QUASI-STATIC THEORY FOR UNIAXIAL CHIRAL OMEGA MEDIA

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Abstract The problem of unbounded uniaxial chiral omega media in the presence of both static electric and magnetic point charges is investigated. For this purpose scalar electric and magnetic potentials in these media are introduced. Using these potentials, the corresponding electric and magnetic fields are determined. The similar problem of static electric and magnetic current sources with the goal of finding the electromagnetic fields is carried out. It is observed that either static electric or magnetic point charges produce both electric and magnetic fields.

Key Words Anisotropy, Uniaxial, Chiral Omega, Vector Potentials, Chiral Media

چکیده محیط نا محدود امگا – کایرال تک محوری با حضور بارهای الکتریکی بررسی شده است. برای این کار پتانسیلهای عددی الکتریکی و مغناطیسی معرفی شده و با استفاده از این پتانسیلها میدانهای الکتریکی و مغناطیسی متناظر تعیین شده است. مساله مشابه برای منابع جریان ثابت الکتریکی و مغناطیسی با هدف بدست آوردن میدانهای الکترومغناطیسی بررسی شده است. مشاهده می‌شود که در این محیطها هر یک از بارهای ساکن الکتریکی یا مغناطیسی هر دو نوع میدانهای الکتریکی و مغناطیسی را ایجاد می‌کنند.

1. INTRODUCTION

In past decades; much attention has been given to the electromagnetic properties of complex materials. Several applications of quasi-static image theory in analyzing EM fields were considered by many authors, among which are: quasi-static image theory for bi-isotropic microstrip structures [1], static image theory for bi-isotropic sphere and isotropic and bi-isotropic cylinders [2,3] and static image theory for bi-isotropic media with parallel interfaces [4]. However an analysis of quasi-static theory is not reported for uniaxial chiral omega medium. Such a medium can be realized by putting two types of particles in a dielectric host medium. One of these is two orthogonal sets of small omega shaped metals with planes normal to x- and y-axes. The other is two orthogonal arrays of small metal helices with their axes along x- and y [5].

Media involving omega shaped metal particles were applied as phase shifters [6]; pseudochiral point-source antennas [7] and nonradiative dielectric waveguides [8]. Recently the constitutive parameters of complex media consisting omega particles were studied [9].

In present article, the problem of static electromagnetic fields due to the presence of both static electric and magnetic point charges in an unbounded uniaxial chiral omega medium is studied. Introducing the appropriate scalar electric and magnetic potentials and using the constitutive relations of the medium identify the corresponding electric and magnetic fields. In the next section a similar problem, but due to arbitrarily oriented static electric and magnetic current sources, with the goal of finding the electromagnetic fields is carried out. It is observed that due to the anisotropic feature of the corresponding parameters of the medium, electromagnetic fields produced by static and magnetic charges are a linear combination of two different scalar functions such that the fields associated with point charges do not have any spherical symmetry in uniaxial chiral omega media. A similar result is obtained for the case of static electric and magnetic current sources, but in this case the fields involve different vector functions.

Generally speaking, we conclude that the physical behavior of electromagnetic fields in uniaxial chiral omega media is properly different from a similar problem but with general isotropic media [4].

Uniaxial chiral omega media, while is best described by its constitutive relations given by 1 and 2 in the next section, can be realized by combining two types of uniaxial media with a common axis. One of these media is called, uniaxial omega (or pseudochiral) and the other, uniaxial chiral media. Uniaxial omega medium with z- axis, can be realized by embedding two orthogonal sets of small omega shaped metals with planes normal to x- and y- axes. In this medium a coupling between electric and magnetic fields along x- and y- axes is present and the electric field along x- axis generates a magnetic field along y- axis and vice versa. Therefore, in this medium, with respect to omega particles, there is no preference in the x – y directions and there exists only one particular direction, normal to x – y plane, i.e., the z direction [8]. The other type of medium namely uniaxial chiral medium along z- axis, can be implemented by arranging small metal helices in two orthogonal arrays with their axes along the x- and y- axes. In this medium, induced electric current in a short helix, while located in a high frequency electric field, produces a magnetic field, which has both longitudinal (along the helix axis) and transverse components. In such a medium the field coupling due to chirality is effective only for the fields in the x – y plane.

Uniaxial chiral omega medium which is a combination of the above mentioned media, can therefore be realized by embedding the above mentioned two types of particles in a dielectric host medium i.e., two orthogonal sets of small omega shaped metals with planes normal to x- and y- axes and two orthogonal arrays of small metal helices with their axes along x- and y [5].

Uniaxial omega media features the coupling between electric and magnetic field components with similar directions in the x – y plane. This property is due to the presence of metal helices in the medium. In addition, due to the existence of omega particles in the medium, another coupling occurs between electric and magnetic field components in the x – y plane that is different from that of metal helices. The electric field along x- axis produces magnetic field along y- axis and vice versa.

2. STATIC SCALAR POTENTIALS

Consider both static electric and magnetic point charges ρ_e and ρ_m located in an unbounded uniaxial chiral omega medium, with the following constitutive relations [5]:

$$\vec{D} = \bar{\bar{\varepsilon}}.\vec{E} + j\sqrt{\varepsilon_0\mu_0}\ (-k_t\,\bar{\bar{I}}_t + k\,\bar{\bar{J}})\,.\vec{H} \tag{1}$$

$$\vec{B} = \bar{\bar{\mu}}.\vec{H} + j\sqrt{\varepsilon_0\mu_0}\ (k_t\,\bar{\bar{I}}_t + k\,\bar{\bar{J}}).\vec{E} \tag{2}$$

Where $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$ refer to permittivity and permeability tensors respectively. These dyadics are uniaxial with transverse component ,t, and the normal component ,n,

$$\bar{\bar{\varepsilon}} = \varepsilon_0\,(\,\varepsilon_t\,\bar{\bar{I}}_t + \varepsilon_n\,\hat{z}\,\hat{z}\,)$$

and (3)

$$\bar{\bar{\mu}} = \mu_0\,(\,\mu_t\,\bar{\bar{I}}_t + \mu_n\,\hat{z}\,\hat{z}\,)$$

Where $\hat{z}$ denotes the unit vector parallel to the axis (orthogonal to the plane including the stems of omega particles). $\bar{\bar{I}}_t = \hat{x}\,\hat{x} + \hat{y}\,\hat{y}$ is the transverse unit dyadic, $\bar{\bar{J}} = \hat{z}\times\bar{\bar{I}}_t = \hat{y}\,\hat{x} - \hat{x}\,\hat{y}$ is 90- degree rotator in the x – y plane and *k* is the magneto-electric coupling parameter (dimensionless) due to omega particles of the medium and k_t is the chirality (dimensionless) due to small metal helices of the medium, which is effective for the fields in the (x-y) plane [5].

The electric and magnetic charge densities satisfy;

$$\nabla.\vec{D} = \rho_e\ , \qquad \nabla.\vec{B} = \rho_m \tag{4}$$

With regard to the coupling between electric and magnetic fields generated due to the omega particles and helices in the uniaxial chiral medium, it is observed that magnetic charge ρ_m produces both electric and magnetic potentials, Φ_e and Φ_m and the corresponding fields.

The static electric and magnetic fields can be expressed in terms of scalar electric and magnetic potentials Φ_e and Φ_m respectively, i.e.;

$$\vec{E} = -\nabla\Phi_e , \qquad \vec{H} = -\nabla\Phi_m \tag{5}$$

With regard to the expressions 5 and making use of static relations, one can derive a system of two coupled partial differential equations in terms of the scalar potentials, Φ_e and Φ_m;

$$\varepsilon_0\varepsilon_t\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_e - jk_t\sqrt{\varepsilon_0\mu_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_m + \varepsilon_0\varepsilon_n\frac{\partial^2\Phi_e}{\partial z^2} = -\rho_e \tag{6}$$

$$jk_t\sqrt{\varepsilon_0\mu_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_e + \mu_0\mu_t\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_m + \mu_0\mu_n\frac{\partial^2\Phi_m}{\partial z^2} = -\rho_m \tag{7}$$

In order to solve Equations 6 and 7, we assume Φ_e and Φ_m as a linear combination of two auxiliary functions Φ_1 and Φ_2.

$$\Phi_e = \Phi_1 + \Phi_2$$
$$\Phi_m = X_1\Phi_1 + X_2\Phi_2 \tag{8}$$

Where X_1 and X_2 are unknown constants depending on the medium parameters. Substituting for Φ_e and Φ_m from Equation 8 in Equations 6 and 7, one can obtain two equations for Φ_1 and Φ_2, i.e.

$$\nabla_1^{*2}\Phi_1 = -\frac{X_2}{\alpha_1^2\varepsilon_0\varepsilon_n(X_2-X_1)}\rho_e + \frac{1}{\alpha_1^2\mu_0\mu_n(X_2-X_1)}\rho_m \tag{9}$$

and

$$\nabla_2^{*2}\Phi_2 = \frac{X_1}{\alpha_2^2\varepsilon_0\varepsilon_n(X_2-X_1)}\rho_e - \frac{1}{\alpha_2^2\mu_0\mu_n(X_2-X_1)}\rho_m \tag{10}$$

where;

$$\alpha_1^2 = \frac{\varepsilon_t - jk_t\eta_0 X_1}{\varepsilon_n} , \qquad \alpha_2^2 = \frac{\varepsilon_t - jk_t\eta_0 X_2}{\varepsilon_n} \tag{11}$$

and

$$\nabla_1^{*2} \equiv \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) + \frac{1}{\alpha_1^2}\frac{\partial^2}{\partial z^2} = \nabla_t^2 + \frac{1}{\alpha_1^2}\frac{\partial^2}{\partial z^2} \tag{12}$$

$$\nabla_2^{*2} \equiv \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) + \frac{1}{\alpha_2^2}\frac{\partial^2}{\partial z^2} = \nabla_t^2 + \frac{1}{\alpha_2^2}\frac{\partial^2}{\partial z^2} \tag{13}$$

With

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} , \qquad X_{1,2} = -j\frac{(\varepsilon_t\mu_n - \varepsilon_n\mu_t) \pm \sqrt{\Delta}}{2\eta_0 k_t \mu_n} \tag{14}$$

and

$$\Delta = (\varepsilon_t\mu_n - \varepsilon_n\mu_t)^2 + 4k_t^2\mu_n\varepsilon_n \tag{15}$$

It is concluded that for any charge distributions ρ_e and ρ_m, Φ_1 and Φ_2 can be obtained from

Equations 9 and 10. Once these auxiliary potentials are known, the potentials Φ_e and Φ_m can be determined from Equation 8 and the electric and magnetic fields follow from Equation 5.

Consider a uniaxial chiral omega medium with the constitutive Relations 1 and 2, in the presence of both static electric and magnetic point charges, i.e.

$$\rho_e = q_e \delta(x)\delta(y)\delta(z)$$

and (16)

$$\rho_m = q_m \delta(x)\delta(y)\delta(z)$$

Equations 9 and 10 are thus transformed to

$$\nabla_1^{*2}\Phi_1 = \left[-\frac{X_2}{\alpha_1^2 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e + \frac{1}{\alpha_1^2 \mu_0 \mu_n (X_2 - X_1)} q_m \right] \delta(x)\delta(y)\delta(z) \tag{17}$$

and

$$\nabla_2^{*2}\Phi_2 = \left[\frac{X_1}{\alpha_2^2 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e - \frac{1}{\alpha_2^2 \mu_0 \mu_n (X_2 - X_1)} q_m \right] \delta(x)\delta(y)\delta(z) \tag{18}$$

By choosing a new Cartesian coordinates in the z direction in the form $z_1' = \alpha_1 z$ and some algebraic manipulations, one can conclude;

$$\nabla_1^2\Phi_1 = \left[-\frac{X_2}{\alpha_1 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e + \frac{1}{\alpha_1 \mu_0 \mu_n (X_2 - X_1)} q_m \right] \delta(x)\delta(y)\delta(z_1') \tag{19}$$

Where

$$\nabla_1^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z_1'^2} \tag{20}$$

Similarly, using $z_2' = \alpha_2 z$, Equation 18 reduces to:

$$\nabla_2^2\Phi_2 = \left[\frac{X_1}{\alpha_2 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e - \frac{1}{\alpha_2 \mu_0 \mu_n (X_2 - X_1)} q_m \right] \delta(x)\delta(y)\delta(z_2') \tag{21}$$

Where

$$\nabla_2^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z_2'^2} \tag{22}$$

The integration of Equations 19 and 21 yields;

$$\Phi_1 = \frac{Q_1}{4\pi r_1'} \tag{23}$$

$$\Phi_2 = \frac{Q_2}{4\pi r_2'} \tag{24}$$

with

$$Q_1 = \frac{X_2}{\alpha_1 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e - \frac{1}{\alpha_1 \mu_0 \mu_n (X_2 - X_1)} q_m \tag{25}$$

Where

$$r_1' = \sqrt{x^2 + y^2 + \alpha_1^2 z^2} \tag{26}$$

and also;

$$Q_2 = -\frac{X_1}{\alpha_2 \varepsilon_0 \varepsilon_n (X_2 - X_1)} q_e + \frac{1}{\alpha_2 \mu_0 \mu_n (X_2 - X_1)} q_m \tag{27}$$

in which

$$r_2' = \sqrt{x^2 + y^2 + \alpha_2^2 z^2} \tag{28}$$

using Φ_1 and Φ_2, the potentials Φ_e and Φ_m and therefore the $\vec{E}$ and $\vec{H}$ fields can be determined from Equation 5.

3. FIELDS OF STATIC CURRENT SOURCES

Consider a uniaxial chiral omega medium with static electric and magnetic current sources $\vec{J}$ and $\vec{J}_m$ respectively. These sources are related to $\vec{E}$ and $\vec{H}$ fields by:

$$\nabla \times \vec{H} = \vec{J} \tag{29}$$

$$\nabla \times \vec{E} = -\vec{J}_m \tag{30}$$

Furthermore, in this case we can write:

$$\nabla \cdot \vec{D} = 0 \quad , \qquad \nabla \cdot \vec{B} = 0 \tag{31}$$

Similar to magnetic charges in section 2, it is observed that magnetic current source $\vec{J}_m$ produces both electric and magnetic fields. Therefore a magnetic current source acts as both magnetic and electric current sources. In addition, one can consider that a magnetic dipole $\vec{J}_m$ acts as a small electric current loop with an identical magnetic moment.

Using the Relations 29 through 31 and constitutive Relations 1 and 2, one can obtain Equations 32 through 35 in Cartesian coordinate system as follows;

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -J_{mx} , \; \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -J_{my} ,$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -J_{mz} \tag{32}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x , \; \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y ,$$
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z \tag{33}$$

$$\varepsilon_0 \varepsilon_t (\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}) - j k_t \sqrt{\varepsilon_0 \mu_0} (\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}) +$$
$$k \sqrt{\varepsilon_0 \mu_0} (\frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x}) + \varepsilon_0 \varepsilon_n \frac{\partial E_z}{\partial z} = 0 \tag{34}$$

$$\mu_0 \mu_t (\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}) + j k_t \sqrt{\varepsilon_0 \mu_0} (\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}) +$$
$$k \sqrt{\varepsilon_0 \mu_0} (\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}) + \mu_0 \varepsilon \mu_n \frac{\partial H_z}{\partial z} = 0 \tag{35}$$

For simplicity, we decompose the vectors and vector operators into longitudinal components along the z axis and transverse components in the plane perpendicular to the z axis in Cartesian coordinates system throughout the analysis. Therefore;

$$\nabla = \nabla_t + \frac{\partial}{\partial z} \hat{z} \tag{36}$$

and

$$\vec{A} = \vec{A}_t + A_z \hat{z} \tag{37}$$

Where;

$$\nabla_t = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} \text{ and } \vec{A}_t = A_x \hat{x} + A_y \hat{y} \tag{38}$$

Applying partial derivative $\frac{\partial}{\partial z}$ in Equations 34 and 35 and making use of Expressions 32-33 and with regard to the operator and vector notations 36 through 38 and tedious algebraic manipulations, for longitudinal components of fields a set of coupled differential equations is obtained; i.e.

$$\varepsilon_0 \varepsilon_t \nabla_t^2 E_z + \varepsilon_0 \varepsilon_n \frac{\partial^2 E_z}{\partial z^2} -$$
$$j k_t \sqrt{\varepsilon_0 \mu_0} \nabla_t^2 H_z = \beta_1 \tag{39}$$

with:

$$\beta_1 = jk_t\sqrt{\varepsilon_0\mu_0}\left(\frac{\partial J_y}{\partial x} - \frac{\partial J_x}{\partial y}\right) + jk\sqrt{\varepsilon_0\mu_0}\,\frac{\partial J_z}{\partial z} + \varepsilon_0\varepsilon_t\left(\frac{\partial J_{my}}{\partial x} - \frac{\partial J_{mx}}{\partial y}\right) \tag{40}$$

and

$$\mu_0\mu_t\nabla_t^2 H_z + \mu_0\mu_n\frac{\partial^2 H_z}{\partial z^2} + jk_t\sqrt{\varepsilon_0\mu_0}\,\nabla_t^2 E_z = \beta_2 \tag{41}$$

with:

$$\beta_2 = jk_t\sqrt{\varepsilon_0\mu_0}\left(\frac{\partial J_{my}}{\partial x} - \frac{\partial J_{mx}}{\partial y}\right) + jk\sqrt{\varepsilon_0\mu_0}\,\frac{\partial J_{mz}}{\partial z} + \mu_0\mu_t\left(\frac{\partial J_x}{\partial y} - \frac{\partial J_y}{\partial x}\right) \tag{42}$$

Using Equations 32 and 33 and 36 through 38 and some algebraic manipulations to eliminate the longitudinal field terms, one can eliminate the tangential field terms and obtain a set of coupled wave equations for the transverse components of the fields.

$$\varepsilon_0\varepsilon_t\nabla_t^2\vec{E}_t + \varepsilon_0\varepsilon_n\frac{\partial^2\vec{E}_t}{\partial z^2} - jk_t\sqrt{\varepsilon_0\mu_0}\,\nabla_t^2\vec{H}_t = \vec{\beta}'_1 \tag{43}$$

and

$$\mu_0\mu_t\nabla_t^2\vec{H}_t + \mu_0\mu_n\frac{\partial^2\vec{H}_t}{\partial z^2} + jk_t\sqrt{\varepsilon_0\mu_0}\,\nabla_t^2\vec{E}_t = \vec{\beta}'_2 \tag{44}$$

Where;

$$\vec{\beta}'_1 = jk_t\sqrt{\varepsilon_0\mu_0}\,\vec{\nabla}_t\times J_z\,\hat{z} + jk\sqrt{\varepsilon_0\mu_0}\,\vec{\nabla}_t J_z + \varepsilon_0\varepsilon_t\vec{\nabla}_t\times J_{mz}\,\hat{z} + \varepsilon_0\varepsilon_n\,\hat{z}\times\frac{\partial\vec{J}_{mt}}{\partial z} \tag{45}$$

and

$$\vec{\beta}'_2 = jk_t\sqrt{\varepsilon_0\mu_0}\,\vec{\nabla}_t\times J_{mz}\,\hat{z} - jk\sqrt{\varepsilon_0\mu_0}\,\vec{\nabla}_t J_{mz} - \mu_0\mu_t\vec{\nabla}_t\times J_z\,\hat{z} - \mu_0\mu_n\,\hat{z}\times\frac{\partial\vec{J}_t}{\partial z} \tag{46}$$

A solution of Equations 39 and 41 as well as the coupled Equations 43 and 44 yields longitudinal and transverse field components respectively. To find the longitudinal components, we assume $\vec{E}_z$ and $\vec{H}_z$ as a linear combination of two auxiliary vector functions f_1 and f_2;

$$E_z = f_1 + f_2, \qquad H_z = X'_1 f_1 + X'_2 f_2 \tag{47}$$

Where X'_1 and X'_2 are unknown constant coefficients. Substituting $\vec{E}_z$ and $\vec{H}_z$ from 47 in 39 and 41, two partial differential equations for f_1 and f_2 are deduced in the following forms

$$\nabla_1^{*2} f_1 = \frac{X'_2 m_1'^2}{\varepsilon_0\varepsilon_n(X'_2 - X'_1)}\beta_1 - \frac{m_1'^2}{\mu_0\mu_n(X'_2 - X'_1)}\beta_2 \tag{48}$$

$$\nabla_2^{*2} f_2 = -\frac{X'_1 m_2'^2}{\varepsilon_0\varepsilon_n(X'_2 - X'_1)}\beta_1 + \frac{m_2'^2}{\mu_0\mu_n(X'_2 - X'_1)}\beta_2 \tag{49}$$

Where

$$m_1'^2 = \frac{1}{\alpha_1^2} = \frac{\varepsilon_0\varepsilon_n}{\varepsilon_0\varepsilon_t - jk_t\sqrt{\varepsilon_0\mu_0}\,X'_1}, \qquad m_2'^2 = \frac{1}{\alpha_2^2} = \frac{\varepsilon_0\varepsilon_n}{\varepsilon_0\varepsilon_t - jk_t\sqrt{\varepsilon_0\mu_0}\,X'_2} \tag{50}$$

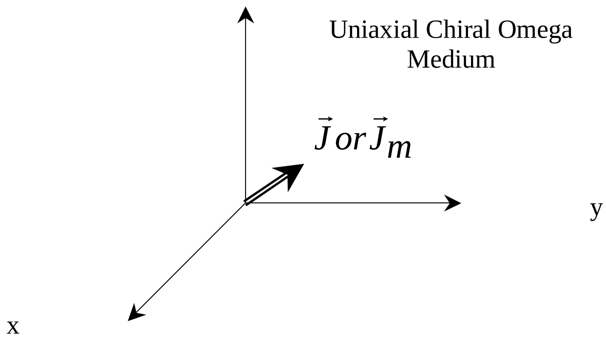

Figure 1. A localized current source at origin in a uniaxial chiral omega medium.

and $X'_{1,2}$ are the same as $X_{1,2}$ in Equation 14. Further more ∇_1^{*2} and ∇_2^{*2} are in Equations 12 and 13.

$$X'_{1,2} = -j\frac{(\varepsilon_t\,\mu_n - \varepsilon_n\,\mu_t) \pm \sqrt{\Delta}}{2\eta_0\;k_t\,\mu_n} \qquad (51)$$

With:

$$\Delta = (\varepsilon_t\mu_n - \varepsilon_n\mu_t)^2 + 4k_t^2\mu_n\varepsilon_n\,,\;\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \qquad (52)$$

and β_1 and β_2 are given in 40 and 42.
It can be seen that for any current source, Equations 48 and 49 together with 40 and 42 can be transformed to conventional Poisson's equation, the solution of which yields, f_1 and f_2. The longitudinal field components, E_z and H_z can therefore be obtained from the corresponding equations in 47.

In a similar manner, to obtain the transverse field components $\vec{E}_t$ and $\vec{H}_t$ from 43 and 44, we specify these components as a linear combination of two auxiliary vector functions $\vec{\Psi}_1$ and $\vec{\Psi}_2$ as follows:

$$\vec{E}_t = \vec{\Psi}_1 + \vec{\Psi}_2\;,\;\vec{H}_t = X'_1\,\vec{\Psi}_1 + X'_2\,\vec{\Psi}_2 \qquad (53)$$

Substituting from 53 in equations 43 and 44 and after similar algebraic manipulations, a set of two differential equations in terms of the unknown functions $\vec{\Psi}_1$ and $\vec{\Psi}_2$ is obtained. i.e.

$$\nabla_1^{*2}\vec{\Psi}_1 = \frac{X'_2\,m_1'^2}{\varepsilon_0\,\varepsilon_n\,(X'_2 - X'_1)}\vec{\beta}'_1 - \frac{m_1'^2}{\mu_0\,\mu_n\,(X'_2 - X'_1)}\vec{\beta}'_2 \qquad (54)$$

$$\nabla_2^{*2}\vec{\Psi}_2 = -\frac{X'_1\,m_2'^2}{\varepsilon_0\,\varepsilon_n\,(X'_2 - X'_1)}\vec{\beta}'_1 + \frac{m_2'^2}{\mu_0\,\mu_n\,(X'_2 - X'_1)}\vec{\beta}'_2 \qquad (55)$$

The operators ∇_1^{*2} and ∇_2^{*2} and the values of X'_1 and X'_2 are given by Equations 12, 13 and 51 respectively. It can be seen that for a current source, Equations 54 and 55 together with 45 and 46 can be transformed to conventional Poisson's equation, the solution of which yields $\vec{\Psi}_1$ and $\vec{\Psi}_2$. The transversal field components, $\vec{E}_t$ and $\vec{H}_t$ can therefore be obtained from the corresponding equations in 53.

4. ELECTROMAGNETIC FIELDS OF LOCALIZED CURRENT SOURCES

Consider a uniaxial chiral omega media with constitutive relations given by 1 and 2 in the presence of constant localized current sources (Figure 1);

$$\vec{J} = (J_x\,\hat{x} + J_y\,\hat{y} + J_z\,\hat{z})\;\delta(x)\;\delta(y)\;\delta(z) \qquad (56)$$

and

$$\vec{J}_m = (J_{mx}\,\hat{x} + J_{my}\,\hat{y} + J_{mz}\,\hat{z})\;\delta(x)\;\delta(y)\;\delta(z) \qquad (57)$$

In order to obtain the EM fields, the functions $f_1, f_2, \vec{\Psi}_1$ and $\vec{\Psi}_2$ should be obtained from 48, 49, 54 and 55 respectively. Using 48 through 52 and 39 through 42 and denoting operators L_1 and L_2 as follows;

$$L_1 = jk_t\sqrt{\varepsilon_0\mu_0}\,(J_y\frac{\partial}{\partial x} - J_x\frac{\partial}{\partial y}) + jk\sqrt{\varepsilon_0\mu_0}\,J_z\frac{\partial}{\partial z} + \varepsilon_0\varepsilon_t\,(J_{my}\frac{\partial}{\partial x} - J_{mx}\frac{\partial}{\partial y}) \tag{58}$$

and

$$L_2 = jk_t\sqrt{\varepsilon_0\mu_0}\,(J_{my}\frac{\partial}{\partial x} - J_{mx}\frac{\partial}{\partial y}) - jk\sqrt{\varepsilon_0\mu_0}\,J_{mz}\frac{\partial}{\partial z} + \mu_0\mu_t\,(J_x\frac{\partial}{\partial y} - J_y\frac{\partial}{\partial x}) \tag{59}$$

After laborious manipulations one can finally find f_1 and f_2 as:

$$f_1 = \left[\frac{X_2' m_1'^2}{\varepsilon_0\varepsilon_n(X_2' - X_1')}L_1 - \frac{m_1'^2}{\mu_0\mu_n(X_2' - X_1')}L_2\right]g_1 \tag{60}$$

$$f_2 = \left[-\frac{X_1' m_2'^2}{\varepsilon_0\varepsilon_n(X_2' - X_1')}L_1 + \frac{m_2'^2}{\mu_0\mu_n(X_2' - X_1')}L_2\right]g_2 \tag{61}$$

Where

$$g_1 = -\frac{1}{4\pi m_1' r_1'} \quad , \quad g_2 = -\frac{1}{4\pi m_2' r_2'} \tag{62}$$

Then the longitudinal field components can be found from the following expressions

$$E_z = f_1 + f_2 \quad , \quad H_z = X_1' f_1 + X_2' f_2 \tag{63}$$

In a similar manner the field components $\vec{E}_t$ and $\vec{H}_t$ require the evaluation of $\vec{\Psi}_1$ and $\vec{\Psi}_2$, functions. Using the source Expressions 56 and 57 together with Expressions 45 and 46 in Equations 54 and 55 and denoting the vector operators $\vec{L}_1'$ and $\vec{L}_2'$ as:

$$\vec{L}_1' = (jk_t\sqrt{\varepsilon_0\mu_0}\;J_z + \varepsilon_0\varepsilon_t J_{mz})\vec{\nabla}_t \times \hat{z} + jk\sqrt{\varepsilon_0\mu_0}\;J_z\vec{\nabla}_t + \varepsilon_0\varepsilon_n\,(J_{mx}\,\hat{y} - J_{my}\,\hat{x})\frac{\partial}{\partial z} \tag{64}$$

$$\vec{L}_2' = (jk_t\sqrt{\varepsilon_0\mu_0}\;J_{mz} - \mu_0\mu_t J_z)\vec{\nabla}_t \times \hat{z} - jk\sqrt{\varepsilon_0\mu_0}\;J_{mz}\vec{\nabla}_t - \mu_0\mu_n\,(J_x\,\hat{y} - J_y\,\hat{x})\frac{\partial}{\partial z} \tag{65}$$

Which finally yields $\vec{\Psi}_1$ and $\vec{\Psi}_2$ as:

$$\vec{\Psi}_1 = \left[\frac{X_2' m_1'^2}{\varepsilon_0\varepsilon_n(X_2' - X_1')}\vec{L}_1' - \frac{m_1'^2}{\mu_0\mu_n(X_2' - X_1')}\vec{L}_2'\right]g_1 \tag{66}$$

$$\vec{\Psi}_2 = \left[-\frac{X_1' m_2'^2}{\varepsilon_0\varepsilon_n(X_2' - X_1')}\vec{L}_1' + \frac{m_2'^2}{\mu_0\mu_n(X_2' - X_1')}\vec{L}_2'\right]g_2 \tag{67}$$

Then the field components $\vec{E}_t$ and $\vec{H}_t$ can be obtained from 53 and the total $\vec{E}$ and $\vec{H}$ fields are expressed as

$$\vec{E} = \vec{E}_t + E_z\,\hat{z} \quad , \quad \vec{H} = \vec{H}_t + H_z\,\hat{z} \tag{68}$$

5. SPECIAL CASES

5.1 Medium With k = 0 In this case, omega particles do not exist and the medium has only chiral helices with chiralities in the $\hat{x}$ and $\hat{y}$ directions. Such a medium can be called anisotropic transverse chiral medium. In view of preceding sections with regard to *k* = 0, we have two types of sources:

1. Electric and magnetic charges: In the presence of only electric and magnetic charges, potential functions Φ_m and Φ_e do not depend on omega parameter k. Thus, all equations and presented solutions in Section 2 corresponding to potential functions Φ_m and

Φ_e and $\vec{E}$ and $\vec{H}$ fields are valid in this case. Although it is observed that omega parameter, k has not any effect on electromagnetic fields in the presence of electric and magnetic charges, but there exists cross coupling between electric and magnetic fields via the chirality parameter k_t.

2. J and J_m as sources: Setting k = 0, in the equations of Sections 3, 4 involving current sources, the equations and solutions for $\vec{E}$ and $\vec{H}$ fields can be obtained. In spite of suppression of magneto-electric cross coupling effects due to the absence of omega particles (k = 0), it is observed that the existing cross coupling between electric and magnetic fields through the chirality parameter of the medium, k_t, remains unchanged.

To verify the analysis, consider the special case, $k_t = 0$ and $k = 0$, in which the medium will transform into a conventional isotropic dielectric medium with $\varepsilon_t = \varepsilon_n = 1$ and $\mu_t = \mu_n = 1$.

With regard to Equations 23 through 28 and 14-15, one can obtain the potentials Φ_e and Φ_m from (8) as

$$\Phi_e = \frac{q_e}{4\pi\varepsilon_0 r} \qquad \text{and} \qquad \Phi_m = \frac{q_m}{4\pi\mu_0 r}$$

with:

$$r = \sqrt{x^2 + y^2 + z^2}$$

with regard to the distance variable, r, the above potentials, Φ_e and Φ_m, are completely in agreement with the well-known static electric and magnetic potential solutions for ordinary isotropic dielectric medium.

5. 2 Medium with k_t = 0 In this case the chiral helices do not exist ($k_t = 0$) and the medium has only omega particles which affect in $\hat{x}$ and $\hat{y}$ directions. Such a medium can be called anisotropic transverse omega medium. By considering the previous sections, and with $k_t = 0$, it is concluded that in a medium with $k_t = 0$ and in the presence of only electric and magnetic charges (ρ_e and ρ_m or Q_e and Q_m), the potential function Φ_e or (Φ_m) depends only on the electric charges and variable r_2' (or only on the magnetic charges and variable r_1'). Where, r_2' and r_1' are completely different. This difference between r_2' and r_1' is due to the fact that r_2' depends only on the medium permittivity tensor $\bar{\bar{\varepsilon}}$ and space coordinates, while r_1' is related only to the medium permeability tensor $\bar{\bar{\mu}}$ and space coordinates. On the other hand, according to the anisotropy present in terms $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$, one can see that similar to section 2, in the presence of electric and magnetic point charges, Φ_e and Φ_m potentials, and consequently $\vec{E}$ and $\vec{H}$ fields do not have any spherical symmetry. Furthermore for $k_t = 0$, potentials Φ_e (and the corresponding $\vec{E}$ field) and Φ_m (and the corresponding $\vec{H}$ field) depend on electric charges and magnetic charges. Consequently there is not any coupling between electric and magnetic fields in this case. This is in contrast to the similar problem in previous case (k = 0), where cross coupling exists between electric and magnetic fields through chirality parameter, k_t.

k_t = 0, with only Current Sources Present (J and J_m) Similarly; either of the fields $\vec{E}$ or $\vec{H}$ depend on both electric and magnetic current sources. This magneto-electric coupling occurs due to omega parameter *k* of the medium through the current sources. In comparison to the similar problem in *k* = 0 case (5.1, part b), in the $k_t = 0$ case, the $\vec{E}$ field depends on a single distance variable r_2', while $\vec{H}$ depends on the distance variable r_1' which is completely different from the similar problem in $k = 0$ case, in which due to the chirality parameter k_t either of the fields $\vec{E}$ and

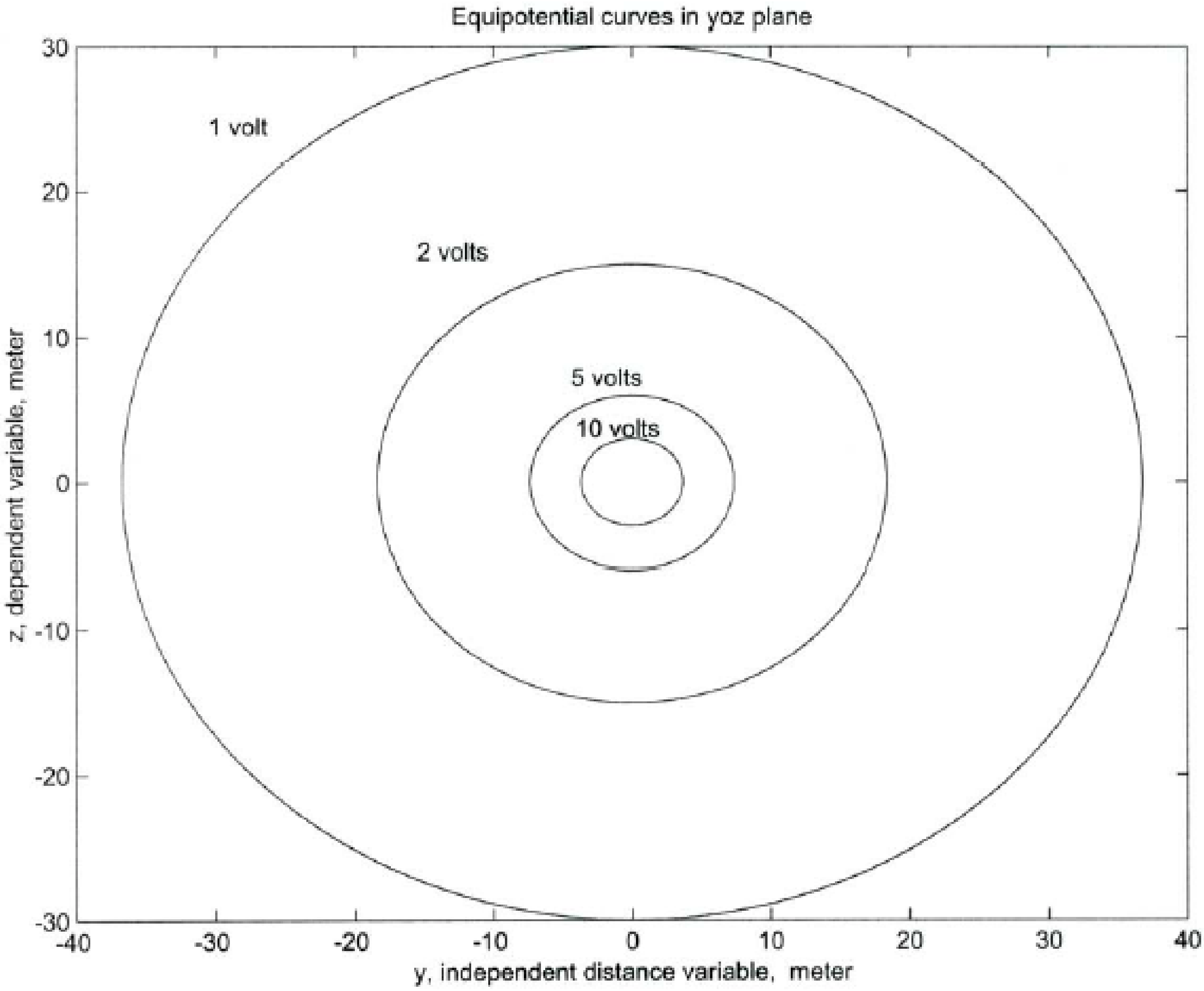

Figure 2. Equipotential contours for $q_e = 0.01\mu C$ in yoz plane for the special case: $k_t = 0, \varepsilon_t = 3, \varepsilon_n = 2, k = 1$, $\mu_n = \mu_t = 1$.

$\vec{H}$ relates to both distance variables r_1' and r_2' simultaneously.

As an example, we determine Φ_e and Φ_m in a special case. By setting $k_t = 0$ and $q_m = 0$ in Equations 23 and 24 and with regard to 25 through 28, 14-15 and 8, one can obtain the potentials Φ_e and Φ_m, as

$$\Phi_e = \frac{q_e}{4\pi\varepsilon_0 \sqrt{\varepsilon_t \varepsilon_n}\, r_e}\ , \ \Phi_m = 0 \qquad (69)$$

With:

$$r_e = \sqrt{x^2 + y^2 + \frac{\varepsilon_t}{\varepsilon_n} z^2}$$

As a numerical example, $k_t = 0, \varepsilon_t = 2, k = 1$, $\mu_n = \mu_t = 1$ and $q_e = 0.01\mu C$, with regard to 69, the equipotential contours for Φ_e and Φ_m in yoz plane are obtained to be elliptic, as is shown in Figure 2. A further investigation reveals, that with increasing the chirality parameter, k_t, of the medium, the magnitude of both electric and magnetic potentials Φ_e and Φ_m will increase. An adequate treatment for these special cases is intended for a future article.

6. CONCLUSION

The electromagnetic fields due to both static electric and magnetic charges and current sources in an unbounded uniaxial chiral omega medium were studied and the special cases of the medium with $k=0$ or $k_t=0$ was briefly pointed out in this work. Two scalar electric and magnetic potentials Φ_e and Φ_m as a linear combination of two different scalar functions Φ_1 and Φ_2 were introduced. The difference between Φ_1 and Φ_2 is due to anisotropic feature of the permittivity $\bar{\bar{\varepsilon}}$ and permeability $\bar{\bar{\mu}}$ and the chirality parameter, k_t. These parameters appear in the distance variables r_1' and r_2' (26 and 28) via α_1 and α_2 in 11. This is in contrast to the general isotropic media, in which, these potentials involve only a single function proportional to (1/r), where r is the distance between the point charge and the observation point [3]. Therefore the fields $\vec{E}$ and $\vec{H}$ associated with point charges do not have spherical symmetry in anisotropic media. With this respect the magneto-electric cross coupling parameter of omega particles of the k medium; has not any effect on the functions Φ_1 and Φ_2 in Equations 23 and 24. According to 11, the terms α_1 and (α_2) which exist in variables r_1' and (r_2') do not depend on the omega parameter k. In the case of current sources, it is observed from 62 that g_1 and g_2 are different. The differences between g_1 and g_2 are due to the terms r_1' and r_2' as before and the terms m_1' and m_2' in 50. Therefore according to 60, 61, 66 and 67, the functions f_1 and f_2 which depend on the functions g_1 and g_2, are different, as well as $\vec{\Psi}_1$ and $\vec{\Psi}_2$. Hence, because of anisotropy of permittivity, permeability, and chirality parameters ($\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}$ and k_t); the physical behavior of $\vec{E}$ and $\vec{H}$ fields are properly different from that of a similar problem but with general isotropic media [4]. Furthermore, it is evident from 26, 28, 50 and 62, that the magneto-electric cross coupling parameter of omega particle k, cannot contribute to the functions g_1 and g_2 via the terms r_1', r_2', m_1' and m_2'. Only omega parameter k, contributes to the fields $\vec{E}$ and $\vec{H}$ through $\beta_1, \beta_2, \beta_1', \beta_2'$ and operators L_1, L_2, L_1' and L_2' in corresponding relations in Sections 3, 4. Therefore this is another significant property of such a medium, which is fundamentally related to identical contributions of parameter k on fields $\vec{E}$ and $\vec{H}$ in constitutive Relations 1 and 2. For the special case (5. 1, part (a)) where k = 0, it can be seen that in the presence of electric and magnetic charges (ρ_e, ρ_m or Q_e, Q_m), all solutions for potentials Φ_e, Φ_m and the fields $\vec{E}$ and $\vec{H}$ given in section (2), are valid. With the existence of only current sources, some terms involving current sources given in Sections 3, 4, such as $\beta_1, \beta_2, \beta_1', \beta_2'$ and operators L_1, L_2, L_1' and L_2' have been changed for k = 0 and other terms related to $\vec{E}$ and $\vec{H}$ such as r_1' and r_2' and the terms m_1', m_2', X_1', X_2', α_1 and α_2 remain unchanged. For this case, cross coupling between electric and magnetic fields occur due to the chirality parameter k_t. For $k_t=0$, in the presence of electric and magnetic charges, potentials Φ_e (Φ_m) and related fields; $\vec{E}$ $(\vec{H})$ depend only on the electric charges and distance variable r_2' (magnetic charges and distance variable r_1').Where r_2' (r_1') is different, with the dependencies of r_2' to permittivity tensor $\bar{\bar{\varepsilon}}$ and r_1' to permeability tensor $\bar{\bar{\mu}}$. In this case, potential Φ_e and the corresponding $\vec{E}$ field depend on electric charges but potential Φ_m and the corresponding $\vec{H}$ field depends on magnetic charges. Therefore there is not any cross coupling between electric and magnetic fields.

For a medium with $k_t = 0$, in the presence of only current sources ($\vec{J}_e$ and $\vec{J}_m$), either field, $\vec{E}$ or $\vec{H}$ depends on both electric and magnetic current sources $\vec{J}_e$ and $\vec{J}_m$. This magneto-electric coupling occurs due to omega parameter k of the medium via current sources. Furthermore for all kinds of sources with $k_t = 0$, $\vec{E}$ depends on a single distance variable r_2', while $\vec{H}$ depends on a single distance variable r_1', which is completely different from the similar problem in the case k = 0, in which case due to chirality parameter k_t, either of fields, $\vec{E}$ or $\vec{H}$ is related to both distance variables r_1' and r_2' simultaneously.

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8. REFERENCES

1. Koivisto, P. K. and Sten, J. C. E., “Quasi-Static Image Method Applied to Bi-Isotropic Microstrip Geometry”, ***IEEE Trans. Microwave Theory Tech.***, 43, (1995), 169-175.
2. Lindell, I. V., “Quasi-Static Image Theory for the Bi-Isotropic Sphere”, ***IEEE Trans. Antennas Propagat.*** 40, (1992), 228-233.
3. Lindell, I. V., “Static Image Method for Layered Isotropic and Bi-Isotropic Cylinders”, ***Microwave and Optical Technology Letters***, 6, (1993), 383-387.
4. Lindell, I. V., “Static Image Theory for Bi-Isotropic Media with Plane Parallel Interface”, ***Microwave and Optical Technology Letters***, 6, (1993), 228-230.
5. Tretyakov, S. A. and Sochava, A. A., “Eigenwaves in Uniaxial Chiral Omega Media”, ***Microwave and Optical Technology Letters***., 6, (1993), 701-705.
6. Saadoun, M. M. I. and Engheta, N., “A Reciprocal Phase Shifter Using Novel Pseudochiral or Ω Medium”, ***Microwave and Optical Technology Letters***, 5, (1992), 184-188.
7. Toscano, A. and Vegni, L., “Novel Characteristics of Radiation Patterns of a Pseudochiral Point-Source Antenna”, ***Microwave and Optical Technology Letters***, 7, (1994), 247-250.
8. Topa, A. L., Paiva, C. R. and Barbosa, A. M., “Full-Wave Analysis of a Nonradiative Dielectric Waveguide with a Pseudochiral Ω-Slab”, ***IEEE Trans. MTT.***, 46, (1998), 1263-1269.
9. Dmitriev, V., “Constitutive Tensors of Omega- and Chiroferrites”, ***Microwave and Optical Technology Letters***, 29, (2001), 201-205.