AERO-THERMODYNAMIC OPTIMIZATION OF TURBOPROP ENGINES USING MULTI-OBJECTIVE GENETIC ALGORITHMS

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Abstract In this paper multi-objective genetic algorithms were employed for Pareto approach optimization of turboprop engines. The considered objective functions are used to maximize the specific thrust, propulsive efficiency, thermal efficiency, propeller efficiency and minimize the thrust specific fuel consumption. These objectives are usually conflicting with each other. The design variables consist of thermodynamic parameters (compressor pressure ratio, turbine temperature ratio, Mach number) and propeller geometric parameters (blade activity factor and integrated design lift coefficient). Main effect of the design variables are calculated to recognize which design variables has the effect on the objective functions. Group method of data handling (GMDH) type neural networks is used for modeling and prediction of propeller efficiency using aerodynamic variables obtained by some experimental data. Relationships among design variables and optimal objectives functions have been obtained by non-dominated sorting genetic algorithm, (NSGA-II) with a new diversity preserving mechanism in multi-objective optimization. The pareto solutions are obtained for both two and five objective optimization processes. For two-objective optimization, different pairs of objectives have been selected. More ever, these objectives have also considered for a five-objective optimization problem. Variables based on this pareto front, indicated the best design point of objective functions. These results also showed that Pareto solutions of five-objective optimization provide more choices for optimal design of turboprop engines.

Keywords: Turboprop, Multi-Objective Optimization, Genetic Algorithms, Pareto Front, GMDH

چکیده دراین مقاله ازالگوریتم ژنتیک جهت بهینه سازی عملکرد موتورهای توربوپراپ استفاده شده است.توابع هدف مورد بررسی جهت بهینه سازی، نیروی تراست ویژه(ST)، مصرف سوخت ویژه(S)، راندمان رانش(η_p)، راندمان حرارتی(η_t) وراندمان ملخ (η_{prop}) درنظرگرفته شده اند .متغیرهای طراحی مساله شامل پارامترهای ترمودینامیکی مانند نسبت فشارکمپرسور(π_C)، نسبت دمای توربین (τ_t) و عدد ماخ(M_0) و همچنین پارامترهای هندسی ملخ ازجمله فاکتورفعالیت تیغه(AF)و ضریب لیفت طراحی(CL_i) می باشند. دربهینه سازی چندهدفی ازروش NSGAII استفاده شده است وارتباط بین متغیرهای طراحی وتوابع هدف به وسیله آن به دست آمده است.روش دسته بندی گروهی داده های عددی(GMDH) برای به دست آوردن مدل راندمان ملخ با استفاده ازداده های تجربی، جهت بهینه سازی به کاررفته است. بهینه سازی با دووپنج تابع هدف انجام گرفته است.دربهینه سازی بادوتابع هدف توابع هدف به دسته های دوتایی تقسیم می شوند ومنحنی های Pareto front برای هرجفت به دست می آید. این منحنی ها بهترین نقاط طراحی برای توابع موردنظر می باشند.این نقاط نسبت به یکدیگر غیربرترند امابرهرنقطه دیگربرتری دارند.دربهینه سازی باپنج تابع هدف، توابع هدف همزمان بهینه می شوند. مقایسه نتایج بهینه سازی پنج هدفی بانتایج بهینه سازی بادوتابع هدف، نشان داده شده است که نتایج حاصل ازبهینه یابی پنج هدفی، نقاط به دست آمده دربهینه ی‌ابی دوهدفی رادربردارد. ازآنجاکه تعداد نقاط بهینه دراین حالت بیشتراز حالت دوهدفی است، درنتیجه حوزه طراحی وسیعی دراختیارطراح قرارمی دهدتاباتوجه به اهمیت مساله، نقطه بهینه راتعیین کند.

1. INTRODUCTION

The turboprop engine plays a major role in short haul commuter aircraft, military transport and patrol aircraft. A turboprop engine is a type of gas turbine engine used in aircraft. It uses a gas turbine core to turn a propeller. Propeller engines develop thrust by moving a large mass of air through a small change in velocity [1].

Aircraft system studies have shown improvement in fuel efficiency and operating cost without compromising productivity for turboprop transport as compared to advanced turbofan transports [2-5]. Recently several optimization methods have been successfully applied to turboprop engines. Cho and Lee developed a numerical optimization technique to determine the optimum propeller blade shape for efficiency improvement of turboprop [6]. An aerodynamic optimization based on the idea of updating the flow variable iterative solutions and the design parameter solutions simultaneously developed by Rizk [7].

Some numerical studies also show that optimization of a modern medium speed turboprop aircraft leads to performance increase by adapting the wing shape [8].

This paper focuses on the optimization of turboprop engines for improvement performance. Optimization theory encompasses the quantitative study of optima and methods for finding them [9]. Thus optimization seeks to improve performance toward some optimal point or points. Genetic algorithms are search algorithms based on the mechanics of natural selection and evolution [10]. It is an example of a search procedure that uses random choices as a tool to guide a highly exploitative search process leading its form of search for minimal energy states [11].

Genetic algorithms were developed by Holland. Although these algorithms emerged simultaneous- ly programming, but; GAs are today the most widely known type of evolutionary algorithms [12].

There are several advantages to the Genetic algorithm. It works well for global optimization especially where the objective function is discontinuous or with several local minima. These advantages lead to potential disadvantages. Since it does not use extra information such as gradients, the Genetic algorithm can be used in both unconstrained and constrained optimization problems. It can be applied to nonlinear programming, stochastic programming [13].

Recently, Genetic algorithms have been applied to the search for multi criteria optima. Multi objective problem (MOP) is an extension of the single-objective optimization. The notion of Genetic search in a multi criteria problem dates back to the early day of GA experimentation. Rosenberg's study contained a suggestion that would have led to multi criteria optimization if he had carried it out as presented. He suggested using multiple properties in his simulation of the Genetics and chemistry of a population of single-called organisms [14].

A practical scheme was developed 17 years later by Schaffer in his vector Evaluated Genetic Algorithm (VEGA) program. He developed the method VEGA which has only one difference with an usual genetic algorithm. It is the way that the selection is carried out for recombination [15].

A French- Italian economist named V. Pareto first developed the principle of multi objective optimization for use in economics. His theories became collectively known as Pareto's optimality concept [16].

Goldberg made a non-dominance ranking to solve the speciation problems of VEGA [10]. Fonseca and Fleming developed this idea into the MOGA method (Multi-objective Genetic Algorithm) [17].

Srinivas and Deb followed ranking method more precisely in their Non-dominated Sorting Genetic algorithm (NSGA) [18].

Zitzler and Thiele developed a multi-objective genetic algorithm called the Strengthen Pareto Evolutionary Algorithm (SPEA) [19].

Knowles and Come introduced the Pareto Archived Evolution Strategy (PAES) [20].

Here, we investigate the Elitist Non-Dominated Sorting Genetic algorithm (NSGAII). It uses a ranking of dominated solution in a population to ensure progress towards the pareto-optimal front and a crowding operator to ensure a good diversity among obtained solutions [21, 22].

Most realistic optimization problems, particularity those in design, require the simultaneous optimization of more than one objective function. Many design problems related to thermal systems and aerodynamic designs that involve multiple conflicting objectives. In thermal systems, a good design is characterized by several factors and objectives. Recently, many studies to develop optimization of thermal systems have been carried out [23-26]. Toffolo and Lazzaretto has been analyzed thermodynamic optimization in a cogeneration power plant in which two conflicting objectives (exergic and economic issues) have been considered [24]. A multi-objective Aerodynamic optimization of supersonic wings has been performed by Sasaki et al. [25]. Homaifar has been studied thermodynamic optimization of turbofan engine with GA [26].

In this paper, the turboprop engines performance optimization using multi objective Genetic algorithms have been considered .The set of design variables, namely, input Mach number $\mathbf{M_0}$, Compressor pressure ratio p_C, turbine temperature ratio t_t, blade activity factor AF and integrated design lift coefficient CL_i, that produces the optimal objective functions are selected as the optimal pareto set. It yields a set of possible solutions.

Multi-objective Evolutionally Algorithms (NSGA-II) with a new diversity preserving mechanism is used for optimization of five important conflicting objectives.

The considered objectives include specific thrust (ST), propulsive efficiency (η_P), thermal efficiency (η_T), propeller efficiency (η_{prop}) and thrust specific fuel consumption (TSFC). Different pairs of these objectives selected for two-objective optimization. Moreover, all of objectives have been optimized simultaneously.

GMDH-type neural networks are used for modeling and prediction of propeller efficiency of turboprop. GMDH algorithm is a self-organizing approach by which gradually complicated models are generated based on the evaluation of their performances on a set of multi-input-single-output data pairs (X_i, Y_i) (i=1, 2, …, M) [27].

2. OPTIMIZATION METHOD

A multi-objective problem is solved in a manner of the single-objective problem. In the MOP, a number of conflicting objective functions are to be optimized simultaneously. Usually there is no single solution for all objectives which are optimal. Therefore, the solution of multi-objective problem comprises a set of solutions which holds that there are no other solutions that are superior considering all objectives. These solutions are called pareto-optimal. Hence, optimizing a multi objective problem is comprised of finding pareto-optimal solutions [28-30].

A general multi-objective design problem is expressed by equations (1) and (2).

$$\min_{x\in C} F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix} \tag{1}$$

Where $n \geq 2$ and

$$C = \{x : h(x) = 0, g(x) \leq 0, a \leq x \leq b \tag{2}$$

Denotes the feasible set constrained by equality and inequality constraints and explicit variable bounds. The space in which the objective vector belongs is called the objective space and image of the feasible set under F is called the attained set.

A solution is said to dominate another solution when it is better on one objective, and not worse on the other objectives. Thus, solution **a** dominates solution **b** if and only if $\exists\ i : O_i(a) < O_i(b)$ and $\forall\ j \neq i\ O_j(a) \leq O_j(b)$. This assumes without loss of generality that the objective functions $O_1, \ldots, O_m$ need to be optimized. A solution is said to be non-dominated if no solution can be found that dominates it. The definition of dominance relation gives rise to the definition of Pareto optimal set, also called the set of non-dominated solutions. This set contains all solutions that balance the objectives in a unique and optimal way.

The NSGAII algorithm is one of most prominent and most successful Evolutionary Multi objective algorithms [21].

In NSGAII, The offspring population Q_t is first created by using the parent population P_t. However, instead of finding the non dominated front of Q_t only, first the two populations are considered together to form R_t of size 2N.Then a non-dominated sorting is used to classify the entire population R_t. Although this requires more effort compared to performing a non-dominating sorting on Q_t alone, it allows a global non-domination check among the offspring and present solutions. Once the non-dominated sorting is over, the new population is filled by solutions of different non-dominated fronts, one at a time the filling starts with the best non-dominated front and continues with solutions of the second non-dominated front, followed by the third non-dominated front and so on. Since the overall population size of R_t is 2N, not all front may be accommodated in N slots available in the new population. All fronts which could not be accommodated are simply detected. When the last allowed front is being considered, there may be exist more solutions in the last front than the remaining slots in the new population. This procedure illustrated in Figure 1. If the non-duplicate solution (feasible solutions) is more than the slots available in the new parent population, crowded comparison operator is used in NSGAII.

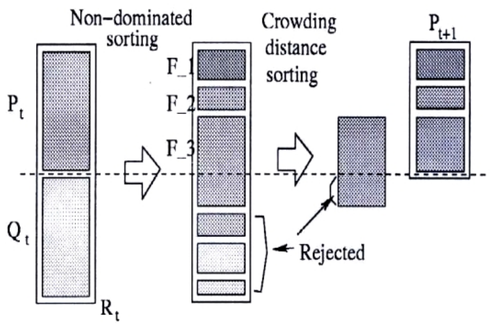

Figure 1. Basies of NSGA-II procedure

The crowding-sorting of the solutions of the last front is performed by using a crowding distance metric, denoting the extent of crowding in the objective space with other population members. The population is arranged in descending order of magnitude of the crowding distance values and required numbers of solutions be chosen from top of list. The crowding distance of a solution i is a measure of the search space around i which is not occupied by any other solution in the population. This quantity is calculated by estimating the perimeter of the cuboid formed by the using the nearest neighbors as the vectors. The crowding distance of the i-th solution in its front is the average side-length of the cuboids [22] (Figure 2).

However, crowding distance algorithm in multi-objective Pareto optimization problem for more than two-objective may not compute the true measure of diversity.

The method presented in this paper for Pareto-based optimization, is NSGAII with a new diversity preserving mechanism, so it is capable of using for more than two objective functions. This method is then used for two and five objective optimization of turboprop engines and results of Pareto solutions are compared with each other.

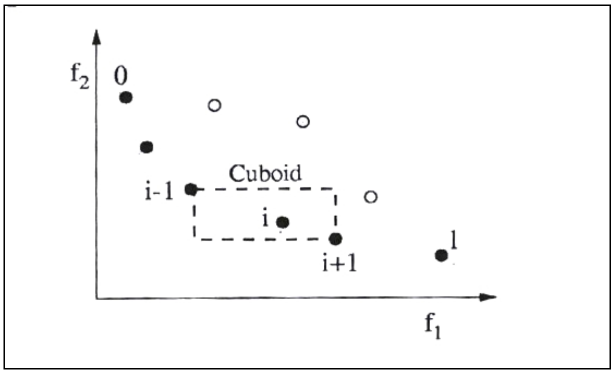

Figure 2. Calculation of crowding distance in NSGAII algorithm

ε-elimination algorithm that based on a pre defined value, ε, as the elimination factor, checks all the individuals in a front and eliminates those within this limit, both in the space of objectives and design variables. Evidently, the eliminated individuals are replaced from the population of new randomly generated individuals. ε-elimination algorithm is shown in the following flowchart

```
pop _ mod = population _ modification (pop)

define  elim _ crit
i=1
until i+1<pop _ size
 j=i+1
  until j< pop _ size
    if (1-elim_crit)*pop( j ) < pop ( i ) < (1+ pop ( j ))
        then pop= pop\ pop( j )
new _ pop = make _  new _ pop
pop _mod=pop U  new_ pop
```

3. MULTI-OBJECTIVE OPTIMIZATION OF TURBOPROP ENGINES

3.1. Model of Turboprop Engines There are two main parts to a turboprop propulsion system, the core engine and the propeller. The core is very similar to a basic turbojet and consists of an intake, compressor, combustion chamber, turbine and propelling nozzle. The only difference is that instead of expanding all the hot exhaust through the nozzle to produce thrust, most of the energy of exhaust is used to turn the turbine. The exhaust velocity of a turboprop is low and contributes little thrust because, most of the energy of the core exhaust has gone in to turning the drive shaft (about 90 percent of the energy of expanding gases is absorbed in turbine portion that drives the propeller, leaving only about 10 percent to accelerate the exhaust jet) [31,32]. The schematic of a turboprop engine is presented in figure 3.

The analyzed real turboprop engine was included component losses, the mass flow rate of fuel through the components and the variation of specific heats. Our analysis still assumes one dimensional flow at the entrance and exit of each component. The variation of specific heats will be approximated by assuming a perfect gas with constant specific heat (C_{pc}) upstream of the main burner (combustor) and a perfect gas with different constant specific heat (C_{pt}) downstream of main burner [31]. This model is sufficient to capture the principles of behavior and interactions among different input and output parameters in a multi-objective optimal sense.

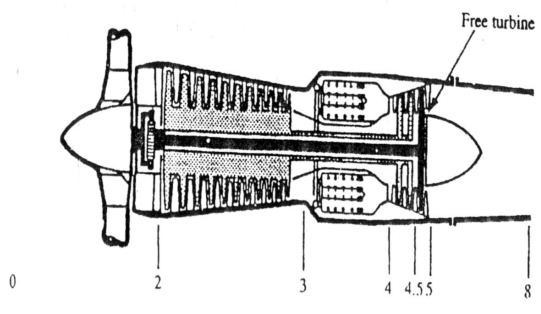

Figure 3. Schematic of turboprop engine

3.2. Design Variables To optimize the performance of the turboprop engine, different parameters were varied to search for the optimum design.

Design variables categorize to thermodynamic and aerodynamic parameters. Input Mach number M_0, Compressor pressure ratio π_c and turbine temperature ratio t_t are used as thermodynamic variables. These design variables are limited as

$4 \le p_c \le 40$

$0.2 \le t_t \le 0.9$

$0.1 \le M_0 \le 0.9$

The activity factor AF and integrated design lift coefficient CL_i are considered as aerodynamic parameters. Activity factor (AF) is introduced with the following definition [32]:

$$AF = \frac{10^5}{D^5} \int_{0.15R}^{1.0R} c r^3 dr = \frac{10^5}{16} \int_{0.15R}^{1.0R} (\frac{c}{D}) x^3 dx \qquad (3)$$

Typical propeller activity factors range from about 70 to 200. The activity factor is closely related to the amount of power which can be absorbed by a propeller. This may be seen from equation (4):

$$P = 4Brp^3 n^3 \bar{c}_d (\frac{D}{10})^5 AF \qquad (4)$$

For a given amount of power, P and for a given propeller diameter, D the number of blades, B and the activity factor, AF cannot be arbitrarily selected because of the activity factor depends on the blade chord, c.

Blade airfoil section and design lift coefficient are related to each other as presented in equation (5) [32]:

$$C_{L_i} = 4 \int_{0.15R}^{1.0R} (c_{l_d}) x^3 dx \qquad (5)$$

c_{l_d} is the section design lift coefficient of the blade that depends on the blade airfoil camber and thickness distribution.

Since propeller thrust is derived from the lift generated by the propeller blades, the average lifting capability of a propeller blade (CL_i) is important. Typical values for CL_i are: $CL_i \approx 0.35 - 0.7$ [32].

3.3. Objective Functions In this study, the design problem has five objective functions. The objective functions considered here are to maximize specific thrust (ST, $N.kg^{-1}.s^{-1}$), propulsive efficiency (η_P), thermal efficiency (η_T) and propeller efficiency (η_{prop}) and minimize the thrust specific fuel consumption (TSFC, $kg.s^{-1}.N^{-1}$).

However, in multi-objective optimization study, some input parameters are already known or assumed as constant parameters. A detailed description of the thermodynamic and aerodynamic analysis and equations [31, 32] of turboprop engines is given in Appendix A.

4. PREDICTION MODEL FOR PROPELLER EFFICIENCY

In order to optimize the propeller performance, a detailed knowledge of aerodynamic factors influencing propeller efficiency is required. The Group method of data handling (GMDH) polynomial neural network to developing a model is discussed [33, 34]. The GMDH was first developed by Ivakhnenko as a multivariate analysis method for complex systems modeling and identification [35].

GMDH network is a learning machine based on the principle of heuristic self-organization unlike classical neural network modeling, GMDH elegantly self determines a network structure of active neurons automatically while synthesizing an analytical model relating the input basis data to the associated output data. By means of GMDH algorithm, a model can be represented as set of neurons in which different pairs of them in each layer are connected through a quadratic polynomial. Thus, they produce new neurons in the next layer. Such representation can be used in modeling to map inputs to outputs.

Using a set of experimental data as input [32], the GMDH algorithm is able to automatically create an algebraic description of the relationship between propeller efficiency and input parameters. The activity factor, integrated design lift coefficient and advanced ratio provided the set of input vectors at constant power coefficient 0.2. The corresponding polynomial representation for η_{prop} is as follows:

$$Y_{23} = 0.01872 + 0.3837\, CL_i + 0.89913\, J_{effective} - 0.1864\, CL_i^{\,2} - 0.21649\, J_{effective}^{\,2} - 0.12086\, CL_i\, J_{effective}$$

$$Y_{13} = -0.27094 + 0.00421\, AF + 0.9499\, J_{effective} - 0.00001\, AF^{2} - 0.21008\, J_{effective}^{\,2} - .00093\, AF\, J_{effective}$$

$$Y_{2311} = -0.80105 + 2.44025\, Y_{23} + 0.00511\, AF - 0.81759\, Y_{23}^{\,2} - 0.00001\, AF^{2} - 0.00313\, Y_{23}\, AF$$

$$Y_{1333} = -0.18824 + 1.28512\, Y_{13} + 0.33345\, J_{effective} + 0.00112\, Y_{13}^{\,2} + 0.021488\, J_{effective}^{\,2} - 0.47396\, Y_{13}\, J_{effective}$$

$$\eta_{prop} = -0.01737 + 0.37919\, Y_{2311} + 0.60994\, Y_{1333} + 5.64345\, Y_{2311}^{\,2} + 4.99573\, Y_{1333}^{\,2} - 10.60329\, Y_{2311}\, Y_{1333}$$

The comparison of GMDH model with experimental data from propeller performance charts that included in Ref. [32] (Generated by Hamilton-standard) is presented in Figure 4. The resulting prediction model appears to correctly incorporate the effect of aerodynamic parameters in propeller efficiency. The obtained polynomial model has been used for optimization as an objective function. It is clearly evident that the evolved GMDH type neural in terms of simple polynomial equations successfully model and predict the output testing data that has not been used during the training process.

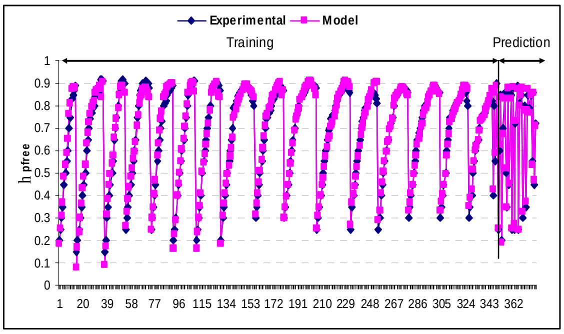

Figure 4. Comparison of prediction model and experimental data

5. PARETO BASED APPROACH

5.1. Two-Objective Optimization Pareto optimality makes use of the concept of dominated and non-dominated solutions. A solution is Pareto optimal if it is not dominated by any other solutions. In order to present tradeoffs between the objectives more clearly, Pareto solutions are considered into the two-dimensional planes.

For two-objective cases, 9 different sets each including two objectives of the output parameters are considered individually. Two conflicting objectives are simultaneously optimized with respect to the design variables.

The system was optimized using a population of 70 individuals and 200 generation with crossover probability p_c, 0.9 and mutation probability p_m, 0.01.

The result of the two-objective optimization is presented in Table 1. For each pair of objectives, the corresponding design variable values are computed and tabulated as well. To investigate the trade-off better, Pareto front of objectives are shown in Figures 5 to13.

It is depicted that some interesting and important relationship as useful optimal design principles involved the performance of turboprop can be discovered by the pareto-based two-objective optimization. Such important optimal principles would not be obtained without the use of multi-objective Pareto optimization. It is clear that choosing appropriate value for objectives to obtain a better value of one objective may cause a worse value of another objective. However, if the set of decision variables is selected based on each of pareto sets, it will lead to the best possible combination of those two objectives.

As it can be observed from Figures 5 and 6 and Table 1, there is a tradeoff between (ST, η_P) and (TSFC, η_P). By accepting a higher propulsive efficiency, the design variables consist of low activity factor, integrated design lift coefficient, pressure ratio and high value of Mach number. In this case, both specific thrust and thrust specific fuel consumption get worse values.

Figures 7 and 8 present the tradeoffs between thermal efficiency with specific thrust and thrust specific fuel consumption, respectively. It can be observed a higher value of thermal efficiency, η_T could be achieved by a high value of pressure ratio(π_c) and Small value of turbine temperature ratio (t_t) in large value of Mach number (M_0) and a propeller with lower activity factor (AF) and integrated design lift coefficient (CL_i).

The obtained Pareto optimal front of propeller efficiency with specific thrust and thrust specific fuel consumption are shown in Figures 9 and 10. These results expressed that the optimal value of propeller efficiency is in accordance with the propulsive efficiency. Therefore, if a better Performance of propeller is desired; better propulsive efficiency could be achieved. The Pareto Solutions for (η_T, η_{prop}) and (η_T, η_p) are depicted in Figures 11 and 12.

It can be seen that these objective have their optimal values in a small ranges with lower activity factor (AF), integrated design lift coefficient (CL_i) for propeller and a small value of pressure ratio and turbine temperature ratio in large value of Mach number for engine performance. The tradeoff between (ST, TSFC) is given in Fig.13. These objectives have a Pareto solutions at minimum value of Mach number, (M_0=0.1) and maximum values of AF and $CL_{i,}$ (AF=20s, CL_i=0.7).

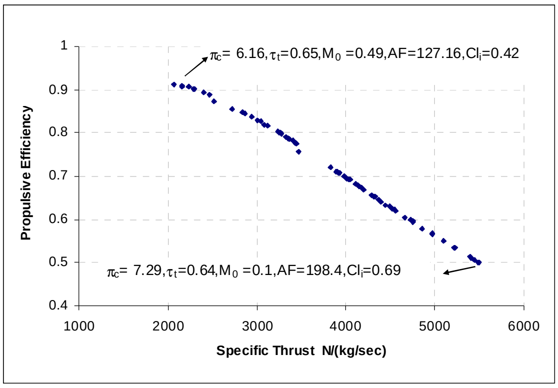

Figure 5. Pareto front for propulsive efficiency and specific thrust in two-objective optimization

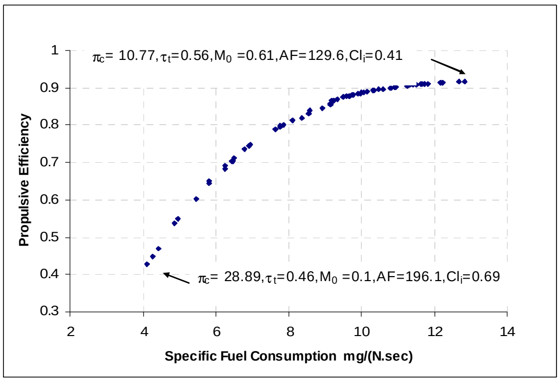

Figure 6. Pareto front for propulsive efficiency and thrust specific fuel consumption in two-objective optimization

TABLE 1.Values of design variables and objective functions in two-objective optimization

(ST, η_p)	(ST, η_T)	(ST, η_{prop})	(η_p, η_T)	$(ST, TSFC)$
$2063.93 \leq ST \leq 5503.32$ $0.4997 \leq \eta_p \leq 0.9116$	$1418.39 \leq ST \leq 5257.88$ $0.177 \leq \eta_T \leq 0.403$	$1345.09 \leq ST \leq 5556.43$ $0.49 \leq \eta_{prop} \leq 0.914$	$0.9146 \leq \eta_p \leq 0.9176$ $0.4068 \leq \eta_T \leq 0.408$	$4918.3 \leq ST \leq 5528.95$ $4.041 \leq S \leq 4.181$
$6.16 \leq \pi_c \leq 11.01$ $0.575 \leq \tau_t \leq 0.664$ $0.1 \leq M_0 \leq 0.49$ $127.16 \leq AF \leq 198.4$ $0.43 \leq CL_i \leq 0.69$	$10.87 \leq \pi_c \leq 30.66$ $0.4498 \leq \tau_t \leq 0.591$ $0.11 \leq M_0 \leq 0.71$ $124.68 \leq AF \leq 199.68$ $0.37 \leq CL_i \leq 0.69$	$8.61 \leq \pi_c \leq 27.85$ $0.4498 \leq \tau_t \leq 0.621$ $0.1 \leq M_0 \leq 0.76$ $135.35 \leq AF \leq 198.09$ $0.37 \leq CL_i \leq 0.7$	$33.71 \leq \pi_c \leq 34.35$ $0.427 \leq \tau_t \leq 0.431$ $0.78 \leq M_0 \leq 0.83$ $126.78 \leq AF \leq 126.84$ $0.34 \leq CL_i \leq 0.37$	$15.26 \leq \pi_c \leq 29.36$ $0.464 \leq \tau_t \leq 0.54$ $M_0 = 0.1$ $AF = 199.9$ $CL_i = 0.7$
$(TSFC, \eta_p)$	**$(TSFC, \eta_T)$**	**$(TSFC, \eta_{prop})$**	**(η_{prop}, η_T)**	
$4.109 \leq TSFC \leq 12.82$ $0.428 \leq \eta_p \leq 0.917$	$4.088 \leq TSFC \leq 14.221$ $0.18 \leq \eta_T \leq 0.408$	$4.31 \leq TSFC \leq 12.825$ $0.468 \leq \eta_p \leq 0.915$	$0.9131 \leq \eta_{prop} \leq 0.9152$ $0.4073 \leq \eta_T \leq 0.4081$	
$10.77 \leq \pi_c \leq 34.14$ $0.442 \leq \tau_t \leq 0.569$ $0.1 \leq M_0 \leq 0.61$ $125.57 \leq AF \leq 196.06$ $0.4 \leq CL_i \leq 0.69$	$25.58 \leq \pi_c \leq 36.396$ $0.435 \leq \tau_t \leq 0.48$ $0.1 \leq M_0 \leq 0.8$ $128.74 \leq AF \leq 200$ $0.4 \leq CL_i \leq 0.69$	$13.43 \leq \pi_c \leq 28.38$ $0.462 \leq \tau_t \leq 0.572$ $0.3 \leq M_0 \leq 0.71$ $131.03 \leq AF \leq 197.27$ $0.41 \leq CL_i \leq 0.7$	$33.56 \leq \pi_c \leq 34.37$ $0.4299 \leq \tau_t \leq 0.4326$ $0.776 \leq M_0 \leq 0.825$ $128.49 \leq AF \leq 130.52$ $0.34 \leq CL_i \leq 0.38$	

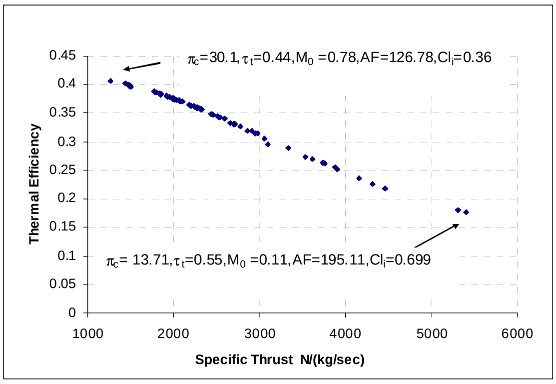

Figure 7. Pareto front for thermal efficiency and specific thrust in two-objective optimization

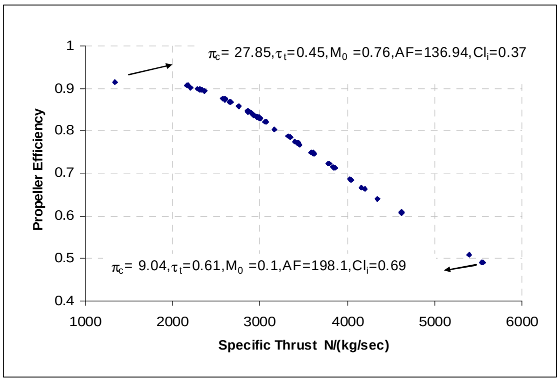

Figure 9. Pareto front for propeller efficiency and specific thrust in two-objective optimization

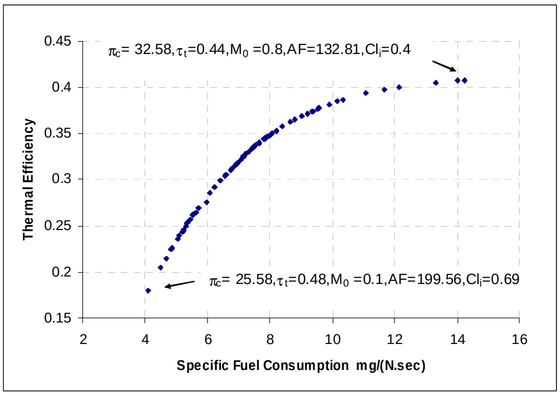

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Figure 8. Pareto front for thermal efficiency and thrust specific fuel consumption in two-objective optimization

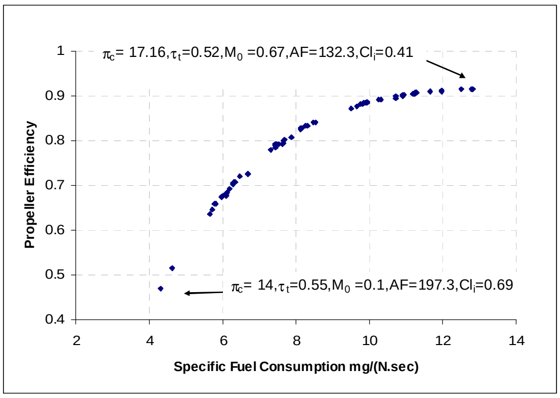

Figure 10. Pareto front for propeller efficiency and thrust specific fuel consumption in two-objective optimization

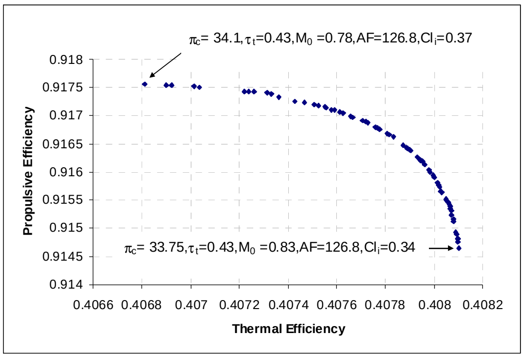

Figure 11. Pareto front for propulsive efficiency and thermal efficiency in two-objective optimization

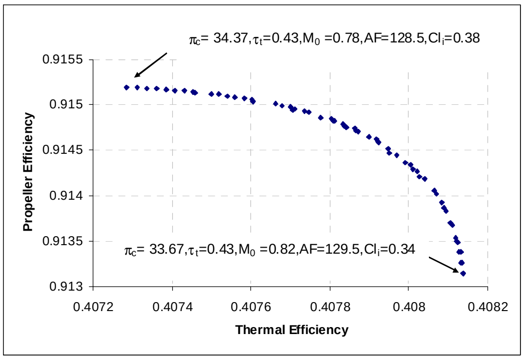

Figure 12. Pareto front for propeller efficiency and thermal efficiency in two-objective optimization

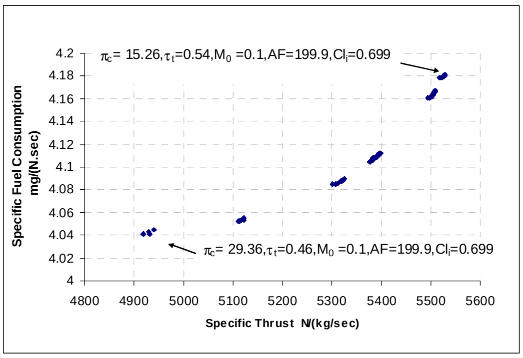

Figure 13. Pareto front for specific thrust and thrust specific fuel consumption in two-objective optimization

In order to choose the final design, the decision-maker has to select concept and then study the tradeoff between the objectives and select a solution point on Pareto frontier.

The effect of variation of AF and CL_i with increasing specific thrust is shown in Figs. 14 and 15, respectively. Figure 14 shows that for a given amount of power, p, the activity factor increases with increasing specific thrust and decreasing propulsive efficiency. The activity factor depends on the blade chord, c, so when a high value of specific thrust is important to the designer, a blade with high value activity factor is required. It should be noted that by selecting a large blade chord, the number of bladed is restricted because the Circumference puts an upper limit on the number of blades [32]. However, with low value of AF and blade chord, c, propulsive efficiency improves.

Fig. 15 demonstrates that higher values of CL_i lead to good specific thrust and low speed performance and lower values of CL_i lead to higher value of propulsive efficiency and cruise performance. Therefore, designing thick blade airfoils tend to have higher lift coefficients at lower speeds. These results are in accordance with Ref. [32] indicating for high speed application, thin blade airfoils with low camber are more appropriate and for takeoff and landing performance, thick blade airfoils with high camber will be useful.

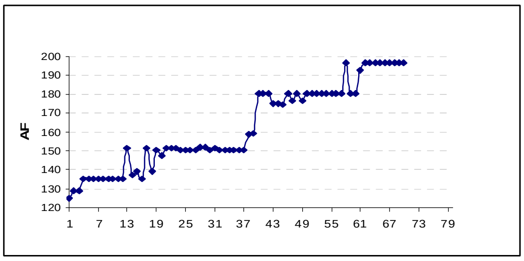

Figure 14. Variation of activity factor in plane of specific thrust and propulsive efficiency (increasing specific thrust)

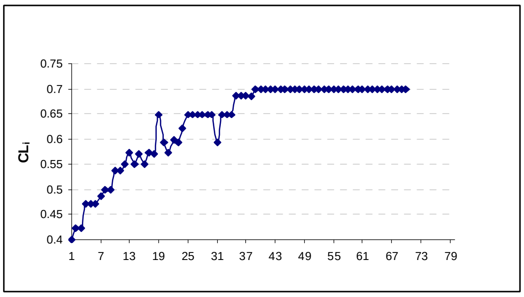

Figure 15. Variation of integrated design lift coefficient in plane of specific thrust and propulsive efficiency (increasing specific thrust)

5.2. Five-Objective Optimization Optimization runs performed for five objectives as well as two objectives. For the five-objective case all objectives are considered simultaneously. The optimization is conducted with a population size of 200 individuals over 200 generations. The crossover probability p_c and mutation probability p_m as 0.9 and 0.01, respectively are used to produce new offspring. For comparison of the obtained results with two-objective optimization, the non-dominated individuals in both 5-objective and two-objective optimization are shown in the plane of (η_p-TSFC) and (η_T-ST) in Figs.16 and 17, respectively. By investigation the result of five-objective optimization, it can be observed that the results of five-objective optimization include the Pareto fronts of each two-objective optimization.

We also tabulated the tradeoff solution of five-objective optimization in range of thrust 1500-2000. Using decision variables, we compute and present the corresponding objective values in Table 2. It can be seen that Pareto front of 5-objective optimization provide more optimal choices for the designer. The resulting Pareto optimal frontier elucidate the advantages of the different concepts and advice the designer which concept to choose depending on his or her preferences.

6. CONCLUSION

In this paper a multi-objective Pareto genetic algorithm (NSGAII) with new diversity preserving mechanism was employed to aero-thermodynamic optimization approach of turboprop engines. This new algorithm is easier to use with a larger number of objectives.

Therefore, this evolutionary multi-objective optimization process is a valuable support for designing. The result of optimization is a set of tradeoff between conflicting objectives.

The analysis of Pareto solutions for propeller blade optimization suggests that low value of activity factor and integrated design lift coefficient provided good propulsive efficiency while higher value of them lead to good specific thrust and thrust specific fuel consumption.

It is also shown that some interesting and important relationship as useful optimal design principles involved in the performance of turboprop engine can be discovered by the Pareto-based multi-objective optimization. It was shown that the results of 5-objective optimization, included those of 2-objective optimization and provided more choices for optimal design.

Furthermore, GMDH model developed to predict the behavior of propeller efficiency was indicated a good agreement between the input data and model prediction results.

Such important optimal design facts would not have been found without the use of multi-objective Pareto optimization.

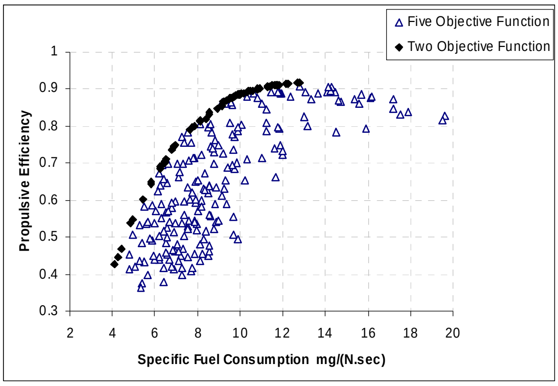

Figure 16. Comparison of the results of 2-objective and 5-objective optimization for propulsive efficiency and thrust specific fuel consumption

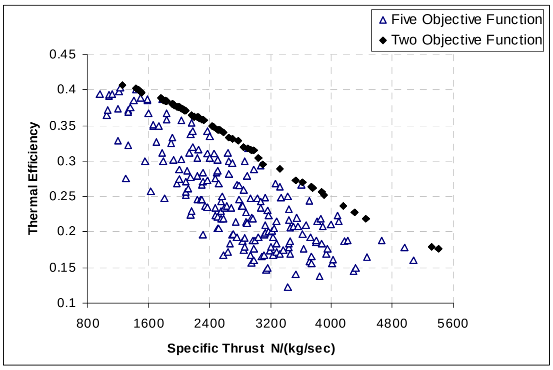

Figure 17. Comparison of the results of 2-objective and 5-objective optimization for thermal efficiency and specific thrust

TABLE 2. Results of multi-objective optimization for five-objective functions (Values of design variables and objective functions)

	ST	TSFC	η_p	η_T	η_{prop}	p_c	t_t	M_0	AF	CL_i
19	1558.177	16.175	0.8788	0.2994	0.8764	7.0681	0.6195	0.6677	84.543	0.3088
20	1582.922	11.874	0.8894	0.385	0.893	28.114	0.4457	0.6302	100.8	0.3088
21	1588.607	11.831	0.8926	0.3864	0.8961	28.114	0.4457	0.6302	99.976	0.533
22	1595.856	13.643	0.8882	0.3671	0.9011	15.376	0.5353	0.6904	96.229	0.485
23	1631.61	15.921	0.7942	0.2579	0.8969	6.1626	0.7016	0.5661	99.404	0.4342
24	1655.459	11.818	0.7919	0.3498	0.8923	25.635	0.5237	0.57	96.229	0.485
25	1662.88	11.765	0.7955	0.3514	0.8973	25.635	0.5237	0.57	157.07	0.5703
26	1707.673	14.238	0.8948	0.327	0.9075	9.0374	0.6023	0.6419	116.3	0.4499
27	1732.379	10.075	0.8039	0.349	0.8072	38.646	0.4217	0.4848	84.543	0.3088
28	1781.856	13.19	0.7992	0.3113	0.9062	11.569	0.6195	0.5661	99.404	0.4342
29	1784.277	10.644	0.8902	0.3863	0.8932	28.413	0.4621	0.5669	134.21	0.4281
30	1801.841	14.117	0.9053	0.2997	0.9045	7.0681	0.6195	0.5833	97.689	0.3355
31	1803.429	14.534	0.7829	0.247	0.9017	6.0308	0.7098	0.4949	155.8	0.6418
32	1833.939	9.5449	0.8077	0.3577	0.8538	38.646	0.4573	0.4707	125.32	0.5632
33	1839.428	11.777	0.8918	0.3679	0.9101	16.862	0.5298	0.5974	132.62	0.368
34	1897.777	9.674	0.7357	0.3248	0.7896	34.295	0.4778	0.4331	84.543	0.3088
35	1919.783	12.354	0.8783	0.3328	0.9134	10.998	0.5921	0.5669	134.21	0.4281
36	1931.264	11.642	0.7404	0.302	0.8696	15.27	0.5969	0.4848	84.543	0.3088
37	1967.637	13.323	0.8725	0.2686	0.8854	6.0308	0.666	0.4934	82.956	0.6684
38	2003.176	11.86	0.7475	0.2874	0.8994	11.569	0.6359	0.4699	146.4	0.4358
39	2005.793	13.067	0.8918	0.2747	0.9066	6.0308	0.666	0.4949	154.59	0.3355
40	2027.996	10.79	0.8745	0.3564	0.889	16.862	0.5298	0.5301	99.976	0.533
41	2036.157	10.322	0.7105	0.3027	0.8222	21.24	0.5538	0.4308	84.543	0.3088
42	2093.566	8.4843	0.6167	0.2707	0.7415	39.921	0.4942	0.3166	161.77	0.3088
43	2095.747	12.003	0.7241	0.2523	0.8671	8.4132	0.6756	0.4175	84.543	0.3088
44	2096.536	11.993	0.7332	0.2558	0.8797	8.4132	0.6756	0.423	177.33	0.368
45	2100.41	9.3327	0.6531	0.2856	0.7453	28.545	0.5168	0.3674	84.543	0.3088

Appendix A. Model of turboprop engine

Input parameters:

$M_0, T_0(K, °R), g_c, c_{pc}\left(\frac{kJ}{kg.K}, \frac{Btu}{lbm.°R}\right), g_t, c_{pt}\left(\frac{kJ}{kg.K}, \frac{Btu}{lbm.°R}\right),$

$h_{PR}\left(\frac{kJ}{kg}, \frac{Btu}{lbm}\right), T_{t4}(K, °R), p_{d\max}, p_b, p_n, e_c, e_{tH}, e_{tL}, h_b, h_g,$

$h_{mH}, h_{mL}, T_{t4}, p_c, t_t$ (if known)

Output parameters:

$ST = \frac{F}{\dot{m}_0}\left(\frac{N}{kg/\sec}, \frac{lbf}{lbm/\sec}\right), \frac{\dot{W}}{\dot{m}_0}\left(\frac{W}{kg/\sec}, \frac{hp}{lbm/\sec}\right), f, SFC\left(\frac{mg/\sec}{N}, \frac{lbm/hr}{lbf}\right),$

$S_P, \left(\frac{mg/\sec}{N}, \frac{lbm/hr}{lbf}\right), h_T, h_P, h_O, C_C, C_{prop}, C_{tot}$

Assumption:

$g_c = 1.4$, $g_t = 1.35$, $T_{t4} = 1370K$, $T_0 = 240K$, $h_{PR} = 42{,}800\, kJ/kg$, $p_d = 0.98$, $p_b = 0.96$, $p_n = 0.99$,

$h_b = 0.99$, $h_{mH} = 0.99$, $h_{mL} = 0.99$, $h_g = 0.99$, $e_c = 0.9$ $e_{tH} = 0.89$, $e_{tL} = 0.91$ $c_{pc} = 1.004\, kJ/(kg.K)$, $c_{pt} = 1.108\, kJ/(kg.K)$,

$B = 3$, $Altitude = 7.6km(25000ft)$

$r = 0.002378\, slugs/ft^3$, $D = 6.192\, ft = 1.887m$,

$C_P = 0.2$ $SHP = 800hp = 220000\ ft.lbf/s = 342.8KW$,

$\dot{m}_{0R} = 14.55kg/\sec$, $T_{t4R} = 1670$, $p_{0R} = 101.3kpa$, $p_{rd} = 1$, $p_{dR} = 0.98$, $p_{cR} = 30$

Equations:

$$R_c = \frac{g_c - 1}{g_c} c_{pc}$$

$$t_r = 1 + \frac{g_c - 1}{2} M_0^2$$

$$R_t = \frac{g_t - 1}{g_t} c_{Pt}$$

$$a_0 = \sqrt{g_c R_c g_c T_0}$$

$$h_c = \frac{p_c^{(g_c - 1)/g_c} - 1}{t_c - 1}$$

$$f = \frac{t_l - t_r t_c}{h_b h_{PR} / (c_{pc} T_0) - t_l}$$

$$t_{tH} = 1 - \frac{t_r (t_c - 1)}{h_{mH} (1 + f) t_l}$$

$$p_{tH} = t_{tH}^{\,g_t / [(g_t - 1) e_{tH}]}$$

$$h_{tH} = \frac{1 - t_{tH}}{1 - t_{tH}^{1/e_{tH}}}$$

$$t_{tL} = \frac{t_t}{t_{tH}}$$

$$p_{tL} = t_{tL}^{\,g_t / [(g_t - 1) e_{tL}]}$$

$$t_l = \frac{c_{pt} T_{t4}}{c_{pc} T_0}$$

$$t_c = (p_c)^{(g_c - 1)/(g_c e_c)}$$

$$h_{tL} = \frac{1 - t_{tL}}{1 - t_{tL}^{1/e_{tL}}} \quad \frac{P_{t9}}{P_0} = p_r p_d p_c p_b p_n p_{tH} p_{tL}$$

$$if \quad \frac{P_{t9}}{P_0} > \left(\frac{g_t + 1}{2} \right)^{g_t / (g_t - 1)} \quad M_9 = 1 \ then$$

$$\frac{p_0}{P_9} = \frac{P_{t9}/P_9}{P_{t9}/P_0} \ and \ \frac{P_{t9}}{P_9} = \left(\frac{g_t + 1}{2} \right)^{g_t / (g_t - 1)}$$

$$else \ M_9 = \sqrt{\frac{2}{g_t - 1} \left[\left(\frac{P_{t9}}{P_9} \right)^{g_t - 1/g_t} - 1 \right]}, \frac{P_{t9}}{P_9} = \frac{P_{t9}}{P_0} \ and \ \frac{p_0}{P_9} = 1$$

$$\frac{V_9}{V_0} = \sqrt{\frac{2 t_l t_{tH} t_{tL}}{g_c - 1} \left[1 - \left(\frac{P_{t9}}{P_9} \right)^{-(g_t - 1)/g_t} \right]}$$

$$C_C = (g_c - 1) M_0 \left((1 + f) \frac{V_9}{a_0} - M_0 + (1 + f) \frac{R_t}{R_c} \frac{T_9/T_0}{V_9/a_0} \frac{1 - P_0/P_9}{g_c} \right) \dot{m}_0 = \dot{m}_{0R} = \frac{P_0 p_r p_d p_c}{(p_0 p_r p_d p_c)_R} \sqrt{\frac{T_{t4R}}{T_{t4}}}$$

$$\dot{W}_{prop} = \dot{m}_0 c_p T_0 h_g h_{mL} (1 + f) t_l t_{tH} (1 - t_{tL})$$

$$C_{prop} = \frac{h_{prop} \dot{W}_{prop}}{\dot{m}_0 c_p T_0} = h_{prop} h_g h_{ml} (1 + f) t_l t_{tH} (1 - t_{tL})$$

$$C_P = \frac{550 \dot{W}_{prop}}{r n^3 D^5}$$

$$J = \frac{V}{nD}$$

$$C_{tot} = C_{prop} + C_C$$

$$ST = \frac{C_{tot} c_{pc} T_0}{M_0 a_0}$$

$$TSFC = \frac{f}{F / \dot{m}_0}$$

$$\frac{\dot{W}}{\dot{m}_0} = C_{tot} c_{pc} T_0$$

$$h_T = \frac{C_{tot}}{f h_{PR} / (c_{pc} T_0)}$$

$$h_P = \frac{C_{tot}}{\dfrac{C_{prop}}{h_{prop}} + \dfrac{g_c - 1}{2} \left[(1 + f) \left(\dfrac{V_9}{a_0} \right)^2 - M_0^2 \right]}$$

$$h_O = h_P h_T$$

NOMENCLATURE

M:	Flight Mach number
T:	Temperature
A:	Velocity of sound
C_{pc}:	Specific heat upstream of the main burner
C_{pt}:	Specific heat downstream of the main burner
e_c :	Polytropic efficiency of compressor
e_t :	Polytropic efficiency of turbine
γ_c :	Ratio of specific heats upstream of the main burner
γ_t :	Ratio of specific heats downstream of the main burner
¦ :	Fuel/air ratio
h_{PR}:	Heating value
V :	Velocity
$\dot{m}$:	Mass flow rate
R :	Gas constant
g_c :	Newton's constant
P :	Pressure
p_t :	Total pressure
p :	Pressure ratio
p_r :	Total static pressure ratio at inlet
t :	Temperature ratio
t_t :	Total static temperature ratio at inlet
t_l :	Burner exit total enthalpy/inlet total enthalpy
C_{prop} :	Work output coefficient of the propeller
C_C :	Work output coefficient of the core stream
C_{tot} :	Work output coefficient for the total turboprop engine
h_t :	Thermal efficiency
h_p :	Propulsive efficiency
h_{prop} :	Propeller efficiency
h_t :	Thermal efficiency
h_q :	Gear efficiency
h_{mH} :	Mechanical efficiency of high pressure turbine
h_{ml} :	Mechanical efficiency of low pressure turbine
ST :	Specific thrust
TSF:	Thrust specific fuel consumption
AF:	Activity factor
CL_i :	Integrated design lift coefficient
b :	Number of blades
c :	Blade Chord
r :	Atmospheric density
D :	Blade diameter
N :	Rotational speed
J :	Advanced ratio

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