TECHNICAL NOTE

HEAT TRANSFER ON PERISTALTIC TRANSPORT WITH SLIP CONDITION IN AN ASYMMETRIC POROUS CHANNEL

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Abstract Simultaneous effects of slip and heat transfer on peristaltic transport of an incompressible electrically conducting viscous fluid in an asymmetric channel is studied under the assumptions of long wavelength and low Reynolds number. The asymmetry is produced by choosing the peristaltic wave train on the walls to have different amplitudes and phases. Exact solutions for stream function, velocity field, temperature profile and pressure gradient are obtained. The effects of different parameters entering into the problem on pressure rise are discussed numerically and explained graphically. Also, pumping and trapping phenomena are discussed by numerical integration.

Keywords Peristalsis, Slip parameter, Permeability parameter, Brinkman number, Heat transfer coefficient

چکیده اثرات همزمان لغزش و انتقال حرارت بر روی انتقال ناهمگن یک سیال تراکم ناپذیر، ویسکوز و هادی جریان الکتریکی در کانال نامتقارن با فرض طول موج بلند و عدد رینولدز پایین مورد مطالعه قرار گرفته است. عدم تقارن با انتخاب سلسله امواج ناهمگن بر روی دیواره‌ها، به منظور داشتن دامنه و فازهای مختلف به وجود امده است. راه‌حل های دقیق برای تابع جریان، میدان سرعت، پروفایل درجه حرارت و گرادیان فشار به دست آمده است. اثر پارامترهای مختلف ورودی بر روی افزایش فشار از نظرعددی مورد بحث قرار گرفته و به صورت گرافیکی توضیح داده شده است. همچنین ، پدیده های پمپاژ و تله گذاری به وسیله انتگرال عددی بحث شده است.

1. INTRODUCTION

Peristalsis is the mechanism of the fluid transport that occurs generally from a region of lower pressure to higher pressure when progressive wave of area contraction and expansion propagate along the flexible wall of the tube. In physiology, peristalsis is an important mechanism for transport of fluid and is used by the body to propel or mix the contents of a tube as in ureter, gastrointestinal tract, bile duct and other glandular ducts. Some biomedical instruments, like the blood pumps in dialysis and the heart lung machine use this principle. The mechanism of peristaltic transport has been exploited for industrial applications like sanitary fluid transport, transport of corrosive fluids where the contact of the fluid with the machinery parts is prohibited. The problem of the mechanism of peristaltic transport has attracted the attention of many investigators since the first investigation of Latham [1]. The fundamental studies on peristalsis were performed by Fung and Yih [2] using laboratory frame of reference and then by Shapiro et al. [3] using wave frame of reference. A number of analytical, numerical and experimental [4-12] studies have been conducted to understand peristaltic action under different conditions with reference to physiological and mechanical situations. However, the interaction of peristalsis and heat transfer has not received much attention which may become highly relevant and significant in several industrial processes. Also, thermodynamical aspects of blood may become significant in processes like oxygenation and hemodialysis [13-16] when blood is drawn out of the body. The combined effects of magnetohydrodynamics and heat transfer on the peristaltic transport have been discussed by Mekheimer, Abd elmaboud and other co-workers [17, 18]. Hayat et al. [19] developed the problem by considering heat transfer effect on peristalsis flow of fluid filling the porous space in an asymmetric channel. Recently Nadeem and Akram [20], Hayat et al. [21] discussed the slip and heat transfer effect on peristaltic flow in an asymmetric channel under different boundary conditions. The aim of the present investigation is to highlight the importance of heat transfer analysis of MHD peristaltic flow in an asymmetric porous channel under the influence of slip conditions in the presence of viscous dissipation terms. The governing equations of momentum and energy have been simplified using long wavelength and low Reynolds number approximations. The exact solutions of momentum and energy equations have been obtained. The features of flow and heat transfer characteristics are analyzed by plotting graphs.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the peristaltic transport of an electrically conducting incompressible viscous fluid in a two dimensional asymmetric channel in the presence of a constant transverse magnetic field B_0 (see Figure1). The channel flow is produced due to different amplitudes and phases of the peristaltic waves with constant speed c along the channel walls.

$$\left.\begin{aligned} H_1(X,t) &= d_1 + a_1 \sin\left[\frac{2\pi}{\lambda}(X-ct)\right], \text{ upper wall} \\ H_2(X,t) &= -d_2 - b_1 \sin\left[\frac{2\pi}{\lambda}(X-ct)+\phi\right], \text{lower wall.} \end{aligned}\right\} \quad (1)$$

where a_1 and b_1 are the amplitudes of the waves, λ is the wave length, $d1+d_2$ is the width of the channel, t is the time and X is the direction of wave propagation. The phase difference φ varies in the range $0 \le \varphi \le \pi$ in which φ=0 corresponds to symmetric channel with waves out of phase, and $\varphi = \pi$ the waves are in phase and a_1, b_1, d_1, d_2 and φ satisfies the condition:

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2 \quad (2)$$

The lower wall of the channel is maintained at temperature T_1 while the upper wall has temperature T_0.

The equations governing the motion for the present problem are [19]:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (3)$$

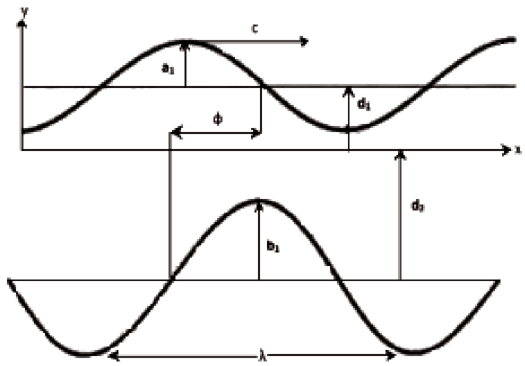

Figure 1. Schematic diagram of the physical model

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \mu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \sigma B_0^2 U - \frac{\mu}{K}U \quad (4)$$

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \mu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\mu}{K}V \quad (5)$$

$$C_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = \frac{\kappa}{\rho}\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \nu\left[2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)^2\right] \quad (6)$$

where U and V are the velocities in the X and Y directions in the fixed frame, P is the pressure, ρ is the density, μ is the coefficient of viscosity of fluid, ν is the kinematic coefficient of viscosity, σ is the electrical conductivity of the fluid, κ is the thermal conductivity, C_p is the specific heat at constant pressure, K is the permeability parameter and T is the temperature of the fluid.

Let us introduce a wave frame (x,y) moving with velocity c away from the fixed frame (X,Y) by the transformation:

$$x = X - ct \ , \ y = Y \ , \ u = U - c \ , \ v = V \ , \ p(x,y) = P(X,Y,t) \tag{7}$$

where u and v are the velocities in the x and y directions in the wave frame and p is the pressure in wave frame.

Introducing the following non-dimensional quantities:

$$\bar{x} = \frac{x}{\lambda},\ \bar{y} = \frac{y}{d_1},\ \bar{u} = \frac{u}{c},\ \bar{v} = \frac{v}{c\delta}, \delta = \frac{d_1}{\lambda},\ \bar{p} = \frac{d_1^{\,2} p}{c\mu\lambda},$$
$$h_1 = \frac{H_1}{d_1},\ h_2 = \frac{H_2}{d_1},\ d = \frac{d_2}{d_1},\ a = \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, \tag{8}$$
$$R = \frac{cd_1}{\nu}, \theta = \frac{T - T_0}{T_1 - T_0}, \mathrm{Pr} = \frac{\rho \nu C_p}{\kappa}, \bar{t} = \frac{ct}{\lambda},$$
$$M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1,\ Ec = \frac{c^2}{C_p(T_1 - T_0)}\ ,\ \bar{\mathrm{K}} = \frac{\mathrm{K}}{\mathrm{d}_1^{\,2}}$$

and defining the stream function ψ by:

$$u = \frac{\partial \psi}{\partial y},\ \ v = -\frac{\partial \psi}{\partial x}$$

we may write the governing Eqs. (3)–(6), using the long wavelength and small Reynolds number assumptions as (after dropping bars):

$$\frac{dp}{dx} = \frac{\partial^3 \psi}{\partial y^3} - (M^2 + \frac{1}{K})\left(\frac{\partial \psi}{\partial y} + 1\right) \tag{9}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + Ec(\frac{\partial^2 \psi}{\partial y^2})^2 = 0 \tag{10}$$

where R is the Renolds number, δ is the wave number, M is the Hartman number, Pr is the Prandtl number and Ec is the Eckert number.

The volume flow rate in the fixed frame is given by:

$$Q = \int_{h_2(X)}^{h_1(X)} U(X,Y,t)\,dY \tag{11}$$

which in the wave frame becomes:

$$q = \int_{h_2(x)}^{h_1(x)} u(x,y)\,dy \tag{12}$$

Using Eq. (7) in Eqs. (11) and (12) we get:

$$Q = q + ch_1 - ch_2 \tag{13}$$

The average volume flow rate over the period $T\left(=\frac{\lambda}{c}\right)$ of the peristaltic wave is defined as:

$$\bar{Q} = \frac{1}{T}\int_0^T Q\,dt \tag{14}$$

Then, using Eq. (13), we obtain:

$$\bar{Q} = q + cd_1 + cd_2 \tag{15}$$

Defining the dimensionless mean flows:

$$\Theta = \frac{\bar{Q}}{cd_1}\ ,\ \ F = \frac{q}{cd_1} \tag{16}$$

in fixed and moving frames, respectively, we may write Eq. (15) as:

$$\Theta = F + 1 + d \tag{17}$$

where F is the dimensionless average flux in the wave frame defined by:

$$F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} dy = \psi(h_1) - \psi(h_2) \tag{18}$$

and the dimensionless peristaltic walls are:

$$\left.\begin{aligned} h_1 &= 1 + a\cos 2\pi x \\ h_2 &= -d - b\cos(2\pi x + \phi) \end{aligned}\right\} \tag{19}$$

in which a , b , d and ϕ satisfy the relation:

$$a^2 + b^2 + 2ab\cos\phi \le (1 + d)^2 \tag{20}$$

Since we are considering the slip on the wall, therefore, the corresponding boundary conditions for the present problem in the wave frame can be written as:

$$\left.\begin{aligned} &\psi = \frac{F}{2}, \frac{\partial \psi}{\partial y} + \beta \frac{\partial^2 \psi}{\partial y^2} = -1, \theta + \gamma \frac{\partial \theta}{\partial y} = 0 \text{ at } y = h_1 \\ &\psi = -\frac{F}{2}, \frac{\partial \psi}{\partial y} - \beta \frac{\partial^2 \psi}{\partial y^2} = -1, \theta - \gamma \frac{\partial \theta}{\partial y} = 1 \text{ at } y = h_2 \end{aligned}\right\} \tag{21}$$

where β is the non-dimensional slip velocity parameter (Knudsen number) and γ is the non-dimensional thermal slip parameter.

3. METHOD OF SOLUTION

The heat transfer analysis of peristaltic flow in a symmetric/asymmetric channel has been studied numerically as well as analytically by many researchers such as Muthuraj and Srinivas [15], Srinivas et al. [16], etc. But, it appears that the analytical solution of this type of problem will be of greater interest. Therefore, the purpose of the present study is to solve the problem of heat transfer on peristaltic flow with slip condition in an asymmetric porous channel analytically and also numerically. The governing equations for stream function and temperature field are solved analytically under long wavelength and low Reynolds number assumptions. Some important basic features of peristaltic motion such as trapping, pumping, etc. are discussed numerically using computation software MATHEMATICA.

4. SOLUTION OF THE PROBLEM

On differentiating from Eq. (13) with respect to y, the compatibility equation is as follow:

$$\frac{\partial^4\psi}{\partial y^4}-(M^2+\frac{1}{K})\frac{\partial^2\psi}{\partial y^2}=0 \tag{22}$$

The closed form solutions for Eqs. (22) and (10) with boundary conditions (21) are:

$$\psi=A_0+A_1y+A_2\cosh Ny+A_3\sinh Ny \tag{23}$$

$$\theta=A_4+A_5y-\frac{1}{8}BrN^2\times \left\{2N^2y^2\left(A_2^2-A_3^2\right)+\left(A_2^2+A_3^2\right)\cosh 2Ny+2A_2A_3\sinh 2Ny\right\} \tag{24}$$

where $N^2=M^2+\frac{1}{K}$, Br (= EcPr) is the Brinkman number.

$$A_0=-\frac{h_1+h_2}{2}A_1,$$

$$A_1=\frac{NF+\left(2+F_0\beta N^2\right)\tanh\frac{1}{2}N\left(h_1-h_2\right)}{N\left(h_1-h_2\right)-\left\{2-\beta N^2\left(h_1-h_2\right)\right\}\tanh\frac{1}{2}N\left(h_1-h_2\right)},$$

$$A_2=\frac{\left(F+h_1-h_2\right)\operatorname{sech}\frac{1}{2}N\left(h_1-h_2\right)\sinh\frac{1}{2}N\left(h_1+h_2\right)}{N\left(h_1-h_2\right)-\left\{2-\beta N^2\left(h_1-h_2\right)\right\}\tanh\frac{1}{2}N\left(h_1-h_2\right)},$$

$$A_3=-A_2\coth\frac{1}{2}N\left(h_1+h_2\right),$$

$$A_4=\frac{1}{8}BrN^2\left\{\left(A_2^2+A_3^2\right)\left(\cosh 2Nh_1+2\gamma N\sinh 2Nh_1\right)+2A_2A_3\left(\sinh 2Nh_1+2\gamma N\cosh 2Nh_1\right)\right\}-A_5(h_1+\gamma),$$

$$A_5=\frac{1}{4}BrN^4\left(A_2^2-A_3^2\right)\left(h_1+h_2\right)+\frac{BrN^2}{4\left(h_1-h_2+2\gamma\right)}\times\left\{\left(A_2^2+A_3^2\right)\sinh N\left(h_1+h_2\right)+2A_2A_3\cosh N\left(h_1+h_2\right)\right\}\times\{\sinh N\left(h_1-h_2\right)+2\gamma N\cosh N(h_1-h_2)\}-\frac{1}{h_1-h_2+2\gamma}$$

The pressure gradient is obtained from the dimensionless momentum equation for the axial velocity as:

$$\frac{dp}{dx}=-\frac{N^3\left(F+h_1-h_2\right)\{1+N\beta\tanh\frac{1}{2}N\left(h_1-h_2\right)\}}{N\left(h_1-h_2\right)-\left\{2-\beta N^2\left(h_1-h_2\right)\right\}\tanh\frac{1}{2}N\left(h_1-h_2\right)} \tag{25}$$

The non-dimensional expression for pressure rise per wavelength ΔP_λ is given by:

$$\Delta P_\lambda=\int_0^1\frac{dp}{dx}dx \tag{26}$$

where $\frac{dp}{dx}$ is defined through Eq.(25).

The heat transfer coefficient Z at the upper wall is given by:

$$\begin{aligned}Z_1&=\left(h_1\right)_x\theta_y\\&=\left(h_1\right)_x[A_4-\frac{1}{4}BrN^3\{2Ny(A_2^2-A_3^2)\\&+(A_2^2+A_3^2)\sinh 2Ny+2A_2A_3\cosh 2Ny\}]\end{aligned} \tag{27}$$

5. NUMERICAL RESULTS AND DISCUSSION

An interesting phenomenon of peristaltic motion in the wave frame is trapping which is basically the formation of an internally circulating bolus of fluid by closed streamlines. This trapped bolus is pushed ahead with the peristaltic wave. The variations of M, K and β on the streamlines are shown in Figure 2. It is observed from Figures 2a-c that the bolus appears in the center region for φ=π/2 and decreases in size as both M and K increase. The effect of slip parameter on the trapping are illustrated in Figures 2a and 2d and it is observed that the number and size of trapped bolus gradually increases with increasing slip velocity parameter. These results are in agreement with the results obtained by Nadeem and Akram [20] and Hayat et al. [21].

The effects of K, M and β on the pressure gradient are plotted in Figure 3. It can be noticed that in the wider part of the channel xε [0, 0.2] and [0.7,1], the pressure gradient is relatively small, that is, the flow can easily pass without imposition of a large pressure gradient. On the other hand, in a narrow part of the channel xε [0.2, 0.7] a much higher pressure gradient is required to maintain the same flux to pass it especially near x=4.5. From

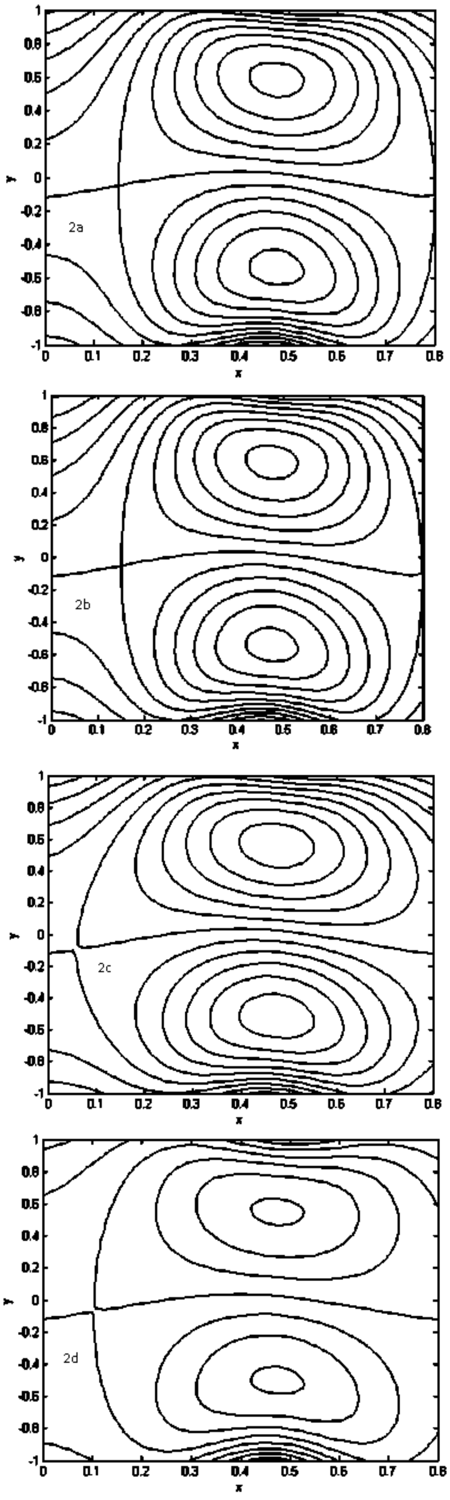

Figure 2. Effect of M, K and β on streamlines when $a=0.25, b=0.4, d=1.1, \Theta=1.9, \varphi=\pi/12$:
(a) $M=0, K=0.5, \beta=0.1$;
(b) $M=2.0, K=0.5, \beta=0.1$;
(c) $M=0, K=2.0, \beta=0.1$;
(d) $M=2.0, K=0.5, \beta=0.0$

Figures 3a and 3b, it may be noted that K decreases the maximum amplitude of the pressure gradient. But the amplitude is increased by increasing M and it rapidly increases for large magnetic field. Figure 3c revels that the pressure gradient decreases with increase in slip parameter β.

The variations of pressure rise ΔP_{λ} per wave length against the mean flow rate Θ of an asymmetric channel are illustrated in Figure 4 for

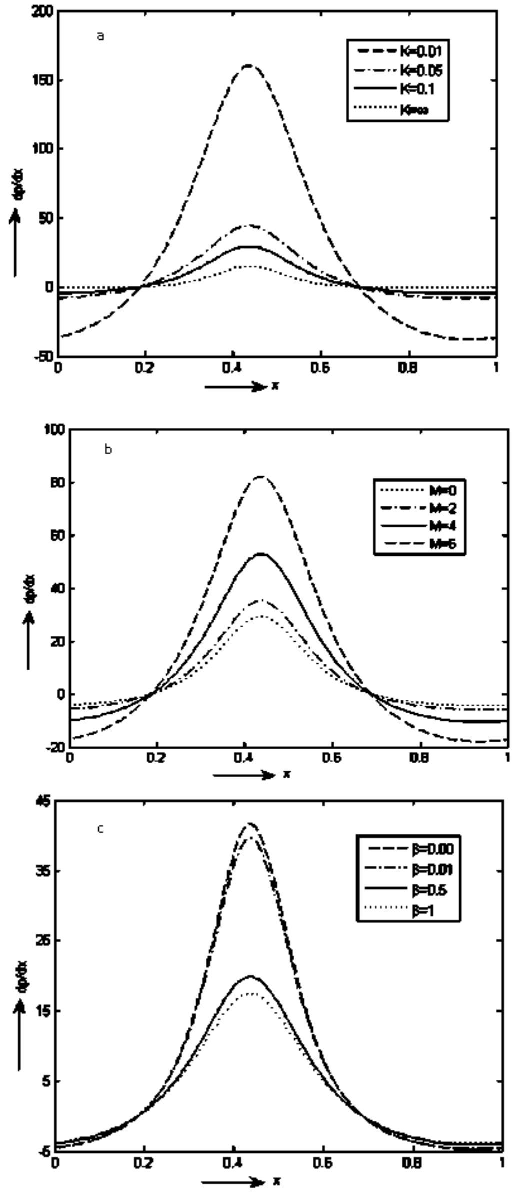

Figure 3. Variation of the pressure gradient with x for different values of (a) K, (b) M, (c) β when : $a=0.25, b=0.4, d=1.1, \Theta=1.9, \varphi=\pi/12$

various physical parameters. In these Figures specific attention is given to the pumping regions, peristaltic pumping (Θ>0, ΔP_{λ} >0), augmented pumping (Θ>0, ΔP_{λ}<0) and retrograde (Θ< 0, ΔP_{λ} >0). Figure 4a shows that the retrograde pumping rate increases with increase of M and decreases as flow rate increases. However, opposite effects are noticed for the case of augmented pumping. Figure 4b gives the effect of K on ΔP_{λ}. It is noted that an increase of K results in decrease of both the

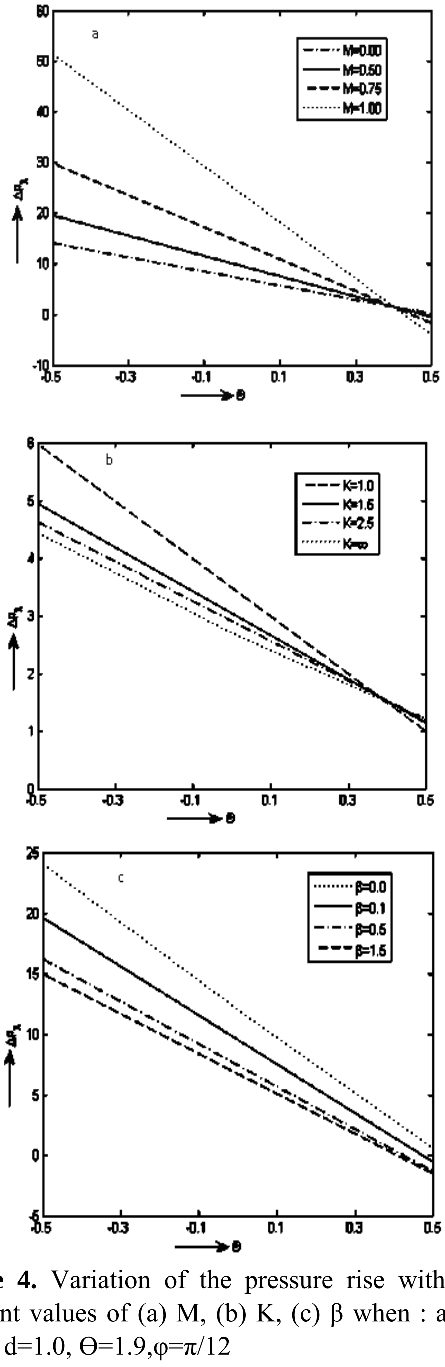

Figure 4. Variation of the pressure rise with x for different values of (a) M, (b) K, (c) β when : a=0.25, b=0.4, d=1.0, Θ=1.9,φ=π/12

retrograde pumping and peristaltic pumping and the effect is negligible for augmented pumping. The effects of β on ΔP_{λ} are shown in Figure 4c. It shows that there is a linear relation between ΔP_{λ} and Θ and an increase of β results in decrease in pumping rate and is uniform for all pumping regions. These results are consistent with the results analyzed in previous studies (Nadeem and Akram [20] and Hayat et al. [21]).

To explicitly see the effects of various parameters, M, K, γ, β and Br on temperature, Eq. (24) has been numerically evaluated and the results are presented in Figure 5. It was observed that the temperature increases with increase of K, γ and Br while it decreases with increasing M, β. Further, it can be noted that the temperature at the lower wall is maximum and it decreases slowly towards the upper wall.

Variations of the heat transfer coefficient at the wall have been presented in Figure 6 for various values of K, γ, M and Br with fixed values of other parameters. One can observe that the heat transfer coefficient decreases with increase of γ. Also, it is noted that there is no quantitative change in the behavior of the heat transfer coefficient near the centre but it increases with increasing Br, K and the effect is reverse for M in the vicinity of the upper wall.

6. CONCLUSION

In this work, the combined effects of slip conditions and heat transfer on MHD peristaltic flow of a viscous incompressible electrically conducting fluid in an asymmetric porous channel are discussed. The closed form analytical solutions of the problem under long wavelength and low Reynolds number approximations are obtained. The results are discussed through graphs and concluded to the following observations:

(i) The number and volume of the trapped bolus decreases by increasing both the M and K but trapped bolus increases in size as β increases.

(ii) The amplitude of pressure gradient decreases with the increase of both β and K, whereas it increases by increase of M.

(iii) The effects of M, K and β on pressure rise are different for different pumping regions.

(iv)The temperature field decreases with the increase in M, while with the increase in γ and K the temperature field increases.

(v) The absolute value of heat transfer coefficient decreases with increasing γ, but it increases with the increase of Br and K in the vicinity of the upper wall.

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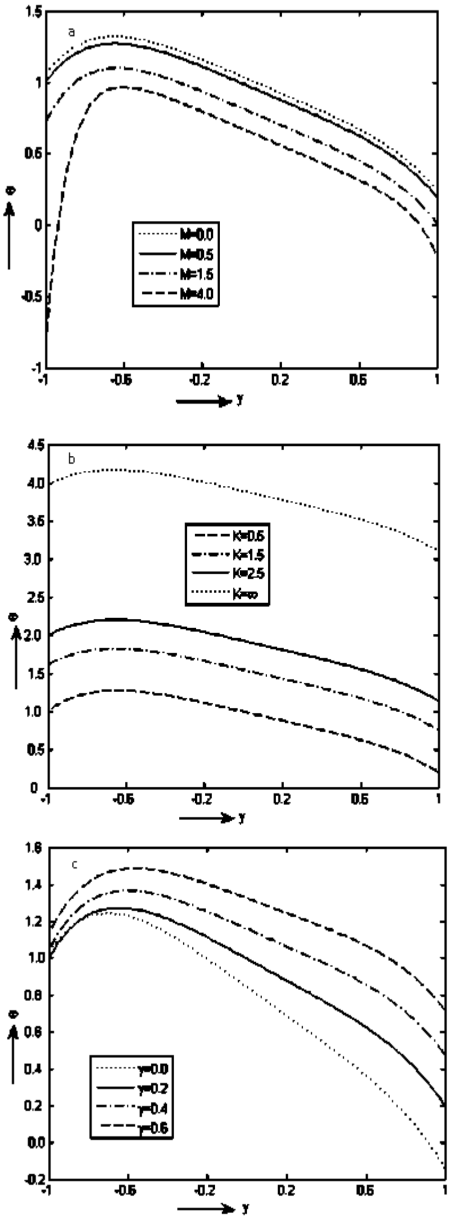

Figure 5. Variation of temperature with y for different values of (a) M, (b) K, (c) γ when : a=0.3, b=0.45, d=1.0, ϴ=1.9,x=0.4, φ=π/12

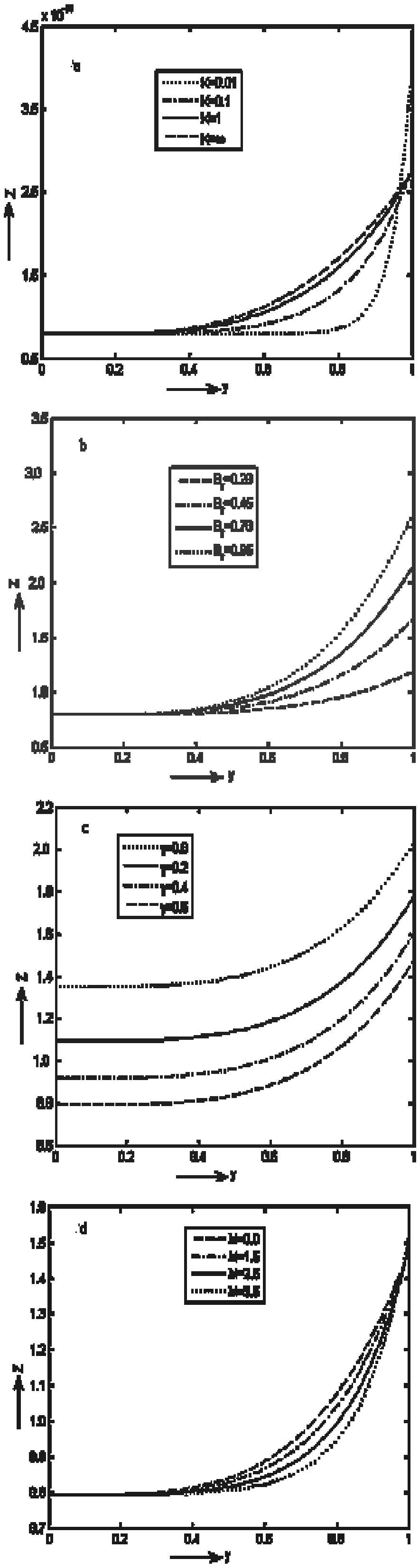

Figure 6. Effect of K, Br, γ and M on heat transfer coefficient

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