RESEARCH NOTE

A NEW APPROACH IN PRELIMINARY DESIGN OF CLOSED LOOP SOLAR THERMAL SYSTEMS

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Abstract In this paper, a model for closed loop solar system is presented and an attempt is made to generalize the model to be utilized for primary design of any solar active thermal system. This model may be used for systems in which gas or a liquid are fluids that flow. Two new parameters, namely, the system heat delivery factor and the system heat absorption factor are introduced in the model. These two factors are fully discussed and some equations are developed for their determination.

Key Words Solar Heating System, Solar Radiation, Closed Loop

چکیده در این مقاله، یک مدل برای سیستم های بسته خورشیدی بیان شده است و سعی شده تا مدل به اندازه کافی عمومیت داشته باشد تا برای طراحی اولیه سیستم های گرمایش خورشیدی فعال مورد استفاده قرار گیرد. این مدل می تواند برای سیستم هایی با سیال عامل مایع یا گاز به کار رود. توسط این مدل، دو پارامتر جدید ضریب مصرف گرمایی سیستم و ضریب جذب گرمایی سیستم نیز معرفی شده اند، این دو پارامتر بطور کامل در مقاله مورد بررسی قرار گرفته اند و معادلاتی در این مورد بدست آمده اند که مقادیر این ضریب را محاسبه می کنند.

1. INTRODUCTION

Since 1970, there has been a surge of interest and activity in solar heating systems and many thousands of active systems have been designed, installed and operated [1]. The general method of measuring collector performance employed by the manufactures is basically conducted in three parts; the first is determination of instantaneous efficiency with beam radiation nearly normal to the absorber surface. The second is determination of effects of angle of incidence of the solar radiation. The third is determination of collector time constant, a measure of effective heat capacity. Comparison between various solar collectors can be made by graphs of collector efficiency versus performance coefficient.

For most system, the Hottel-Whillier model [2] is used to determine the efficiency of the solar collector where certain parameters such as number of covered plates, absorption covers, number of pipes, etc. are considered and related to obtain new parameters, such as transmittance-absorptance product, overall heat removal factor and overall loss coefficient. This paper, introduces a new method for calculation of collector efficiency by utilizing the heat exchanger theory.

Yanadori [3] and Gauthier [4] employed this heat exchanger theory to estimate the performance of solar collectors where NTU (Number of Transfer Units) was employed for simplification of their results. The method employed in this paper is different from those mentioned above since the heat transfer surfaces

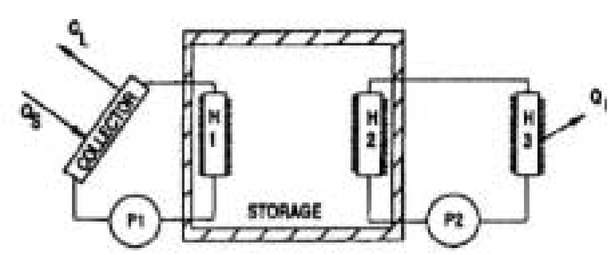

Figure 1. Schematic diagram of thermal system.

are considered independent.

2. ANALYTICAL MODEL

The schematic diagram of a solar thermal system is shown in Figure 1. The pump (P_1) carries a fluid between a collector and a heat exchanger (H_1). The product of mass flow rate and a specific heat ($\dot{m}c_p$) for this circuit is C_1 where the heat exchanger (H_1) transfers heat from fluid into the storage system. The pump (P_2), which receives heat from the storage tank, transfers a fluid from the heat exchanger (H_2) into the heat exchanger (H_3) where heat is utilized for a specific thermal process.

Mass flow-rate specific heat-product for this circuit is C_2. Each of the three heat exchangers employed in this system is specified by a coefficient, E, and each exchanger can be replaced by considering an appropriate effectiveness new factor. The model derived in here is based on the following assumptions:

1. The overall area of collector (A_a) and the area of receiver (A_r) are considered to be equal.
2. The temperature of the storage system is considered to be uniform and homogeneous, even though stratification phenomenon in real storage system occurs.
3. The whole solar heat system receives a constant heat load (Q_p) during process at temperature (T_p) while the system also receives some amount of time dependent solar heat (Q_s) at an ambient temperature (T_a).
4. Solar irradiance (the rate at which radiant energy is incident on a surface, per unit area of surface) is considered to start at sunrise, t = 0, and to reach its maximum value at mid-day and goes back to zero at sunset ($t = t_s$).
5. The pump (P_1) is assumed to be operational from sunrise until the sunset and remains non-operational for the rest of design period.
6. The pump (P_2) can be on or off depending on the receivers heat process (Q_p).
7. The model is developed for an active solar system.

In this model, α' is defined as the overall heat absorption factor and U_L is the overall heat loss coefficient. The collector system is specified by an effective factor, E, where $E = \frac{F_R A_c U_L}{\dot{C}}$ and F_R = heat removal factor which is estimated from Hottel-Whillier model. Here $\dot{C}_1 = (\dot{m}c_p)$ = mass flow rate - heat capacity product. Also, C_s is defined as the product of mass and specific heat of the storage system showing its sensible heat.

3. FORMULATION OF THE ACTIVE SYSTEM

Forced Circulation T_i is assumed to be the collector entrance temperature and the rate of heat transferred from collector to the storage tank. From the energy balance equation for the collector, we have:

$$Q_u = F_R A_c [S - U_L (T_i - T_a)] \qquad (1)$$

It is useful to write the energy balance equation in terms of the heat exchanger effectiveness factor (E_c) as below:

$$E_c = F_R A_c \frac{U_L}{\dot{C}_1} \quad (2)$$

From Equations 1 and 2:

$$Q_u = \dot{C}_1 E_c \left(\frac{S}{U_L} + T_a - T_1\right) \quad (3)$$

Substituting for α', as defined before, into Equation 3:

$$Q_u = \dot{C}_1 E_c \left(\frac{\alpha' A_a}{A_r U_L} q_s + T_a - T_1\right) \quad (4)$$

From the energy balance for heat exchanger (H_1):

$$Q_u = \dot{C}_1 E_1 (T_2 - T_s) \quad (5)$$

where:

E_1 = The heat exchanger effectiveness factor (up to the entrance of the tap).

T_2 = The fluid temperature leaving the storage tank.

T_s = The temperature of the storage tank.

The heat transferred from the collector to the storage tank can be written as:

$$Q_u = \dot{C}_1 (T_2 - T_1) \quad (6)$$

From Equations 5 and 6:

$$Q_u = \dot{C}_1 \left(\frac{E_1}{1 - E_1}\right)(T_1 - T_s) \quad (7)$$

Also, from Equations 7 and 4:

$$Q_u = \frac{1}{R_c}(\alpha' A_a R_L q_s + T_a - T_s) \quad (8)$$

where:

$$R_c = \frac{E_c + E_1 - E_c E_1}{\dot{C}_1 E_c E_1} \quad (9)$$

and

$$R_L = \frac{1}{A_r U_L} \quad (10)$$

For the next step, it is assumed that all heat loss from the storage tank is utilized as a heat process. So, Equation 11 can be developed:

$$(mc_p)_s \frac{dT_s}{dt} = Q_u - L - (UA)_s (T_s - T_a) \quad (11)$$

By assuming $(\dot{m}c_p)_s = C_s$ and $Q_p = L - (UA)_s (T_s - T_a)$, Equation 11 can be expressed as:

$$C_s \frac{dT_s}{dt} = Q_u - Q_p \quad (12)$$

The value of Q_u can be obtained from Equation 8, during the storage time from t = 0 to $t = t_s$. For the rest of the design period (from $t = t_s$ up to $t = t_d$), we have Q = 0 and, therefore, Equation 13 can be written as:

$$C_s \frac{dT_s}{dt} = \begin{cases} \frac{1}{R_c}(\alpha' A_a R_L q_s + T_\infty - T_s) - Q_p \\ -Q_p \end{cases} \quad (13)$$

The first equation in 13 is for $0 \le t \le t_s$ and the second one is for $t_s \le t \le t_d$.

Let's assume that t_o is the minimum temperature of the storage tank, which allows Q_p leaving the storage tank without receiving any heat from other sources except the sun. The energy balance for the heat exchanger H_2 results in:

$$Q_p = \dot{C}_2 E_2 (T_o - T_3) \quad (14)$$

T_3 is the temperature of the fluid that leaves the heat exchanger H_3 and enters the heat exchanger

H_2. Also, from energy balance for the heat exchanger H_3, we have:

$$Q_p = C_2 E_3 (T_4 - T_p) \tag{15}$$

Here T_4 is the fluid temperature leaving the heat exchanger H_2 and entering the heat exchanger H_3. Also, we know that:

$$Q_p = \dot{C}_2 (T_4 - T_3) \tag{16}$$

Employing Equation 16 to eliminate T_4 from Equation 15:

$$Q_p = \frac{\dot{C}_2 E_3}{1 - E_3}(T_3 - T_p) \tag{17}$$

Now, use Equation 17 to eliminate T_3 from Equation 14:

$$Q_p = \frac{1}{R_p}(T_o - T_p) \tag{18}$$

where

$$R_p = \frac{E_2 + E_3 - E_2 E_3}{\dot{C} E_2 E_3} \tag{19}$$

By utilizing the above equations, a period can be designed under steady state conditions in which a constant heat load is undertaken and heat is only supplied from solar energy.

The minimum allowed temperature of the storage tank during design procedure is T_o. So, from Equation 18:

$$T_s(0) = T_p + R_p Q_p \tag{20}$$

$$T_s(t_d) = T_p + R_p Q_p \tag{21}$$

Equation 20 can be employed as an initial condition for Equation 13 and Equation 21. These can be used to determine the maximum amount of heat that the process undergoes (Q_p). Rewriting Equation 21:

$$Q_p = \frac{1}{R_p}[T_s(t_d) - T_p] \tag{22}$$

Now. the following integration variables are defined:

$$H_s = A_a \int_0^s q_s dt \tag{23}$$

$$H_p = \int_0^d Q_p dt \tag{24}$$

$$Y_p = \int_0^s T_p dt \tag{25}$$

$$Y_\infty = \int_0^s T_\infty dt \tag{26}$$

These integration variables can be employed for the following dimensionless parameters:

$$\theta_s = \frac{T_s t_s - \gamma_\infty}{\alpha' R_L H_s} \tag{27}$$

$$\theta_p = \frac{T_s t_s - \gamma_\infty}{\alpha' R_L H_s} \tag{28}$$

$$\phi_s = \frac{A_a q_s t_s}{H_s} + \frac{T_\infty t_s - \gamma_s}{\alpha' R_L H_s} \tag{29}$$

$$\phi_p = \frac{H_p}{\alpha' H_s} \tag{30}$$

$$F_c = \frac{R_L}{R_c} \tag{31}$$

$$F_p = \frac{R_L}{R_p} \tag{32}$$

$$G = \frac{R_L C_s}{t_s} \tag{33}$$

$$\tau = \frac{t}{t_s} \tag{34}$$

$$\beta = \frac{t_d}{t_s} \tag{35}$$

Writing Equations 13, 20 and 22 in dimensionless form, a complete formula can be obtained as follows:

$$G = \frac{d\theta_s}{dr} = \begin{cases} F_c = (\phi_s - \theta_s) - \phi_p \beta \\ \qquad -\phi_p / \beta \end{cases} \tag{36}$$

in which the first term is for $0 \le \tau \le 1$; while the second is for $1 \le \tau \le \beta$.

$$\theta_s(0) = \frac{\phi_p}{\beta F_p} \tag{37}$$

$$\phi_p = \beta F_p [\theta_s(\beta) - \theta_p] \tag{38}$$

4. THE SOLUTION METHOD

Integrating Equation 36 from $\tau = 0$ to τ for $0 \le \tau \le 1$, the following equation is obtained:

$$\theta_s(\tau) = [\theta_s(0) + \frac{\phi_p}{F_c} - \Psi(0)] \exp(\frac{-F_c \tau}{G}) - \frac{\phi_p}{\beta F_c} + \Psi(\tau) \tag{39}$$

In the above equation, $\Psi(\tau)$ is a specific solution for Equation 40:

$$\frac{G}{F_c} \frac{d\Psi}{d\tau} + \Psi = \phi_s(\tau) \tag{40}$$

Using Equations 37 and 40 for $0 \le \Psi \le 1$, one can obtain:

$$\theta_s(\tau) = [\theta_P + \frac{1}{\beta}(\frac{1}{F_P} + \frac{1}{F_c})\phi_P - \Psi(0)]. \mathrm{Exp}(\frac{-F_c \tau}{G}) - \frac{\phi_P}{\beta F_c} + \Psi(\tau) \tag{41}$$

Now integrating Equation 3 from $\tau = 1$ to τ for $1 \le \tau \le \beta$:

$$\theta_s(\tau) = \theta_s(1) - \frac{\phi_P}{\beta G}(\tau - 1) \tag{42}$$

Using Equation 41 for calculating $\theta_s(1)$ when $1 \le \tau \le \beta$, gives:

$$\theta_s(\tau) = [\theta_P + \frac{1}{\beta}(\frac{1}{F_P} + \frac{1}{F_c})\phi_P - \Psi(0)]. \mathrm{Exp}(\frac{-F_c}{G}) + \Psi(1) - \frac{\phi_P}{\beta}(\frac{1}{F_c} + \frac{\tau - 1}{G}) \tag{43}$$

Equation 43 can be employed to evaluate $\theta_s(\beta)$. Equation 38 then becomes:

$$\phi_P = \beta F_P ([\theta_P + \frac{1}{\beta}(\frac{1}{F_P} + \frac{1}{F_c})\phi_P - \Psi(0)]. \mathrm{Exp}(\frac{-F_c}{G}) + \Psi(1) - \frac{\phi_P}{\beta}(\frac{1}{F_c} + \frac{\beta - 1}{G}) - \theta_P) \tag{44}$$

Solving Equation 44 for ϕ_P:

$$\phi_P = F_u(\alpha_s - \theta_P) \tag{45}$$

$$F_u = \beta [\frac{1}{F_P} + \frac{1}{F_c} + \frac{\beta - 1}{G[1 - Exp(-F_c / G)]}]^{-1} \tag{46}$$

$$\alpha_s = \frac{\Psi(1) - \Psi(0)}{1 - \mathrm{Exp}(-F_c / G)} + \Psi(0) \tag{47}$$

Here, $\Psi(\tau)$ is a specific solution of 3. Rewriting Equation 45 in dimensional form, an expression for the whole received heat process during the specified period can be obtained:

$$H_P = F_u(\alpha_s \alpha' H_s - A_r U_L(\gamma_P - \gamma_\infty)) \quad (48)$$

Equation 48 gives two variables for the new system, F_u and α_s, which are the system heat delivery factor and the heat absorption factor, respectively. Equation 46 defines the system heat delivery factor; but the solution for α_s is rather complicated and needs to be explained more.

5. THE SYSTEM HEAT ABSORPTION FACTOR

To obtain a numerical value for α_s, it is necessary to know variation of solar irradiance and ambient temperature with time. By knowing $S(t)$ and $T_a(t)$, from Equation 26, the value of $\phi_s(\tau)$ can be calculated. With these data, Equation 3 can be integrated and then by considering some suitable primary conditions, $\Psi(1)$ is obtainable. By substituting for $\Psi(1)$ and $\Psi(0)$ in Equation 47, α_s can be found. It should be noted that α_s is independent of $\Psi(0)$ and $\phi_s(\tau)$ is unique. So, α_s is only a function of the one-dimensional variable $\frac{G}{F_s}$. The above method can be utilized to calculate α_s numerically, even though it is possible to evaluate α_s analytically if $\phi_s(\tau)$ is in a suitable form. For example, if solar irradiance is in sinusoidal form for $0 \le t \le t_s$, then:

$$q_s(\tau) = q_a \sin(\omega t) \quad (49)$$

t_s is the time in which the solar irradiance approaches zero:

$$\omega = \frac{\pi}{t_s} \quad (50)$$

By substituting from Equation 49 and 50 into Equation 23, one can obtain:

$$H_s = \frac{2A_a q_a t_s}{\pi} \quad (51)$$

Also, from Equation 26, when the ambient temperature is constant:

$$\gamma_\infty = T_\infty t_s \quad (52)$$

By substituting Equations 49, 50, 51 and 52 into Equation 29:

$$\phi_s(\tau) = \frac{\pi}{2}\sin(\pi\tau) \quad (53)$$

Now substituting Equation 53 into Equation 2:

$$\frac{G}{F_c}\left(\frac{d\Psi}{d\tau}\right) + \Psi = \frac{\pi}{2}\sin(\pi\tau) \quad (54)$$

The solution of Equation 55 is:

$$\Psi(\tau) = \frac{m^2\pi\sin(\pi\tau) - \pi^2 m\cos(\pi\tau)}{2(m^2+\pi^2)} + \left[\Psi(0) + \frac{\pi m}{2(m^2+\pi^2)}\right]\mathrm{Exp}(-m\tau) \quad (55)$$

where $m = \frac{F_c}{G}$, $\Psi(1)$ can be calculated $(\tau = 1)$ from Equation 55:

$$\Psi(1) = \frac{\pi^2 m}{2(m^2+\pi^2)}[1+\mathrm{Exp}(-m)] + \Psi(0)\mathrm{Exp}(-m) \quad (56)$$

By employing Equation 56 in Equation 47:

$$\alpha_s(G/F_s) = \frac{\pi^2(G/F_c)[1+\mathrm{Exp}(-F_c/G)]}{2[\pi^2 + (F_c/G)^2][1-\mathrm{Exp}(-F_c/G)]} \quad (57)$$

Equation 57 is used whenever the variation of solar irradiance on collector is in sinusoidal form. Phillipes [5] showed that the results of numerical solution

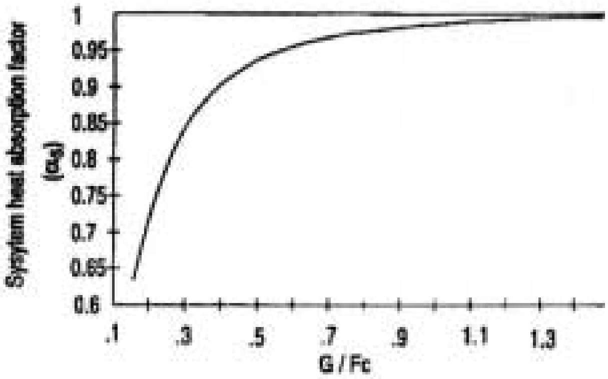

Figure 2. Variation of the system heat absorption factor versus G/Fc.

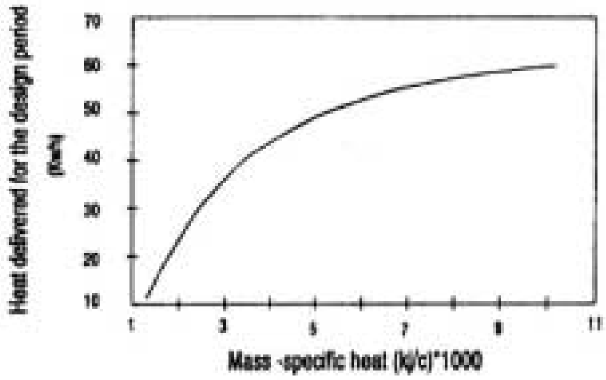

Figure 3. Heat delivered versus mass-specific heat product.

for most of solar irradiance variations are nearly the same as those obtainable from Equation 57. Therefore, if the variable data is not enough, then Equation 57 can be employed to calculate the system absorption factor (Figure 1).

6. PRESENTATION OF THE RESULTS

Utilization equations developed for the system for a given geometry is shown in Figures 2, 3, and 4. Figure 2 shows variation of the system heat absorption factor versus G/F_c which is in dimensionless form. This graph is applicable to all systems of this kind and it shows sharp slope up to the point where $G/F_c = 0.6$ and $\alpha_s = 0.95$. After this point, the slope slows down while the system heat absorption factor approaches its final value, asymptotically. It can be seen that in order to get the value of $\alpha_s \succ 0.95$, G/F_c which is related to mass-specific heat product in the storage tank, should be more than 0.6. Therefore, in design procedure, the geometrical conditions of the system should vary in a way that it achieves the value of $G/F_c \succ 0.6$.

Figure 3 is plotted for a specific system; but it is generally applicable to all similar systems. It shows that if the mass-specific heat of tank product exceeds a certain amount, heat load delivery becomes almost constant. So, for maximum efficiency point and economical optimization point, the mass-specific heat product should be kept at a certain value. This is achieved by the use of a computer program. For example this value is about 9000 KJ/c for a sample design (Figure 3).

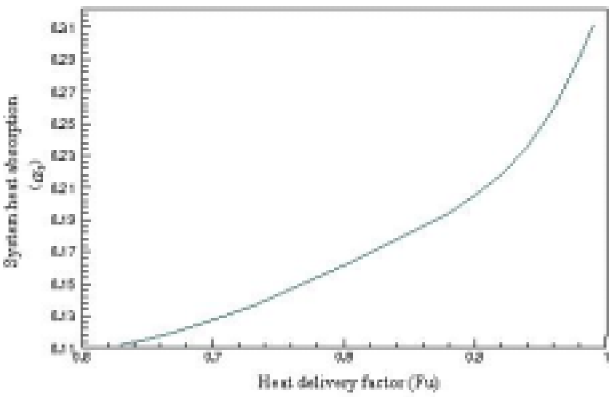

Figure 4. Variation of the system heat absorption factor versus heat delivery factor.

Figure 4 is an alternative way of showing Figure 2 in a dimensionless form. This graph shows the variation of system heat absorption factor versus F_u, expressed by Equation 46. It can be seen that the system heat absorption factor, F_u tends to one and if the value of mass-specific heat product increases more than a certain level, then, F_u variation is not significant. The desired value of mass-specific heat product can be found for this certain value of F_u.

The design procedure adopted here is to achieve the value $G / F_c \succ 0.6$ and then to obtain F_u from Figure 4. Knowing the system heat absorption factor (F_u) and using Equation 48, the value of delivered heat can then be achieved.

7. CONCLUSIONS

A new method was presented for the preliminary design of a closed loop solar thermal system. A computer program, CSHS, was employed to calculate the main parameters affecting the system performance, namely, the system heat absorption factor (α_s), mass-specific heat product, heat delivery factor and G / F_c. The results show that for maximum efficiency, the mass-specific heat product should be kept at a certain value:

$$[G / F_c \succ 0.6 \text{ and } \alpha_s \succ 0.95].$$

The method adopted in this paper, shows an economical quick way of designing an active solar heating system, especially when the delivered heat load is high.

8. NOMENCLATURE

A_a	overall area of collector
A_c	collector area
A_r	area of receiver
A_s	area of storage tank
$C_{1,2,s}$	product of mass and specific heat capacity
c_p	specific heat at constant pressure
$\dot{C}$	mass flow rate-heat capacity product
E	effective factor
E_s	heat exchanger effective factor
F_c	collector heat removal factor
F_p	heat removal factor for design period
F_u	system heat delivery factor
G	dimensionless parameter
$H_{1,2,3}$	heat exchanger
H_p, H_s	integration variables
L	load
$P_{1,2}$	pump
Q_p	constant heat load
Q_s	solar heat
Q_u	rate of heat transfer from collector to storage tank
S	solar irradiance
T_a	ambient temperature
T_i	collector entrance temperature
T_o	temperature during design period
$T_{1,2,3,4}$	fluid temperature
t	time
t_d	design time
t_s	storage time
U_1	overall heat loss coefficient
Y_p, Y_∞	integration variables
α'	overall heat absorption factor
α_s	heat absorption factor
ϕ_p, ϕ_s	dimensionless parameters
θ_p, θ_s	dimensionless parameters

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