# TRANSMISSION DYNAMICS OF MALARIA IN HUMAN HOST: A NEURO-FUZZY APPROACH

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**Abstract** A neuro-fuzzy approach is proposed for modeling the malaria transmission in a human host. Three dynamic models (i) consistent host preference model (ii) switching behavior model and (iii) increasing preference model, are developed to understand the equilibrium and stability. These models allow us to estimate biting preference by mosquitoes with respect to infection in their host. We have focused on the dynamics of malaria transmission by considering the aspects of non-random host choice. In determining the levels of malarial infection, it is shown that mosquitoes are preferentially attracted to an infected host even when biting does not occur. Numerical results are obtained using both analytical and neruo-fuzzy approach to explore the equilibrium behavior.

**Key Words** Malarial Parasites, Infected Host, Biting Function, Neuro-Fuzzy, Host Preference, Switching-Behavior, Equilibrium

چکیده یک رویکرد فازی – عصبی برای مدلسازی انتقال مالاریا در یک میزبان (بدن) انسانی ارایه شده است. سه مدل پویا شامل (۱) مدل ارجحیت میزبان سازگار، (۲) مدل تغییر رفتاری و (۳) مدل افزایش ارجحیت برای درک تعادل و پایداری توسعه داده شده اند. این مدل ها به ما اجازه میدهند تا بتوانیم ارجحیت گزندگی توسط پشه ها را با توجه به آلودگی در میزبان ایشان تخمین بزنیم. ما با در نظر گرفتن جنبه های انتخاب میزبان غیر تصادفی بر پویایی انتقال مالاریا تمرکز میکنیم. در تعیین سطوح آلودگی مالاریا، نشان داده میشود حتی اگر گزندگی رخ ندهد باز هم پشه ها با ترجیح زیاد و علاقه به طرف میزبان آلوده میروند. نتایج عددی از دو رویکرد تحلیلی و فازی – عصبی برای کشف رفتار تعادلی ارائه میگردد.

### **1. INTRODUCTION**

The dynamics of malaria transmission incorporates non-random feeding behavior of the mosquito. Infected proportions of the human host and mosquito vector populations are the type of dynamic variable. It appears that when the level of host infection is low, preference of infected hosts is low. It allows a stable infected equilibrium. It is considered that there are three types of nonrandom host choices: a consistent preference for an infected host, an increasing preference for an infected host and a switching behavior for mosquito vector. A consistent preference does not

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alter the basic properties of the system, relative to random host choice at all levels of host infection. An increasing preference increases the infection level of the host. In a switching behavior the mosquito switches preference from uninfected to infected host and also increases the infection level of the host. The consistent preference is easier to maintain a stable infection in the host population and increases the equilibrium level of infection in the vector population. The state space is divided into a lower uninfected region and an upper infected region. Various dynamic models have been studied to analyze the predator-prey systems in which the predator changes its preference or

aggregation behavior. This behavior also occurs in some other epidemiological models.

Previous studies demonstrate that non-random host choice by mosquitoes with respect to host infection could have important quantitative and qualitative effects on the dynamics of malaria infection. Aron and May [1] suggested that the mosquito do not feed randomly. Non-random feeding may be expressed at three different stages: attraction and penetration, probing and the location of blood intake. Mosquitoes are preferentially attracted to infected hosts even when biting does not occur. Berding et al. [2] presented the population dynamics of acquired immunity, for the helminthes infections. Mosquito host choice and the epidemiology of malaria was studied by Kingsolver [3]. Anderson [4] illustrated the epidemiology of malaria infection by considering the variable incubation and infectious periods. Born and Dierzk [5] examined the dynamics of parasite population within a dynamic host population. Nasell [6] considered the quasistationary distribution of the Ross malaria model. They noted that, mosquitoes need not directly assess the prevalence of infection in the host population, as required by the increasingpreference and switching models. Hellriegel [7] suggested a model for immune response to malaria with an ecological concept, which is short-term behavior against long term equilibrium.

Gatton et al. [8] developed a model to estimate the duration for which malaria antibody levels in the blood remain high in a closed population. Gravenor et al. [9] estimated the parasite for the population dynamics in cerebral malaria. This estimate can be used to calculate the transmission rate within a region. Herbert and Isham [10] discussed stochastic host-parasite interaction models. The simple nonlinear stochastic model for the evolution of the parasite load of a single host is extended to allow three parasite stages (larval, mature and offspring), and to allow durations of these stages to be non-exponentially distributed. Luchsinger [11] considered the long-term behavior of a model for parasitic infection. The behavior of the deterministic models is analogous to the stochastic ones. Mohtashemi and Levins [12] illustrated mathematical models of the dynamics of infectious diseases and consistently focused on understanding the long-term behavior of the interacting components. Hoshen et al. [13] suggested the mathematical model for the withinhost dynamics of plasmodium falciparum. Deterministic extinction effect of parasites on host populations has been studied by Hwang and Kuang [14]. Lloyd and Jansen [15] discussed the dynamics of epidemics in cases of synchrony in metapopulation models. The linear stability of spatially homogeneous solutions are provided. Experimental studies have shown that parasites can reduce host density and make host population to extinction.

The present study on dynamics of malaria transmission incorporates biting preferences by mosquitoes with respect to infection in their host. It also includes the non-random host choice depending on the interaction of human hosts and mosquito vectors. In order to determine the levels of malarial infection, it is considered that mosquitoes are preferentially attracted to infected hosts. It is noted that the mosquitoes need not directly assess the prevalence of infection in the host population as required by the increasing preference and switching models.

We develop mathematical model of the outcome of a complex feeding process involving habitual choice, attraction behavior of the host, probing and blood location, blood intake, and parasite transmission. The analytical results for transmission dynamics are compared with those obtained by the neuro fuzzy systems (NFS) approach. The NFS approach is one of the key soft computing approaches, which combines artificial neural networks (ANNs) and fuzzy systems (FS). These systems resemble the nervous systems, where the ANNs are analogous to neural cells which are low-level perceptives, responsible for the signal integration, while FS is equivalent to the brain which provides high level reasoning and linguistic abilities. In this study, a special class of neuro-fuzzy systems, i.e. Adaptive Network-based Fuzzy Interface system (ANFIS) is used to identify parameters by applying a supervised learning method. A survey on fusion technology of fuzzy logic and neural networks was done by Takagi [16]. Jang and Sun [17] studied the learning algorithms of adaptive network-based fuzzy inference systems. More detailed descriptions of adaptive neuro fuzzy systems can be found in Cornelius and Leondes [18] and Tettamanzi and

Tomassini [19].

The rest of the paper is organized as follows: In Section 2, we give an introduction to adaptive network-based fuzzy inference systems (ANFIS) and defuzzification related to our study. Mathematical formulation and description of the problem are discussed in Section 3. In Section 4, we illustrate the numerical results which are shown in a graph. The conclusion is drawn in Section 5.

## 2. ADAPTIVE NETWORK-BASED FUZZY INFERENCE SYSTEMS (ANFIS)

ANFIS is a network representation of the Tagaki-Sugeno-Kang (TSK) type fuzzy systems with learning capabilities. TSK is a special type of fuzzy rule-based systems in which the rules are:

IF  $x_1$  is  $A_1$  AND  $x_2$  is  $A_2$  ... AND  $x_n$  is  $A_n$  then  $y = f(x_1, x_2, ..., x_n)$ 

where f is usually a linear combination of the input variables, i.e.

$$f(x_1, x_2, ..., x_n) = w_0 + w_1 x_1 + w_2 x_2 + ... + w_n x_n$$
(1)

Here  $w_0$ ,  $w_1$ ,  $w_2$ , ...,  $w_n$  are real constants which are part of the rule specification. The result of applying the rules of a TSK system is a crisp number, which is computed as the average of the outputs of the single rule weighed by the degree of truth of their antecedents. This is a particular case of that which has been weighed by average method of defuzzification.

**2.1 Defuzzification** This is a technique for recalled outputs. The recall fit-vector output B equals a weighted sum of the individual recall vectors  $B'_{k}$ :

$$B = \sum_{k=1}^{m} w_k B'_k$$
(2)

where  $w_k$  is the non negative weight, which

summarizes the strength of the  $k^{th}$  fuzzy associative memories (FAM) rule ( $A_k$ ,  $B_k$ ). Generally we choose  $w_1 = w_2 = \ldots = w_m = 1$  as a default.

The natural alternative is the fuzzy centroid defuzzification scheme. We directly compute the real-valued output as a normalized convex combination of fit values, the fuzzy centroid B' of fit-vector B with respect to output space  $Y = (y_1, y_2, ..., y_p)$ 

$$B' = \frac{\sum_{j=1}^{p} y_{j} m_{B}(y_{j})}{\sum_{j=1}^{p} m_{B}(y_{j})}$$
(3)

where  $m_B(y_i)$  is a membership function, j = 1,

2, ..., p. The fuzzy centroid is unique and uses all the information in the output distribution B. The schematic representation of fuzzy associative memory rule has been done in Figure 1.

#### **3. MODEL DESCRIPTION**

A more complete model includes a mechanical description of each stage of the feeding process. There is only a stable equilibrium at any time and a single condition governs whether the stable equilibrium is zero (uninfected) or non-zero (infected). For the purpose of model formulation, let us consider the size of the human population and the size of the female mosquito population as N and M respectively; x and y are the size of the human host population and mosquito vector population, respectively. Let the number of female mosquito per human host be denoted by m = (M/N), k the population of infected biting on human hosts that produce an infection; r the per capita rate of recovery for human hosts,  $\mu$  the per capita morality rate for mosquitoes. The Ross Macdonald (R-M) model (cf. Kingsolver, 1987) for the dynamics of malaria infection is as follows:

$$\frac{dx}{dt} = k y \beta_{u} (x) \frac{M}{N} - r x$$
(4)

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Figure 1. Schematic diagram for fuzzy associative memories (FAM) rule.

$$\frac{dy}{dt} = \beta_{i}(x) (1-y) - \mu y$$
(5)

where  $\beta_i(x)$  and  $\beta_u(x)$  are the functions describing the rates of bites per female mosquito on the infected hosts and uninfected hosts, respectively. Equations 4 and 5 are the pair of ordinary differential Equations describing the time course of x and y.  $\beta$  is the total rate of biting per unit of time, so that

$$\beta = \beta_{i}(x) + \beta_{u}(x) \tag{6}$$

Then the biting rate functions for this model are given by

$$\beta_{i}(x) = \beta x \tag{7}$$

$$\beta_{u}(x) = \beta(1-x) \tag{8}$$

In this model bites are distributed between infected and uninfected hosts. The biting rate functions  $\beta_i(x)$  and  $\beta_u(x)$  describe non-random host choice by the mosquito with respect to host infections. The main conclusion from the R-M model is the identification of a parameter (R) that governs the model's behavior, where R = km  $\beta^2/\mu$  r > 1, and R is the net reproductive rate of the parasites.

An infection will persist and lead to a stable equilibrium at which an infection is maintained in

both the host and vector populations. To examine the dynamics of malaria infection, we consider three types of non-random host choices:

- (i) A consistent preference for an infected host
- (ii) A increasing preference for an infected host
- (iii) A switching behavior for mosquito vector

**3.1 Consistent Preference Model for Infected Host** This model describes the infection at all levels of hosts. For this model, P is the preference for infected hosts, then from Kingsolver (1987) the biting rate functions for infected hosts, are as follows:

$$\beta_{i}(x) = \beta \left[ \frac{Px}{1 + (P-1)x} \right]$$
(9)

$$\beta_{u}(x) = \beta \left[ 1 - \frac{Px}{1 + (P-1)x} \right]$$
(10)

Alternative biting functions can be defined as

$$\beta_{i}(x) = \beta \left(1 - e^{-c x^{\alpha}}\right)$$
(11)

$$\beta_{u}(x) = \beta - \beta_{i}(x) = \beta e^{-cx^{\alpha}}$$
(12)

where  $\alpha$  is the non-linearity parameter corresponding to infected host; c is the positive

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constant that reflects the intensity of the preference. When c increases, the biting rate of infected the host increases for all values of x. The parameter c must be chosen so that  $\beta_i(x)$  approaches  $\beta$  as x approaches 1.

Now, Equations 4 and 5 of the R-M model can be modified as

$$\frac{dx}{dt} = \beta k m y e^{-cx^{\alpha}} - rx$$
(13)

$$\frac{dy}{dt} = \beta \left( 1 - e^{-cx} \alpha \right) (1 - y) - \mu y$$
(14)

Here, we consider the y null-cline, along which  $\frac{dy}{dt} = 0$ , and the x null-cline, along which  $\frac{dx}{dt} = 0$ . Then we get **x** null-cline and y null-cline as follows:

$$f_{x}(x) = \frac{r x}{k m \beta e^{-c x} \alpha}$$
(15)

$$f_{y}(x) = \frac{\beta \left(1 - e^{-c x^{\alpha}}\right)}{\mu + \beta \left(1 - e^{-c x^{\alpha}}\right)}$$
(16)

From Equations 15 and 16, we note that  $f_x(x)$  and  $f_y(x)$  both are non-negative and increasing functions of x. This can be verified from Figures 2-5.  $f_x(x)$  is convex as  $d^2(f_x) / dx^2 > 0$ , whereas  $f_y(x)$  is concave as  $d^2(f_y) / dx^2 < 0$ . The point ( $\underline{x} = 0, \underline{y} = 0$ ) is always an equilibrium point. The condition for a second, nonzero equilibrium is given by  $R_1 = ckm\beta^2/\mu r > 1$ . The uninfected equilibrium (0,0) is stable if  $R_1 < 1$ . This can firms the results of Kingsolver [3]. Thus the uninfected equilibrium is stable when it is the

only equilibrium point and unstable when a second, nonzero equilibrium exists.

**3.2 Switching Behavior Model for Mosquito Vector** The following biting rate functions for this model are taken from Kingsolver [3] which are stated below:

$$\beta_{i}(x) = \beta \left[ \frac{P_{1}x}{1 + (P_{1} - 1)} (1 - x) + \frac{P_{2}x}{1 + (P_{2} - 1)x} \right]$$
(17)

$$\beta_{1}(x) = \beta \left[ 1 - \frac{P_{1}x}{1 + (P_{1} - 1)} (1 - x) + \frac{P_{2}x}{1 + (P_{2} - 1)x} \right]$$
(18)

Alternative biting functions that exhibit switching behavior are:

$$\beta_{i}(\mathbf{x}) = \frac{\beta \left(1 - e^{-c \mathbf{x}^{\alpha}}\right)}{1 + a e^{-c \mathbf{x}^{\alpha}}}$$
(19)

and

$$\beta_{u}(x) = \beta - \beta_{i}(x) = \frac{(a+1)\beta e^{-cx^{\alpha}}}{1+ae^{-cx^{\alpha}}}$$
 (20)

Here a, c, and  $\beta$  are all positive constants. The biting functions given by Equations 19 and 20 are called switching biting functions, and have the same effect as Equations 11 and 12. Here  $\beta_i(1) = \beta$  only when  $c = \infty$ .

Equation 17 has the desirable property that  $\beta_i(1) = \beta$ , for any realistic values of  $P_1$  and  $P_2$ . For this case of switching behavior, we observe that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{(a+1)\,\mathrm{k}\,\mathrm{m}\beta\,\mathrm{y}\,\mathrm{e}^{-c\,\,\mathrm{x}\,\,\alpha}}{1+a\,\mathrm{e}^{-c\,\,\mathrm{x}\,\,\alpha}} - \mathrm{r}\,\,\mathrm{x} \tag{21}$$

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$$\frac{dy}{dt} = \frac{\beta \left(1 - e^{-c x \alpha}\right) (1 - y)}{1 + a e^{-c x \alpha}} - \mu y$$
(22)

In the case of biting functions given in 21 and 22, we can find the x null-cline  $\left[f_x(x)\right]$  and y null-cline  $\left[f_y(x)\right]$  as follows:

$$f_{x}(x) = \frac{r x \left(1 + a e^{-c x^{\alpha}}\right)}{\beta k m (a+1) e^{-c x^{\alpha}}}$$
(23)

$$f_{y}(x) = \frac{\beta \left(1 - e^{-c x \alpha}\right)}{\mu \left(1 + a e^{-c x \alpha}\right) + \beta \left(1 - e^{-c x \alpha}\right)}$$
(24)

For this switching model, we note from Equations 23 and 24 that both  $f_x(x)$  and  $f_y(x)$  are non negative, increasing functions of x;  $f_x(x)$  is convex as  $d^2(f_x)/dx^2 > 0$ ;  $f_y(x)$  is sigmoid when  $\beta_i(x)$  approaches 1 as x approaches 1 and c + 2  $\beta < 2$  a  $\mu$ , the point ( $\underline{x} = 0, \underline{y} = 0$ ) is always an equilibrium point, called the zero or uninfected equilibrium. These conditions are verified from the Figures 6-10. The conditions for the existence of nonzero (infected) equilibrium point is  $R_2 = c \text{ km}\beta^2/\mu \text{ r}$  (a+1) > 1. The uninfected equilibrium is stable if  $R_2 < 1$ , i.e. it is stable either when it is the only equilibrium point or when there are two nonzero equilibria.

**3.3 Increasing Preference Model for Infected Host** Biting rate functions for this model are taken as follows:

$$\beta_{i}(x) = \beta \left[ x(1-x) + \frac{P x^{2}}{1+(P-1)x} \right]$$
(25)

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$$\beta_{u}(x) = \beta \left[ 1 - x (1 - x) - \frac{P x^{2}}{1 + (P - 1)x} \right]$$
(26)

Alternative biting functions that exhibit increasing preference behavior are

$$\beta_{i}(x) = \beta x \left[ 2 - x^{2} - e^{-c x^{\alpha}} \right]$$
(27)

$$\beta_{u}(x) = \beta \left[ 1 - 2x + x^{2} + x e^{-c x^{\alpha}} \right]$$
 (28)

The above Equations 27 and 28 demonstrate the same affect as Equations 11 and 12. Here  $\beta_i(1) = \beta$  only when  $c = \infty$ . For P = 1, the model is identical to the R-M model. For this case, we observe that the Equations 4 and 5 of R-M model can be expressed as

$$\frac{dx}{dt} = \beta k m y (1 - 2 x + x^{2} + e^{-c x^{\alpha}}) - r x$$
(29)

$$\frac{\mathrm{d}\,\mathrm{y}}{\mathrm{d}\,\mathrm{t}} = \beta \,\mathrm{x} \left( 2 - \mathrm{x}^2 - \mathrm{e}^{-\,\mathrm{c}\,\mathrm{x}\,\alpha} \right) \,(1 - \mathrm{y}) - \mu \,\mathrm{y} \qquad (30)$$

Here, we get  $\mathbf{x}$  null-cline and y null-cline as follows:

$$f_{x}(x) = \frac{rx}{k \, m\beta \, (1 - 2x + x^{2} + e^{-c \, x^{\alpha}})}$$
(31)  
$$f_{y}(x) = \frac{\beta \, x \left(2 - x^{2} - e^{-c \, x^{\alpha}}\right)}{\mu + \beta \, x \left(2 - x^{2} - e^{-c \, x^{\alpha}}\right)}$$
(32)

From this model, we note that,  $f_x(x)$  and  $f_y(x)$  both are non negative, increasing functions of x; the point ( $\underline{x} = 0$ ,  $\underline{y} = 0$ ) always exists and is an equilibrium point. The condition for being locally stable is given by  $R = km\beta^2/\mu r < 1$ . The uninfected equilibrium (0,0) is stable if R < 1.

Thus the uninfected equilibrium is stable when it is the only equilibrium point and unstable when a second, nonzero equilibrium exists.

Here we have considered the host choice behavior in which preference depends on the relative configuration of the population of infected and uninfected hosts. Uninfected hosts are preferred at relatively low levels of infected hosts whereas infected hosts are preferred at relatively high levels. The biting rate functions chosen examine the consequences of switching behavior for the dynamics of infection and also for the dynamics of non-random host choice by the mosquito with respect to host infection. There are several important differences between the consistent preference and switching models. The conditions for instability of the uninfected equilibrium for the switching model are different from those for the preference model, under the equivalent situations  $R_2 = R_1$  (a+1). The most important result is that of multiple stable equilibrium that can occur for both the increasing preference and the switching-behavior models.

## **4. NUMERICAL RESULTS**

In order to validate the analytical results, a numerical experiment is performed. To explore the effect of various parameters, the numerical results are exhibited graphically. A numerical program is developed through MATLAB 6.0 software and run on Pentium III. We display the Phase-plane of the infection proportion of the vector population as y-axis (vertical), and the host population as x-axis (horizontal) for the consistent preference model. The solid line presents x null-cline and y null-cline.

For the validity of the analytical results, we employed the adaptive network-based fuzzy inference systems (ANFIS). The outputs of the analytical method are the data for the fuzzy system. For the analytical method, secondary data has been taken into consideration from Kingsolver [3].

The neuro-fuzzy approach is employed for computing numerical results for all models. The ANFIS networks (for approximating the nonzero



**Figure 2**. Phase-plan plot of the infected population of the vector population vs. infected host for consistent host preference model at (a) c = 3.0, (b) c = 5.0.

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**Figure 3**. Phase-plan plot of the infected population of the vector population vs. infected host for consistent host preference model at (a)  $\mu = 10$ , (b)  $\mu = 50$ .

equilibrium point), are denoted by a dashed line for x-null cline and y-null cline. The different parameters are treated as the linguistic variables in the context of fuzzy systems. When we build the respective ANFIS networks, these parameters are taken as input values. The Gaussian function is used for describing the membership functions for various input parameters. The numerical results obtained for various parameters are also compared with neuro-fuzzy results.

Figures 2-5 depict the phase plane plot between the host and vector populations for consistent host preference model acceding to various parameters. Figure 2(a-b) illustrates the x null-cline and y nullcline (obtained analytically) (ANFIS) by solid lines (dashed line) for different values of preference (c). For fixed parameter values  $\beta = 10$ ,  $\mu = 3.0$ ,  $\alpha =$ 0.9, and Km/r = 1, we note that, as preferences increase, a stable nonzero (infected) equilibrium point appears. In Figure 3(a-b), we examine equilibrium points for different values of  $\mu$  and fixed parameter values  $\beta = 10$ , c = 3.0,  $\alpha = 0.9$ , and Km/r = 1. In this case, as preferences increase, the equilibrium level of infection in the host population decreases. For fixed parameter values  $\mu = 10$ , c = 3.0,  $\alpha = 0.9$ , and Km/r = 1, the equilibrium points for different values of  $\beta$  are displayed in Figure 4(a-b). It is noted that, as the biting rate increases, the equilibrium levels of infection in the host and vector population increase. Figure 5(a-b) exhibits the equilibrium points for different values of  $\alpha$  for fixed parameters  $\mu = 10$ , c = 3.0,  $\beta = 10$ , and Km/r = 1.

In Figures 6-10, we demonstrate the effect of parameters c,  $\mu$ ,  $\beta$ ,  $\alpha$ , a, respectively by fixing Km/r = 1 for the phase plane plot in case of the switching behavior model. Figure 6(a-b) illustrates the x null-cline (obtained analytically) (ANFIS) through a solid line (dashed line) for different values of preference (c). For  $\beta = 10$ ,  $\mu = 100$ ,  $\alpha = 0.9$ , a = 10 and Km/r = 1, we note that as preferences increase, a stable nonzero (infected) equilibrium point appears.

In Figure 7(a-b), for different values of  $\mu$  and  $\beta$  = 10, c = 15,  $\alpha$  = 0.9, a = 10 and Km/r = 1, we exhibit the equilibrium points. In this case, as preferences increase, the equilibrium level of

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**Figure 4**. Phase-plan plot of the infected population of the vector population vs. infected host for consistent host preference model at (a)  $\beta = 10$ , (b)  $\beta = 30$ .



**Figure 5**. Phase-plan plot of the infected population of the vector population vs. infected host for consistent host preference model at (a)  $\alpha = 0.5$ , (b)  $\alpha = 0.7$ .

infection in the host population decreases. Figure 8(a-b) shows equilibrium points for different

values of  $\beta$  values of  $\beta$  and  $\mu = 100$ , c = 15,  $\alpha = 0.9$ , a = 10 and Km/r = 1. It is noted that, as the

biting rate increases, the equilibrium levels of infection in the host and vector population increase. Figure 9(a-b) exhibits the equilibrium points for different values of  $\alpha$ , and  $\beta = 10$ , c = 15, a = 10,  $\mu = 100$  and Km/r = 1. It is seen that when  $\alpha$  increases, the switch point of preference increases. The equilibrium points for different values of a, and fixed parameters  $\beta = 10$ , c = 15,  $\alpha = 0.9$ ,  $\mu = 100$  and Km/r = 1 are indicated in Figure 10(a-b). It is seen that, as constant a increases, the switch point of preference increases, too.

In conclusion, the numerical results using ANFIS are quit close to the results, which are obtained analytically. We observe from the phase plane plot that the biting rate functions are a phenomenological description of the outcome of the feeding process which involves the attraction, defensive behavior of the host, probing and blood location, blood intake, and parasite transmission. The consistent preference model or the switching behavior model is a result of nonrandom host distribution and mosquito vector aggregation. But the basic property of the system does not change in the consistent preference model. Both models can maintain the stable infection in certain circumstances. These models are the mechanistic description of each stage of the vector-hostparasite interaction.

#### **5. SENSITIVITY ANALYSIS**

In this section, we present the sensitivity analysis between the host and vector populations for both the consistent preference and switching behavior models. Figures 2-5 are for phase plan plot between the infected vector population and infected host population for consistent the preference model whereas Figures 6-10 are for the phase plan plot between the infected vector population and infected host population of the switching behavior model. The phase plane plot between the infected population of the vector population and infected host population for the consistent preference model is presented in Figure 2(a) at c = 3.0 and 2(b) at c = 5.0. It is noted that the equilibrium point increases as we increase the



**Figure 6**. Phase-plan plot of the infected population of the vector population vs. infected host for switching behavior model at (a) c = 9.0, (b) c = 11.0.

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Figure 7. Phase-plan plot of the infected population of the vector population vs. infected host for switching behaviour model at (a)  $\mu = 100$ , (b)  $\mu = 120$ .



**Figure 8**. Phase-plan plot of the infected population of the vector population vs. infected host for switching behaviour model at (a)  $\beta = 20$ , (b)  $\beta = 40$ .

value of c. Figure 3(a) at  $\mu = 10$  and 3(b) at  $\mu = 50$  stands for the phase plan plot of the infected

individuals between the vector population and host population. From these figures, it is clear that the

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Figure 9. Phase-plan plot of the infected population of the vector population vs. infected host for switching behaviour model at (a)  $\alpha = 0.5$ , (b)  $\alpha = 0.7$ .



Figure 10. Phase-plan plot of the infected population of the vector population vs. infected host for switching behaviour model at (a) a = 10, (b) a = 30.

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equilibrium level of infection increases as the equilibrium level of infection increases as the value of  $\mu$  is increased. Figure 4(a) at  $\beta = 10$  and 4(b) at  $\beta = 30$  depicts that the labels of equilibrium increase as we increase the value of  $\beta$ . Figure 5(a) at  $\alpha = 0.5$  and 5(b) at  $\alpha = 0.7$  shows the equilibrium point for the infected population between the host and vector populations. It is noted that the label of the equilibrium point decreases slightly when we increase the value of  $\alpha$ .

Figure 6 is for the phase plan plot between the infected vector population and infected host population at c = 9.0 and c = 11.0, respectively. From this figure, it is noted that the equilibrium label increases as we increase the value of c. Figure 7(a) at  $\mu = 100$  and 7(b) at  $\mu = 120$ shows that as we increase the value of  $\mu$ , the equilibrium label decreases. Figure 8 is for the phase plan plot between infected vector population and infected host population for  $\beta =$ 20 and  $\beta = 40$  respectively. It is noted that the point of equilibrium increases when we increase the value of  $\beta$ . Form Figure 9(a) at  $\alpha = 0.5$  and 9(b) at  $\alpha = 0.7$ , it is observed that the equilibrium point does not change when we increase the value of  $\alpha$ . The effect of a is shown in Figure 10(a) at a = 10 and 10(b) at a = 30 for phase plan plot between the infected population of vector and host populations. It noted that the equilibrium point decreases when we increase the value of a, in all figures. It is seen that the ANFIS results are quite close to the results which are obtained by the analytical method. So, we can say that our results are more accurate in comparison to other existing results.

# 6. CONCLUSION

An increasing preference biting function could result from a preferential attraction to infected hosts coupled with a "giving-up time" whereby the mosquito would bite the next host encountered. The qualitative and quantitative features of equilibrium and stability for the consistent host preference and the switching model have been studied. The purpose of the present investigation is to introduce biting rate functions,  $\beta_i(x)$  and  $\beta_u(x)$  in terms of host infection. These models are useful in describing the important aspects of the transmission dynamics of malaria by taking the concepts of non-random host choice, which affect qualitative aspects of the interaction of human hosts and mosquito vectors in determining the levels of malaria infection. Both the random choice and consistent-preference models predict either a stable uninfected state or a stable persistence of the infection.

The important observation that can be made from our study is that the increasing-preference and switching models lead to qualitatively identical results. These results demonstrate that nonrandom host choice by mosquitoes with respect to host infection which could have important quantitative effects on the dynamics of malaria infection. Both the random choice and consistence-preference models predict either a stable uninfected state or a stable persistence of the infection for these models. The increasing consistent host preference makes it easier to maintain a stable infection, relative to the random choice model. However, increasing consistent host preference can either increase or decrease the equilibrium level of infection in the host population. Therefore increment in the strength of the consistent performance increases the equilibrium level of infection of the host population when the equilibrium value is small, but decreases the equilibrium level when the equilibrium value is greater.

The results obtained by the neuro-fuzzy techniques are at par with the numerical results obtained by analytical formulae. We conclude that artificial neuro-fuzzy controllers can be easily developed to provide an easy and fast solution technique for the concerned study.

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