
TECHNICAL NOTE

DETERMINATION OF PRINCIPAL PERMEABILITIES IN TWO-DIMENSIONAL FISSURED ROCKS USING PERMEABILITY CIRCLE

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Abstract Among the many parameters affecting the design of hydraulic structures on fissured rocks are orientation of principal permeability axes and degree of anisotropy of fissured rocks. Geological explorations rarely provide this type of information. If at all the information is available, methods employed are computationally cumbersome and inefficient. A new concept of simple equations and geometrical method termed as Permeability Circle similar to Mohr's circle of stresses has been developed for determination of principal permeability and their orientation from the known characteristics of two-dimensional fissured rock mass.

Key Words Secondary Permeability, Principal Permeability, Geological explorations, Degree of Anisotropy, Permeability Circle

چکیده جهت محورهای اصلی نفوذ پذیری و درجه ناپیکنواختی یک تخته سنگ، پارامترهای مهمی هستند که بر طرح ساختارهای هیدرولیک مربوط به صخره های ترک خورده تاثیر می گذارند. اکتشافات زمین شناسی بندرت این نوع اطلاعات را به دست می دهند. حتی اگر این اطلاعات موجود باشد هم روش های بکار گرفته شده از نظر محاسباتی طاقت فرسا و ناکار آمد هستند. در این تحقیق ایده جدیدی بنام "دایره نفوذ پذیری" توسعه یافته که با استفاده از معادلات ساده و روش هندسی مشابه دایره مور، از روی مشخصات جرم صخره ترک خورده دو بعدی مقدار و جهت نفوذ پذیری اصلی را معین می کند.

1. INTRODUCTION

The uplift force on the base of a hydraulic structure and flow nets in the flow field depend on the orientation of principal permeability axes and degree of anisotropy of fissured rocks, Verma [1]. Hence it is very important to know the orientation of principal axes and degree of anisotropy of fissured rocks. Very cumbersome processes are required to measure directly these permeability parameters in the field.

The present paper suggests simple equations and a rational geometrical approach in the form of Permeability Circle for determination of directions and magnitudes of principal permeability using opening, spacing and orientation of fissures. These characteristics of fissures can be obtained from the

geological explorations.

2. PERMEABILITY OF ROCK MASSES

Majority of the studies like Scheidegger [2], Sharp [3] and Sharp and Maini [4] have stated that the flow through fissures is in the laminar regime and is governed by Darcy's law. On the basis of analytical solution of the Navier-Stoke's equation for the steady state flow between the two parallel plates, the permeability through an individual fissure (P') with opening of fissure (e) can be obtained as

$$P' = \frac{\gamma_w}{\mu} \frac{e^2}{12} \quad (1a)$$

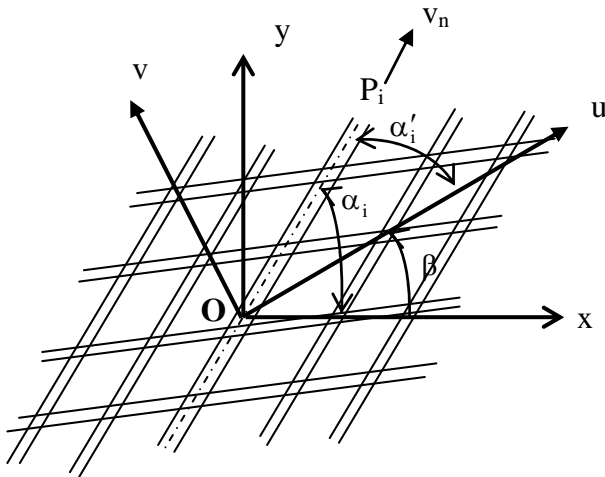


Figure 1. A set of parallel fissures and rotation of co-ordinate system.

where γ_w and μ are specific weight and dynamic viscosity of the flowing fluid respectively. Fissures usually form families or set each having approximately the same orientation and characteristics of spacing, aperture, infilling material and roughness. For a set of fissures in a rock mass with spacing (d) between the fissures and an opening for each fissure as (e), effective permeability through the set of fissures (P) is

$$P = P' \frac{e}{d} \quad (1b)$$

Other than Equations 1a and 1b there are also other experimental, theoretical and numerical methods available to estimate the permeability, see for example Snow [5], Rocha and Francise [6], Hudson and Pointe [7], Long, Remer, Wilson and Witherspoon [8] and Dienes [9].

3. COMPONENTS OF PERMEABILITY TENSOR

In an anisotropic medium, except in the directions of principal permeability, the direction of the gradient and the velocity do not coincide. Thus the permeability in any direction can be defined as directional permeability at a given

point, Scheidegger [2]. Consider a rock mass intersected by 'n' number of sets of fissures whose orientation and characteristics are known from the geological explorations. Let P_i be the permeability of i^{th} set of parallel fissures, which makes an angle of α_i with x direction as shown in Figure 1. Here P_i can be found using Equations 1a and 1b. If a unit hydraulic gradient is applied in x- direction, the velocity, v_x , in x- direction will be equal to the permeability in that direction, k_{xx} as per Darcy's law.

$$v_x = k_{xx} \quad (2a)$$

Similarly if v_n and $\frac{\partial h}{\partial n}$ are the flow velocity through i^{th} set of fissure and hydraulic gradient in the direction of fissure respectively (see Figure 1) then,

$$v_n = P_i \frac{\partial h}{\partial n} \quad (2b)$$

Also from Figure 1

$$v_x = v_n \cos \alpha_i \quad (2c)$$

and for unit hydraulic gradient along x-direction

$$\frac{\partial h}{\partial n} = \cos i \quad (2d)$$

The use of Equations 2a, 2b, 2c and 2d and their simplification lead to $k_{xx} = P_i \cos^2 \alpha_i$. Thus for n set of fissures, permeability in x-direction is

$$k_{xx} = \sum_{i=1}^n P_i \cos^2 \alpha_i \quad (3a)$$

Similarly, if k_{yy} is defined in y-direction by applying a unit hydraulic gradient in y-direction, the following relationship can be obtained as explained for k_{xx} .

$$k_{yy} = \sum_{i=1}^n P_i \sin^2 \alpha_i \quad (3b)$$

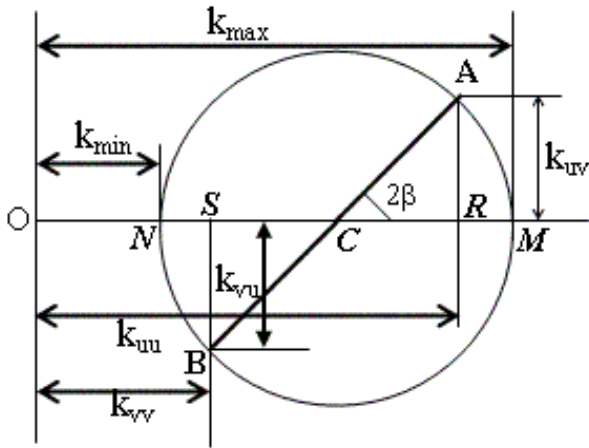


Figure 2. Permeability Circle.

When the unit hydraulic gradient is applied in x -direction its contribution to velocity in y -direction is defined as k_{xy} , and if unit hydraulic gradient is applied in y -direction, its contribution to velocity in x direction is defined as k_{yx} . Consequently the relationship for these can be obtained as below

$$k_{xy} = k_{yx} = \sum_{i=1}^n P_i \cos \alpha_i \sin \alpha_i \quad (3c)$$

The permeability tensor, k_{ij} (Oda [10] and Oda and Hatsuyama [11]) is written as

$$k_{ij} = \begin{vmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{vmatrix} \quad (4)$$

4. ROTATION OF CO-ORDINATE AXES

Consider a new set of co-ordinate axes u, v as shown in Figure1. The co-ordinate system x, y is rotated through an angle β about the origin O such that $\alpha'_i = (\alpha_i - \beta)$. Now the new permeability tensor, k'_{ij} can be written as

$$k'_{ij} = \begin{vmatrix} k_{uu} & k_{uv} \\ k_{vu} & k_{vv} \end{vmatrix} \quad (5)$$

having components as

$$k_{uu} = \sum_{i=1}^n P_i \cos^2 \alpha'_i \quad (6a)$$

$$k_{vv} = \sum_{i=1}^n P_i \sin^2 \alpha'_i \quad (6b)$$

$$k_{uv} = k_{vu} = \sum_{i=1}^n P_i \cos \alpha'_i \sin \alpha'_i \quad (6c)$$

Substituting the value of α'_i in Equation 6a and simplifying, we get

$$k_{uu} = \frac{k_{xx} + k_{yy}}{2} + \frac{k_{xx} - k_{yy}}{2} \cos 2\beta + k_{xy} \sin 2\beta \quad (7a)$$

Similarly rearranging Equations 6b and 6c, one gets

$$k_{vv} = \frac{k_{xx} + k_{yy}}{2} - \frac{k_{xx} - k_{yy}}{2} \cos 2\beta - k_{xy} \sin 2\beta \quad (7b)$$

$$k_{uv} = k_{vu} = -\frac{k_{xx} - k_{yy}}{2} \sin 2\beta + k_{xy} \cos 2\beta \quad (7c)$$

Equations 7a, 7b and 7c are the parametric equations of a circle, which means that if we choose a set of rectangular axes and plot a point $A(k_{uu}, k_{uv})$ and $B(k_{vv}, k_{vu})$ as shown in Figure 2, for any given value of the parameter β , all the points thus obtained will lie on a circle.

To establish this property β can be eliminated from Equations 7a, 7b and 7c and rearranged to Equations 8a and 8b

$$|(k_{uu} - k_{av})|^2 + |(k_{uv} - 0)|^2 = R^2 \quad (8a)$$

or,

$$|(k_{vv} - k_{av})|^2 + |(k_{vu} - 0)|^2 = R^2 \quad (8b)$$

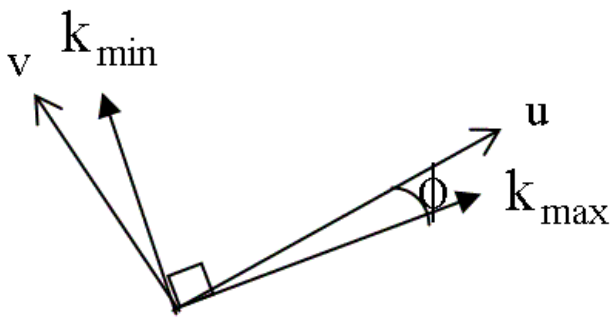


Figure 3. Directional Permeability.

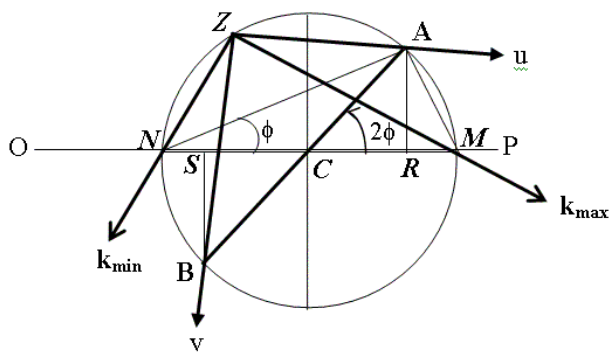


Figure 4. Construction of Permeability Circle.

where

$$k_{av} = \left| \frac{k_{xx} + k_{yy}}{2} \right|$$

and

$$R = \sqrt{\left| \frac{k_{xx} - k_{yy}}{2} \right|^2 + |k_{xy}|^2} .$$

Equation 8a or Equation 8b is an equation of a circle of radius R centered at point $C (k_{av}, 0)$ as shown in Figure 2. The point 'M' corresponds to maximum value of permeability k_{max} and point 'N' to minimum value of permeability k_{min} . The degree of anisotropy for the medium is defined as the ratio k_{max}/k_{min} .

Let β_m be the particular values of the angle β which corresponds to the points M and N of Figure 2, which may be obtained by setting k_{uv} or $k_{vu} = 0$ in Equation 7c, hence

$$k_{uv} = k_{vu} = -\frac{k_{xx} - k_{yy}}{2} \sin 2\beta_m + k_{xy} \cos 2\beta_m = 0$$

$$\tan 2\beta_m = \frac{2k_{xy}}{k_{xx} - k_{yy}}$$

(9)

Equation 9 defines two values of $2\beta_m$ which are 180° apart. These values of β_m are 90° apart. Thus the two axes are perpendicular to each other. Depending on their magnitudes, these axes are called major and minor principal permeability axes of the medium. The corresponding values are k_{max} and k_{min} respectively as given by Equation 10.

$$k_{max, min} = \frac{k_{xx} + k_{yy}}{2} \pm \sqrt{\left| \frac{k_{xx} - k_{yy}}{2} \right|^2 + |k_{xy}|^2}$$

(10)

Therefore if orientation and characteristics of fissured rock mass are known, the direction and magnitudes of principal permeability can be calculated using Equations 9 and 10.

5. CONSTRUCTION OF PERMEABILITY CIRCLE

If u and v axes are at angle of ϕ with principal directions k_{max} and k_{min} as shown in Figure 3, the components of permeability tensor k_{uu} , k_{vv} , k_{uv} and k_{vu} can be written using Equations 6a, 6b and 6c as

$$k_{uu} = k_{max} \cos^2 \phi + k_{min} \sin^2 \phi \quad (11a)$$

$$k_{vv} = k_{max} \sin^2 \phi + k_{min} \cos^2 \phi \quad (11b)$$

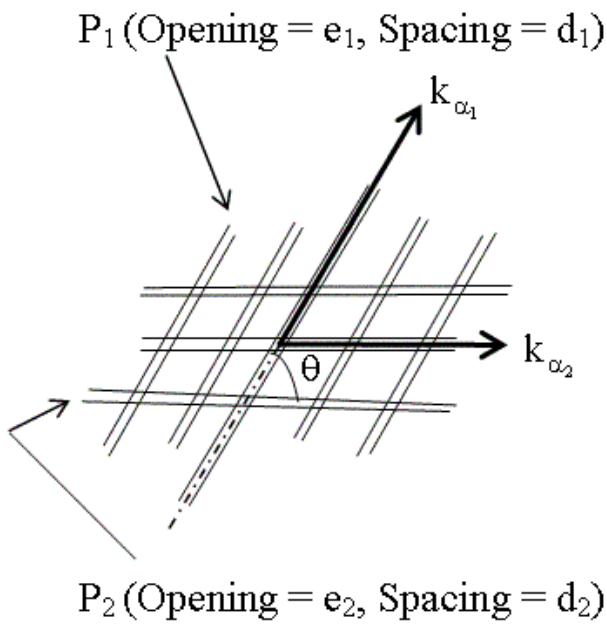


Figure 5. Directional Permeability with Direction of P_1 .

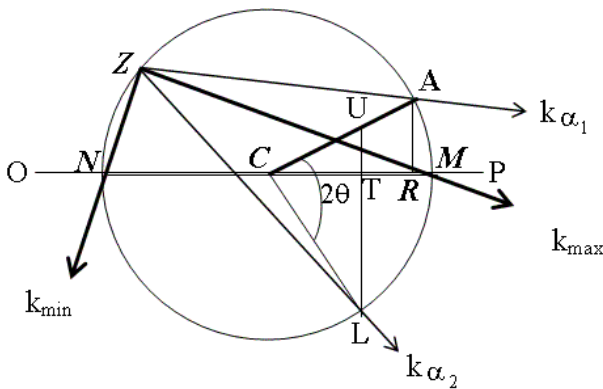


Figure 6. Determination of k_{max} and k_{min} .

$$k_{uv} = k_{vu} = -(k_{max} - k_{min}) \sin \phi \cdot \cos \phi \quad (11c)$$

A Permeability Circle can be constructed similar to Mohr's circle of stresses as shown in Figure 4 as follows: Draw a line OP. Locate the center of Permeability Circle, C at a distance equal to $\frac{k_{max} + k_{min}}{2}$ from O. Draw a circle taking C as a center and the difference of $k_{max} - k_{min}$, MN as diameter. Choose a point A on the circle such that

line AB makes an angle of 2ϕ at C with line OP. If a perpendicular is drawn from A on the line OP as AR, the value of OR and AR will be equal to k_{uu} and k_{uv} respectively. Similarly if a perpendicular line is drawn from B on OP, OS and SB will correspond to k_{vv} and k_{vu} respectively.

6. APPLICATION OF PERMEABILITY CIRCLE

To illustrate the applicability of Permeability Circle a case of known Permeability (k_{α_1} and k_{α_2} , where $k_{\alpha_1} > k_{\alpha_2}$) of two sets of fissures intersecting at an angle are presented to determine principal permeability and their orientation.

Let the two families of fissures of a rock mass (with negligible primary permeability) intersect at an acute angle of θ as shown in Figure 5. The secondary permeability of the fissures sets are P_1 and P_2 respectively which can be found using Equations 1a, 1b, 3a, 3b and 3c.

$$k_{\alpha_1} = P_1 + P_2 \cos^2 \theta$$

$$k_{\alpha_2} = P_2 + P_1 \cos^2 \theta$$

Figure 6 illustrates the requirements to be met for the construction of Permeability Circle in this case. Let OR and OT represent the magnitudes of k_{α_1} and k_{α_2} respectively.

With C as center, a circle is drawn using some radius. A point A is chosen on the circle. To locate a point U on the line, CA is divided such that

$$\frac{CA}{CU} = \frac{CR}{CT} = \frac{k_{\alpha_1} - (P_1 + P_2)/2}{k_{\alpha_2} - (P_1 + P_2)/2}$$

The Point U is joined to L to get a line UL. This circle should intersect at points A and L on the perpendiculars drawn through points R and T, respectively so that the angle LCA is equal to 2θ . A line is drawn perpendicular to UL and passing through C. On this line a point O is marked so that the length $OC = \frac{P_1 + P_2}{2}$. The scale for this is

obtained by the value of $TR = k_{\alpha_1} - k_{\alpha_2}$. Lengths OM and ON are measured to obtain the principal permeability k_{max} and k_{min} . A point Z is marked on the circle as pole with respect to direction of k_{α_1} and k_{α_2} . Since pole is known on the circle the direction of principal permeability can also be known.

7. CONCLUSIONS

For a two-dimensional fissured rock mass with negligible primary permeability, if the orientations, spacing, openings and homogeneity of fissure sets are obtained from geological studies, the magnitudes and directions of principal permeability and hence degree of anisotropy can be determined using simple equations and a geometrical method known as Permeability Circle.

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