OPTIMUM AGGREGATE INVENTORY FOR SCHEDULING MULTI-PRODUCT SINGLE MACHINE SYSTEM WITH ZERO SETUP TIME

Alireza Haji and Rasoul Haji

Department of Industrial Engineering, Sharif University of Technology
Tehran, Iran, ahaji@sharif.edu - haji@sharif.edu

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Abstract In this paper we adopt the common cycle approach to economic lot scheduling problem and minimize the maximum aggregate inventory. We allow the occurrence of the idle times between any two consecutive products and consider limited capital for investment in inventory. We assume the setup times are negligible. To achieve the optimal investment in inventory we first find the idle times which minimize the maximum aggregate inventory for a given sequence of production runs and for any arbitrary cycle time T . Then, we show that these values of idle times, for the given sequence, are also optimal idle times for any other sequence. The result is an easy-to-apply rule that greatly simplifies the task of scheduling to achieve minimum required investment in inventory.

چکیده در این مقاله با روش دور مشترک برای مساله برنامه ریزی اقتصادی دسته ای، حداکثر موجودی جمعی محصولات کمینه شده است. در این تحقیق زمانهای بیکاری ماشین در بین محصولات و محدودیت سرمایه درگیر در موجودی در نظر گرفته شده و فرض شده است که زمانهای آماده سازی صفر است . برای تعیین مقدار بهینه سرمایه در گیر در موجودی، ابتدا مقادیر زمانهای بیکاری برای حداکثر موجودی جمعی در یک توالی معلوم تولید و یک دور تولید اختیاری بدست آمده است. سپس، نشان داده شده که مقادیر بهینه زمانهای بیکاری، به توالی تولید بستگی ندارد . نتیجه این تحقیق یک قاعده آسان کاربردی ارائه می دهد به طوری که کار برنامه ریزی را برای نیل به کمینه سرمایه مورد نیاز در موجودی به طور قابل توجهی ساده می سازد.

INTRODUCTION

The Economic Lot Scheduling Problem (ELSP) arises frequently in industry and research. The problem is to economically schedule lots (i.e., product runs) of one or more products on one multipurpose machine. The problem of determining economic production quantity (EPQ) for the case of one product is well known. However, in the case of two or more products, treating each item as independent does not work. For, in general, the economic production quantities so determined can not be feasibly scheduled, and the phenomenon of interference will occur sooner or later- that is the facility will be required to produce more than one item at the same time which is physically impossible [9].

There is no universal solution procedure available which solves this problem optimally. This NP- hard problem is notoriously difficult to solve [3,10]. Several different types of approaches, either analytical approach to a restricted problem or heuristic approaches to the entire problem, have been presented in the literature (See, for instance, [1,4,5,13]).

One approach, common cycle approach (CC) Hanssman [8] proposed to solve the problem is to schedule exactly one lot of each product in a time interval called the 'Common Cycle' or T. This approach has the advantage of always finding a feasible schedule and it consists of a very simple procedure. The CC approach also requires much less computational effort than the other approaches [2,11]. Jones and Inmans [11] have shown that the CC scheduling approach produces optimal or near optimal schedules in many realistic situations.

In this research we adopt the CC approach and consider the importance of limited resources in managing the aggregate inventory of a multi-product single machine system. In a real-word situation, due to various reasons such as corporate policy or limitation on amount insured, the resources (e.g., capital or space) are limited. Parsons [12], Haji [6], and Haji and Mansouri [7] have considered the total investment in inventory in CC approach. But Parsons' method unrealistically assumes the aggregate inventory is equal to the sum of the values of individual product lot sizes. In fact, in practice, the maximum aggregate inventory must be evaluated based on the sequence in which the products are produced and the amount of idle times allocated between consecutive products. Haji [6], and Haji and Mansouri [7], in their papers consider the sequence in which the products are produced in the cycle; but they impose the restriction that the total idle time in a cycle extends from the end of production run of the last product produced in the cycle up to the start of the next cycle where the production of the first product of the next cycle starts.

In this paper we consider limited investment and relax the above restriction, i. e., we allow the occurrence of idle time between production of any two consecutive products. This can provide some flexibility for performing certain tasks such as preventive maintenance. Also it may provide operators more rest times resulting in lower number of accidents and higher quality products. Then assuming the setup time is zero we minimize the maximum aggregate inventory for any common cycle time T (including optimal cycle time). To do this, we first develop a simple and easy- to- apply rule that finds the values of idle times which minimizes the maximum aggregate inventory for a given sequence of production runs of products and for any arbitrary cycle time T. Then, we show that these values of idle times for the given sequence are also optimal for any other sequence. That is, optimal idle times, and therefore, the corresponding optimal investment in inventory is independent of the production sequence.

MINIMIZATION OF MAXIMUM AGGREGATE INVENTORY

In this paper, all the standard assumptions of the general ELSP hold true. The most relevant of the assumptions used in this paper are as follows:

1. There are N products, all of which must be produced on a single machine. Machine can make only one product at the time.
2. Demand rates for all products are constant, known, and finite.
3. Production rates for all products are constant, known, and finite.
4. The time horizon is infinite.
5. All demands must be filled immediately. So, no shortages are permitted.

In addition, we consider the CC Scheduling approach, which implies:

6. In each common cycle, T, all of the products will be produced.
7. Each product is produced only once in each cycle T.

Furthermore it is assumed that:

8. During the production time of any product, the aggregate inventory increases. That is, the production rate of any product (in units of money per unit time) is greater than the aggregate demand rate (in units of money per unit time).

The following notations are used in this paper:

N Number of products.

P_j: Production rate of product j, in units of money per time unit.

D_j Demand rate of product j, in units of money per time unit.

D Aggregate demand in units of money per time unit ($\sum_{j=1}^{N} D_j$).

T Common cycle time in units of time.

t_j Production run-time of product j, j=1,2,…N.

X_j Duration of idle time occurring just before the production run of product j, j=1,2,…N.

E_j Aggregate inventory in units of money at the end of production run of product j, j=1,2,…N.

$I_j(t)$ Inventory level of product j at time t, in units of money.

I_{max} Maximum aggregate inventory in units of money.

Following the above notation, I_{max} can be written as: $I_{max} = \max_{0 \le t \le T} \sum I_j(t)$

The value of I_{max} depends on the order of the production of products [6].

AGGREGATE INVENTORY AT THE END OF A PRODUCTION RUN

Consider an arbitrary production schedule of N products, which repeats itself every T time units. Choose a cycle T that begins just at the start of the production run of a particular product and ends at the start of the next production run of the same product.

Now designate the given sequence by {1},{2},…,{N}, where {1} represents the particular product and {j} represent the product that has j-th position in the sequence. In each cycle time T the aggregate inventory increases at the rate ($P_{\{j\}}$ –D) during $t_{\{j\}}$ and decreases at the rate D during $X_{\{j\}}$, j = 1, 2, …, N, Figure 1.

To find $E_{\{j\}}$, the aggregate inventory level at the end of production of product {1}, note that

$$E_{\{1\}} = \sum_{k=1}^{N} I_{\{k\}} \tag{1}$$

where

$I_{\{k\}}$ = the inventory level of product {k} at the end of production run of product {1}, k=1,2,…, N.

Clearly

$$I_{\{1\}} = (P_{\{1\}} - D_{\{1\}})t_{\{1\}} \tag{2}$$

To find $I_{\{j\}}$, j ≥ 2, let $u_{\{j\}}$ = the length of time from the end of production run of product {1} to the start of production run of product {j}, Figure 1. Thus

$$u_{\{2\}} = X_{\{2\}}, \quad u_{\{j\}} = \sum_{k=2}^{j-1} t_{\{k\}} + \sum_{k=2}^{j} X_{\{k\}}.$$

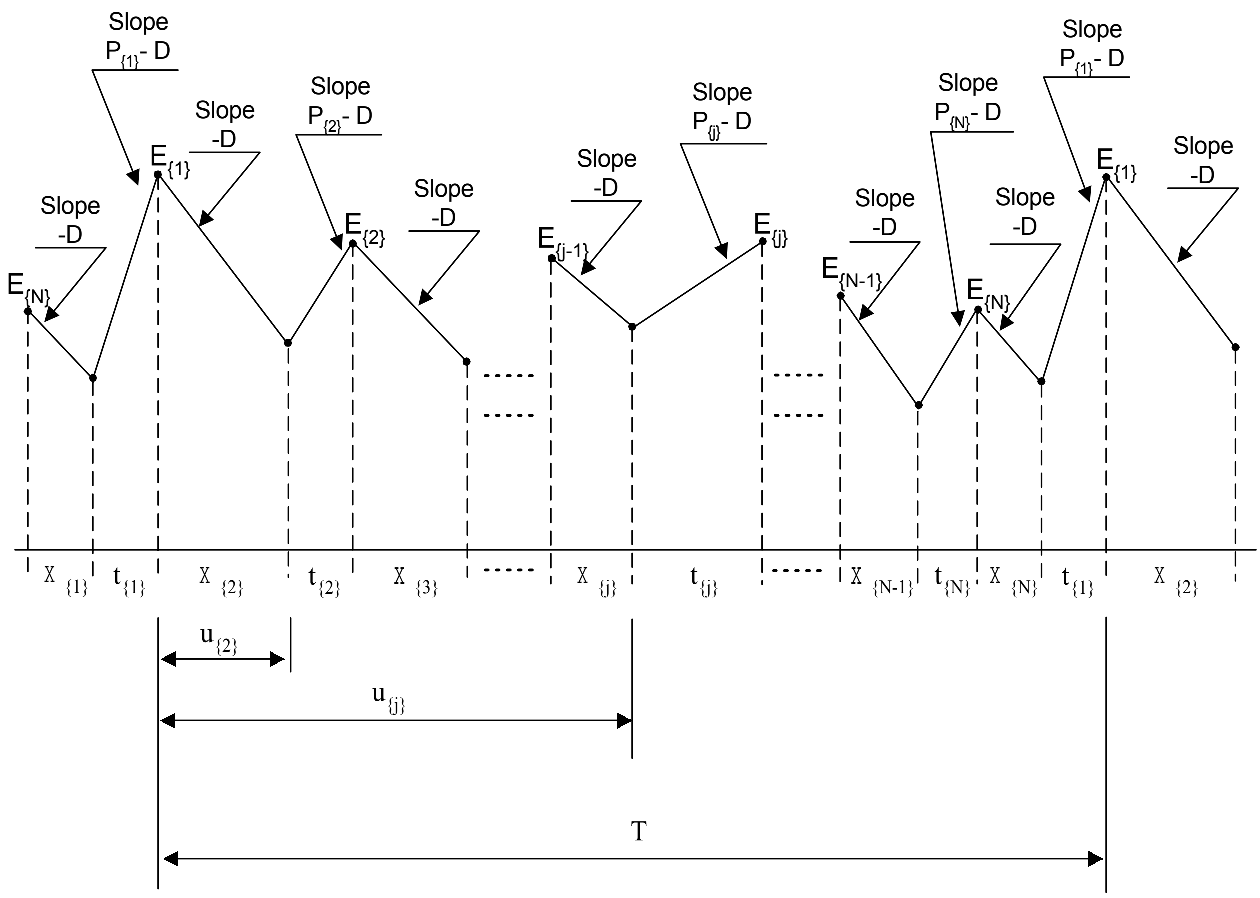

Figure 1. Aggregate inventory over time.

Since no shortage is permitted, $I_{\{j\}}$, the inventory level of product {j} at the end of production of product {1}, is equal to demand for that product during $u_{\{j\}}$, j=1,2,…,N. That is, $I_{\{j\}} = D_{\{j\}} u_{\{j\}}$. Hence, for j = 2

$$I_{\{2\}} = D_{\{2\}} X_{\{2\}} \qquad j = 2 \qquad (3)$$

and for j ≥ 3

$$I_{\{j\}} = D_{\{j\}} \sum_{i=2}^{j} X_{\{i\}} + D_{\{j\}} \sum_{i=2}^{j-1} t_{\{i\}} \qquad j \geq 3$$

or equivalently adding and subtracting $D_{\{j\}}\ t_{\{j\}}$, we can write:

$$I_{\{j\}} = D_{\{j\}} \sum_{i=2}^{j} X_{\{i\}} + D_{\{j\}} \sum_{i=2}^{j} t_{\{i\}} - D_{\{j\}} t_{\{j\}} \qquad j \geq 2 \qquad (4)$$

The aggregate inventory at the end of production run of product {1}, form relations (1), (2), and {4), is

$$E_{\{1\}} = (P_{\{1\}} - D_{\{1\}}) t_{\{1\}} + \sum_{j=2}^{N} \left(D_{\{j\}} \sum_{i=2}^{j} X_{\{i\}} + D_{\{j\}} \sum_{i=2}^{j} t_{\{i\}} - D_{\{j\}} t_{\{j\}} \right) \qquad (5)$$

Since the aggregate inventory decreases at the rate D during $X_{\{j\}}$ and increases at the rate ($P_{\{j\}}$ – D) during $t_{\{j\}}$, j=1, 2, …, N, we can write for j =2

$$E_{\{2\}} = E_{\{1\}} - DX_{\{2\}} + (P_{\{2\}} - D) t_{\{2\}} \qquad j = 2 \qquad (6)$$

and for all j ≥ 2

$$E_{\{j\}} = E_{\{1\}} - D \sum_{k=2}^{j} X_{\{k\}} + \sum_{k=2}^{j} (P_{\{k\}} - D) t_{\{k\}} \qquad j \geq 2 \qquad (7)$$

Lemma 1 For a given schedule and known $X_{\{j\}}$, j = 1, 2, …, N, if we decrease $X_{\{2\}}$ by an amount y, increase $X_{\{3\}}$ by the same amount, and fix all other $X_{\{j\}}$, (j ≠ 2and 3), then,

(*i*) $E_{\{j\}}$, (j ≠ 2) will decrease by $D_{\{2\}}y$

and

(*ii*) $E_{\{2\}}$ will increase by $(D - D_{\{2\}})y$

Proof To prove (*i*) denote the new value of $X_{\{j\}}$ by $X'_{\{j\}}$, thus:

$$\begin{cases} X'_{\{2\}} = X_{\{2\}} - y \\ X'_{(3\}} = X_{\{3\}} + y \\ X'_{\{j\}} = X_{\{j\}} & \forall\ j \neq 2 \text{ and } 3 \end{cases} \qquad (8)$$

Clearly,

$$X'_{\{2\}} + X'_{\{3\}} = X_{\{2\}} + X_{\{3\}} \qquad (8a)$$

and

$$\sum_{i=2}^{j} X'_{\{i\}} = \sum_{i=2}^{j} X_{\{i\}} \qquad j \geq 3 \qquad (8b)$$

Now we denote the new value of the inventory of product {j}, at the end of production run of product {1}, by $I'_{\{j\}}$ j = 1,2,…, N. Then, as we derived (2) and (3), we can write

$$I'_{\{1\}} = (P_{\{1\}} - D_{\{1\}}) t_{\{1\}} \qquad j = 1 \qquad (9a)$$

which implies

$$I'_{\{1\}} = I_{\{1\}}$$

and

$$I'_{\{2\}} = D_{\{2\}} X'_{\{2\}}$$

or from (3) and (8)

$$I'_{\{2\}} = D_{\{2\}} (X_{\{2\}} - y) \qquad (9b)$$

which implies

$I'_{\{2\}} = I_{\{2\}} - D_{\{2\}} y$, j = 2 and, as we derived (4),

$$I_{\{j\}} = D_{\{j\}} \sum_{i=2}^{j} X'_{\{i\}} + D_{\{j\}} \sum_{i=2}^{j} t_{\{i\}} - D_{\{j\}} t_{\{j\}} \qquad j \geq 2$$

and from (4), and (8b)

$$I'_{\{j\}} = I_{\{j\}} \qquad j \geq 3 \qquad (9c)$$

Let $E'_{\{j\}}$ be the new value of aggregate inventory at the end of production run of product {j}. Then,

$$E'_{(1)} = \sum_{j=1}^{N} I'_{(j)}$$

and from (9a), (9b), and (9c), we can write

$$E'_{\{1\}} = \sum_{j=1}^{N} I_{\{j\}} - D_{\{2\}} y$$

or from (1), we have

$$E'_{\{1\}} = E_{\{1\}} - D_{\{2\}} y \qquad (10)$$

To find $E'_{\{j\}}$ for $j \geq 2$, as we derived (7), we can write

$$E'_{\{j\}} = E'_{\{1\}} - D\sum_{k=2}^{j} X'_{\{k\}} + \sum_{k=2}^{j} (P_{\{k\}} - D)\, t_{\{k\}} \qquad j \geq 2 \quad (11)$$

Thus, for $j \geq 3$, from (8b) and (10), we can write (11) as

$$E'_{\{j\}} = (E_{\{1\}} - D_{\{2\}} y) - D\sum_{k=2}^{j} X_{\{k\}} + \sum_{k=2}^{j} (P_{\{k\}} - D) t_{\{k\}}\, , \; j \geq 3$$

or from (7)

$$E'_{\{j\}} = E_{\{j\}} - D_{\{2\}} y \qquad j \geq 3 \; , \; y > 0 \quad (12)$$

which proves the part (*i*) of the lemma.
To prove part (*ii*), note that for j = 2, from (11) we have

$E'_{\{2\}} = E'_{\{1\}} - DX'_{\{2\}} + (P_{\{2\}} - D) t_{\{2\}}$ or from (8) and (10)

$$E'_{\{2\}} = (E_{\{1\}} - D_{\{2\}} y) - D(X_{\{2\}} - y) + (P_{\{2\}} - D) t_{\{2\}}$$

$$E'_{(2)} = E_{(1)} - DX_{(2)} + (P_{(2)} - D) t_{(2)} + (D - D_{(2)}) y \qquad (13)$$

Now, from (6), we can write (13) as

$$E'_{\{2\}} = E_{\{2\}} + (D - D_{\{2\}}) y \qquad (14)$$

which proves the part (*ii*) of the lemma.

Theorem 1. For any arbitrary common cycle T and for a given production sequence the maximum aggregate inventory is minimized if the following rule is used

$$X_{\{j\}} = X^{*}_{\{j\}} = \frac{P_{\{j\}} - D}{D} t_{\{j\}} \qquad j = 1, 2, \ldots, N \quad (15)$$

Note: The above condition states that:

$$X_{\{j\}} D = (P_{\{j\}} - D) t_{\{j\}} \qquad j = 1, 2, \ldots, N \qquad (16)$$

which means that the duration of the idle time that occurs immediately before production of product {j} should be long enough so that $DX_{\{j\}}$, the decrease in total inventory level during the idle time, $X_{\{j\}}$, be exactly equal to $(P_{\{j\}}-D)t_{\{j\}}$,the increase in total inventory during the production run of product {j} $t_{\{j\}}$, j = 1 ,…,N. As a result, the theorem states that for the optimal solution all of the $E_{\{j\}}$ quantities for all of the products are equal. That is, $E_{(1)} = E_{(2)} = \cdots = E_{(N)}$.

Proof. To prove the theorem, suppose there exist an optimal solution for which all $E_{\{j\}}$ are not equal. That is, for the given production sequence, in the optimal solution, there exist 2 consecutive products, denote them by i1 and i2, for which

$$E_{i1} > E_{i2}$$

Now, choose a cycle T that begins with the start of production run of product i1. In this cycle, for the given production schedule, i1 has position {1}, i2 has position {2}, the next product to be produced after i2 has position {3}, and so on. Thus

$$E_{i1} = E_{\{1\}}\, , \quad E_{i2} = E_{\{2\}}\, , \quad \text{and} \quad E_{\{1\}} > E_{\{2\}} \qquad (17)$$

Let

$$I_{max} = \max_{1 \leq j \leq N} E_{j}$$

or from (17), for the given solution

$$I_{max} = \max_{1 \leq j \leq N \;,\; j \neq 2} E_{\{j\}}\, , \qquad (E_{\{1\}} > E_{\{2\}}) \qquad (18)$$

Now we will show that if we decrease $X_{\{2\}}$ (i.e., X_{i2}) by an amount y, where

$$0 < y < \frac{E_{\{1\}} - E_{\{2\}}}{D} \qquad (19)$$

increase $X_{\{3\}}$ by the same amount, and fix all other $X_{\{j\}}$'s ($j \neq 2$ and 3), then the new maximum aggregate inventory, denoted by I'_{max}, will be less than its pervious value I_{max} contradicting the assumption that I_{max} is optimal.

To show this, note that from part (*ii*) of the lemma

$$E'_{\{2\}} = E_{\{2\}} + (D - D_{\{2\}})y$$

and from (19)

$$E'_{\{2\}} < E_{\{2\}} + (E_{\{1\}} - E_{\{2\}}) - D_{\{2\}}y \quad \text{or}$$

$$E'_{(2)} < E_{(1)} - D_{(2)}y, \quad y > 0 \qquad (20)$$

Now, from (10) and (20)

$$E'_{\{2\}} < E'_{\{1\}} \qquad (21)$$

Also from part (*i*) of the lemma we have

$$E'_{(j)} = E_{(j)} - D_{(2)}y, \qquad y > 0, \qquad j \neq 2$$

That is,

$$E'_{\{j\}} < E_{\{j\}}, \qquad j \neq 2 \qquad (22)$$

The new maximum aggregate inventory is

$$I'_{max} = \max_{1 \le j \le N} E'_{(j)}$$

or, from (21)

$$I'_{max} = \max_{1 \le j \le N,\ j \neq 2} E'_{\{j\}}, \qquad (E'_{(2)} < E'_{(1)}) \quad (23)$$

Thus, from (18), (22), and (23), we can write

$$I'_{max} = (\max_{j \neq 2} E'_{(j)}) < (\max_{j \neq 2} E_{(j)}) = I_{max}$$

As it was to be shown.

Theorem 2. Given $X^*_{\{j\}} = \frac{P_{\{j\}} - D}{D} t_{\{j\}}$, $j = 1, 2, \ldots, N$, any sequence of production runs of products is optimal.

Proof: Let, for $j = 1, 2, \ldots, N$

$E^*_j =$ Aggregate inventory at the end of production run of product j when

$$X_j = X^*_j = \frac{P_{\{j\}} - D}{D} t_{\{j\}}$$

Now, from theorem 1, for an arbitrary sequence, denoted by {1}, {2}, , …, {N}, the optimal value of maximum aggregate inventory is equal to

$$E^*_{\{1\}} = E^*_{\{2\}} = \cdots = E^*_{\{N\}}$$

We prove the theorem by showing that the optimum value of maximum aggregate inventory of a given production sequence, denoted by I^*_{max}, or equivalently $E^*_{\{1\}}$ is constant and is independent of the production sequence. To do this we note that by replacing $X_{\{i\}}$ in Equation 5 by

$$X^*_{(j)} = \frac{P_{(j)} - D}{D} t_{(j)}$$

we can write

$$E^*_{(1)} = (P_{(1)}t_{(1)} - D_{(1)}t_{(1)}) + \sum_{j=2}^{N} D_{(j)} \sum_{i=2}^{j} \left(\frac{P_{(i)} - D}{D} t_{(i)} + t_{(i)} \right) - \sum_{i=2}^{N} D_{(j)}t_{(j)}$$

or

$$E^*_{(1)} = P_{(1)}t_{(1)} + \sum_{j=2}^{N} D_{(j)} \sum_{i=2}^{j} \frac{P_{(i)}t_{(i)}}{D} - \sum_{i=1}^{N} D_{(j)}t_{(j)} \qquad (24)$$

Since, no shortage are allowed, the total production of product {j} during its production run time, $t_{\{j\}}$, must be equal to its demand during the cycle T, i. e.,

$$P_{(j)}t_{(j)} = D_{(j)}T \qquad j = 1, 2, \ldots, N \qquad (25)$$

Thus, from (25), and the fact that $\sum_{j=1}^{N} D_{(j)} t_{(j)}$ is a constant number, denoted by C_1, we can write (24) as:

$$E^*_{\{1\}} = D_{\{1\}}T + \frac{T}{D}\sum_{j=2}^{N} D_{\{j\}} \sum_{i=2}^{j} D_{\{i\}} - C_1$$

or

$$E^*_{(1)} = D_{(1)}T + \frac{T}{D}\sum_{j=2}^{N} D_{(j)}\left[\sum_{i=1}^{j-1} D_{(i)} + D_{(j)} - D_{(1)}\right] - C_1 \tag{26}$$

Hence,

$$E^*_{(1)} = D_{(1)}T + \frac{T}{D}\left[\sum_{j=2}^{N} D_{(j)}\sum_{i=1}^{j-1} D_{(i)} + \sum_{j=2}^{N} D_{(j)}(D_{(j)} - D_{(1)})\right] - C_1$$

Note that, the first term in the brackets is constant, denoted by C_2. That is:

$$C_2 = \sum_{j=2}^{N} D_{\{j\}} \sum_{i=1}^{j-1} D_{\{i\}} = \frac{1}{2}\left[\left(\sum_{j=1}^{N} D_{\{j\}}\right)^2 - \sum_{j=1}^{N} D^2_{\{j\}}\right] \tag{27}$$

and is independent of the production sequence. Also the second term in the brackets can be written as

$$\sum_{j=2}^{N} D_{\{j\}}(D_{\{j\}} - D_{\{1\}}) = \sum_{j=1}^{N} D_{\{j\}}(D_{\{j\}} - D_{\{1\}})$$

$$= \sum_{j=1}^{N} D^2_{\{j\}} - D_{\{1\}} \sum_{j=1}^{N} D_{\{j\}}$$

$$= C_3 - D_{\{1\}}D \tag{28}$$

where

$$C_3 = \sum_{j=1}^{N} D^2_{\{j\}} = \sum_{j=1}^{N} D^2_j$$

and

$$D = \sum_{j=1}^{N} D_{\{j\}} = \sum_{j=1}^{N} D_j$$

which are independent of production sequence. Thus, from (27) and (28) we can write (26) as

$$E^*_{(1)} = D_{\{1\}}T + \frac{T}{D}[C_2 + C_3 - D_{\{1\}}D] - C_1$$

$$= \frac{T}{D}(C_2 + C_3) - C_1$$

which is constant and independent of production sequence.

CONCLUSION

In this paper, we considered the scheduling problem of a multi-product single machine system in which the occurrence of idle time between any two consecutive products is allowed. Applying the common cycle approach to this problem and considering the limited capital for investment in aggregate inventory, a simple and an easy-to-apply rule for minimizing the maximum aggregate inventory of all products has been obtained. It is shown that this rule is sequence independent and it can be used for any length of a common cycle time, including the optimal cycle time.

By changing one or more of the assumptions adopted in the problem considered in this paper one can easily define a number of new problems, each of which is worth for further research to obtain the optimal aggregate inventory level.

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