

# TRANSIENT TWO-DIMENSIONAL (r-z) CYCLIC CHARGING/DISCHARGING ANALYSIS OF SPACE THERMAL ENERGY STORAGE SYSTEMS

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(Received: October 30, 2000 – Accepted in Revised Form: January 5, 2002)

**Abstract** A two-dimensional transient axi-symmetric model was developed to study the effects of various thermal and geometric parameters on cyclic heating and cooling modes of a phase-change thermal energy storage system. The high-temperature thermal energy storage device utilizes LiH for heat sink applications to store the waste heat generated during power-burst periods. The stored heat is then discharged into space through radiators during the off-peak periods. The enthalpy method is developed to analyze a shell and tube configuration where the phase-change material is contained in an annulus by an inner tube and an outer shell. The development ignores the energy source, pressure variation, and external work done. The Gauss-Seidel iterative method with successive over-relaxation is used to solve the non-linear simultaneous difference equations. The charging period is about 1000 seconds (0.27777 hours). Various effective liquid thermal conductivities have significant effects on heating periods and various inner tube and outer shell radius have significant effects on cooling periods.

**Key Words** Enthalpy Method, Thermal Energy, Charge/Discharge, Radial-Axial, Gauss-Seidel

**چکیده** یک مدل دو بعدی ناپایای متقارن برای مطالعه اثرات پارامترهای مختلف گرمایی و هندسی روی حالات سیکلهای حرارتی و سرمایی یک سیستم ذخیره انرژی گرمایی با تغییر فاز توسعه یافته است. این سیستم از LiH برای کاربرد در چشمه حرارتی به منظور ذخیره اتلاف حرارت تولیدی در مدت زمان توانهای ناگهانی (انفجاری) استفاده می کند. حرارت ذخیره شده در زمانهای غیر ماکزیمم توسط رادیاتورها در فضا تخلیه می شود. روش انتالپی برای آنالیز یک شکل پوسته و لوله ای که مواد تغییر فاز دار آن در یک حلقه بین لوله درونی و پوسته بیرونی قرار می گیرد، استفاده شده است. در این آنالیز از منبع انرژی، تغییرات فشار، و کار خروجی صرف نظر می شود. روش تکراری گاس - سایدل با ضریب همگرایی برای حل سری معادلات دیفرانسیل غیر خطی استفاده شده است. زمان شارژ ۱۰۰۰ ثانیه (۰/۲۷۷۷۷ ساعت) است. به منظور مشاهده تاثیرات توزیع محوری، مدل یک بعدی نیز مطالعه شده است. تغییرات ضریب حرارتی موثر مایع اثرات قابل توجهی در محدوده زمانی سرد کردن دارد.

## 1. INTRODUCTION

Storage techniques that employ phase change material as thermal energy storage medium for space applications are very attractive. Large amounts of heat per unit mass of storage material could be stored or released in the form of latent heat. These storage units may be used for powering devices during the time a satellite is in the eclipse of the sun. Most phase change material problems are

solved using numerical methods.

Rathjen and Jiji [1] studied the two dimensional free boundary problem of solidification of a liquid initially at a uniform temperature Seigel [2] applying conformed mapping made an analysis of the two dimensional solidification of an ingot being cooled and withdrawn vertically downward from a mold consisting of parallel walls of finite length. Shamsunder and Sparrow [3] using the temperature and enthalpy method solved the two-dimensional

solidifying liquid problem that was initially at the fusion temperature. The effect of various geometric and thermal parameters for solving multidimensional phase change heat transfer problem using enthalpy method is used in [4,5]. Sadeghipour and Alborzi [6] used the Galerikin finite element method to study axial conduction effect in the transient laminar freezing of liquids in the convectively cooled tubes. To solve a moving boundary phase change problem on a fixed FEM grid, Voller and Swaminathan [7] general source based method is generalized to calculate convective heat transfer effects. Rady [8] by mathematical modeling of fluid flow during phase change showed the great effect of natural convection in the liquid phase for describing growth velocity in the solid and shape of liquid-solid interface. Banaszek et al. [9] developed a two-dimensional conjugate heat transfer numerical model in the composite domain of a spiral thermal energy storage (STES) mid-height cross section. The model is based on local energy balances over a set of curvilinear control volumes and the enthalpy approach to account for latent heat effect. Domenski et al. [10] by use of experimental set up on STES system determined the quantitative thermal characteristics of the unit and the effect of cyclic transient work of the unit on charging and standstill time on discharging.

The primary objective of this investigation was to study the effects of various thermal and geometric parameters on cyclic heating and cooling modes of a two-dimensional transient axi-symmetric phase-change thermal energy storage model. For this purpose a tube and shell configuration is used. The phase change material (LiH) is contained in an annulus. Heat is supplied/extracted by a secondary fluid at the inner surface of the tube (convection) while the outer shell surface is insulated. To account for the convective effects, higher effective thermal conductivity values were used in the liquid region.

## 2. THEORY AND NUMERICAL METHODS USED

The energy equation is applied once over the complete domain covering both phases, tube and shell materials. The location of the solid/liquid interface of the phase change material is eliminated from the formulation and is obtained as one of the

results after the temperature is found. The development assumes no energy source, no pressure variation, and no external work done on the control volume. Applying the law of conservation of energy to the control volume, the relation between the enthalpy and temperature may be written as;

$$\frac{d}{dt} \iiint_V \rho h \, dv = \iint_A k \nabla T \cdot n \, dA \quad (1)$$

Enthalpy and temperature for different phases of a pure substance are related in the following manner

$$h - h_s = C(T - T_F) = \frac{K}{\rho\alpha} (T - T_F) \text{ for } T < T_F$$

$$h - h_s = \frac{K}{\rho\alpha} (T - T_F) + h_{s1} \text{ for } T > T_F \quad (2)$$

Dimensionless temperature and enthalpy for various ranges of values of enthalpy are related as follow;

$$H = \frac{1}{\rho_s \delta v} \int_V \int \frac{\rho(h - h_s)}{h_{s1}} dv = \frac{K_f (T - T_F)}{\alpha_f \rho_s h_{s1}} f \delta v$$

$$+ \frac{K(T - T_F)}{\alpha \rho_s h_{s1} \delta v} (1 - f) \delta v = \frac{K_f \alpha_s}{\alpha_f K_s} f \theta + \frac{K \alpha_s}{\alpha K_s} (1 - f) \theta \quad (3)$$

that:

$$\theta = \frac{H}{DF - f(DF - DFR)} \text{ for } H < 0 \text{ (solid)}$$

$$\theta = 0 \text{ for } 0 \leq H \leq 1 \text{ (during phase change )} \quad (4)$$

$$\theta = \frac{H + f - 1}{DF - f(DF - DFR)} \text{ for } H > 1 \text{ (liquid)}$$

where

$$DF = \frac{K \alpha_s}{\alpha K_s} \quad DFR = \frac{K_f \alpha_s}{\alpha_f K_s} \quad (5)$$

The above equations are written in finite difference

form. The representation for the solid region is written as

$$\left\{ K_s \frac{(\delta R)^2}{\delta \tau} + \frac{1}{D_{i,k}^{m-1}} \left[ \left(1 + \frac{\delta R}{2R}\right) K_{1i,k}^{m-1} + \left(1 - \frac{\delta R}{2R}\right) K_{2i,k}^{m-1} + \left(\frac{r_1 \delta R}{L \delta Z}\right)^2 K_{5i,k}^{m-1} + \left(\frac{r_1 \delta R}{L \delta Z}\right)^2 K_{6i,k}^{m-1} \right] \right\}$$

$$H_{i,k}^m = K_s \frac{(\delta R)^2}{\delta \tau} H_{i,k}^m + \left(1 + \frac{\delta R}{2R}\right) K_{1i,k}^{m-1} \theta_{i+1,k}^m$$

$$+ \left(1 - \frac{\delta R}{2R}\right) K_{2i,k}^{m-1} \theta_{i-1,k}^m + \left(\frac{r_1 \delta R}{L \delta Z}\right)^2 K_{5i,k}^{m-1} \theta_{i,k+1}^m$$

$$+ \left(\frac{r_1 \delta R}{L \delta Z}\right)^2 K_{6i,k}^{m-1} \theta_{i,k-1}^m$$

and  $\theta_{i,k}^m = \frac{H_{i,k}^m}{D_{i,k}^m}$  for  $H_{i,k}^m < 0$  (6)

For other regions and boundary conditions the governing equations are expressed in a similar manner.

The above non-linear simultaneous finite difference equations are solved using the Gauss-Seidal iterative scheme with successive over-relaxation. When the enthalpy and temperature distributions have been calculated with sufficient accuracy, the position of the solid/liquid interface is determined from the enthalpy distribution.

In order to verify the numerical solutions obtained using the enthalpy approach, an approximated overall balance method was used. At any time the change in internal energy of the system must be equal to the total energy supplied/extracted at the surface up to the corresponding time.

### 3. DISCUSSION OF RESULTS

Numerical results for evaluation of cyclic heating and cooling mode studies of two-dimensional axisymmetric configuration are shown in Figures 1 to.7. In this two-dimensional shell and tube configuration, a secondary fluid supplies heat at the inner surface of the tube. The temperature of the secondary heating fluid is varied from 1150 K at the tube inlet to 1100 K at the tube outlet. For the discharge mode, the secondary fluid temperature is varied from 250 K at the tube inlet to 290 K at

the tube outlet. In this study the system is charged for a period of 1000 seconds and then discharged until all the charged heat is extracted. In all figures where it has not been mentioned, the film heat transfer coefficient is 61325.25 Kj/hr-sq.m-K, liquid effective thermal conductivity is 9.0435 Kj/hr-m-K, initial system temperature is 300 K, tube diameter is 0.05 m, shell diameter is 0.1 m, and tube length is one meter.

Figure 1 shows the charged/discharged-stored heat versus the time for various liquid effective thermal conductivity values. For two-dimensional case these effects for a specific time of charging and discharging are shown in Figures 2 and 3. For very high value of K the rate of heat transfer is so high that at a short period of time of charging the whole solid material of LiH is melted and so, there is no interface radius. Figure 4 shows the effect of different inner tube and outer shell diameters on cyclic heating and cooling heat storage of the system. Increase of volume of phase-change material results in increase of more stored heat and longer time period of discharging. Ignore of axial

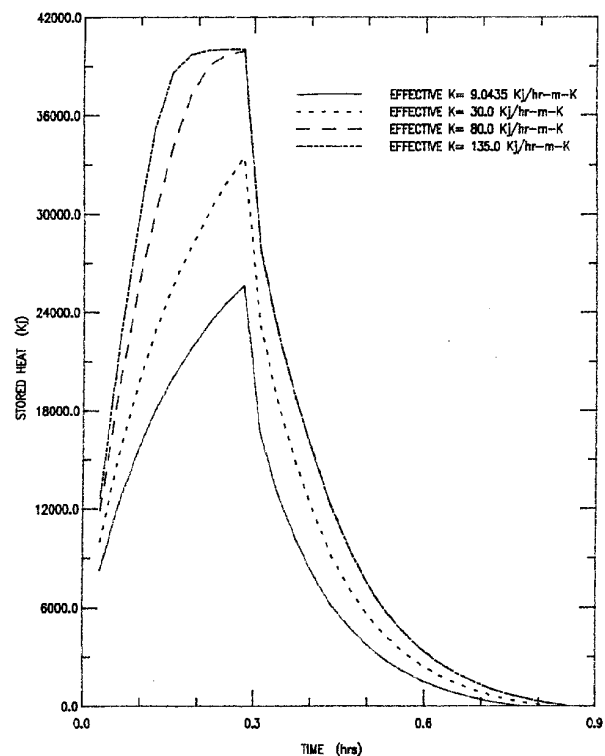


Figure 1. Stored heat vs. time for various liquid effective thermal conductivity values (two-dim.).

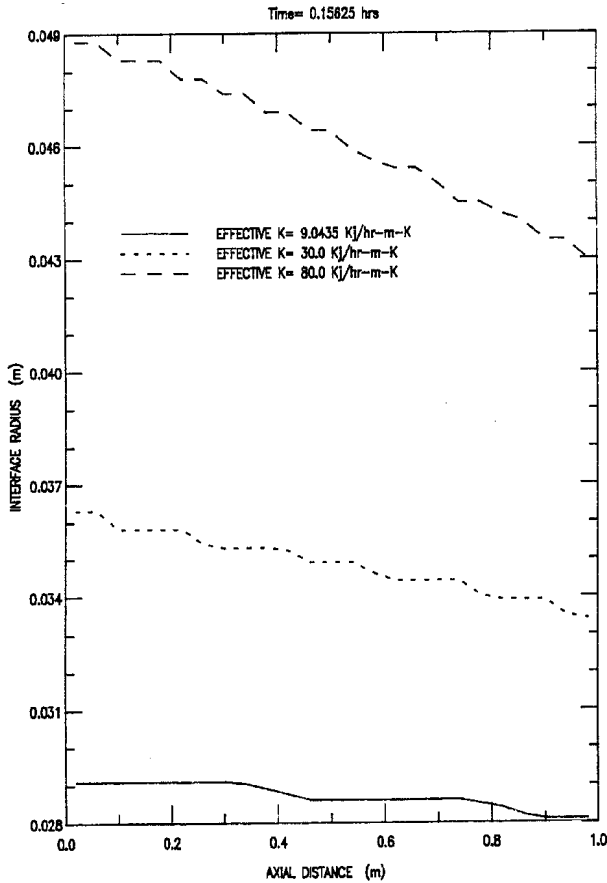


Figure 2. Melting front depth vs. axial distance for various liquid effective thermal conductivity values (two-dim.).

variation make the amounts of stored heat and discharging period higher. For this same case Figures 5, 6, and 7 show axial melting front depth and radial temperature distribution at some specific time of heating and/or cooling period. As shown in Figure 5 the difference in interface radius through axial distance is more obvious at higher times of charging (similar behavior could be seen in [6]). This is because of quicker increase of temperature in phase change material at lower outer diameters (i.e. lower amount of heat stored). Shorter periods of discharging have direct effects on radial temperature distributions as shown in Figure 6. In [12] internal radiation effect on solidification (discharging) is shown. Similar path of the curves as could be seen in Figure 7 are represented which are comparable to last graph. This is mainly due to the similarity in approach, geometry, and boundary conditions used in both cases.

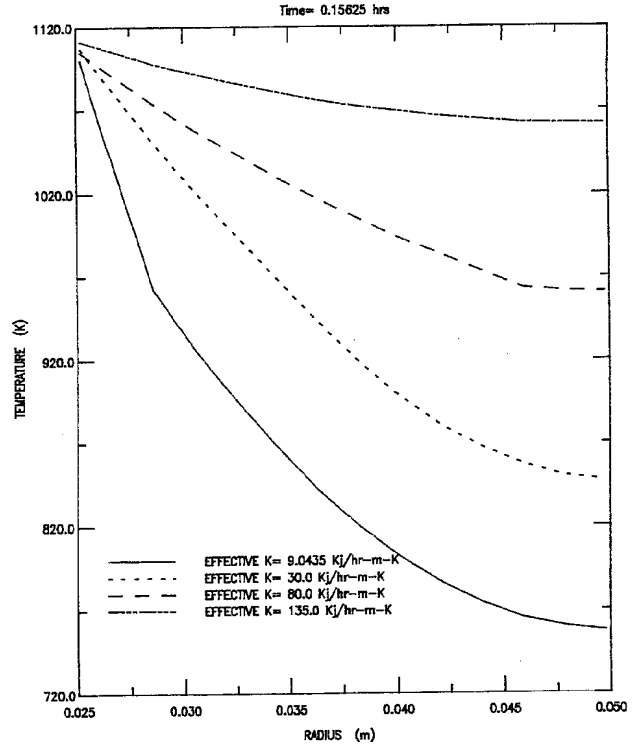


Figure 3. Temperature distribution vs. radial distance at axial dist.= 0.5 m for various effective thermal conductivity values (two-dim.).

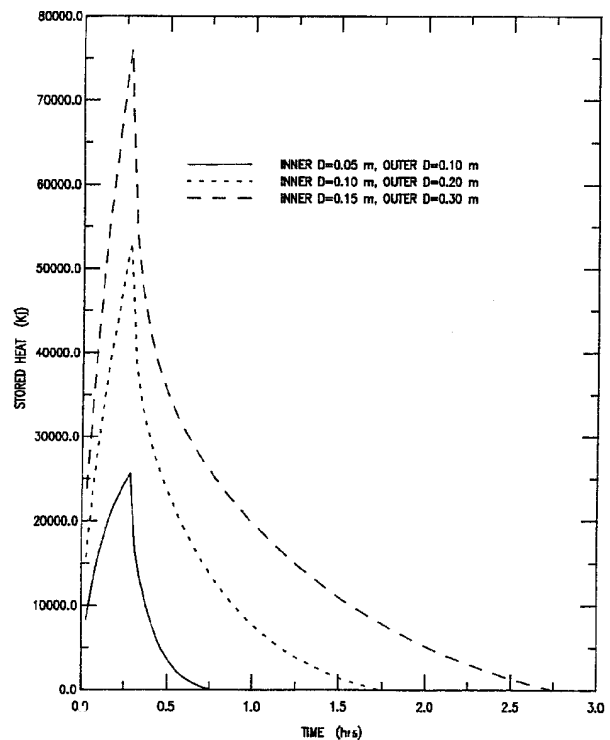


Figure 4. Stored heat vs. time for various tube and shell diameters (two-dim.).

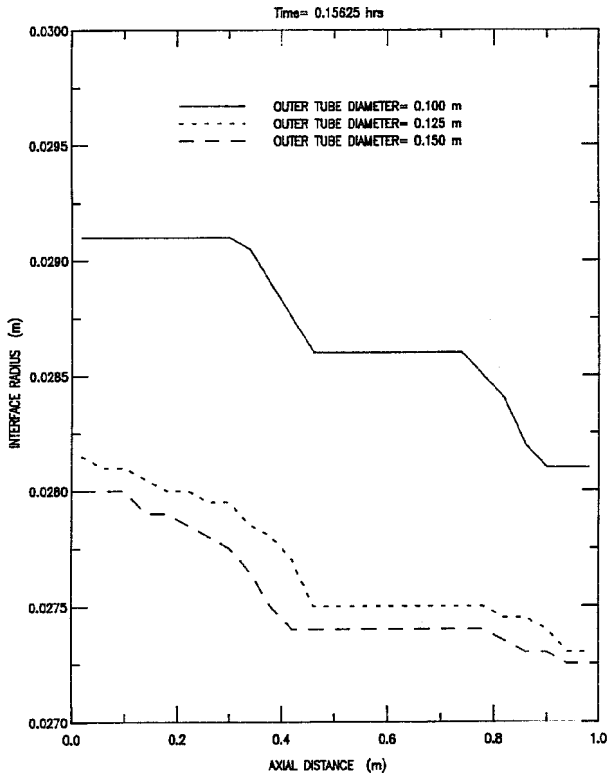


Figure 5. Melting front depth vs. axial distance for various outer tube (shell) diameters (two-dim.).

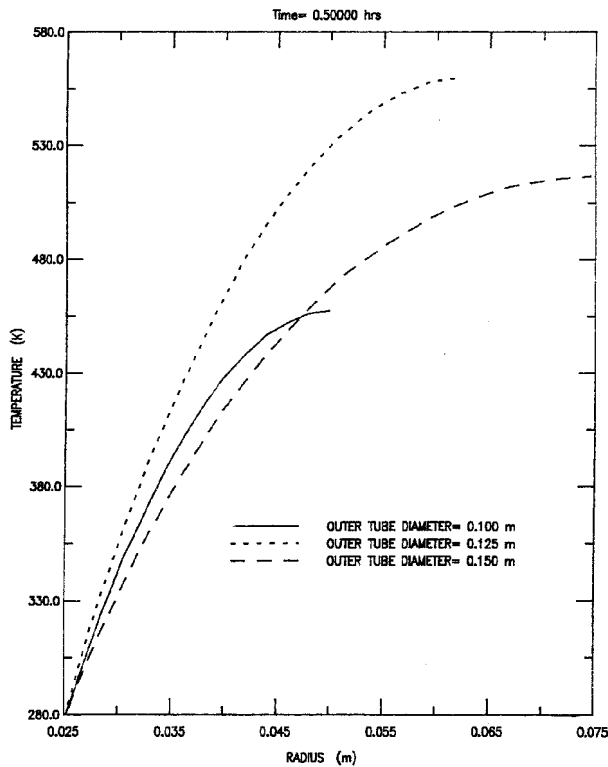


Figure 6. Temperature dist. vs. radial distance at axial dist.=0.5 m for various outer tube (shell) diameters (two-dim.).

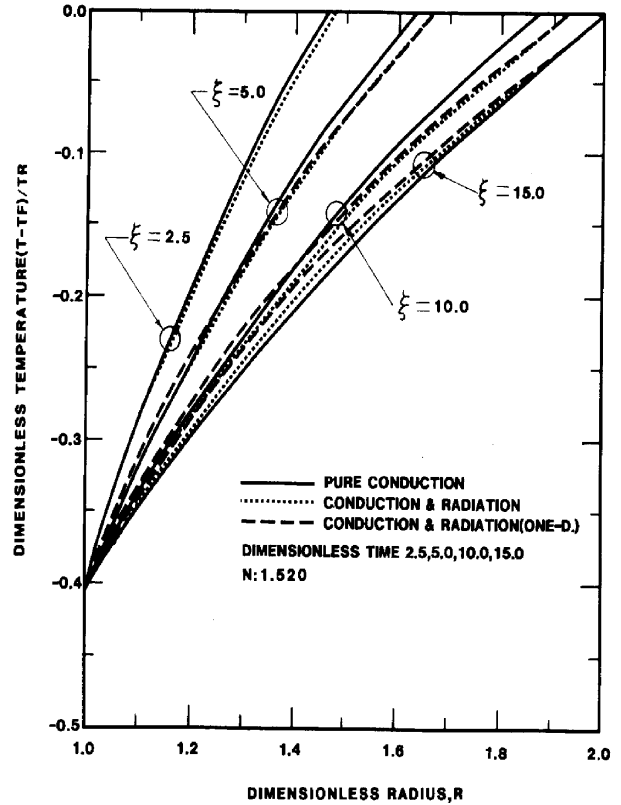


Figure 7. Dimensionless temp. vs. dimensionless radius [12].

#### 4. CONCLUSIONS

An analytical two-dimensional radial-axial model was developed to study the cyclic heating and cooling modes of different geometric and thermal parameters on the performance of a thermal energy storage system. The system is used for space applications with LiH as phase change material. Numerical parametric studies show that:

- Amount of stored heat, charging rates, and axial melting front depth at a given time increase with increasing effective thermal conductivity of melted medium caused by accelerating forces. However, rate of radial temperature distribution is slow.
- The effect of liquid effective thermal conductivity is not very significant during the discharging period, specifically at higher values.
- Amount of stored heat, charging rates, and axial melting front depth at a given time increase with increasing inner tube diameter and/or outer shell radius (i.e. increase of the volume space of phase change material). This would increase the total mass of the system and the space occupied. There might be some optimized values.

- d. Temperature distribution rate at a given charging period and radius is increased with increase of outer shell radius.
- e. The effect of increase of the volume of phase change material is significant and has direct effect on discharging period.
- a. At small time steps of charging, variation of outer shell diameter have insignificant effects on axial melting front depth.

$\alpha_s$	thermal diffusivity of solid LiH
$\theta$	dimensionless temperature
$\theta_I$	dimensionless initial temperature
$\theta_w$	dimensionless tube inner wall temp
$\theta_\infty$	dimensionless secondary fluid temp
$\rho$	density
$\rho_s$	density of solid LiH
$\tau$	dimensionless time

## 5. NOMENCLATURE

A	area
C	specific heat
D	equivalent to DF
f	volume fraction of container material in an element
H	dimensionless enthalpy
h	specific enthalpy
$h_s$	specific enthalpy of solid phase
$h_{sl}$	latent heat of fusion
K	thermal conductivity
$K_f$	thermal conduct. of container material
$K_s$	thermal conductivity of solid phase
$K_1, K_2$	thermal conductivity of surface elements
$K_5, K_6$	at $R = R_i + \delta R/2$ , $R = R_i - \delta R/2$ , $Z = Z_i + \delta Z/2$ , $Z = Z_i - \delta Z/2$ , respectively
k	thermal conductivity
L	axial length
m	time level
n	unit normal vector
R	dimensionless radius
r	radial distance
$r_I$	radius of inner tube
$r_o$	radius of outer shell
$T_F$	fusion temperature of LiH
$T_w$	tube inner wall temperature
$T_\infty$	secondary working fluid temperature
$T_1$	temperature in solid phase
$T_2$	temperature in liquid phase
t	time
V	integration volume
v	volume
Z	dimensionless axial distance
z	axial distance

## Greek Letters

$\alpha$	thermal diffusivity
$\alpha_f$	thermal diffusivity of container material

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