

# RESERVOIR OPERATION DURING DROUGHTS

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**Abstract** Drought is an inevitable part of the world's climate. It occurs in wet as well as in dry regions. Therefore, planning for drought and mitigating its impacts is essential. In this study, a hedging rule is developed using the zero/one mixed integer-programming approach. Furthermore, some procedures are introduced to ease the computational burden inherent in integer programming. Hedging rules are developed using three, two, and one-year historical droughts. Moreover, yield model (YM) along with the standard operating policy (SOP) are also formulated for comparison purposes. Simulations are carried out using 40 years of monthly historical data along with 20 series of synthetically generated inflows of the same length. The Karadj reservoir located in the northwest of Tehran is the major source of the capital's municipal water supply. It also provides a substantial portion of the irrigation demand of the Karadj Valley. Synthetic data are generated using single and multi-variate autoregressive modeling approaches. Models are compared using important reservoir operation criteria including reliability, resiliency, and vulnerability. As compared to the well-known SOP model, it is noticed that the application of the hedging rule and the yield model substantially reduces the system reliability as well as its vulnerability, however it increases the resiliency. Moreover, hedging rules developed using longer drought periods tend to have lower vulnerability and reliability, and higher resiliency.

**Key Words** Drought, Hedging Rule, Reservoir Management, Reservoir Operation, Karadj Reservoir, Yield Model, Zero/One Programming, Water Deficit Management

**چکیده** امروزه خشکسالی یکی از موضوعات اساسی در برنامه ریزی منابع آب به شمار می آید. بطوریکه هرگونه برنامه ریزی در این زمینه بدون لحاظ نمودن خشکسالی ناقص بوده و نمی تواند پاسخگوی معضلات ایجاد شده در اثر خشکسالی ها باشد. از طرفی خشکسالی یک پدیده طبیعی و اجتناب ناپذیر آب و هوایی حتی در بخش های پر باران جهان می باشد. بنابراین برنامه ریزی برای مقابله با آن و کاستن اثرات آن لازم و ضروری می باشد. در سیستم های آبرسانی برای کاستن اثرات کمبود آب، از مدل های جیره بندی استفاده می کنند. این عمل وقتی توجیه پذیر می باشد که مصارف آب دارای توابع خسارت غیرخطی باشند، بطوریکه کمبودهای بزرگتر خسارات خیلی بیشتری را باعث شوند. در این تحقیق از یک مدل بهینه سازی جیره بندی گسسته دو مرحله ای اصلاح شده برای مدیریت مخزن در یک دوره خشک استفاده شد. در این روش اصلاح شده که به صورت گام به گام حل می شود، امکان اجرای مدل جیره بندی برای تعداد سالهای بیشتر وجود دارد. زمان اجرای مدل اصلاح شده نیز به طور قابل ملاحظه ای کاهش می یابد. نتایج اجرای مدل بر روی سیستم سد مخزنی کرج نشان داد که به ازای تعداد دوره های جیره بندی مرحله دوم مشابه، مدل اصلاح شده در مقایسه با مدل اولیه نتایج مطلوبتری نشان می دهد. یعنی اینکه تعداد دوره های بدون جیره بندی آن بیشتر می باشد. اما این وضعیت با افزایش تعداد دوره های جیره بندی مرحله ۲ برعکس می شود. اجرای مدل برای دوره های خشک یک الی سه ساله نشان می دهد که با افزایش دوره خشکسالی در مدل، معیارهای ارزیابی همچون اعتماد پذیری و آسیب پذیری کاهش یافته و برگشت پذیری افزایش می یابد. مدل از جنبه های مختلف مورد تحلیل و بررسی قرار گرفته و با یک مدل آبدهی و مدل بهره برداری استاندارد مقایسه گردیده است. نتایج نشان می دهد که مدل جیره بندی برتری محسوسی در کاهش خسارات ناشی از خشکسالی دارد. بکارگیری چنین ابزاری می تواند کمک شایانی در مواقع بحران آب به مدیران در اتخاذ تصمیمات مناسب بنماید.

## 1. INTRODUCTION

Hedging is based on the fact that having more frequent droughts of lower intensity is preferred

to the fewer ones with higher intensities. For example, the cost of having two droughts each with a deficit equal to, say 5 units, is less than a single drought with 10 units of shortage. In other

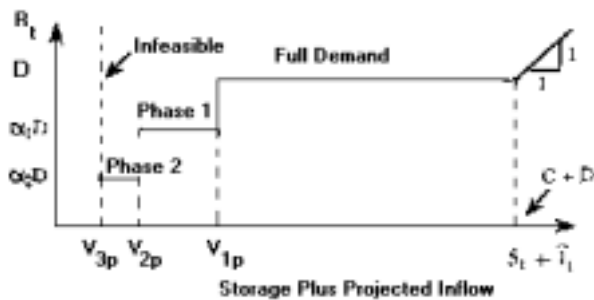


Figure 1. A two-level discrete hedging model.

words, losses due to droughts are not linear. On the other hand, uncertainties on the future events prevent us from a perfect allocation of resources during a drought. Therefore, it is inevitable to search for models that could help us in these situations. These models must rely on the trend of the past events. They usually consist of two types; optimization and simulation. Simulations are typically used to study and verify the results of optimization models.

Despite of the fact that it is an inevitable part of today's world, there has been less effort and research on the planning of resources during droughts. Drought is a very general term with no unique definition. However, in water resources we may define it as a situation in which due to lower river flows, demands are not met. Alike to its definition, the extension and variety of studies carried out are numerous. In this paper, we have focused on the approaches that are used to manage a reservoir system during some critical drought periods. Hedging is an approach that is used for reservoir operation during droughts. Shih and Revelle [1] introduced and developed a continuous hedging rule for a single reservoir operation. They used a Zero/One programming to linearize the nonlinear functions. Darlane [2] used a different procedure to solve the nonlinear functions and showed that the solution algorithm employed by Shih and Revelle [1] is not efficient. He also proposed a revision on the objective function to include the importance of different values of the water released in different periods. Bayazit and Unal [3] investigated the impact of hedging on a reservoir operation. They concluded that the reliability and vulnerability of the system is reduced when hedging is applied.

Srinivasan and Philipose [4] and [5] studied the sensitivity of a system to the changes in hedging thresholds. Shih and Revelle [6] extended their previous work and developed a discrete hedging rule. Neelakantan and Pundarikanthan [7] studied the discrete hedging rule by optimization and simulation models.

## 2. MODEL DEVELOPMENT

In this paper, we employed the discrete hedging model originally developed by Shih and Revelle [6]. Our objective is to develop a practical reservoir operation model during drought periods. Figure 1 shows the model with two hedging levels.

Application of the model developed by Shih and Revelle [6] indicated very high computer execution time and memory utilization. In fact, for some cases of their example, we never reached a solution within acceptable time consumption. These are the cases where the phase 2 frequencies are low enough to force the model to extensively search for optimal solutions. Therefore, we moved the spilling constraint into the objective function. This eliminated 36 zero/one variables responsible to control spills from the reservoir, resulting in substantial reduction of computer time and memory utilization. Despite these changes, it was not yet possible to reach any solution using a Pentium II computer. The followings are the revised objective function and constraints of the model.

Objective:

$$\text{MAX} \sum_{n=1}^N \sum_{t=1}^T (Y1_{n,t} - \omega_{sp} \times SP_{n,t}) - \omega \sum_{t=1}^T (V1_t + V2_t + V3_t) \quad (1)$$

Constraints:

$$Y1_{n,t} \geq ((S_{n,t} + Q_{n,t}) - (V1_t - \epsilon)) / M \quad (2)$$

$$Y1_{n,t} \leq 1 - (V1_t - (S_{n,t} + Q_{n,t})) / M \quad (3)$$

$$Y2_{n,t} \geq ((S_{n,t} + Q_{n,t}) - (V2_t - \epsilon)) / M \quad (4)$$

$$Y2_{n,t} \leq 1 - (V2_t - (S_{n,t} + Q_{n,t})) / M \quad (5)$$

**TABLE 1. Computer Execution Time Required for Different  $N_2$ 's – Method 1.**

Number of phase 2 events ( $n_2$ )	34	32	30	28
Execution time (min: sec)	1:24	20:05	41:53	52:55
Iterations	26832	296136	738455	942043

$$R_{n,t} = (1 - a_1) \times TD_t \times Y1_{n,t} + (a_1 - a_2) \times TD_t \times Y2_{n,t} + a_2 \times TD_t \quad (6)$$

$$S_{n,t+1} = S_{n,t} + Q_{n,t} - R_{n,t} - SP_{n,t} \quad (7)$$

$$SF = S_{N,T} + Q_{N,T} - R_{N,T} - SP_{N,T} \quad (8)$$

$$S_{n,t} \leq CAP \quad (9)$$

$$S_{1,1} \leq SF \quad (10)$$

$$V1_t \geq 1.05 \times V2_t \quad (11)$$

$$V2_t \geq 1.05 \times V3_t \quad (12)$$

$$V3_t \geq a_2 \times TD_t \quad (13)$$

$$S_{n,t} + Q_{n,t} \geq V3_t + \epsilon \quad (14)$$

$$Y1_{n,t-1} + Y1_{n,t+1} \leq 1 + Y1_{n,t} \quad (15)$$

$$Y1_{n,t} \leq Y2_{n,t+1} \quad (16)$$

$$\sum_{n=1}^N \sum_{t=1}^T Y2_{n,t} = N \times T - n_2 \quad (17)$$

Where  $Y1_{n,t}$  and  $Y2_{n,t}$  are zero/one variables. They are both equal to 1 for no hedging and 0 for phase 2 hedging level. In phase 1,  $Y1$  is 0 and  $Y2$  is equal to 1.  $\omega_{sp}$  is weight less than one used for controlling the spills. Through a trail and error it was noticed that a value of 0.01 for  $\omega_{sp}$  would satisfy our goal of minimizing the spills while not affecting the primary objective of maximizing the number of full demand supply periods.  $TD$ ,  $S$ ,  $Q$ ,

$R$ , and  $SP$  are respectively the total demand, storage, inflow, release, and spills from the reservoir.  $V1$ , and  $V2$  are the hedging thresholds for the beginning of phase 1 and phase 2, respectively.  $V3$  indicates the minimum reservoir operation threshold during droughts where no further release is possible. Second term in the objective function is used to avoid variable solutions.  $\omega$  is a weight similar to  $\omega_{sp}$  and was found by trail and error.  $n_2$  is the number of phase 2 hedging.  $N$  and  $T$  are number of years and seasons considered in the analysis, respectively.  $M$  and  $\epsilon$  are very big and small numbers, respectively.  $\alpha_1$  and  $\alpha_2$  are fraction of the demand to be met in phase 1 and 2, respectively.

Constraints 2 through 5 along with 11 and 12 are used to logically determine the hedging threshold levels. Constraint 6 is used to set the hedging rule. Reservoir continuity and capacity constraints are stated by Constraints 7, 8, and 9. Constraint 10 is used to avoid the consumption of initial storage during drought. This constraint may be altered depending on a specific case. Constraints 15 and 16 are used to enforce smooth transitions in hedging levels. Finally, Constraint 17 is set to control the number of phase 2 hedging.

Shih and Revelle used an IBM 3090-600J super computer to run their model. As mentioned earlier, even using these revisions, it was not possible to reach a solution in lower values of  $n_2$  by a Pentium II computer. Therefore, further improvements were required before the model could be practically applied using the publicly available computers.

Assuming  $\alpha_1 = 0.75$  and  $\alpha_2 = 0.60$  for the example in Shih and Revelle [6], the model was executed using different phase 2 frequencies. It was noticed that as  $n_2$  is reduced, the computer execution time is exponentially increased (Table 1).

Further investigation of the solutions revealed that some of the results stay the same from one  $n_2$  level to another. In fact, as Table 2 shows solutions

**TABLE 2. Changes in the Solution of Y2 with Different  $N_2$  Levels.**

	$n_2 = 34$	$n_2 = 32$	$n_2 = 30$	$n_2 = 28$
Y2( 3, 4)	0	0	0	0
Y2( 3, 5)	0	0	0	1
Y2( 3, 6)	0	0	0	1
Y2( 3, 7)	0	0	1	1
Y2( 3, 8)	0	0	1	1
Y2( 3, 9)	0	1	1	1
Y2( 3, 10)	0	1	1	1
Y2( 3, 11)	1	1	1	1
Y2( 3, 12)	1	1	1	1

of Y2 that are equal to 1 stay the same with reduced  $n_2$  levels. This is obvious, since as the frequency of phase 2 hedging is reduced, number of phase 1 hedging events must increase. If we call  $\Delta n$  to be the difference between the two subsequent assumed  $n_2$  levels, then as  $n_2$  is reduced  $\Delta n$  periods must take either a phase 1 or full demand commitment level. In either case, they will have Y2 equal to 1. In this process,  $\Delta n$  periods of phase 2 must be raised to phase 1 level. To do this some of the full commitment levels must be changed to phase 1, so that the extra water gained from this transaction could be used to rise  $\Delta n$  periods from phase 2 to phase 1. Therefore, reducing  $n_2$  will usually increase frequency of phase 1 (cases with  $Y_2 = 1$  and  $Y_1 = 0$ ) and decrease frequency of full success (cases with  $Y_1 = 1$  and  $Y_2 = 1$ ).

To further reduce the computer execution time and memory, we used the above-mentioned results. Starting from a high  $n_2$  level, such as 34 in this case, we may easily reach a solution. In the next step, we set  $n_2$  equal to a lower value such as 32, and transfer those results of Y2 from the previous step that were found to be 1. Computer execution time of this method will be much less than the earlier method. By continuing this procedure, we

would easily solve the model for values of  $n_2$  as small as 10. It is possible to apply this technique for solutions of more sophisticated models within acceptable time and memory spans. It is also practically possible to further increase the number of hedging phases. Table 3 show the execution time and iterations required for the same problem when the new procedure is applied. Figure 2 clearly shows the superiority of the proposed method.

In Figure 3 results of Shih and Revelle [6] and the proposed method are compared. It is noticed that in overall the proposed method has higher frequencies of full demand commitments. This is clearly true for lower levels of  $n_2$ , where proper allocation of available water is crucial.

### 3. CASE STUDY

The revised method is applied to Karadj reservoir located in northwest of Tehran. The reservoir is the major source of municipal water supply for Tehran, the capital. It is also used to provide irrigation water needs of downstream valley. Table 4 shows Tehran's mean monthly water demand from Karadj reservoir. Note that orders of the months are based on typical Iranian water year. It starts on the first day of each fall and ends on the last day of each summer.

In our study, we used 40 years of monthly historical inflow as well as 20 series of synthetically generated data of the same length. For this purpose, a single and a multi-variate AR model were used to generate 10 series of reservoir inflow each. Table 5 shows a summary of long-term parameters of historical and generated data.

The model was prepared and the proposed method was applied using  $\alpha_1 = 0.75$  and  $\alpha_2 = 0.60$ . For this purpose, droughts with three, two, and one-year long durations were identified from the historical data and used to develop the hedging rules. Frequency of different hedging levels for a

**TABLE 3. Computer Execution Time Required for Different  $N_2$  Levels Using the Proposed Method.**

Number of phase 2 events ( $n_2$ )	34	32	30	28
Execution time (min: sec)	1:24	2:14	1:20	1:07
Iterations	26832	40904	26045	21916

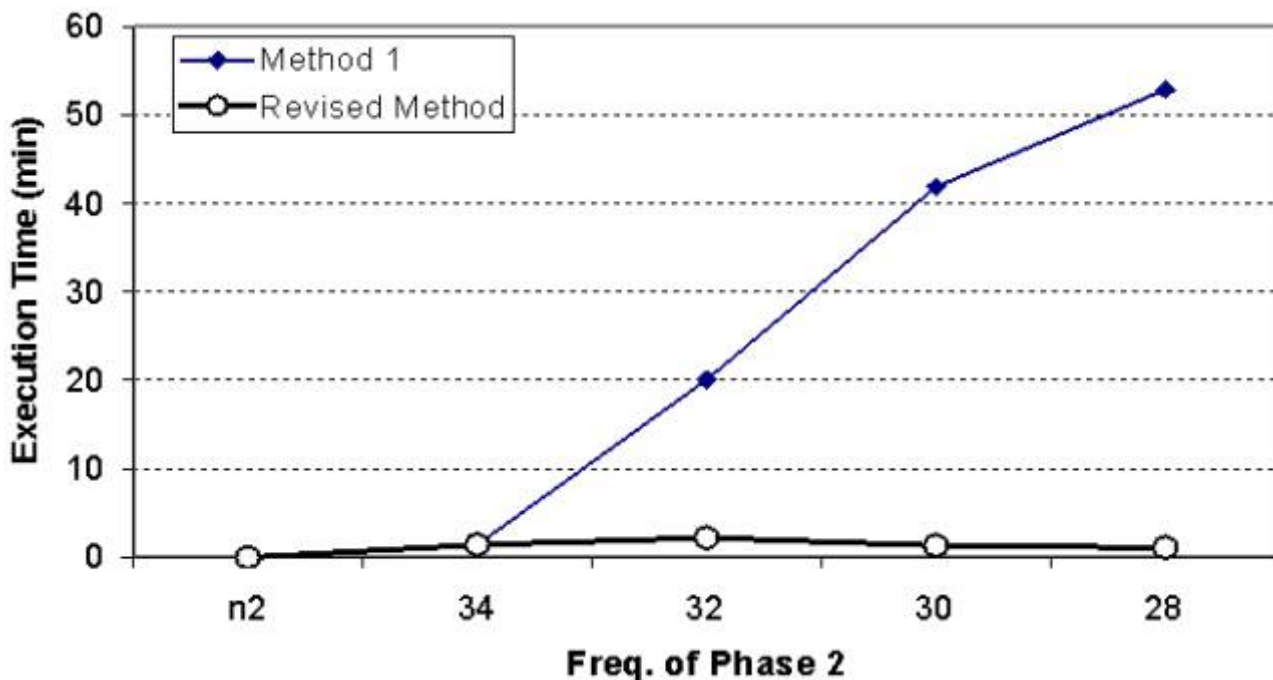


Figure 2. Comparison of execution time of the methods.

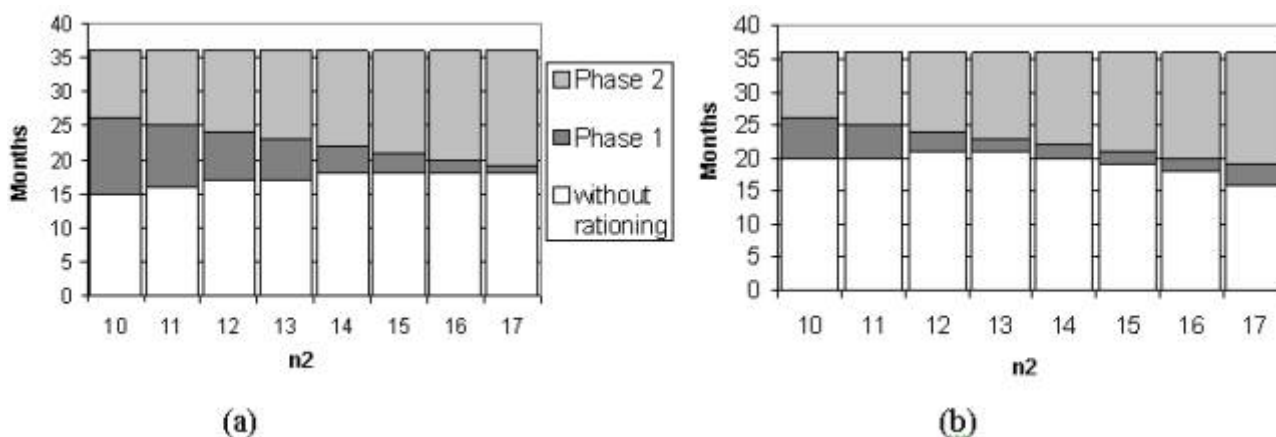


Figure 3. Frequencies of different hedging levels (a) Shih and Revelle and (b) proposed method.

three-year drought is illustrated in Figure 4. Similar charts were depicted for two and one-year droughts. In Figure 5 rule curves analogous to the well-known form of reservoir rule curves are shown. These curves are simply derived from the hedging threshold levels.

In the next step, suitability of the hedging rules was investigated through simulation of the system by historical and synthetic data. Meanwhile, a

yield model and standard operating procedure (SOP) were also used for comparison purposes. Table 6 shows the summary of the results. It should be noted that the values shown under the artificial columns are the average of all series. The yield model (YM) is developed using firm and secondary yields equal to 60 and 15 percent of total demand. The initial storage for all models is assumed to be 100 mcm. It is noticed that YM is

TABLE 4. Monthly Municipal Water Demand of Tehran from Karadj Reservoir (mcm).

month	1	2	3	4	5	6	7	8	9	10	11	12
demand	34.4	30.2	25.4	25.1	23.3	24.2	26.8	46.8	57.6	52.8	45.7	41.3

TABLE 5. Monthly Parameters of Historical and Generated Reservoir Inflow.

Data Type	Parameter	1	2	3	4	5	6	7	8	9	10	11	12
Historical	Avg. (mcm)	15.1	17.3	15.5	13.8	14.7	26.5	61.8	102.3	87.0	50.3	26.6	17.6
	Stdev.	3.4	8.1	8.2	4.8	4.1	13.5	23.2	33.5	31.8	20.7	9.3	4.9
	r	0.70	0.79	0.76	0.68	0.33	0.68	0.73	0.82	0.96	0.96	0.94	0.66
Generated ARMV	Avg. (mcm)	26.4	21.9	18.1	15.9	14.8	23.4	50.9	80.9	80.2	50.2	40.2	34.1
	Stdev.	6.3	6.5	6.5	6.5	5.7	29.5	57.2	34.3	28.1	15.3	9.8	8.1
	r	0.99	0.98	1.00	0.97	0.26	0.23	0.53	0.96	0.98	0.96	0.98	0.46
Generated AR	Avg. (mcm)	15.0	17.0	15.6	14.0	15.0	26.2	62.7	101.5	87.4	50.2	26.5	17.5
	Stdev.	3.2	6.2	5.9	4.1	4.1	10.5	25.5	34.4	32.3	19.7	8.8	4.4
	r	0.73	0.80	0.87	0.70	0.51	0.55	0.73	0.82	0.96	0.96	0.94	0.73

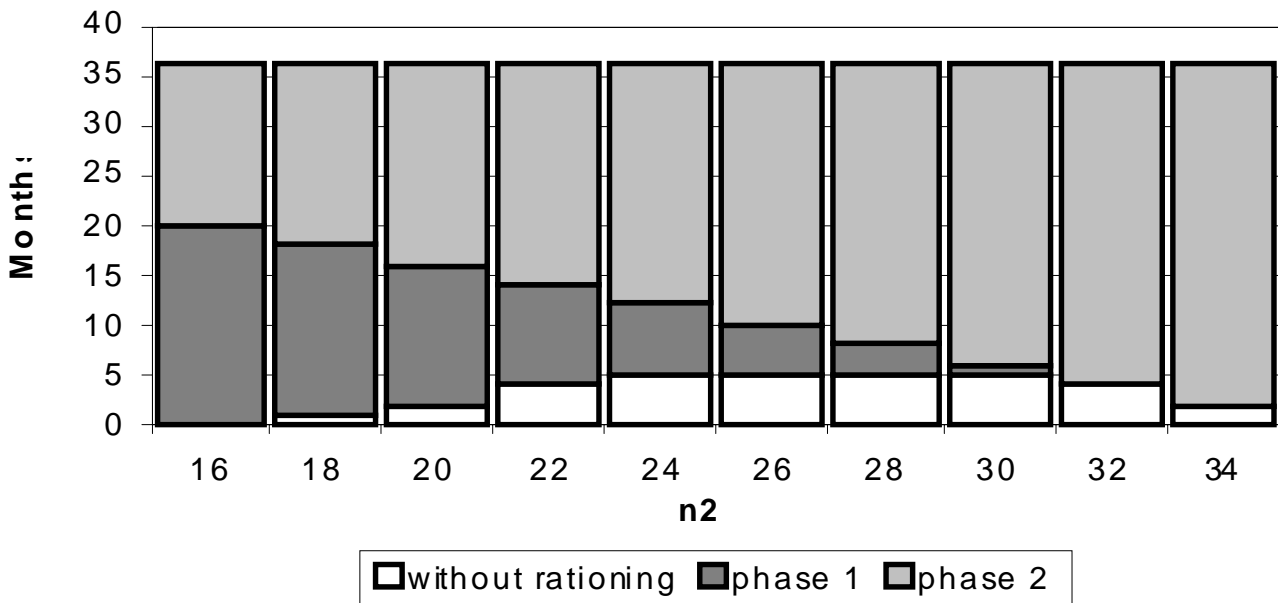


Figure 4. Frequency of different hedging level – Karadj reservoir.

not successful in making full demand commitments and its resiliency is high, however its maximum vulnerability is much less than any other rule. On the other hand, SOP has the highest

frequency of full success, but it also exerts the highest maximum vulnerability among the others with the exception of 1-year hedging rule. In fact, the SOP and 1-year hedging rules behave similar in

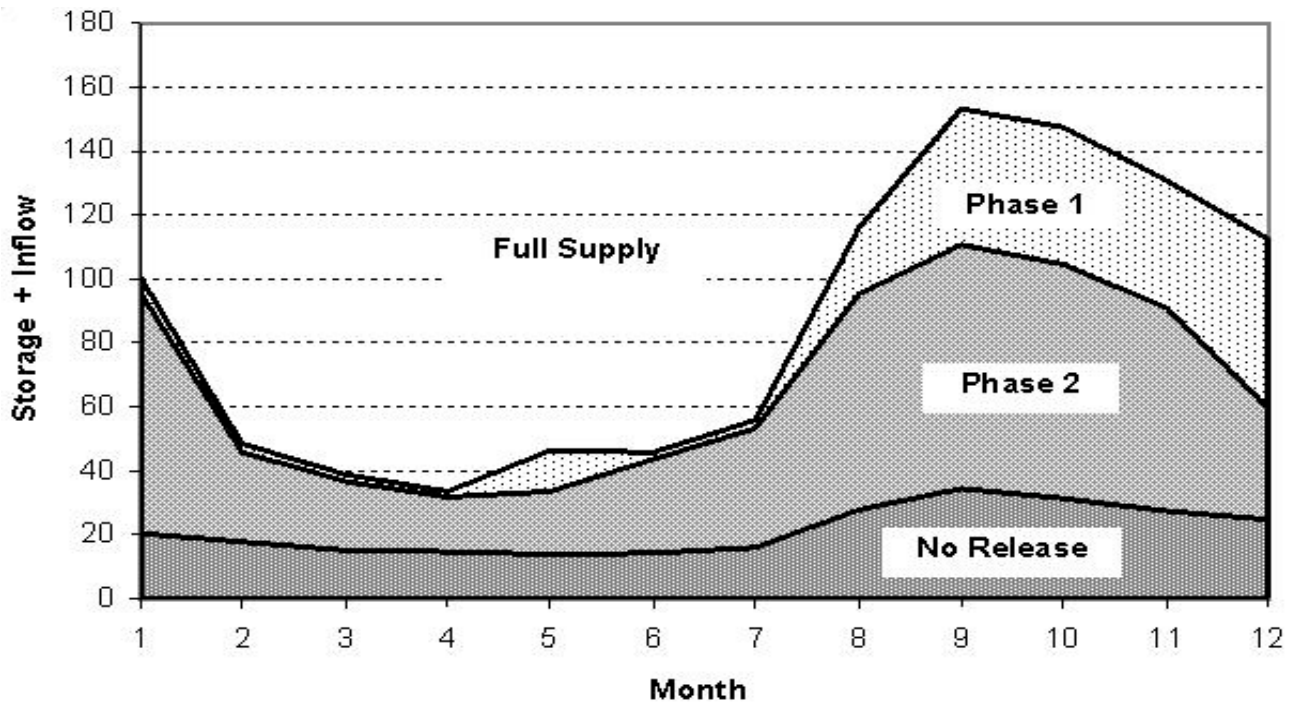


Figure 5. Hedging rule curves for Karadj reservoir.

TABLE 6. Summary of Results.

Criteria	Yield		SOP		Hedging 3-yr		Hedging 2-yr		Hedging 1-yr	
	His.*	Arti.**	His.	Arti.	His.	Arti.	His.	Arti.	His.	Arti.
Freq. of Full	286	310	417	406	394	391	398	392	415	409
Freq. of Phase 1	185	157	-	-	39	40	20	26	15	20
Freq. of Phase 2	4	6	-	-	46	45	60	58	41	40
Time Reliability	0.60	0.65	0.87	0.84	0.82	0.81	0.83	0.82	0.86	0.85
Quantity Reliability	0.91	0.92	0.95	0.94	0.94	0.94	0.95	0.94	0.95	0.94
Resiliency (%)	9.61	7.98	3.01	2.56	4.81	5.03	3.46	3.63	3.90	4.23
Avg. Vulnerability	3.14	3.07	1.71	2.03	2.04	2.35	1.95	2.25	1.77	2.09
Max. Vulnerability	17.8	21.2	29.0	28.3	23.0	26.4	25.1	30.9	30.2	36.4
Avg. annual Spill	5.24	4.88	0.00	0.00	4.13	3.69	4.05	4.12	3.88	3.97
Avg. Annual Deficit	3.1	3.1	1.7	2.0	2.0	2.3	1.9	2.25	1.8	2.09
% of Time Full	14.4	15.1	9.0	12.2	11.5	10.7	11.0	12.8	9.0	12.3
% of Time Empty	0.0	0.1	13.1	15.6	0.0	0.0	0.0	0.0	0.0	0.1
Avg. Monthly	124	127	96	98	109	107	106	108	99	101
Final Storage	156	112	147	66	156	95	156	85	148	78

\* historical data.      \*\* artificial data.

most aspects. SOP also faces more empty reservoir storages than any other model.

Although the 3-year hedging rule indicates slightly lower frequencies of full success, however

its maximum vulnerability is much less than SOP. The results also indicate that as we shift from 3-year to 1-year modeling of droughts we slightly gain more frequency of success, however the systems maximum vulnerability rises greatly. Although not presented in here, both models of synthetic data generation show similar results. The values indicated in the Table are the average of all 20 series.

#### 4. CONCLUSIONS

The hedging model as developed by Shih and Reville [6] is not suitable for practical applications. It requires supercomputers to run. A simple method is developed that greatly reduces computer execution time and memory requirements. The proposed method allows the model application with the publicly available computers. It also makes further increase in the number of hedging levels or extension of time horizon practically possible. It was also compared with the original model based on the example from Shih and Reville [6]. The results indicated higher frequencies of full success by the new method.

To further illustrate the method, it was applied to the Karadj reservoir system in Iran. The results show that a 3-year hedging rule has slightly lower

frequency of full release commitment, however its maximum vulnerability is much less than SOP. When compared to the yield model, its reliability is much higher. The yield model has the lowest maximum vulnerability among all of the models experienced in this paper.

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