VIBRATION ANALYSIS OF ROTATING SHAFT WITH LOOSE DISK

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Abstract In this paper energy method is used to calculate rotor response with loose rotating disk on it. System equation of motion is obtained based on energy method and Lagrange equation. Mathematical modeling of loose disk in a rotor bearing system has resulted in terms similar to unbalance and gyroscopic effect in the system equation of motion. The effect of loose disk axial position and orthotropic bearing has been considered in this investigation. By assuming that shaft and loose disk are always in contact, the results of these study shows that clearance between loose disk and shaft, shaft speed, mass and mass moment of inertia of disk have a major effect on a rotor response and beating phenomena.

Key Words Loose Rotating Part, Rotor Dynamics, FEM, Machine Diagnostic

چکیده در این مقاله روش انرژی برای محاسبه پاسخ روتور با دیسک لق دوار واقع بر آن بکار گرفته شده است. معادله حرکت با استفاده از روش انرژی و معادله لاگرانژ بدست آمده است. نتیجه مدل ریاضی دیسک لق در یک سیستم روتور – یاتاقان پیدایش ترمهایی شبیه نامیزانی و اثر ژیروسکوپیک درمعادله حرکت سیستم می‌باشد. اثر موقعیت طولی دیسک لق و یاتاقانهای نامتقارن (اورتوتروپیک) در این تحقیق در نظر گرفته شده است. با فرض تماس دائم شفت و دیسک لق، نتایج مقاله نشان می‌دهد که تلرانس لقی بین دیسک و شفت، سرعت شفت، جرم و ممان اینرسی جرمی دیسک اثر عمده‌ای در پاسخ روتور و پدیده ضربان دارا می‌باشند.

1. INTRODUCTION

Eventual involvement in rotor bearing systems is one of the machine common malfunctions. Loose disks, Looseness in thrust collars on rotating shafts, and mechanical looseness in pedestal or bearing housing are the main causes for eventual involvement.

The supper harmonics of shaft rotational speed in frequency analysis of the system response, and high phase change when measuring in different points on the system, are used to detect this malfunction [1].

Muszyneska [2] studied the effect of loose rotating parts on the dynamic behavior of rotor bearing systems. An Experimental model along with a theoretical approach was used in the investigation. The experimental and analytical results obtained by Muszyneska show the existence of subsynchronous self-excited vibrations with frequencies related to shaft rotating speed.

Muszyneska and Goldman [3] investigated the dynamic behavior of rotor-bearing-stator systems with stationary or rotational joint looseness. They used chaos theory for studying the machine behavior due to mechanical looseness and concluded that harmonic and sub harmonic responses, as well as chaotic patterns of vibrations are the main characteristics of such systems.

Goldman and Muszyneska [4] developed an analytical algorithm for investigating local nonlinear effects in rotor systems. They used a specially developed variable transformation which smoothes discontinuities, and then applied an averaging technique. Their results show good agreement with experimentally observed typical behavior and orbits

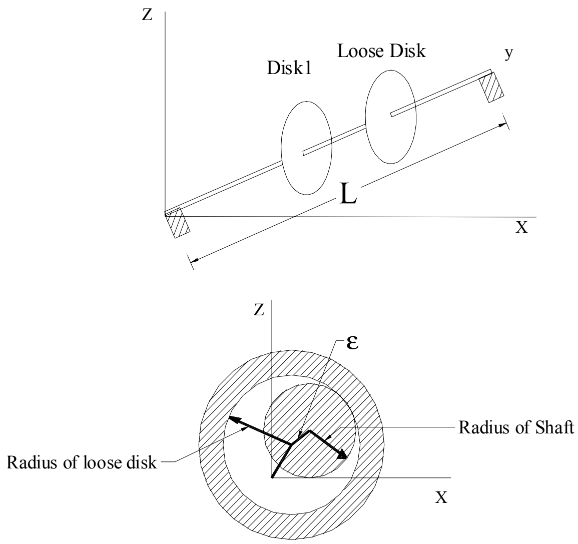


Figure 1. Rotor with loose disk.

of rubbing rotors.

In this paper, the energy equation is used to calculate the system response due to loose rotating parts. By using this method any axial position for loose item, and isotropic or orthotropic supports can be handled.

2. MATHEMATICAL MODEL OF ROTOR PARTS

In the study by Muszyneska [2] the system with loose rotating part was divided into a simple rotor and a loose part. The translatory equation of motion for rotor and loose part in their coordinate systems was derived.

The normal force between loose part and rotor can be calculated from rotational equation of motion of loose Part. Finally, a geometric constraint is used for deriving the equation of motion of system with loose part in a constant rotating speed.

In this investigation the governing equation of bearing, shaft, and disk are calculated. For a rigid disk with small rotation about x and z-axis the kinetic energy is as follows:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left(I_{Dx}\omega_x^2 + I_{Dz}\omega_z^2 + I_{Dy}\omega_y^2\right) \tag{1}$$

For a symmetric disk with constant rotative speed and $I_{Dx} = I_{Dz}$ one has:

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2} I_{Dx}\left(\dot{\theta}^2 + \dot{\psi}^2\right) + \frac{1}{2} I_{Dy}\left(\omega^2 + 2\omega\dot{\psi}\theta\right) \tag{2}$$

The term $\frac{1}{2} I_{Dy}\omega^2$ represents the energy of rotating disk and $I_{Dy}\omega\dot{\psi}\theta$ is due to Coriolis Effect. The energy of a rotating shaft is the sum of kinetic and strain energies. The kinetic energy of a shaft can be calculated by integrating the kinetic energy of a disk over the shaft length. For a shaft with length L this equation is:

$$T_s = \frac{\rho s}{2}\int_0^L \left(\dot{u}^2 + \dot{w}^2\right) dy + \frac{\rho I}{2}\int_0^L \left(\dot{\psi}^2 + \dot{\theta}^2\right) dy + \rho I L \omega^2 + 2\rho I \omega \int_0^L \dot{\psi}\theta \, dy \tag{3}$$

By neglecting the axial force effect and for small deflection, the shaft strain energy becomes:

$$U_s = \frac{EI}{2}\int_0^L \left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)^2\right] dy \tag{4}$$

A linear bearing is assumed in the system and the bending effect is neglected. The virtual work of the forces acting on the shaft in bearing can be written in matrix form as:

$$\begin{Bmatrix} F_u \\ F_w \end{Bmatrix} = -\begin{bmatrix} K_{xx} & K_{xz} \\ K_{zx} & K_{zz} \end{bmatrix}\begin{Bmatrix} u \\ w \end{Bmatrix} - \begin{bmatrix} C_{xx} & C_{xz} \\ C_{zx} & C_{zz} \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{w} \end{Bmatrix} \tag{5}$$

3. CALCULATION FOR A SIMPLE MODEL

A simple symmetric rotor of length L with two simply supported ends, a symmetric disk and mass unbalance at $y=L_1$ is considered. The displacements in x and z directions, and the angular displacements about x and z axis are considered in the following form.

$$u(y,t) = f(y)q_1 \qquad \theta = \frac{\partial w}{\partial y} = g(y)q_2$$

$$w(y,t) = f(y)q_2 \qquad \psi = -\frac{\partial u}{\partial y} = -g(y)q_1 \tag{6}$$

The second derivatives of the displacement for the strain energy is:

$$\frac{\partial^2 u}{\partial y^2} = h(y)q_1$$
$$\frac{\partial^2 w}{\partial y^2} = h(y)q_2 \tag{7}$$

By using Equations 6 and 7 the energy equations for disk and shaft can be calculated as:

$$T_D = \frac{1}{2}\left[M_D f^2(L_1) + I_{Dx} g^2(L_1)\right](\dot{q}_1^2 + \dot{q}_2^2) - I_{Dy}\omega g^2(L_1)\dot{q}_1 q_2 + \frac{1}{2} I_{Dy}\omega^2 \tag{8}$$

$$T_s = \frac{\rho s}{2}\int_0^L f^2(y)dy(\dot{q}_1^2 + \dot{q}_2^2) + \frac{\rho I}{2}\int_0^L g^2(y)dy(\dot{q}_1^2 + \dot{q}_2^2) - 2\rho I\omega\int_0^L g^2(y)dy\dot{q}_1 q_2 + \rho I\omega^2 L \tag{9}$$

$$U_s = \frac{EI}{2}\int_0^L h^2(y)dy(q_1^2 + q_2^2) \tag{10}$$

The force component in the support can be written as:

$$\delta W = F_1(t) f(L_2)\delta q_1 + F_2(t) f(L_2)\delta q_2 \tag{11}$$

By using the Lagrange equation, the system equation of motion can be written in the following form:

$$\frac{d}{dt}\left(\frac{\partial(T_D + T_s)}{\partial \dot{q}_1}\right) - \frac{\partial(T_D + T_s)}{\partial q_1} + \frac{\partial u_s}{\partial q_1} = F_{q1}$$
$$\frac{d}{dt}\left(\frac{\partial(T_D + T_s)}{\partial \dot{q}_2}\right) - \frac{\partial(T_D + T_s)}{\partial q_2} + \frac{\partial u_s}{\partial q_2} = F_{q2} \tag{12}$$

4. LOOSE DISK

A loose disk at distance L_3 is considered on the shaft. By using Equations 2 and 12, the equation of motion of the loose disk in its own coordinate system can be written in the following form:

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{Bmatrix} \ddot{q}_1' \\ \ddot{q}_2' \end{Bmatrix} + \begin{bmatrix} 0 & -G \\ G & 0 \end{bmatrix}\begin{Bmatrix} \dot{q}_1' \\ \dot{q}_2' \end{Bmatrix} + [C]\begin{Bmatrix} \dot{q}_1' \\ \dot{q}_2' \end{Bmatrix} = \begin{Bmatrix} f_1' \\ f_2' \end{Bmatrix} \tag{13}$$

[C] is the damping matrix that includes, the component of damping elements between rotor and loose part. A geometrical constraint is used for coupling the equation of motion of loose disk and rotating shaft [2].

5. FINITE ELEMENT MODEL

The shaft, disk and bearing elements along with mass unbalance are considered in the finite element analysis of the rotor. The shaft is modeled as a beam element with two nodes and four degrees of freedom per node (two translations and two rotations). The shape functions used in this study are those used by Nelson and McVaugh [6,7,8].

Using Lagrange equations for a shaft element one has:

$$\frac{d}{dt}\left(\frac{\partial Ts}{\partial \dot{q}}\right) - \frac{\partial Ts}{\partial q} = (M + M_S)\ddot{q} + c\dot{q} \tag{14}$$

Similarly, by using strain energy the stiffness of shaft can be included. By integrating along the shaft length, the mass, gyroscopic and stiffness matrices can be calculated from equation (14).

TABLE 1. Shaft and Disk Data.

	M_D [kg]	I_{DX} [kgm^2]	I_{Dy} [kgm^2]	L [m]	ρ [kg/m^3]	R_{out} [m]	R *in* [m]	ν	E [N/m^2]	M *u* [kg]	r_u [m]
Disk	16.47	9.427 e-2	0.1861	-	-	-	-	-	-	-	-
Shaft	-	-	-	0.4	7800	0.01	0	0.3	2e11	-	-
Mass unbalance	-	-	-	-	-	-	-	-	-	10^{-4}	0.015

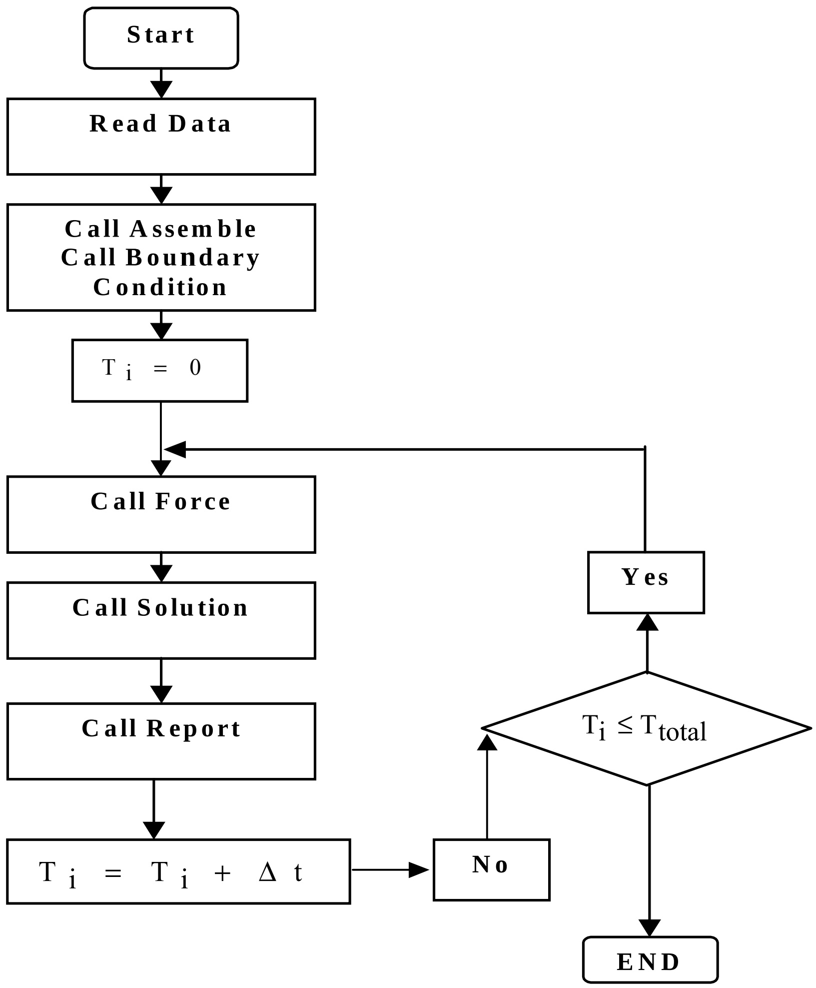


Figure 2. Flow chart used for FEM analysis of rotor with loose disk.

In this study a rigid disk is positioned at a given node on a shaft element. Each disk has four degrees of freedom. Then the mass and gyroscopic effects of the disk are obtained by using Lagrangian equation.

At the bearing locations, the effects of slopes are omitted and only the forces are included in the

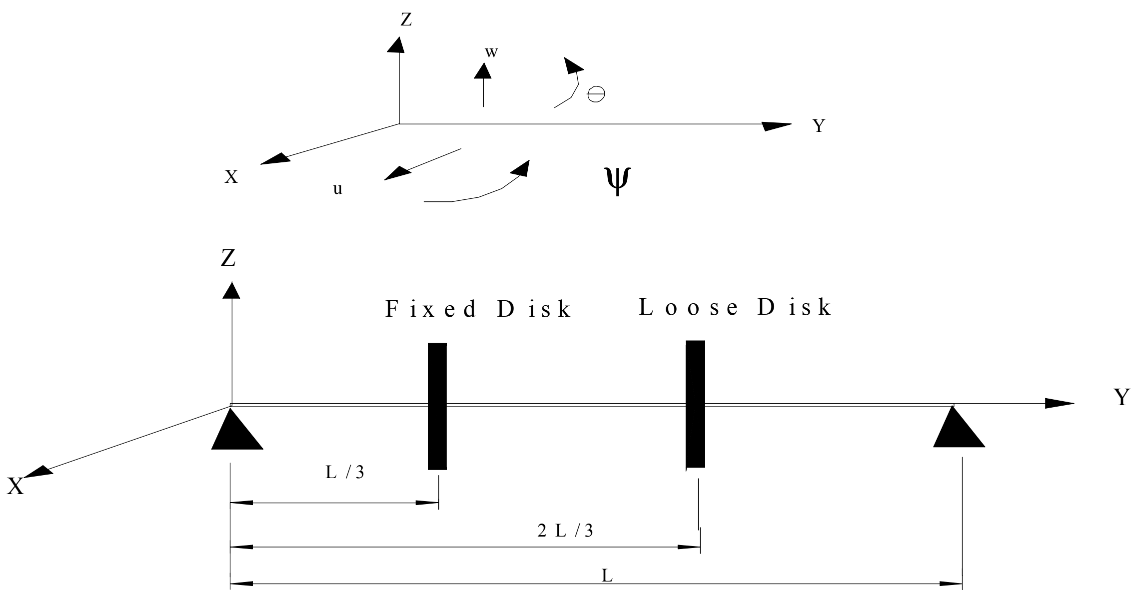


Figure 3. Rotor-Bearing system with loose disk.

equations.

The general expression of forces due to mass unbalance is:

$$\begin{Bmatrix} f_u \\ f_w \end{Bmatrix} = m_u r_u \omega^2 \begin{Bmatrix} \sin(\omega t + \alpha) \\ \cos(\omega t + \alpha) \end{Bmatrix} \tag{15}$$

The total equation of motion for the rotor bearing system will be:

$$M\ddot{q} + C\dot{q} + k\dot{q} = F(t) \tag{16}$$

In the presence of mass unbalance and loose disk, equation (16) can be derived and solved in time domain by using Newmark-β method. The algorithm is shown in Figure 2.

6. ANALYSIS FOR A SIMPLE ROTOR WITH A LOOSE DISK

For investigating the effects of mass unbalance and loose disk on the rotor bearing response a model shown in Figure 3 is considered in this study. The numerical data for this model is given in Table 1. The second disk as a loose disk is considered as the same disk on the system.

Now by using Equations 8, 9, and 10 the kinetic and strain energy of shaft, disk, and mass unbalance is obtained as follows:

$$\begin{aligned} T_D &= 6.902(\dot{q}_1^2 + \dot{q}_2^2) - 2.87\,\omega \dot{q}_1 q_2 \\ T_s &= 0.2454(\dot{q}_1^2 + \dot{q}_2^2) - 1.512e-3\omega\, \dot{q}_1 q_2 \\ T_u &= 1.299 \times 10^{-5} \omega(\dot{q}_1 \cos \omega t - \dot{q}_2 \sin \omega t) \\ U_s &= 5.977 \times 10^5 (q_1^2 + q_2^2) \end{aligned} \tag{17}$$

The system equation of motion can be given as:

$$\begin{bmatrix} 14.29 & 0 \\ 0 & 14.29 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \left[\begin{bmatrix} 0 & -2.871\,\omega \\ 2.871\omega & 0 \end{bmatrix} + \underline{[C_e]} \right] \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} 1.195e6 & 0 \\ 0 & 1.195e6 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = (1.299 \times 10^{-5} \omega^2) \begin{Bmatrix} \text{Sin}\,\omega\,t \\ \text{Cos}\,\omega\,t \end{Bmatrix} \tag{18}$$

The underlined matrix C_e in Equation 18 is due to the external damping that acts on the rotor in the loose disk location. Now one can obtain the equation of motion for a loose disk located at $L_3 = \frac{2}{3}L$ as follows:

$$\begin{bmatrix} 13.806 & 0 \\ 0 & 13.806 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1' \\ \ddot{q}_2' \end{Bmatrix} + \begin{bmatrix} 0 & -2087\,\omega' \\ 2.87\,\omega' & 0 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1' \\ \ddot{q}_2' \end{Bmatrix} - [C] \begin{Bmatrix} \ddot{q}_1' \\ \ddot{q}_2' \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{19}$$

where ω' is the constant rotative speed of the loose disk. The following geometrical constraint is used to convert the prime system on the loose disk to the fixed frame:

$$\begin{Bmatrix} q_1' \\ q_2' \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} - \varepsilon \begin{Bmatrix} \sin \omega' t \\ \cos \omega' t \end{Bmatrix} \tag{20}$$

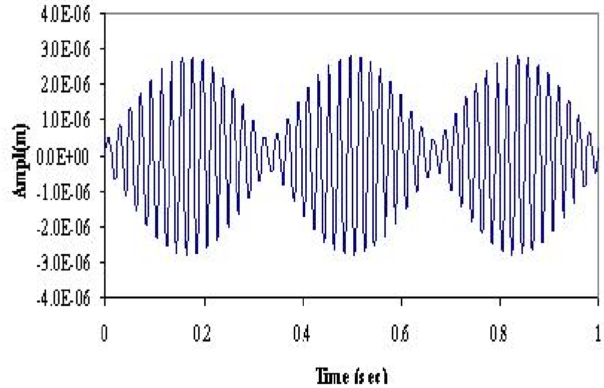
ε is the clearance between the loose disk and the rotating shaft and considered 1.0e-6 (m). By calculating the first and second derivatives of Equation 20, and then substituting into Equation 19 one can obtain the equation of motion for loose disk in fixed reference system as follows:

$$\begin{bmatrix} 13.809 & 0 \\ 0 & 13.809 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} 0 & -2.87\omega' \\ 2.87\omega' & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + [C] \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} = -\varepsilon(13.806)\omega'^2 \begin{Bmatrix} \mathrm{Sin}\omega' t \\ \mathrm{Cos}\omega' t \end{Bmatrix} - \begin{Bmatrix} (-2.807\varepsilon\omega')\mathrm{Sin}\omega' t \\ (2.807\varepsilon\omega')\mathrm{Cos}\omega' t \end{Bmatrix} \tag{21}$$

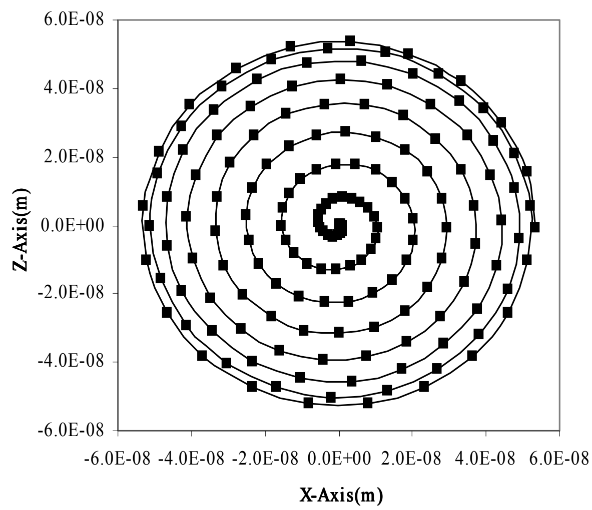
The equation of motion for rotor - bearing system with mass unbalance and loose rotating disk is the sum of Equation 21 and 18 as follows:

$$\begin{bmatrix} 28.096 & 0 \\ 0 & 28.096 \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} 0 & -(2.871\omega + 2087\,l\omega') \\ 2.871\omega + 2.871\omega' & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix} + \begin{bmatrix} 1.195\times10^6 & 0 \\ 0 & 10195\times10^6 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = (1.229\times10^{-5}\omega^2) \begin{Bmatrix} \mathrm{Sin}\omega t \\ \mathrm{Cos}\omega t \end{Bmatrix} - (13.806\varepsilon\omega'^2) \begin{Bmatrix} \mathrm{Sin}\omega' t \\ \mathrm{Cos}\omega' t \end{Bmatrix} - \begin{Bmatrix} -2.807\varepsilon\omega' & \mathrm{Sin}\omega' t \\ 2.807\varepsilon\omega' & \mathrm{Cos}\omega' t \end{Bmatrix} \tag{22}$$

The matrix [C] in both Equations 18 and 21 is the external damping in the loose disk location between the shaft and the loose disk and cancels out when adding Equations 18 and 21. It is clear from Equation 22 that the effects of loose disk is an additional mass unbalance and a forcing that depends on the rotative speed of the loose disk. The Equation of motion 22 can be solved separately for each forcing function.



(a)



(b)

Figure 4. (a) System response due to unbalance and gyroscopic effect and (b) shaft orbit for analytical solution.

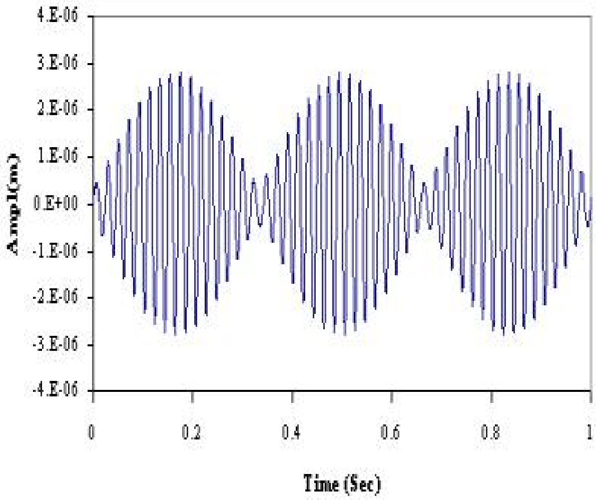


Figure 5. The system response due to unbalance effect for analytical solution.

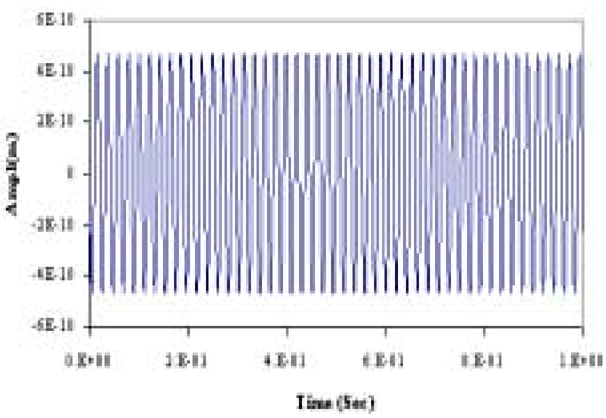


Figure 6. System response due to gyroscopic effect.

7. SOLUTION OF THE EQUATION OF MOTION

Solving Equation 22 needs to superpose the results for three parts including initial mass unbalance, gyroscopic effect and additional mass unbalance due to loose disk. Figure 4a shows the time response for this case from zero to one second by 0.001 second time step. The rotative speed of shaft and loose disk are 50Hz and 47Hz respectively and the clearance between loose disk and shaft is 1.e-6 (m). Shaft orbit for this example is given in Figure 4b. In Figure 5 the gyroscopic effect of loose disk is neglected. From Figure 5 one can see the negligible effect of gyroscopic terms for this example. For comparing gyroscopic and mass unbalance effects the system response are shown in Figures 5 and 6 separately. It can be seen that the gyroscopic effect is less than unbalance response in this example. A finite element model with only three elements shown in Figure 7 is used to study the effect of loose disk at Node 3. The time responses for both unbalance and combination of unbalance and gyroscopic effect are shown in Figures 8 and 9 on Node 2. From these two figures, one can conclude that the gyroscopic term has no major effect on the system response. In these figures, the response for Node 2 has been calculated. Comparing Figures 8a and 9 with Figures 4a and 5 one can see an excellent agreement between the results of analytical and finite element methods. However, Figures 4a and 5 are the results of analytical solution with two degrees of freedom while Figures 8a and 9 are the results of finite element model with 4 nodes. This will result in a slightly different shape of results at different locations.

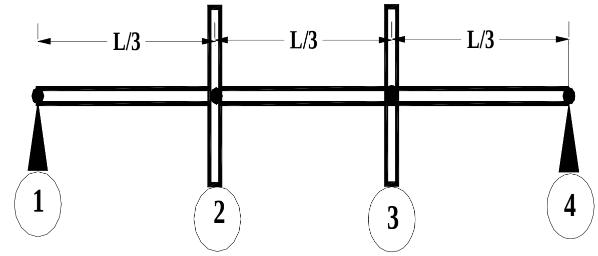
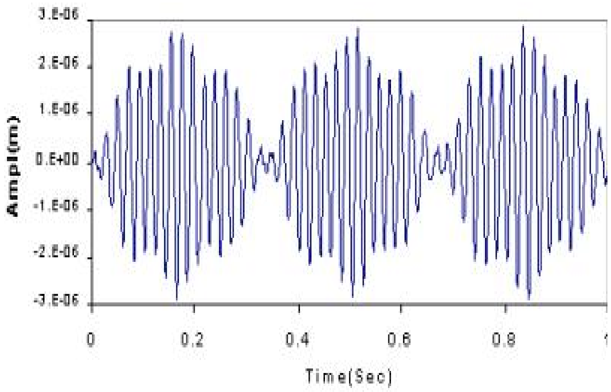


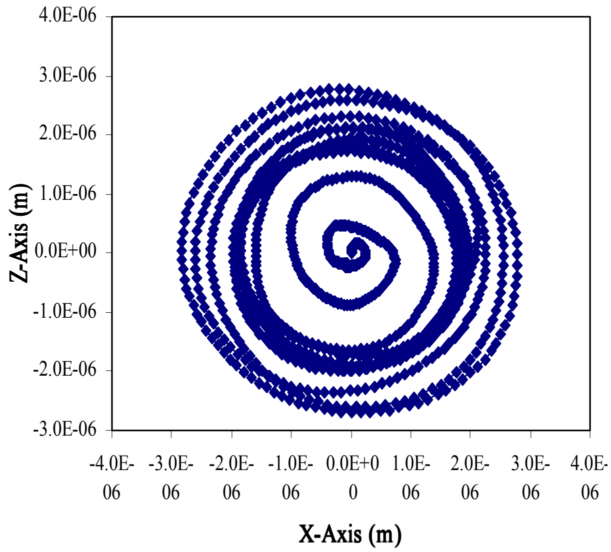
Figure 7. Finite element model of an unbalance rotor with loose disk.

8. CONCLUSION

The energy method is used for studying the effects of loose disk on rotor response. The rotor has been divided into three elements including supports or bearings, disks, and shafts. The energy equation for each element is written in the fixed coordinate system. By implementing the Lagrange equation,



(a)



(b)

Figure 8. (a) System response due to unbalance and gyroscopic effects at Node 2 for numerical solution; (b) shaft orbit for numerical solution at Node 2.

system equation of motion is derived. A loose disk is considered at an arbitrary position along the shaft. Similar to a fixed disk on shaft, the equation of motion for loose disk is written in its plane. By assuming that the loose disk is in contact with shaft during rotation, a geometrical constrain is used for converting the equation of motion of loose disk to

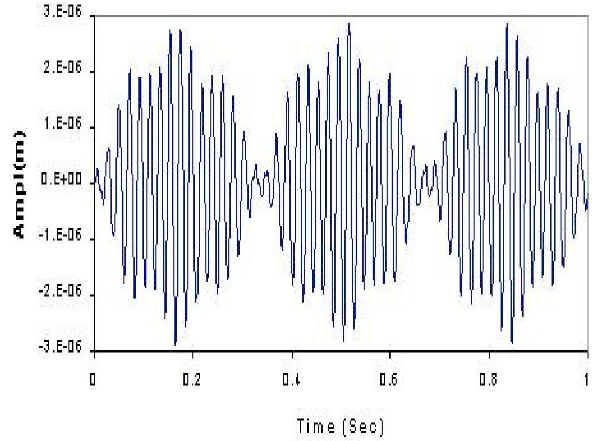


Figure 9. System response due to unbalance at Node 2 for numerical solution.

the fixed reference system. During this transformation, it has been observed that loose disk on a shaft has similar terms as gyroscopic effect and unbalance.

Assuming constant speed for loose disk and shaft results of this study shows that rotor response and beating phenomena are a function of measurement location, loose disk mass and inertia, $\frac{\omega}{\omega_L}$ ratio, and clearance between loose disk and shaft.

9. ACKNOWLEDGMENT

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10. NOMENCLATURE

C_{ij}	Damping in ij direction
D	Disk diameter
E	Young modulus
$F_{u,w}$	Generalized force component in u and w directions

[G]	Gyroscopic matrix
I	Area moment of inertia about neutral axis
I_{ij}	Mass moment of inertia in ij direction
K_{ij}	Stiffness in ij direction
L	Length
M_D	Mass
m_u	Mass of unbalance
Q	General displacement coordinates
$q_{1,2}$	Generalized independent coordinate system
R_{in}	Inner radius
R_{out}	Outer radius
R_u	Radius of unbalance mass
r_u	Unbalance mass radius
s	Cross sectional area
S	Shaft
t	Time (second)
u,v,w	Displacement in x, y and z directions
.	Derivatives with respect to time
ω	Rotational frequency of shaft
ω	Rotational frequency of loose disk
ε	Clearance between loose disk and rotating shaft
ν	Poisson ratio
ψ, θ, ϕ	Rotation about z, x and y directions

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