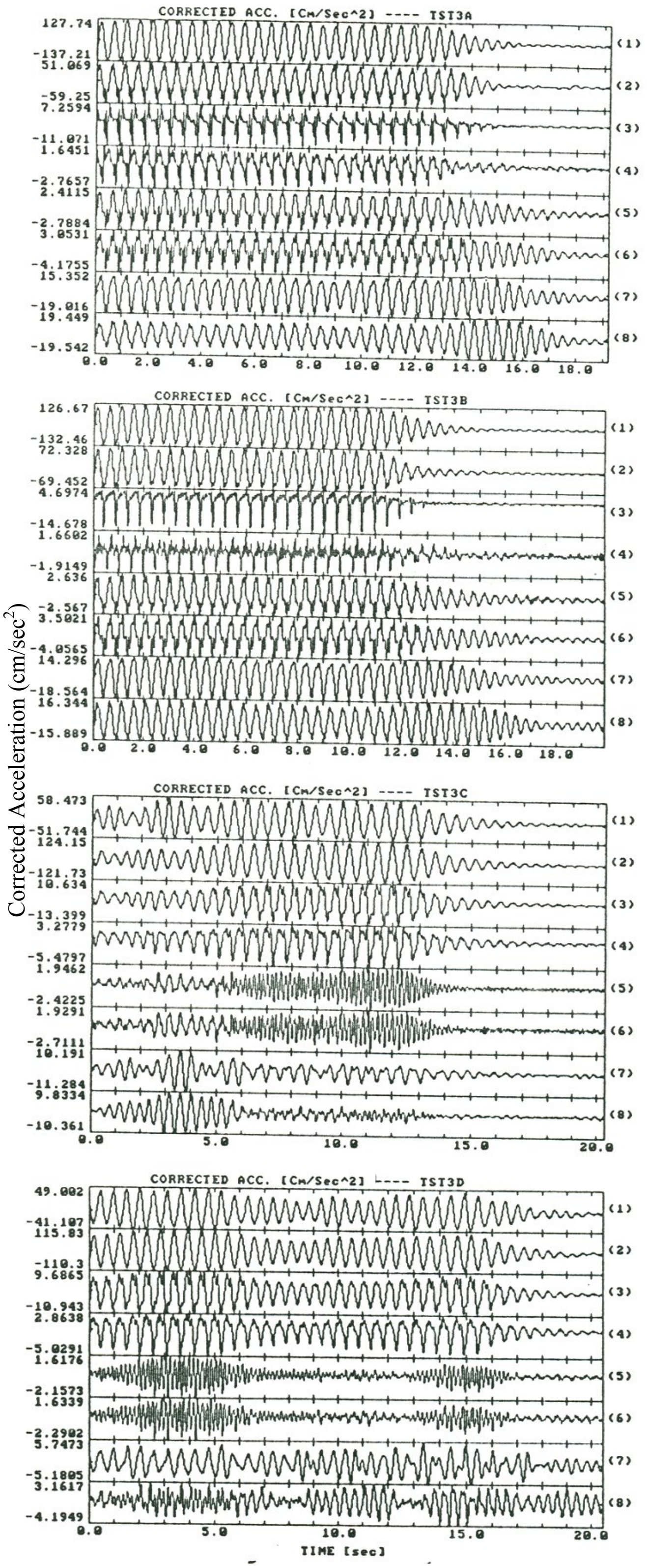


Figure 9. Forced vibration acceleration time histories recorded from Test 3

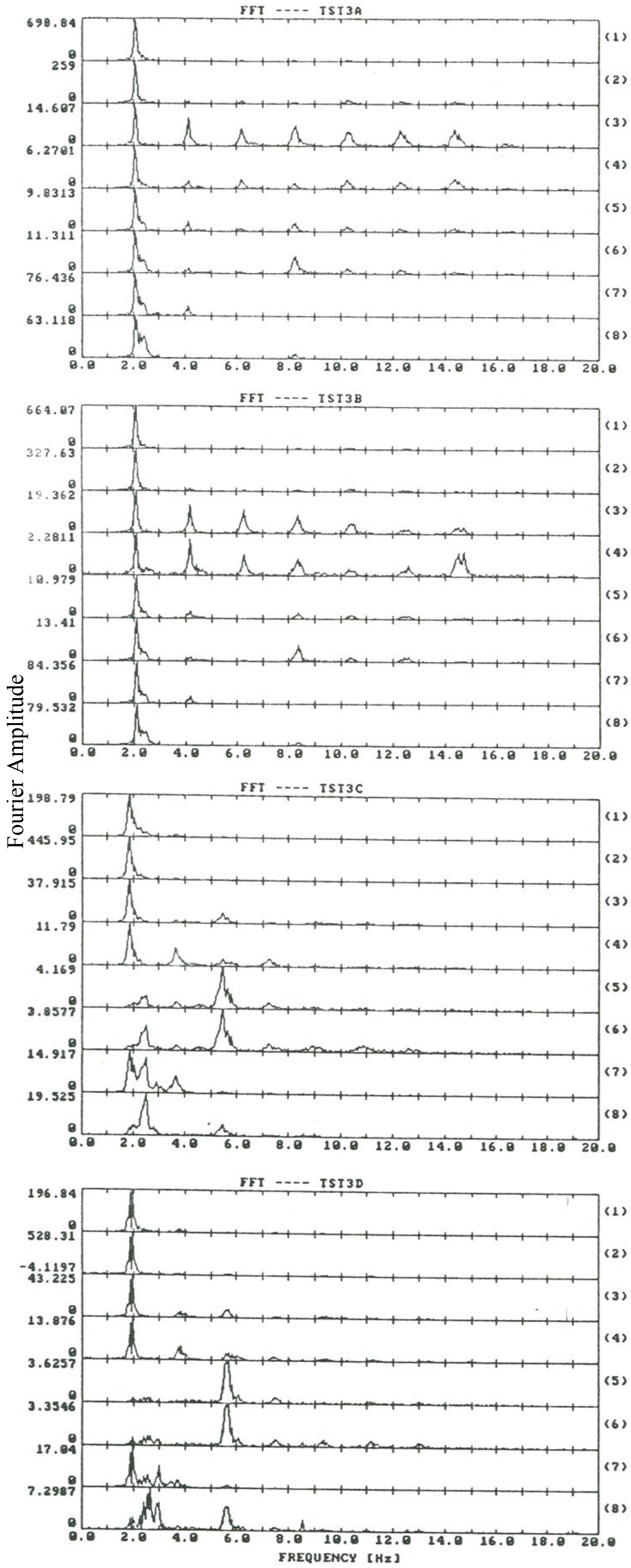


Figure 10. Forced vibrations Fast Furrier Transformation (FFT) of Test 3

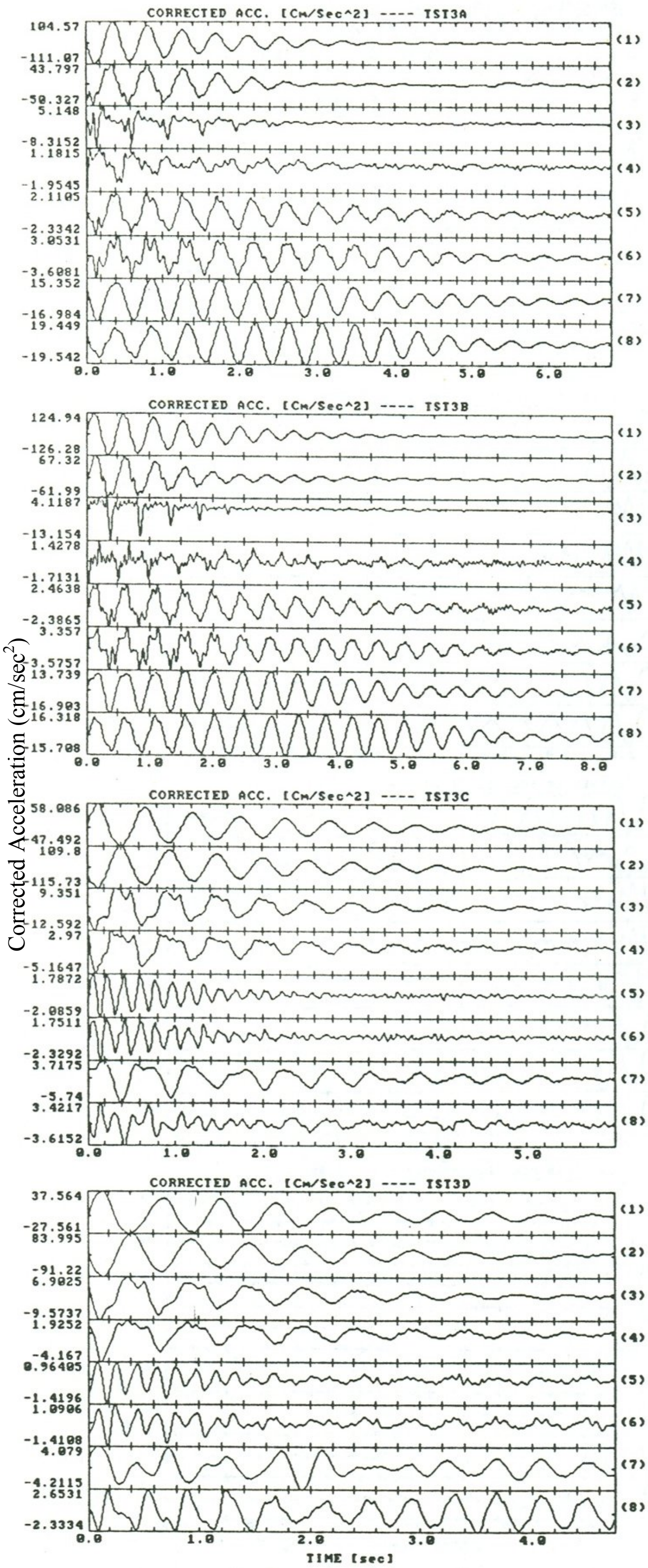


Figure 11. Free vibration acceleration time histories recorded from Test 3

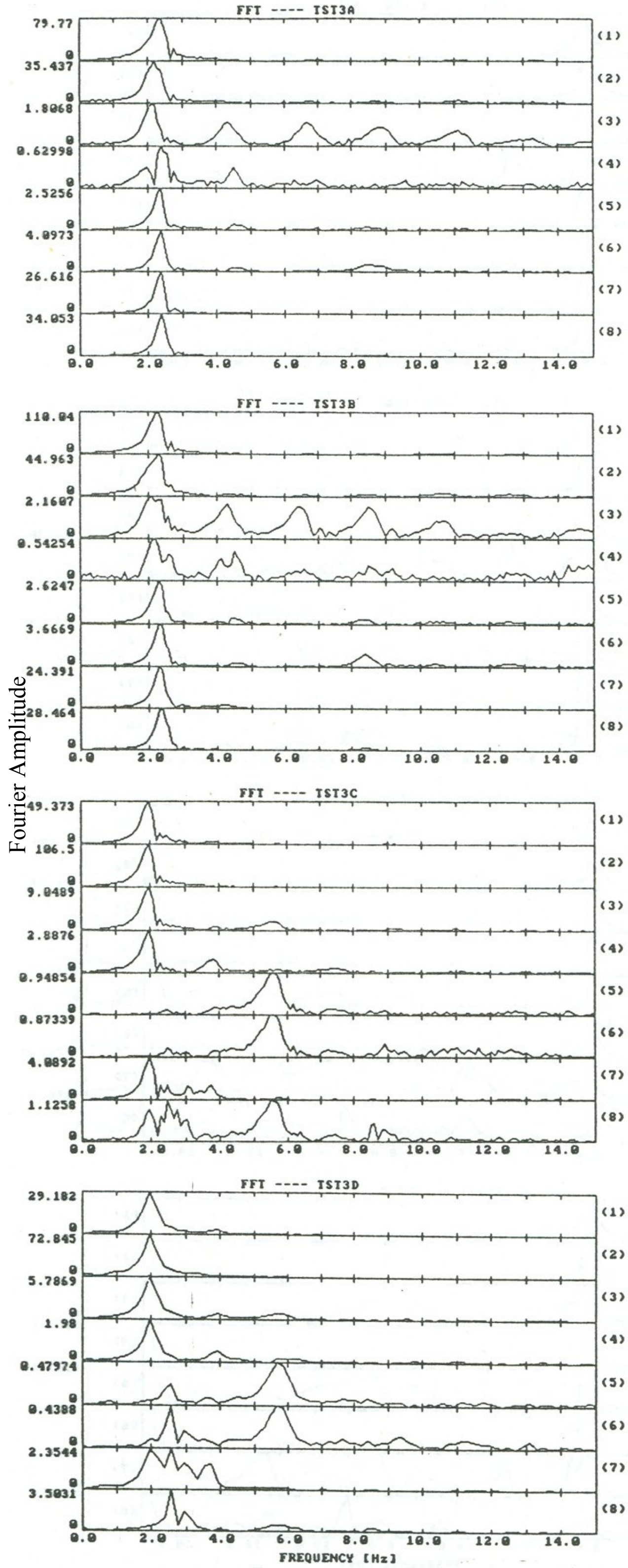


Figure 12. Free vibrations Fast Furrier Transformation (FFT) of Test 3

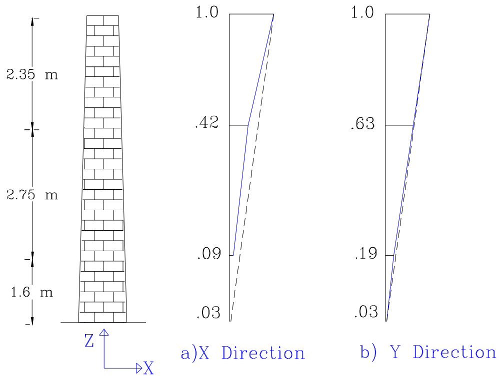
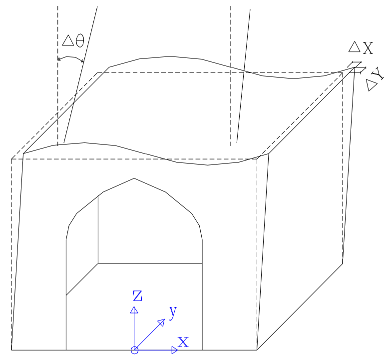
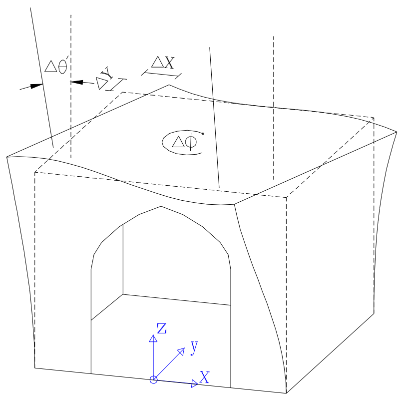


Figure 13. First vibration mode of left tower in a) X direction and b) Y direction



a) First mode (lateral-bending)



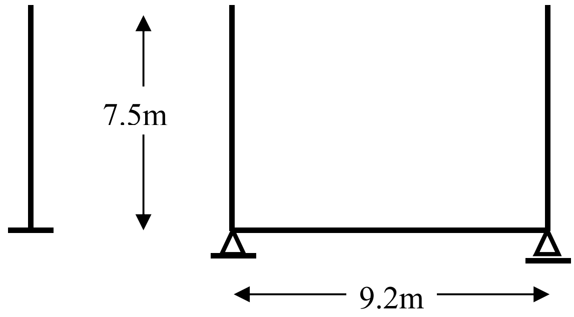
b) Second mode (lateral-torsional-bending)

Figure 14. Monument main vibration mode shapes includes a) First mode and b) Second mode

direction is slightly greater than that in the X-direction. It seems that the performance of existing joints is effective in reducing the towers vibration frequencies and produces partially vibration isolators.

6. ANALYTICAL STUDIES

Two analytical models have been studied using SAP2000 software as shown in Figure 15. Analytical results are used for further assessment of the experimental findings and determination of other properties. The first model includes a cantilever model of one tower. The second model is a U frame consisting of two column elements linked by a beam element at bottom. The beam element is considered as the roof flexibility. The unit weight of the masonry material γ=1800kg/m^3 and elastic-modules of the masonry material E= 6000 kg/cm^2 are assumed for non cracked masonry column and beam elements.



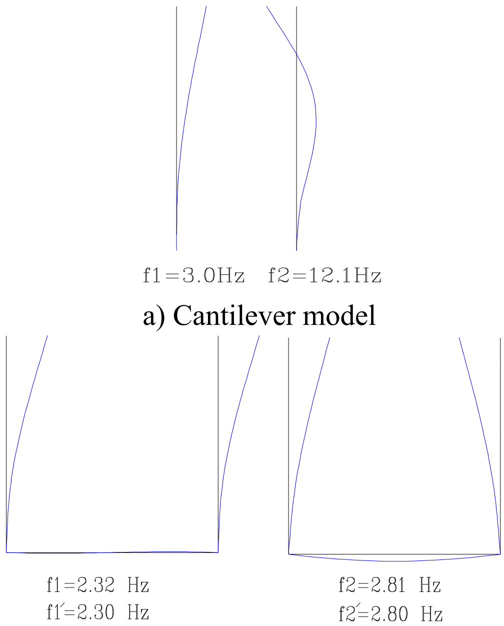
a) Cantilever model b) U frame model

Figure 15. Analytical models of towers as a) cantilever model, and b) U frame model

Modal analyses of the cantilever and U frame models are presented in Figure 16. First and second mode frequencies of the cantilever model are f1=3.0Hz and f2=12.1Hz, respectively. Similar results for U frame model without joints are f1=2.32Hz and f2=2.81Hz, respectively. These analytical results indicate good agreements with the test results. The frequencies of f1′=2.30Hz and f2′=2.801Hz belongs to the model with joints (reduced cross section). The dominant frequency of U frame model is about 30% lower than the cantilever tower model.

A concentric force equal to 5KN applied as an equivalent static force at the top of the excited tower in cantilever and U frame models and the resultant lateral displacements is shown in Figures 17. Equations (6) and (7), a dynamic system with %5 damping ratio indicates a response factor equal to 10 at the resonance frequency. Therefore, by assuming the amplitude of the induced harmonic force (by a person) equal to 50kg, the equivalent static force at the top of the tower will be 500kg (or 5KN).

Results of static analysis indicate that the maximum lateral displacement of the cantilever model subjected to 5KN concentric force is about 1.1cm. This value for U frame without joints is 4.93 cm and with joints is 4.96 cm. These values are comparable with maximum top displacements (about ± 5 cm) obtained from the dynamic excitation tests. Also, this simple analysis indicates that %80 of the top lateral displacements of the towers results from the bending deformations of the roof, while the remaining %20 displacements result from the lateral deformations of the two towers. Another considerable observation of these simple analyses is that the joints have no important effects on the lateral deformations of the towers. However, joints are modeled as a reduced cross-section of the beam element in this study.

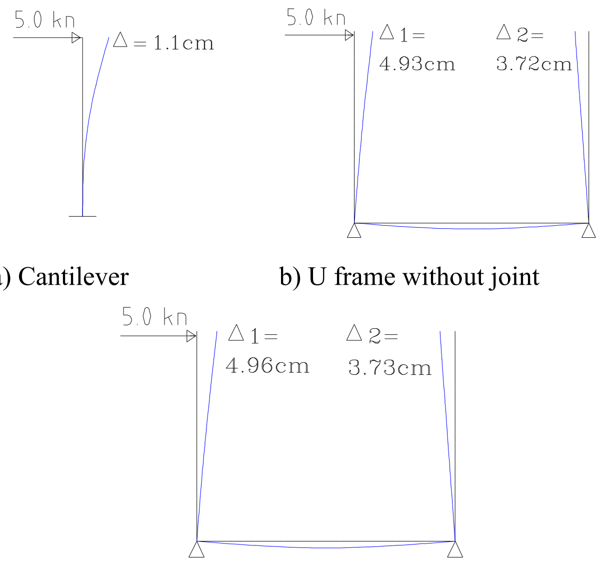


a) Cantilever model

b) U frame model with and without joint

Figure 16. Frequencies of the first and the second modes of a) cantilever models, and b) U frame model with and without joint

A refined and detailed finite element model of the *Menar-Jonban* monument using ANSYS has been also developed and analyzed by De Roeck [6] as shown in Figure 18. According to the results of this study, it is necessary to choose an elastic module of E=8000 kg/cm^2 in order to obtain the natural frequencies comparable with the forced vibration tests. This study also indicates that joints do not have any considerable effects on the natural frequencies and the vibration mode shapes. It should be noted that joints have been modeled as a reduced cross-section and there is a need for more and exact studies about the mechanism of joints.



a) Cantilever

b) U frame without joint

c) U frame with joint

Figure 17. Lateral displacements of different models a) cantilever, and b) U frame without joint and c) U frame with joint

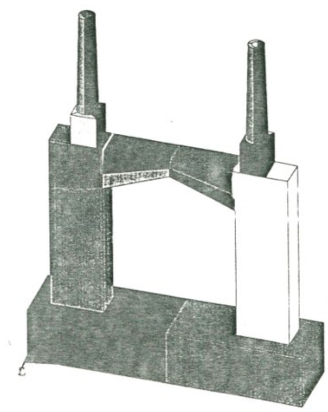


Figure 18. Finite Element Model studied by De Roeck [6]

7. CONCLUSINS

Experimental and analytical investigations have been performed on the 13th century Iranian monument of *Menar-Jonban* located in the famous city of Esfahan. This investigation indicates that the ancient Iranian architects and engineers that mainly designed and constructed the amazing and wonder structures similar to this monument had a good knowledge of the structural dynamics, resonance phenomena, and vibration isolations. However, based on the forced and free vibration test results, and also analytical studies performed on this historical monument, the following conclusions can be derived:

1. The natural frequency of the first vibration mode of the *Menar-Jonban* system is about 2Hz. This is the effective natural frequency of two similar towers. A person at the top of one tower can vibrate it up to 10cm ($\pm$5cm) in amplitude only by producing harmonic forces at the resonance frequency.
2. The natural frequency of the second vibration mode of the *Menar-Jonban* system is about 5.5Hz. This is a lateral-torsional mode of whole monument structural system which is far from the first mode of the vibration.
3. From the structural dynamics point of view, the vibration mechanism of the *Menar-Jonban* monument can be considered similar to the equivalent pendulums. Two towers play the role of two similar pendulums and the roof between two towers acts as a connecting medium to vibration transmission. Experimental and analytical results indicate that about %80 of the top lateral displacements of both towers result from the bending deformations at the roof while the remaining %20 is a result of the lateral deformations of the towers.
4. Any changes in the frequency balance of the towers or lack of the vibration transmission system causes that the resonance phenomenon not to occur. Test results indicate some minor differences (less than 5%) in the natural frequencies of the two towers. Therefore, about 35% of the amplitude of the excited tower is transmitted to the other (non excited) tower.
5. From the forced and free vibration test results it can be observed that the natural free vibration frequency of the towers is about 10% greater than the natural forced vibration frequency. This may be resulting from the nonlinear behavior of the masonry materials, and also large deformations of the excited tower (P-□ effects). These nonlinear effects can be considerable in frequency unbalancing of the excited and non excited towers and therefore vibration transmission between them.
6. From the test results, the damping ratio of the system in natural frequency is about 5%. Therefore, a complete vibration transmission between two towers is impossible.
7. Considering the amount of E= (6000~8000) kg/cm^2 for masonry materials with special mortar, a good analytical and experimental agreement will be provided. These studies also states that joints do not have any considerable effect on the natural frequencies and the vibration mode shapes.

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