RESEARCH NOTE

ANALYSIS AND DESIGN OF A SIMPLE SURGE TANK

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(Received: December 9, 2003 – Accepted in Revised Form: October 14, 2004)

Abstract In a hydroelectric power plant or in a pumping station in order to avoid sudden large increase of pressure due to instantaneous valve closure sometimes a surge tank is installed. The height of surge tank is designed by the highest possible water level during the operation. The theoretical treatment of oscillation in a surge tank is difficult because of the non-linearity of friction term in the governing differential equation of the system. The present study attempts to find a general solution for the surge oscillation in a simple surge tank in terms of non-dimensional parameters. Equations for the highest and the lowest water level in the tank, which are very important in the design of a surge tank have been found.

Key Words Surge Tank, Pressure, Surge Oscillation, Non-Linear Differential Equation

چکیده در نیروگاههای آبی به منظور اجتناب از افزایش فشار بعلت بسته شدن ناگهانی دریچه گاهی اوقات تانکهای موجگیر کاهش فشار نصب می گردد. بررسی تئوری نوسان سطح آب در این تانکها به علت غیر خطی بودن قسمت اصطکاکی در معادله اصلی سیستم مشکل می باشد. در این تحقیق سعی گردیده که یک راه حل کلی برای نوسان موج در یک تانک موجگیر ساده با استفاده از پارامترهای بدون بعد پیدا شود. همچنین معادلاتی برای بالاترین و پایین ترین سطح آب در تانک که در طراحی پارامترهای بسیار مهمی می باشند ارائه گردیده است.

1. INTRODUCTION

When the valve in a hydroelectric power plant is suddenly completely closed, because of its small inertia the water in the penstock stops almost at once [1,3]. The water in the pipeline, with large inertia retards slowly. The difference in flows between pipeline and penstock causes a rise in the water level in the surge tank. The water level rises above the static level of the reservoir water, producing a counter-pressure so that water in the pipeline flows towards the reservoir and the level of water in the surge tank drops [4].

In the absence of damping, oscillation would continue indefinitely with the same amplitude. The extent of damping is governed by roughness condition, restricted orifice, and so on. The flow into the surge tank and water level in the tank at any time during the oscillation depends on the dimension of the pipeline and tank and on the type of valve movement. The main functions of a surge tank are [5]:

1. It reduces the amplitude of pressure fluctuations by reflecting the incoming pressure waves;
2. It improves the regulation characteristic of a hydraulic turbine.

Depending upon its configuration, a surge tank may be classified as simple, orifice, differential, or closed. A simple surge tank is defined as a tank or shaft of constant horizontal cross sectional area that connects the conduit of a hydroelectric power plant for preventing the pressure surges entering into it (Figure 1). The maximum amplitude of water level (maximum surge) can be observed when a full load is suddenly rejected [6].

2. MAIN CONSIDERATIONS IN THE DESIGN OF A SURGE TANK

In order to accomplish its mission most effectively,

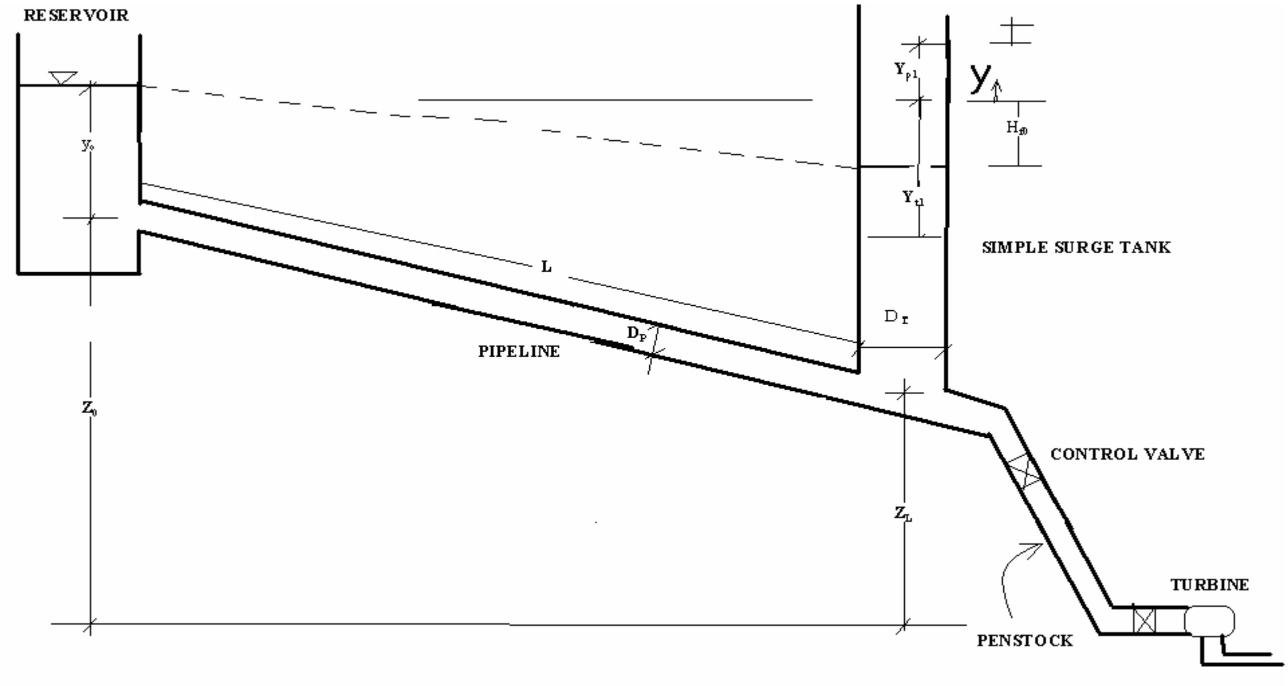

Figure 1. Simple surge tank system.

the surge tank dimensions and location are based on the following considerations [6], [7]:
1. The surge tank should be located as close to the power or pumping plant as possible;
2. The surge tank should be of sufficient height to prevent overflow for all conditions of operation;
3. The bottom of surge tank should be low enough that during its operation the tank is drained out and admit air into the turbine penstock or pumping discharge line; and
4. The surge tank must have sufficient cross sectional area to ensure stability.

The height of a surge tank is governed by the highest possible water level that can be anticipated during its operation. All available methods are based on a linearized resistance relationship, since the resistance law flow varies as Reynolds number and relation roughness [2]. These equations describe approximate values of peaks and downsurges.

3. ANALYSIS OF SURGES IN SIMPLE SURGE TANK

In a simple surge tank, there is very little head loss between the surge tank and the pipeline, also the reservoir is considered so large that its level remains constant [8].

3.1. Derivation of Governing Equation To simplify the derivation of dynamic and continuity equations that describe the oscillations of the water level in the tank, it has been assumed that
(i) the conduit walls are rigid;
(ii) the water is incompressible; and
(iii) the effect of entrance loss in comparison with the friction loss has been neglected.
The equation of motion is written as [9], [12] :

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = g(S_0 - S_f) \tag{1.a}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = -g\left(\frac{\partial Z}{\partial x} + \frac{\partial h_f}{\partial x}\right) \tag{1.b}$$

Integration (1.b) with respect to x between the limits x=0, x=L (see Figure 1) and simplifying, one gets

$$L\frac{dV}{dt} + gh_f + gy = 0 \tag{2}$$

From continuity condition between tank and pipe, it can be shown that

$$V = \left(\frac{D_T}{D_P}\right)^2 \frac{dy}{dt} \tag{3}$$

That with use of 3 in 2 the following equation can be found:

$$L\left(\frac{D_T}{D_P}\right)^2 \frac{d^2y}{dt^2} + gh_f + gy = 0 \tag{4}$$

With initial condition at t=0

$$y = -h_{f0} \tag{5.a}$$

$$\frac{dy}{dt} = \left(\frac{D_P}{D_T}\right)^2 V_0 \tag{5.b}$$

Equation 4 is the governing differential equation for surge oscillation in a simple surge tank.

3.2. Adopted Methods for the Solution Depending on the nature of friction loss, the following particular cases arise:

3.2.1. Frictionless Flow In this case $h_f = -h_{f0} = 0$. Thus Equation 4 changes to

$$\frac{d^2y}{dt^2} + \frac{gD_P^2}{D_T^2 L} y = 0 \tag{6}$$

Solving 6 for the initial condition 5.a, 5.b one gets

$$y = V_0 \frac{D_T}{D_P}\sqrt{\frac{L}{g}} \sin t \tag{7}$$

Equation 7 describes sinusoidal oscillations[10].

3.2.2. Laminar and Turbulent Flow In this case head loss is expressed by Darcy-Weisbach equation

$$h_f = \frac{fLV^2}{2gD_P^2} \tag{8}$$

Combining 3 and 8 one gets

$$h_f = \frac{fLV^2}{2gD_P^2} = \frac{fLD_T^2}{2gD_P^4}\left(\frac{dy}{dt}\right)^2 \tag{9.a}$$

With the change of flow direction, the direction of friction also changes. Hence $\left(\frac{dy}{dt}\right)^2$ occurring in 9.a should be split to $\frac{dy}{dt}\left|\frac{dy}{dt}\right|$. Thus 9.a changes to

$$h_f = \frac{fLD_T^2}{2gD_P^4}\frac{dy}{dt}\left|\frac{dy}{dt}\right| \tag{10}$$

In which 8[11]

$$f = \left[\left(\frac{64}{R_e}\right)^8 + \frac{9.5}{\left(Ln\left(\frac{3.7\varepsilon}{D_P} + \frac{5.74}{R_e^{0.9}}\right) - \left(\frac{2500}{R_e}\right)^6\right)^{16}}\right]^{0.12} \tag{11}$$

Substituting 10 in 4 one gets

$$\frac{d^2y}{dt^2} + \frac{fD_T^2}{2D_P^3}\frac{dy}{dt}\left|\frac{dy}{dt}\right| + \frac{gD_P^2}{D_T^2 L} y = 0 \tag{12}$$

The initial conditions prescribed on 12 are at t=0

$$y = -h_{f0} = -\frac{f_0 L V_0^2}{2gD_P^2} \tag{13.a}$$

$$\frac{dy}{dt} = \left(\frac{D_P}{D_T}\right)^2 V_0 \tag{13.b}$$

In order to reduce the number of parameters, the following dimensionless groups are formed:

$$y_* = y\frac{D_T}{D_P V_0}\sqrt{\frac{g}{L}} \tag{14.a}$$

TABLE 1. Coefficients of Equation 20.

S. No	S	C	k_1	k_2	k_3	Max.Error %
1	y_{p1*}	1	(20.a)	(20.b)	(20.c)	4
2	t_{p1*}	$\pi/2$	(20.d)	(20.e)	0.587	3
3	y_{t1*}	-1	(20.f)	(20.g)	(20.h)	5
4	t_{t1*}	$3\pi/2$	1.9	1.49	0.653	4.2
5	y_{t01*}	-1	(20.1)	0.914	-0.946	4.4
6	t_{p2*}	$5\pi/2$	3.0	1.362	0.7	4.6

$$t_* = t\frac{D_P}{D_T}\sqrt{\frac{g}{L}} \tag{14.b}$$

The conduit velocity may be expressed as

$$V = V_0\frac{dy_*}{dt_*} \tag{15}$$

Substituting 15 in 11 one gets

$$R_e = R_0\frac{dy_*}{dt_*} \tag{16}$$

$$R_0 = \frac{V_0 D_P}{\nu} \tag{17}$$

in which R_0 is the initial Reynolds number defined as using non-dimensional variables 12 reduces to:

$$\frac{d^2y_*}{dt_*^2} + \frac{f}{f_0}h_{f0*}\frac{dy_*}{dt_*}\left|\frac{dy_*}{dt_*}\right| + y_* = 0 \tag{18}$$

The initial conditions for sudden valve closure at t=0

$$\frac{dy_*}{dt_*} = 1 \tag{19.a}$$

and the initial conditions for sudden valve opening are

$$\frac{dy_*}{dt_*} = -1 \tag{19.b}$$

Being non-linear it will be very difficult to find a closed form solution of 18. A numerical solution, using a fourth order Runga Kutta method, can be attempted. From this numerical solution one can see that the various properties of the solution like y_{p1*}, t_{p1*}, y_{t1*}, and t_{t1*} are functions of R_0, $\frac{\varepsilon}{D_P}$, and h_{f0*}.

With varying $0 \le \frac{\varepsilon}{D_P} \le 0.02$ the following empirical formula for finding the mentioned values has been suggested**:**

$$S = C\left[1 + \left(\frac{h_{f0*}}{k_1}\right)^{k2}\right]^{k3} \tag{20}$$

In which the values of S, C, k_1, k_2, k_3 can be found from the Table 1.

$$k_1 = \frac{1.85\left(1 + 0.35\left(\frac{\varepsilon}{D_P}\right)^{0.144}\right)}{\left(1.32 + R_0^{0.024}\right)} \tag{20.a}$$

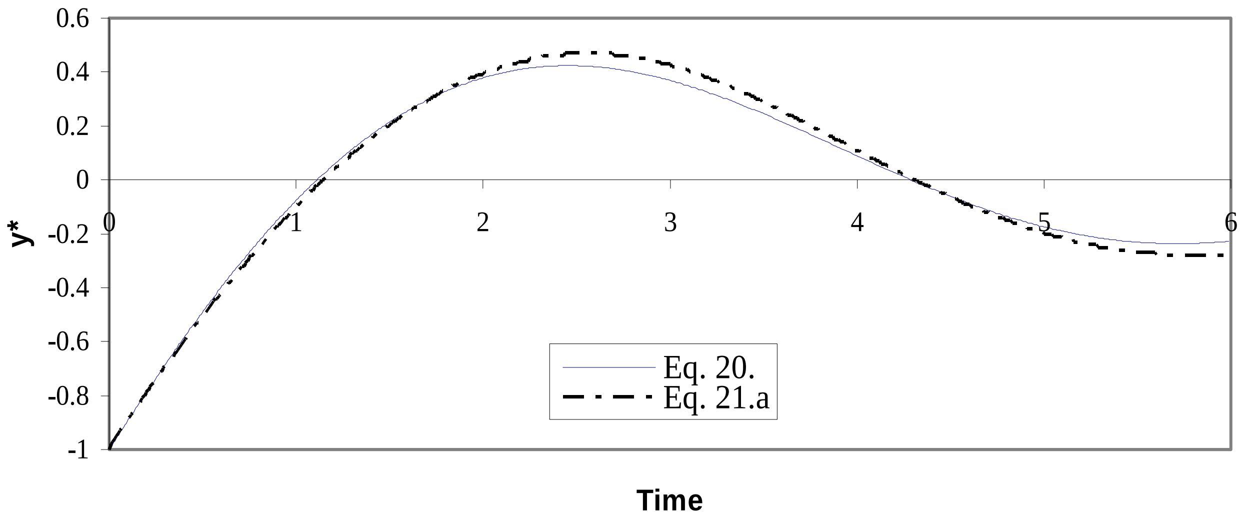

Figure 2. Surge Oscillation in Simple Surge Tank.

$$k_2 = 1.896 R_0^{-0.011} \quad (20.b)$$

$$k_3 = 0.946 R_0^{-0.01} \quad (20.c)$$

$$k_1 = 0.786 + \frac{15600}{\left(1+\left(\frac{\varepsilon}{D_P}\right)^{0.179}\right)\left(10^5 + R_0\right)} \quad (20.d)$$

$$k_2 = 1.73 + \frac{740}{\left(1+19.61\left(\frac{\varepsilon}{D_P}\right)^{0.65}\right)\left(3162 + R_0^{0.7}\right)} \quad (20.e)$$

$$k_1 = \frac{1.36\left(1+0.05\left(\frac{\varepsilon}{D_P}\right)^{0.016}\right)}{1.5 + R_0^{0.035}} \quad (20.f)$$

$$k_2 = \frac{0.462\left(1.92 + R_0^{0.024}\right)}{1+0.6\left(\frac{\varepsilon}{D_P}\right)^{0.083}} \quad (20.g)$$

$$k_1 = 1.66 + 1.227\left(\frac{\varepsilon}{D_P}\right)^{0.287} \quad (20.h)$$

$$k_3 = \frac{0.57\left(1+1.6R_0^{-0.041}\right)}{1+0.054\left(\frac{\varepsilon}{D_P}\right)^{0.037}} \quad (20.i)$$

Figure 2 shows a typical plot of surge oscillation for sudden valve closure obtained by solution Equation 18 using fourth order Runga Kutta method. Analysis of a large number of surge oscillation curves suggest the following empirical equation for $h_{f0*} < 1.5$:

$$y_* = \frac{h_{f0*}}{2\sin\phi}\left(\exp(-\alpha t_*) + \exp(-\beta t_*)\right)\sin(\omega_* t_* - \phi) \quad (21.a)$$

in which

$$\omega_* = \frac{2\pi}{t_{p2*} - t_{p1*}} \quad (21.b)$$

$$\phi = \frac{\pi}{2}\frac{5t_{p1*} - t_{p2*}}{t_{p2*} - t_{p1*}} \tag{21.c}$$

Putting $t_* = t_{p1*}$ in Equation 21.a and equating it to y_{p1*} one gets:

$$\exp(-\alpha t_{p1*}) + \exp(-\beta t_{p1*}) = \frac{2y_{p1*}\sin\phi}{h_{f0*}} \tag{22}$$

Also by putting $t_* = t_{t1*}$ in Equation 21.a and equating it to y_{t1*} one gets:

$$\exp(-\alpha t_{t1*}) + \exp(-\beta t_{t1*}) = \frac{2y_{t1*}\sin\phi}{h_{f0*}} \tag{23}$$

The quantities of α and β can be obtained by solving Equations 22 and 23, simultaneously [1]. Figure 2 shows the result obtained by solving Equation 21.a and compares it with the result obtained by numerical solution of differential Equation 18 by Runga Kutta method. The y_* versus t_* curve can be converted to a y versus t curve via using Equations 14.a and 14.b.

4. CONCLUSION

In the present study a general solution for surge oscillation in a simple surge tank and an optimal design of system has been discussed. The main conclusions are as follows:

1. Equations for maximum surge height and corresponding time of occurrence have been obtained.
2. Equations for minimum downsurge and the corresponding time of occurrence have been developed.
3. An equation for the occurrence of a second peak of the surge oscillation has been obtained.
4. An equation for surge oscillation has been obtained.

5. NOTATIONS

D_P	Diameter of Conduit
D_{P0}	Minmum Conduit Diameter
D_T	Diameter of Simple Surge Tank
D_{TS}	Stable Diameter of Tank
f	Friction Factor
f_0	Friction Factor for Initial Velocity
g	Gravitational Acceleration
h_1	Free Board in Tank
h_2	Cushion Level in Tank
h_f	Head Loss
h_{f0}	Initial Head Loss
h_{f0*}	Non-Dimensional Initial Head Loss
H_0	Desired Head
H_T	Height of Tank
K_P	Cost Parameter for Conduit
K_T	Cost of Tank per Unit Area
L	Length of Conduit
m	Cost Parameter for Conduit
P_0	Desired Power
Q_0	Initial Discharge
R	Reynolds Number
R_0	Initial Reynolds Number
t	time
t_*	Non-Dimensional Time Parameter
t_{p1}	The Occurrence Time of First Peak
t_{p1*}	Non-Dimensional Time of Occurrence First Peak
t_{p2*}	Non-Dimensional Time of Occurrence Second Peak
t_{t1}	The Occurrence Time of First Downsurge
t_{t1*}	Non-Dimensional Time of First Downsurge
V	Velocity
V0	Initial Velocity
y	Height of Surge
y_*	Non-Dimensional Height of Surge
y_{p1}	Height of First Peak
y_{p1*}	Non-Dimensional Height of First Peak
y_{t1}	Height of First Downsurge
y_{t1*}	Non-Dimensional Height of First Downsurge
Z_L	Bottom Level of Tank
Z_0	Level of Reservoir

ε	Roughness of Conduit
η_0	Steady State Efficiency
μ	Viscosity of Water
ν	Kinematic Viscosity of Water
ρ	Mass Density of Water

6. REFERERENCE

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