PREDICTION OF SULFATE SCALE DEPOSITIONS IN OILFIELD OPERATIONS USING ARITHMETIC OF LR FUZZY NUMBERS

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Abstract In this study fuzzy arithmetic is presented as a tool to tackle the prediction of the amount of barium, strontium and calcium sulfates scales in oilfield operations. Since the shape of fuzzy numbers' membership functions is a spread representative of the whole possible values for a special model parameter, fuzzy numbers are able to consider the uncertainties in parameter determinations and thus give more real results than crisp values. Solubility product models and other required Equations for scale prediction contain uncertain parameters and therefore application of fuzzy numbers can be useful. LR fuzzy numbers and related primary arithmetical operations based on Zadeh's extension principle have been introduced and their use in predicting scale depositions has been investigated. Parameters such as solubility products, free sulfate concentration and scale mass have been determined as fuzzy numbers. As a case study, scale depositions of barium and strontium sulfate resulted from mixing two incompatible waters have been obtained and compared with none fuzzy approach. Fuzzy computations are able to predict the maximum scale mass with respect to existing information.

Key Words Scale Prediction, Fuzzy Numbers, Sulfates, Uncertainty

چکیده در این تحقیق از جبرفازی برای پیش بینی رسوب سولفاتهای باریم، استرانسیم و کلسیم در مخازن و تجهیزات بهره برداری نفت استفاده شده است. از آنجاییکه اعداد فازی با شکل تابع عضویت خود، گستره مقادیر ممکن برای یک پارامتر مدل را با درجه عضویت آن مشخص می‌کنند، عدم قطعیتهای موجود در تعیین پارامتر مدل را بطور کامل درنظر می‌گیرند و لذا نتایج واقعی تری را نسبت به حالت غیرفازی بدست می‌دهند. مدلهای پیش بینی رسوب دارای پارامترهایی با عدم قطعیت می‌باشند و بهمین دلیل کاربرد اعداد فازی می‌تواند مفید باشد. اعداد فازی LR و اعمال جبری اولیه بر اساس اصل گسترش Zadeh معرفی شده اند و کاربرد آنها برای پیش بینی رسوب ناشی از اختلاط دو آب سازندی و آب دریا مورد بررسی قرار گرفته است. پارامترهایی مانند حاصلضرب حلالیت، غلظت سولفات آزاد و جرم رسوب بصورت اعداد فازی بدست آمده اند. برای مثال میزان رسوب سولفاتهای باریم و استرانسیم ناشی از اختلاط دو آب ناسازگار تعیین شده و با روش غیرفازی مقایسه گردیده است. محاسبات فازی قادر است حداکثر رسوب ممکن را با توجه به اطلاعات موجود بدست دهد.

1. INTRODUCTION

Inorganic scale deposition is a common problem in oilfield operations, production wells, equipment, and transportation pipes. The general usage of the term scale denotes hard, adherent deposits of inorganic mineral constituents of water that formed in these places [1]. Barium, strontium, and calcium sulfate scale occurrence is considered a potentially serious problem that causes formation damage near the production well-zone. Sulfate scale may result from changes in temperature and/or pressure while water flows from one location to another, but the major cause of sulfate scaling is the chemical incompatibility between the injected sea water which is high in sulfate ion, and the formation water, which originally contains high concentrations of barium, calcium, and/or strontium ions [2]. An accurate, convenient, and fast model capable of predicting such scaling problems can be useful in planning water flooding schemes and selecting appropriate chemical inhibitors in terms of scale type and its potential severity. A large number of equations and super saturations indices have been developed in order to predict such scaling tendencies. These Equations estimate solubility products with respect to the effective parameters such as temperature, pressure and ionic strength. But the model parameters obtained from experiments and mathematical formulations may contain substantial uncertainties.

In general, reliable results for the numerical solution of engineering problems can be achieved, provided exact values for the parameters of the problem equations are available. In practice, however, these exact values may often not be achieved. Model parameters usually show variability, e.g. due to special difficulties with sample collection and existing uncertainties in analysis. Thus, the results obtained for solutions that just use some specific crisp values for the uncertain parameter cannot be perceived as the representative of the whole spread of possible results. To eliminate this limitation, application of fuzzy set theory, first introduced by Zadeh in 1965, proves to be a practical and realistic approach. Again, the uncertainties in the model parameters can be treated by fuzzy numbers which represent the effect of scatter with their shape derived from experimental data. Fuzzy arithmetical operations, which are the generalized ones for fuzzy numbers, can theoretically be defined by means of Zadeh's extension principle [3-4]. Using fuzzy arithmetic, initially assumed uncertainties can be processed through the computation procedure leading to fuzzy results that show the reliability of problem solution.

2. FUZZY ARITHMETIC

The concept of fuzzy numbers originates from the fact that many qualitative phenomena in the real world cannot be expressed by precise and certain numbers (e.g. about 10. 25 or around 7). Basically, fuzzy numbers can be considered as a special class of fuzzy sets showing some specific properties [3]. The fuzzy sets, themselves, result from a generalization of conventional sets by allowing elements of a universal set not only to belong or not to belong entirely to a specific set, but also to belong to the set to a certain degree [4-5]. Crisp sets can be defined by using the concept of characteristic function as following

$$\chi_A(x)=\begin{cases}0 & if\ x\notin A\\ 1 & if\ x\in A\end{cases} \tag{1}$$

where A is a subset of the universe set U. Extending the domain of the characteristic function from the set {0-1} to the interval [0-1] lead to the emerging of a new type of sets renowned as fuzzy sets. Each element of the universal set has a membership degree in the arbitrary subset A or set A, for short. The characteristic function is then $\mu_A(x)\in[0,1]$ specified by a certain mathematical function.

Fuzzy numbers are convex fuzzy sets over the universal set U with their membership functions $\mu(x)\in[0,1]$ where solely one single value $x=m\in U$ has the membership degree of unity. Convexity of fuzzy set A is achieved when

$$\begin{aligned}&\forall\, x,\ y\in U,\ \exists\ \lambda\in(0,1):\\ &\mu(\lambda x+(1-\lambda)y)\geq\mu(x)\wedge\mu(y)\end{aligned} \tag{2}$$

As a fuzzy number, Gaussian shapes are defined by the membership function

$$\mu_{\tilde{a}}(x)=e^{-\frac{(x-m_a)^2}{2\sigma_a^{\,2}}} \tag{3}$$

where m_a and σ_a are the mean value and the standard deviation of the Gaussian distribution, respectively. (Figure 1)

The membership function for triangular fuzzy numbers is also determined as

$$\mu(x)=\max\left(\min\left(\frac{x-a}{b-a},\frac{c-x}{c-b}\right),0\right) \tag{4}$$

Arithmetical operations with fuzzy numbers can be

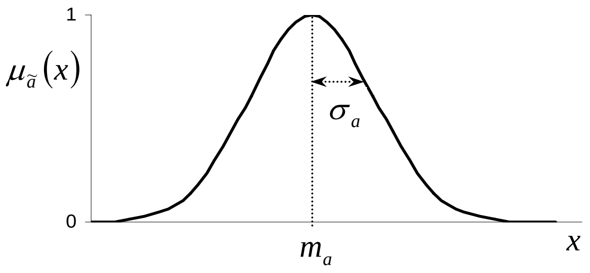

Figure 1. Gaussian distribution of the fuzzy number $\tilde{a}$.

defined according to Zadeh's extension principle [3]. On this basis, if $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers with the membership functions $\mu_{\tilde{a}}(x), x \in U$ and $\mu_{\tilde{b}}(y), y \in U$, then the result of the binary operation:

$$\tilde{c} = f\left(\tilde{a}, \tilde{b}\right) \tag{5}$$

for an arbitrary function f is determined by:

$$\mu_{\tilde{c}}(z) = \sup_{z=f(x,y)} \min \left\{\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)\right\} \tag{6}$$

Since performing engineering calculations with Equation 6 are sometimes tedious and time-consuming, LR fuzzy numbers have been defined by Dubios and Prade [6]. The membership

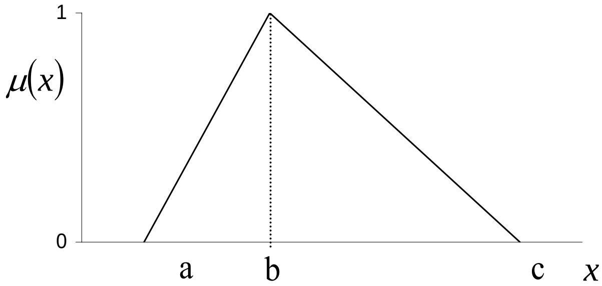

Figure 2. Triangular fuzzy number with threeParameters.

functions of these fuzzy numbers are parameterized, in which arithmetical operations can effectively be performed through a certain pattern.

2.1 Definition If fuzzy number $\tilde{a}$ has a membership function of the form

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left[\dfrac{m-x}{\alpha}\right] & x \le m \\ R\left[\dfrac{x-m}{\beta}\right] & x > m \end{cases} \tag{7}$$

where L and R are none ascending functions from $\Re^{+}$ to [0,1] and $L(0) = R(0) = 1$, then $\tilde{a}$ is an LR fuzzy number that can now be denoted as $\tilde{a} = (m, \alpha, \beta)_{LR}$ where m is the mean value [3, 6]. Numbers α and β determine the width of distribution on the left and right, respectively; and in the case of symmetric fuzzy numbers are equal.

Parameterization of the Gaussian and triangular fuzzy numbers make them LR ones, e.g. the basis functions L and R for the latter are of the form a, b, c

$$L(x) = R(x) = \begin{cases} 1-x & for \quad 0 \le x \le 1 \\ 0 & else \end{cases} \tag{8}$$

Now, using the above definition and the extension principle, one can determine the results of arithmetical operations for fuzzy numbers. For example, the fuzzy addition of $\tilde{a} = (m, \alpha, \beta)_{LR}$ and $\tilde{b} = (n, \delta, \gamma)_{LR}$ is determined as $\tilde{a} \oplus \tilde{b} = (m+n, \alpha+\delta, \beta+\gamma)_{LR}$.

3. SCALE PREDICTION

Prediction of scaling tendency is a complex task due to many effective parameters such as hydrodynamic factors, change in temperature and pressure, ionic strength, pH, impurities, commingling of incompatible waters, etc. Various methods have been used to predict scaling phenomenon in oilfield operations. Vetter [7] proposed a competitive model for predicting sulfate scale forming minerals. Yuan and Todd [2] improved the Vetter model by an iterative algorithm which used the Pitzer equation for ion activity coefficients. Correcting the solubility of sulfates for temperature, pressure, and ionic strength, Oddo and Tomson [8] presented a new saturation index to predict when the scale forms.

The atkinson [9-10] method rests on the special case of the Pitzer equation and standard thermodynamical ones. These scale prediction models usually have deficiencies that restrict their applications for precise estimation of potential scale problems in water flooding operations. Therefore, there is a necessity for an improved model, which considers cardinal scale forming factors of the various locations within the formation, production wells, and surface equipment. In such cases, the use of fuzzy numbers can be helpful. Since fuzzy numbers completely consider the uncertainties of model parameters, the results obtained from the fuzzy calculations can be more reliable and realistic.

3.1. Ordinary Equations of Solubility Products

The equations extracted by Oddo and Tomson [8] from data in the literature were used to calculate the sulfates solubility products. These equations are of the following functional form

$$pK = a + bT + cT^2 + dP + eI^{0.5} + fI + gI^{0.5}T \tag{9}$$

where pK is negative logarithm of solubility, T = temperature (F), P = pressure (psi), and I = ionic strength (M). Required parameters can be specified according to Table 2. To quantify the scale mass, the effect of the common ion must also be taken into account. In scale calculations, hence, the free sulfate concentration should be determined.

The free sulfate ion concentration can be calculated using Equation 9.

$$[SO_4^{2-}]_{free} = \frac{C_{SO_4} - C_{Ca} - C_{Mg} - K' + \left[(C_{SO_4} - C_{Ca} - C_{Mg} - K')^2 + 4K'C_{SO_4}\right]^{0.5}}{2} \tag{10}$$

where, C_{SO_4}, C_{Ca}, C_{Mg} are total sulfate (M), total $K' = 1/K_{MgSO_4}$ calcium (M), total magnesium (M), respectively [8].

3.2. Fuzzy Equations of Solubility Products

Using data in the Table 1, the possibility (membership) distribution functions of the respective parameters of the required Equations were determined. To simplify the calculations, these Gaussian functions were approximated to the triangular ones. The original fuzzy number $\tilde{a}$ with the membership function $\mu_{\tilde{a}}(x)$ can be approximated by a symmetric triangular fuzzy $\tilde{a}_t$ with the membership function $\mu_{\tilde{a}_t}(x)$ that can be obtained by postulating:

$$\mu_{\tilde{n}_t}(m_n) = \mu_{\tilde{n}}(m_n) = 1 \tag{11}$$

TABLE 1. Coefficients and Standard Deviations of The Solubility Constants [8].

	σ	Coefficients of $MgSO_4$		σ	Coefficients of $SrSO_4$		σ	Coefficients of $BaSO_4$
a	0.037	1.858	a	0.086	6.105	a	0.062	10.025
b	0.0000437	0.000451	b	0.00091	0.00198	b	0.00047	-0.0047
c	0.000001342	-0.0000001173	c	0.00000252	0.000006379	c	0.000000772	0.0000011411
d	0.000002317	0.0000010658	d	0.0000063	-0.000004573	d	0.0000024	-0.00004750
e	0.046	-2.378	e	0.087	-1.887	e	0.076	-2.616
f	0.017	0.583	f	0.039	0.667	f	0.0033	0.889
g	0.000015	-0.00133	g	0.000030	-0.00188	g	0.00014	-0.00203

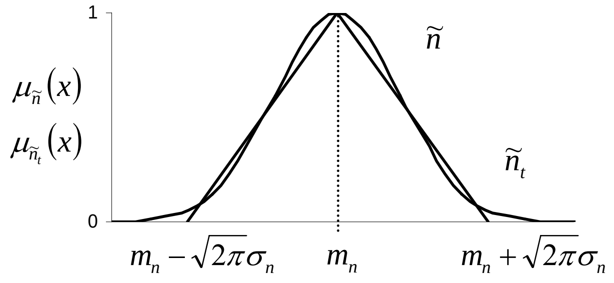

Figure 3. Original fuzzy number $\tilde{a}$ with a Gaussian shape and its linear approximation to a triangular fuzzy number $\tilde{a}_t$.

and

$$\int_{-\infty}^{+\infty} \mu_{\tilde{n}_1}(x)\, dx = \int_{-\infty}^{+\infty} \mu_{\tilde{n}_1}(x)\, dx \qquad (12)$$

Equation 10 guarantees that the converted membership function remains convex and Equation 11 approximates the equality of Gaussian area of uncertainty to that of a triangular one. The membership function $\mu_{\tilde{a}_t}(x)$ is then given by Equation 12.

$$\mu_{\tilde{n}_t}(x) = \max\left\{0,\ 1 - \frac{|x - m_n|}{\sqrt{2\pi}\sigma_n}\right\} \qquad (13)$$

Obviously, the use of triangular fuzzy numbers has two main advantages. First, the very simple way of implementation with linear functions and solely three parameters that can even be reduced to two parameters in the case of symmetric fuzzy numbers. And second, the quite uncomplicated realization of the elementary fuzzy arithmetical operations leads again to triangular fuzzy numbers. Using Equation 13 and data in Table 1, one can determine the membership function of the solubility product of the sulfates as follows

$$p\tilde{K} = \langle a + bT + cT^2 + dP + eI^{0.5} + fI + gI^{0.5}T,\ \sqrt{2\pi}\left(\sigma_a + T\sigma_b + T^2\sigma_c + P\sigma_d + I^{0.5}\sigma_e + I\sigma_f + I^{0.5}T\sigma_g\right)\rangle_{TFN} \qquad (14)$$

where the first parameter denotes the mean value and the second one shows the right and left spreads.

4. CASE STUDY

In this part of the study, the possibility of scale formation resulting from mixing of the formation and sea water is discussed. Ion concentrations of two water types have been listed in Table 2. As mentioned before, $p\tilde{K}$ is a symmetric triangular fuzzy number that can be specified by the Equation 3 or 13. Using the extension principle, $\tilde{K}$ or the membership function of the one dimensional operator $10^{-p\tilde{K}}$, can be determined as follows

$$\mu_{10^{-p\tilde{K}}}(x) = \begin{cases} 0 & \text{if } x < 10^{-c} \\ \dfrac{-\log_{10} x - c}{b - c} & \text{if } 10^{-c} \le x \le 10^{-b} \\ \dfrac{-\log_{10} x - a}{b - a} & \text{if } 10^{-b} < x \le 10^{-a} \\ 0 & \text{if } 10^{-a} < x \end{cases} \qquad (15)$$

Also, $\tilde{SO}_4$ mirrors the free sulfate concentration as a fuzzy number obtained by using the extension principle. As an example, these membership functions for barium sulfate are shown in Figure 4, 5, and 6.

Equation 15 is used to calculate the amount of barite or celestite that can precipitate from a

TABLE 2. Composition of The Formation Water (Ninian field) and Seawater (North sea) [7].

Ion species	Concentration in formation water (mg/l)	Concentration in sea water (mg/l)
Na	8511	10500
K	160	380
Mg	25	1350
Ca	151	400
Sr	44	8.1
Ba	20	0.03
Cl	12660	19354
SO_4	14	2712
HCO_3	1430	142
Calculated	0.4	0.7

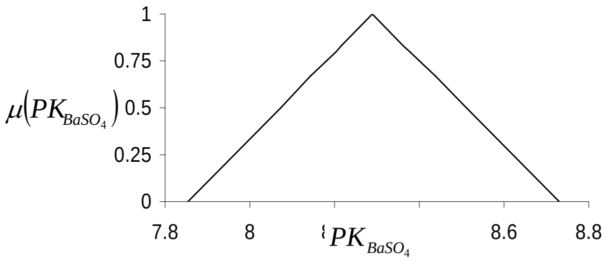

Figure 4. Negative logarithm of solubility product of barium sulfate, $p\widetilde{K}_{BaSO_4}$, at T = 77° F, P = 14.5 psi and 10% seawater as a triangular fuzzy number.

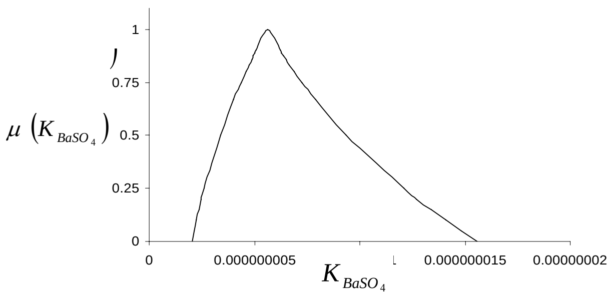

Figure 5. Application of the extension principle to predict solubility product of barium sulfate, $\widetilde{K}_{BaSO_4}$, at T = 77° F, P = 14.5 psi and 10% seawater.

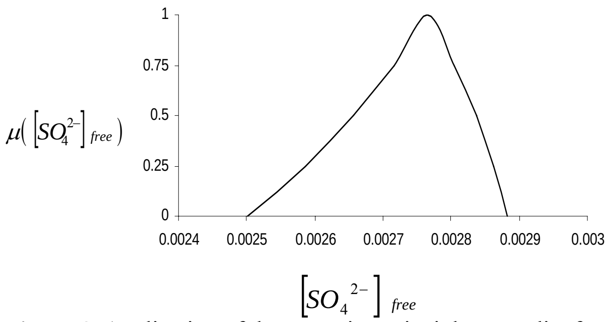

Figure 6. Application of the extension principle to predict free concentration of sulfate, $\widetilde{SO}_4$, at T = 77° F, P = 14.5 psi, and 10% seawater.

solution of known composition [11]

$$ppt_{Salt} = \frac{[Metal] + [SO_4^{2-}]_{free} - x}{2} \qquad (16)$$

where

$$x = \left\{ \left([\mathrm{Metal}] + [\mathrm{SO}_4^{2-}]_{\mathrm{free}} \right)^2 - 4\left([\mathrm{Metal}][\mathrm{SO}_4^{2-}]_{\mathrm{free}} - \mathrm{K}_{\mathrm{sp}} \right) \right\}^{0.5}$$

In non-fuzzy computations, however, free concentration of sulfate, $[SO_4]_{free}$, and K_{sp} have only a single crisp value that eventually leads to one crisp value as the representative of scale mass whereas in fuzzy computations they are some fuzzy numbers and the use of the extension principle leads to some fuzzy numbers showing the respective scale mass. Figure 7, 8, 9 and 10 show these fuzzy numbers that predict the amount of barite and celestite that can deposit in various temperature, pressure and ionic strength.

Whitening gray or fuzzy numbers is a procedure that is called defuzzification. In the present study, the mean of area defuzzification method was used. In this regard, however, the value which divides the area covered by a fuzzy number into two equal sections is selected as the representative of the related fuzzy number [12]. Figure 11, 12, and 13 show the amount of the scale of barite and celestite predicted by the fuzzy and non-fuzzy methods.

5. RESULTS AND DISCUSSION

Fuzzy numbers represent the whole spectrum of possible values for a specific model or equation parameter by their shape and consequently, show the spread of possible results recurring from arithmetical operations as a fuzzy number. The great advantage of application of fuzzy numbers and their arithmetic is to conserve the set of available information so that human experts can have a wider perspective over the problem. Also the degree of possibility of a crisp value occurrence as a response of the solution is determined by the membership function of the respective fuzzy number.

Once the problem solution is achieved, there is a need for a single crisp value to make a comparison between the fuzzy and none fuzzy procedure.

Although the process of reducing the final fuzzy

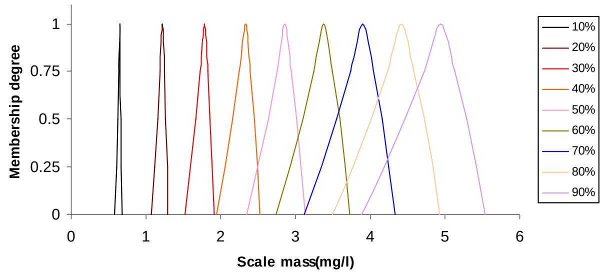

Figure 7. Scale mass of barite obtained from fuzzy calculations at room temperature, pressure and various seawater in mixture.

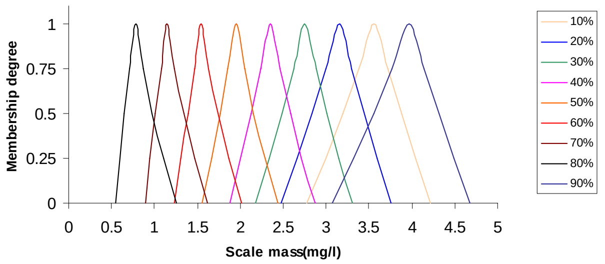

Figure 8. Scale mass of celesite obtained from fuzzy calculations at room temperature, pressure and various seawater in mixture.

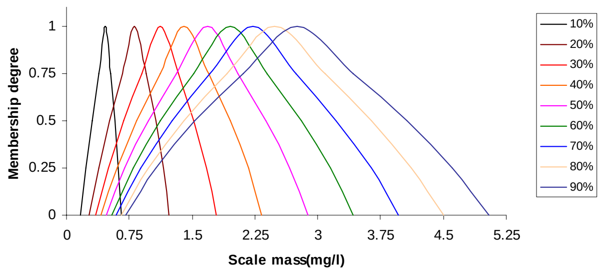

Figure 9. Scale mass of barite obtained from fuzzy calculations at T = 212° F, P= 4000 psi and various seawater in mixture.

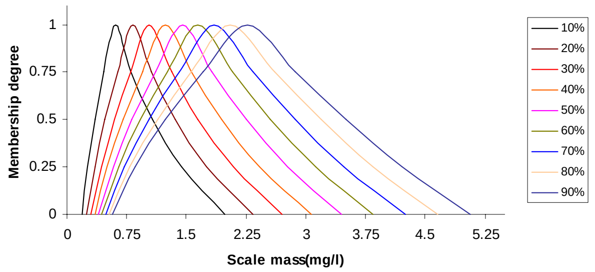

Figure 10. Scale mass of celesite obtained from fuzzy calculations at T = 212° F, P = 4000 psi and various seawater in mixture.

set to a crisp value does seem appropriate for some cases such as control problems, much information is lost by doing this and further work needs to be done on how to use the information available in the solution fuzzy set.

6. CONCLUSIONS

In this study, prediction of sulfates scale mass was performed by using the arithmetic of LR fuzzy numbers. From the results of this study, it can be pointed out that:

1. Parameters such as solubility product constant, free sulfate concentration and amount of barium and strontium sulfate, can effectively be determined by means of fuzzy numbers and their arithmetical operations defined by Zadeh's extension principle.
2. As a case study, the sulfate scale mass, which can be deposited as a result of mixing incompatible waters, calculated by both crisp and fuzzy numbers.
3. The fuzzy number based solution gives more reliable and realistic results. Furthermore, calculation of the maximum scale mass in fuzzy procedure can be helpful in planning the water flooding scheme and selecting suitable chemical inhibitors.

7. ACKNOWLEDGMENTS

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8. NOMENCLATURE

C_i	total concentration of component i, molar
I	ionic strength, molar
K	equilibrium constant
K_s	solubility product
M	molar unit
$[Metal]$	metal concentration, molar

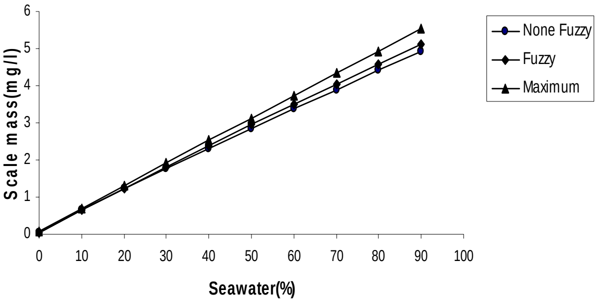

Figure 11. Comparison of the amount of barite predicted by the fuzzy and non-fuzzy methods (T = 77° F, P = 14.5 psi).

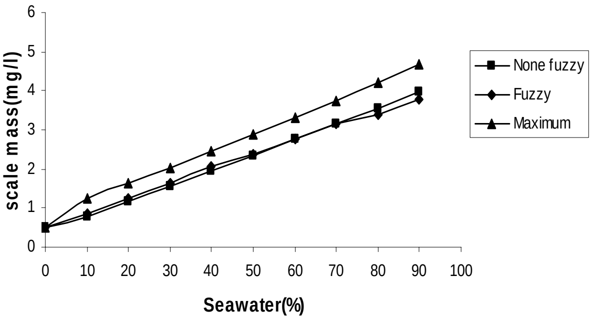

Figure 12. Comparison of the amount of celestite predicted the by fuzzy and non-fuzzy methods (T = 77° F, P = 14.5 psi).

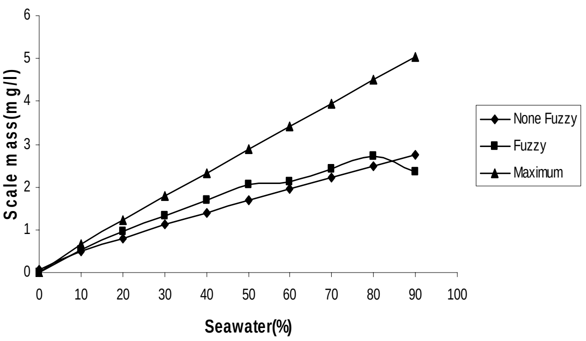

Figure 13. Comparison of the amount of barite predicted by the fuzzy and non-fuzzy methods (T = 212° F, P = 4000 psi).

P	pressure, psi
pK	negative logarithm of equilibrium constant
ppt_{Salt}	amount of precipitated salt, mole/kg H_2O
$\left[SO_4^{\,2-}\right]_{free}$	free concentration of sulfate ion, molar

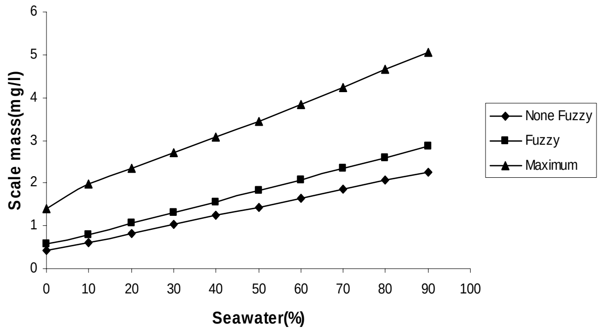

Figure 14. Comparison of the amount of celestite predicted by the fuzzy and non-fuzzy methods (T = 212° F, P = 4000 psi).

T	temperature, Fahrenheit
$\mu(x)$	membership function of parameter x

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