VERTICAL AND ROCKING IMPEDANCES FOR SURFACE RIGID FOUNDATION RESTING ON A TRANSVERSELY ISOTROPIC HALF-SPACE

M. E. Ghadi

Dept. of Civil Engineering, Mazandaran University of Science and Technology
Babol, Iran

A. Noorzad

Dept. of Civil Engineering, Tehran University
P.O. Box 11365-4563 Tehran, Iran

K. Konagai

Inst. of Industrial Science, University of Tokyo
Minato-ku, Tokyo 106, Japan

(Received: October 28, 1999 – Accepted in Final Form: July 19, 2001)

Abstract The vertical and rocking impedances of a rigid foundation resting on a semi-infinite transversely isotropic medium are obtained in the frequency domain. In the present approach, the contact pressure distribution on the soil foundation-interface is approximated by a linear combination of known pressure patterns. It is shown herein that the approximate solutions of spatial displacement distributions satisfy quite well the boundary conditions for this mixed boundary problem.

Key Words Vertical and Rocking Impedances, Rigid Foundation, Transversely Isotropic, Frequency Domain, Mixed Boundary

چکیده در این مقاله توابع امپدانس قائم و چرخشی حول محور افقی در فضای فرکانسی برای شالوده صلب دایره‌ای که برروی خاک (محیط) نیمه‌متناهی با رفتار ایزوتروپ جانبی قرار گرفته، بدست می‌آید. در روش این مقاله، توزیع فشار تماسی در سطح مشترک شالوده و خاک در فضای خطی از توابع مشخص و معلوم در نظر گرفته می‌شود. در هر مورد (قائم یا چرخشی) فضای خطی با چهار تابع مبنا ساخته می‌شود. در این مقاله نشان داده می‌شود که تقریب ساخت جواب در این فضا در شرایط مرزی مختلط مساله با دقت مناسب صدق می‌کند.

1. INTRODUCTION

Many researchers have extensively studied various problems related to wave propagation due to concentrated and/or distributed loadings on the surface of elastic half-space. When a load is applied through a rigid circular disk to the medium, the problem is rigorously described in terms of dual integral equations. Reissner and Sagoci [1] solved the torsional motion of a rigid foundation by describing the problem in the oblate spherical coordinates. Arnold et. al. [2] obtained an approximate solution by tentatively assuming that the dynamic contact stress distribution is about identical to the static distribution pattern. This assumption has been taken by a number of researchers including Bycroft [3] for instance. Awoboji and Grootenhuis [4] solved the rigorous vertical and torsional motions of a circular rigid body as well as vertical and rocking motions of a rigid strip foundation on a semi infinite half-space. Gladwell [5] showed that the dual integral equation describing the problem is reduced to the second kind of Fredholm integral equation by using Noble's method [6]. He solved the Fredholm equation by using a numerical method.

As for an isotropic material, however, quite a few studies have been conducted. They include the works by Fabrikant [7] and Hanson [8]. Fabricant [7] studied the elastic field caused by a rigid flat punch in normal and rotational directions to the material surface. Hanson studied the same problem taking into account the effect of shear traction.

In this paper, the vertical and rocking impedances of a rigid circular foundation resting on a semi-infinite transversely isotropic medium are obtained in the frequency domain. In the present approach, the normal contact pressure distribution on the soil-foundation interface is approximated by a linear combination of known pressure patterns.

2. STATEMENT OF THE PROBLEM

By ignoring body forces, the time-harmonic governing equations of a three-dimensional elastic medium are written in the following form as:

$$C_{ijkl}u_{k,jl} = -\rho\omega^2 u_i. \qquad (i=1,2,3) \tag{1}$$

where C_{ijkl}, ρ, ω and u_i are the elasticity coefficients, mass density, circular frequency and displacement components in $x_i\,(i=1,2,3)$ direction, respectively, and comma (,) denotes differential operator with respect to spatial coordinate.

In a transversely isotropic material C_{ijkl} is reduced to the following five elastic constants:

$$A_{11} = C_{1111}, \quad A_{12} = C_{1122}, \quad A_{33} = C_{3333}, \quad A_{13} = C_{1133}, \quad A_{44} = C_{2323} \tag{2}$$

and

$$A_{66} = 2(A_{11} - A_{12}) \tag{3}$$

Since the strain energy stored up within the material must be positive, the coefficient tensor $|C_{ijkl}|$ should also be positive. This condition calls for:

$$A_{11} > |A_{12}|, \qquad (A_{11} + A_{12})A_{33} > 2A_{13}^2, \qquad A_{44} > 0 \tag{4}$$

The authors have uncoupled the equations of motion, and obtained the rigorous solutions of both vertical displacement u_z and the stress σ_{zz} due to vertical harmonic force applied to the surface of a transversely isotropic half space. In their approach, Fourier series and Hankel transform were utilized in respective circumferential and radial directions. The solutions are as follows:

$$u_z(r,\theta,z) = \sum_{m=-\infty}^{+\infty} e^{im\theta}$$

$$\left[\int_0^\infty \varsigma\left(\left[\rho_0\omega^2 - \varsigma^2(1+\alpha_1)\right]\left[c_1 e^{-\alpha_2' z} - c_2 e^{-\alpha_1' z}\right] + \alpha_2\left[c_1\alpha_2'^2 e^{-\alpha_2' z} - c_2\alpha_1'^2 e^{-\alpha_1' z}\right]\right)\frac{p_{zm}^m}{g(\varsigma)} J_m(\varsigma r)\,d\varsigma\right]$$

$$\sigma_{zz}(r,\theta,z) = \sum_{m=-\infty}^{+\infty} e^{im\theta}$$

$$\left[\int_0^\infty \varsigma\left(\left(\alpha_3 A_{13}\varsigma^2 + A_{33}\left[\rho_0\omega^2 - \varsigma^2(1+\alpha_1)\right]\right)\left[-c_1\alpha_2' e^{-\alpha_2' z} + c_2\alpha_1' e^{-\alpha_1' z}\right] + \alpha_2 A_{33}\left[-c_1\alpha_2'^3 e^{-\alpha_2' z} + c_2\alpha_1'^3 e^{-\alpha_1' z}\right]\right.\right.$$

$$\left.\left.\frac{p_{zm}^m}{g(\varsigma)} J_m(\varsigma r)\,d\varsigma\right]$$

where

$$c_1 = \alpha_1'\alpha_3 + \rho_0\omega^2 - \varsigma^2(1+\alpha_1) + \alpha_1'^2\alpha_2 \tag{7}$$

$$c_2 = \alpha_2'\alpha_3 + \rho_0\omega^2 - \varsigma^2(1+\alpha_1) + \alpha_2'^2\alpha_2 \tag{8}$$

$$\alpha_1' = S_1\sqrt{\frac{\rho}{\alpha_2}\omega^2 - \varsigma^2} \tag{9a}$$

$$\alpha_2' = S_2\sqrt{\frac{\rho}{1+\alpha_1}\omega^2 - \varsigma^2}$$

$$\alpha_1 = \frac{A_{11}}{A_{66}}, \quad \alpha_2 = \frac{A_{11}}{A_{66}}, \quad \alpha_3 = \frac{A_{11}}{A_{66}}, \quad \rho_0 = \frac{\rho}{A_{66}} \tag{9b}$$

and S_1^2 and S_2^2 are the roots of the following equation:

$$A_{33}A_{44}S^4 + (A_{13}^2 + 2A_{13}A_{44})S^2 + A_{11}A_{44} = 0. \tag{10}$$

In Equations 5 and 6, $J_m(\varsigma r)$ is the first kind Bessel function of the mth order, p_{zm}^{m} is the mth order Hankel transform of $p_{zm}(r)$ and $g(\varsigma)$ is given as:

$$g(\varsigma) = \left[\alpha'_2\alpha_3 + \rho_0\omega^2 - \varsigma^2(1+\alpha_1) + \alpha'^2_2\alpha_3\right]$$

$$\left[\alpha'_1\left(\alpha_3 A_{13}\varsigma^2 + A_{33}\left[\rho_0\omega^2 - \varsigma^2(1+\alpha_1)\right]\right) + \alpha_2\alpha'^3_1 A_{33}\right]$$

$$-\left[\alpha'_1\alpha_3 + \rho_0\omega^2 - \varsigma^2(1+\alpha_1) + \alpha'^2_1\alpha_3\right]$$

$$\left[\alpha'_2\left(\alpha_3 A_{13}\varsigma^2 + A_{33}\left[\rho_0\omega^2 - \varsigma^2(1+\alpha_1)\right]\right) + \alpha_2\alpha'^3_2 A_{33}\right] \quad (11)$$

The term $p_{zm}(r)$ is the mth term of the Fourier series of the load $p_z(r,\theta)$ with respect to circumferential direction. Rayleigh pole is the root of equation $g(\varsigma) = 0$.

When a rigid circular foundation of radius R is concerned, contact pressure distribution $p_z(r,\theta)$ is unknown. Ignoring the friction between the foundation and the medium, the boundary conditions of rigid foundation subjected to vertical motion of Δ and rocking motion of ϕ are given as (Figure 1):

$$u_z(r,\theta,0) = \Delta \qquad \forall\theta,\ 0 \le r < R \qquad (12a)$$

$$\sigma_{zz}(r,\theta,0) = 0 \qquad \forall\theta,\ r > R \qquad (12b)$$

and those for rocking motion of ϕ as:

$$u_z(r,\theta,0) = r\phi \qquad \forall\theta,\ 0 \le r < R \qquad (13a)$$

$$\sigma_{zz}(r,\theta,0) = 0 \qquad \forall\theta,\ r > R \qquad (13b)$$

respectively. Substituting Equations 5 and 6 into Equations 12 and 13 yields dual integral equations.

3. SOLUTION

We obtain the solutions of the dual integral Equations 12 and 13 by making an N-dimensional function space and expressing the normal contact pressure distribution in this space. The solution for vertical and rocking motions are to be introduced separately.

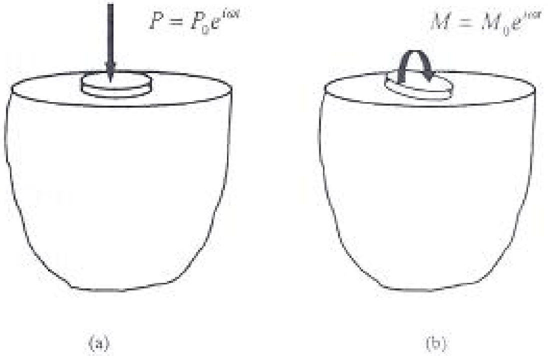

Figure 1. Configuration of the Problem (a) Vertical Motion and (b) Rocking Motion.

3.1 Vertical Motion of the Foundation As for the vertical motion of the foundation all variables are independent of circumferential coordinate, and the load function can be expressed by the following functional expansion function space [9]:

$$p_z(r) = \begin{cases} \sum_{j=1}^{N} \alpha_j f_j(r) & r \le R \\ 0 & r > R \end{cases} \qquad (14)$$

where $f_j(r)(j = 1,2,\ldots)$ are basic functions which are determined by:

$$f_j(r) = \begin{cases} \dfrac{p+1}{\pi R^2}\left[1 - \left(\dfrac{r}{R}\right)^2\right]^p & r \le R \\ 0 & r > R \end{cases} \qquad (15)$$

with $p = j - \frac{1}{2}$.

In Equation 14, α_j is the coordinate of function p_z in the jth basic and is a complex number in the frequency domain. Since the basic functions satisfy

$$\int_0^\infty r f_j(r)\,dr = \frac{1}{2\pi} \qquad (16)$$

the total force acting on the surface of the half-space is given by:

$$F_z = \sum_{j=1}^{N} \alpha_j \tag{17}$$

The contact pressure distribution p_z is axi-symmetric and this condition calls for m=0 in Equations 5 and 6. Thus, the necessary procedure is eventually to obtain the zeroth order Hankel transform of $f_j(r)$:

$$f_j^0 = \frac{p+1}{\pi} \frac{2^p \Gamma(p+1)}{(\varsigma R)^{p+1}} J_{p+1}(\varsigma R) \tag{18}$$

where Γ is the Gamma function.

Substituting Equation 18 in Equation 5, the displacement $u_z(r,z)$ can be expressed as:

$$u_z(r,z) = \sum_{j=1}^{N} \alpha_j u_{z,j}(r,z) \tag{19}$$

where

$$u_{z,j}(r,z) = \int_0^\infty \varsigma \Big(\big[\rho_0 \omega^2 - \varsigma^2 (1+\alpha_1)\big]\big[c_1 e^{-\alpha_2' z} - c_2 e^{-\alpha_1' z}\big] + \alpha_2 \big[c_1 \alpha_2'^2 e^{-\alpha_2' z} - c_2 \alpha_1'^2 e^{-\alpha_1' z}\big] \Big) \left[\frac{p+1}{\pi} \frac{2^p \Gamma(p+1)}{(\varsigma R)^{p+1}} J_{p+1}(\varsigma R)\right] \frac{J_0(\varsigma r)}{g(\varsigma)} d\varsigma \quad j = 1,2,\ldots,N \tag{20}$$

Since the foundation is rigid, u(r,0) must be equal to Δ over the entire extent of the foundation-medium interface. In this paper, approximation is made in a four-dimensional function space i.e. N=4. N (= 4) unknown constant $\alpha_j (j = 1,2,3,4)$ are determined in such a way that the constants α_j allows the approximate expression of $u_z(r,z)$ described in Equation 19 to best fit its rigorous value Δ within the contact area. Thus, four points ($r_k = 0.1R, 0.4R, 0.7R$ and $1.0R$) are taken within the contact area, and total N=4 (Equation 19) at these particular points eventually make up a set of N simultaneous equations:

$$\sum_{j=1}^{4} \alpha_j u_{z,j}(r_k, 0) = \Delta \quad (k = 1,2,3,4) \tag{21}$$

Solving the simultaneous Equation 21, the constants α_j are obtained. With all α_j, Equation 17 gives total force applied to the foundation and the impedance function K_{zz} is obtained as:

$$K_{zz} = \frac{F_z}{\Delta} \tag{22}$$

3.2 Rocking Motion of the Foundation In this case, $p_z(r,\theta)$ can be written in the following form as:

$$p_z(r,\theta) = \begin{cases} \sum_{j=1}^{N} \beta_j g_j(r,\theta) & r \le R \\ 0 & r > R \end{cases} \tag{23}$$

where β_j is the coordinate of function p_z in the *j*th basic and $g_j(r,\theta) (j = 1,2,\ldots)$ are the basis of the N-dimensional function space expressed as [10]:

$$g_j(r) = \begin{cases} \frac{2(p+1)(p+2)}{\pi R^4} r \left[1 - \left(\frac{r}{R}\right)^2\right]^p \cos\theta & r \le R \\ 0 & r > R \end{cases} \tag{24}$$

with $p = j - \frac{1}{2}$.

This inversely symmetric problem of contact pressure distribution calls for m=1 in Equations 5 and 6, the functions $p_{z1}(r)$ and $p_{z1}^1(\varsigma)$ are respectively given as:

$$p_{z1}(r) = \begin{cases} \sum_{j=1}^{N} \beta_j \dfrac{2(p+1)(p+2)}{\pi R^4} r\left[1-\left(\dfrac{r}{R}\right)^2\right]^p & r \le R \\ 0 & r > R \end{cases} \quad (25)$$

$$p_{z1}^1(\varsigma) = \beta_j \frac{2(p+1)(p+2)}{\pi R} \frac{2^p \Gamma(p+1)}{(\varsigma R)^{p+1}} J_{p+2}(\varsigma R) \quad (26)$$

Substituting Equation 26 into Equation 5, the displacement $u_z(r, \theta, z)$ is expressed as:

$$u_z(r, \theta, z) = \sum_{j=1}^{N} \alpha_j u_{z,j}(r, z) \cos\theta \quad (27)$$

where

$$u_{z,j}(r, z) =$$

$$\int_0^\infty \varsigma\Big(\left[\rho_0 \omega^2 - \varsigma^2 (1+\alpha_1)\right]\left[c_1 e^{-\alpha_2' z} - c_2 e^{-\alpha_1' z}\right] +$$

$$\alpha_2\left[c_1 \alpha_2'^2 e^{-\alpha_2' z} - c_2 \alpha_1'^2 e^{-\alpha_1' z}\right]\Big) \quad (28)$$

$$\left[\frac{(p+1)(p+2)}{\pi R} \frac{2^{p+1}\Gamma(p+1)}{(\varsigma R)^{p+1}} J_{p+2}(\varsigma R)\right]$$

$$\frac{J_1(\varsigma r)}{g(\varsigma)} d\varsigma \qquad j = 1, 2, \ldots, N$$

Since the foundation is rigid, the derivative of the displacement u_z, with respect to the radial coordinate, (i.e. $\varphi = \dfrac{\partial u_z(r, \theta, 0)}{\partial r}$) should be equal to one at any point on the foundation-medium interface. For this motion, the same as that of vertical direction, using a four-dimensional function space and choosing four points ($r_k = 0.1R$, $0.4R$, $0.7R$ and $1.0R$) on the foundation-medium interface, the four unknown coefficients $\beta_j\ (j = 1, 2, 3, 4)$ are given by solving the system of the algebraic equations as:

$$\sum_{j=1}^{4} \beta_j \varphi_j(r_k, 0) = 1 \qquad (k = 1, 2, 3, 4) \quad (29)$$

where

$$\varphi_j(r_k, 0) = \frac{\partial u_{z,j}(r_k, 0)}{\partial r} \qquad (k = 1, 2, 3, 4) \quad (30)$$

In the case of rigid foundation, the system of Equations 29 is exactly equal to the following equations:

$$\sum_{j=1}^{4} \beta_j u_{z,j}(r_k, 0) = r_k \qquad (k = 1, 2, 3, 4) \quad (31)$$

Since $\varphi(r, 0) = 1$ at any point on the foundation-medium interface, the impedance function, $K_{\varphi\varphi}$, is given as:

$$K_{\varphi\varphi} = \frac{M}{\varphi} = M \quad (32)$$

where M is equal to the total bending moment which is given by:

$$M = \int_0^R \int_0^{2\pi} (r\cos\theta)\, p_z(r, \theta)\, r\, d\theta\, dr =$$

$$\int_0^R \int_0^{2\pi} \sum_{j=1}^{4} \beta_j \frac{2(p+1)(p+2)}{\pi R^4} r^2 \cdot$$

$$\left[1-\left(\frac{r}{R}\right)^2\right]^p \cos^2\theta\, d\theta\, dr$$

or

$$M = \sum_{j=1}^{4} \beta_j \quad (33)$$

For obtaining Equation 33, the following expression has been used:

$$\int_0^R \int_0^{2\pi} \frac{2(p+1)(p+2)}{\pi R^4} r^2 \left[1-\left(\frac{r}{R}\right)^2\right]^p \cos^2\theta\, d\theta\, dr = 1 \quad (34)$$

Substituting Equation 33 into Equation 32, $K_{\varphi\varphi}$ is expressed as:

$$K_{\varphi\varphi} = \sum_{j=1}^{N} \beta_j \quad (35)$$

TABLE 1. Material Constants.

Material	$\frac{A_{11}}{A_{44}}$	$\frac{A_{12}}{A_{44}}$	$\frac{A_{13}}{A_{44}}$	$\frac{A_{33}}{A_{44}}$	$A_{44} \times 10^4 \frac{N}{mm^2}$
Isotropic	3.00	1.00	1.00	3.00	1.00
Layered soil	2.11	0.43	0.47	2.58	1.40
Beryl rock	4.13	1.47	1.01	3.62	1.00

TABLE 2. Comparison of the Results of This Study with that of Luco and Mita [11].

Vertical impedance function	$\mathrm{Re}(k_{zz})$
Present study	6.03
Luco and Mita (1987)	5.97

Same as the case of vertical excitation, the inverse of this impedance function is as:

$$F_{\varphi\varphi} = \frac{1}{K_{\varphi\varphi}} \tag{36}$$

where $F_{\varphi\varphi}$ is the flexibility function for rocking motion of the foundation.

4. NUMERICAL RESULTS

In the previous section, by using a four-dimensional function space, the vertical displacement u_z and the impedance functions K_{zz} and $K_{\varphi\varphi}$ have been expressed in terms of four complex coefficients for vertical and rocking motions, separately. In this section the numerical evaluation of the displacement and then the impedance functions are given. For this purpose, three kinds of transversely isotropic materials as well as isotropic one considered. These four materials are (1) isotropic medium, (2) limestone/sandstone layered soil and (3) Beryl rock (Ragapakse and Wang, 1993). The Poisson's ratio of the isotropic material is equal to 0.25 and the mechanical properties of the materials are listed in Table 1.

For numerical evaluation, the dimensionless frequency $a_0 = R\omega\sqrt{\frac{\rho}{A_{66}}}$, real part of impedance function $\mathrm{Re}(k_{ii}) = \mathrm{Re}(K_{ii})/A_{66}$ and imaginary parts of impedance function $\mathrm{Im}(k_{ii}) = \mathrm{Im}(K_{ii})/(\alpha_0 A_{66})$ $(i = z,\varphi)$ are introduced.

By solving the algebraic Equations 21, the coefficients α_j $(j=1,2,3,4)$ are obtained. Putting α_j in Equation 22, the impedance function k_{zz} is obtained.

In order to provide a proper perspective on the accuracy of the present method, the impedance of the disk for vertical motion is first obtained setting α_0 at 0.1, and compared in Table 2 with the rigorous solution by previous authors. Only 1% error validates the present approximation.

Figures 2 to 4 show the spatial variation of vertical displacements u_z of the four materials with respect to the radial distance r/R for different values of dimensionless frequency $a_0 = 0.1$, 1.0 and 3.0, respectively. In general, outwardly propagating waves exhibits less attenuation with increasing frequency. Among the waves through these three materials, the wave traveling away through the isotropic medium is the least attenuated in the higher frequency range $a_0 = 3.0$. Figures 5 to 7 show the variations of vertical displacements u_z with the depth z/R for the different frequencies. In these figures also, amplitudes of the waves are less decreased with increasing depth as the frequencies increase. However, differing from Figure 5, the wave traveling down through the isotropic medium (Figure 7, $a_0 = 3.0$) exhibits a sharp rise of its amplitude shortly beneath the disk, and as the wave propagates further down, its amplitude is reduced faster than that of other waves through the transversely isotropic materials. This fact indicates that transversely isotropic features of the material affect the directivity of the waves through a material.

Figures 8 to 10 show the spatial variations of displacements u_z due to the rocking motion of the

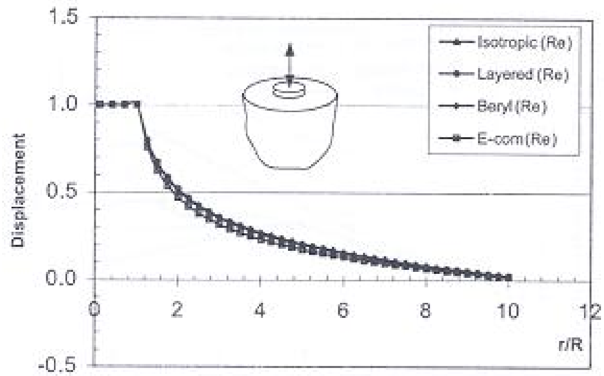

Figure 2. Vertical displacements versus radial distance due to vertical excitation (a_0=0.1). The imaginary parts are equal to zero.

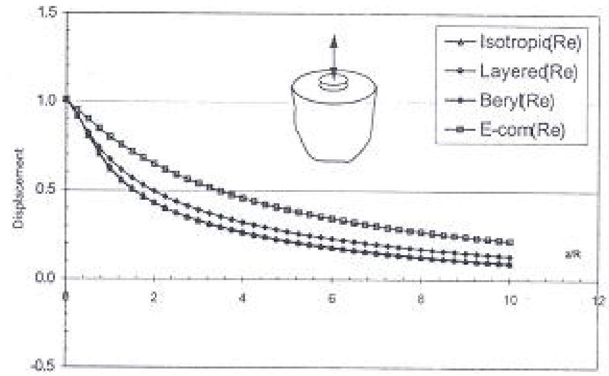

Figure 5. Vertical displacements versus depth due to vertical excitation (a_0 = 0.1). The imaginary parts are equal to zero.

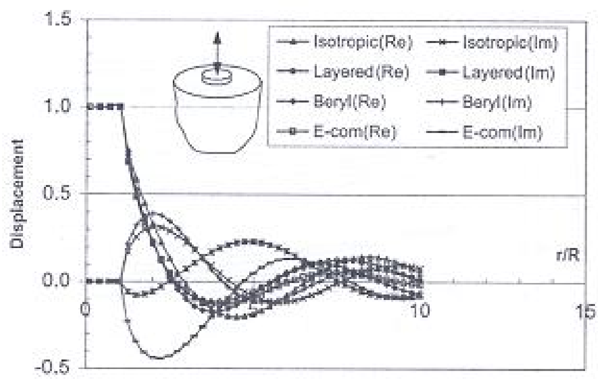

Figure 3. Vertical displacements versus radial distance due to vertical excitation (a_0 = 1.0).

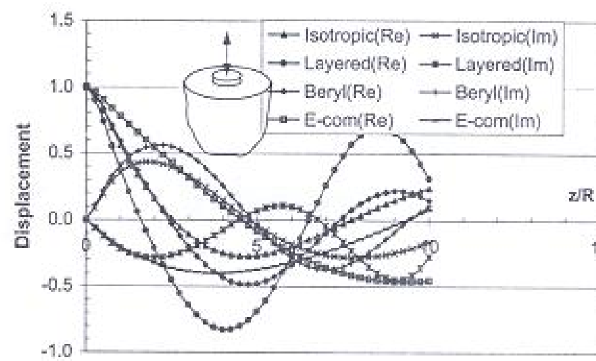

Figure 6. Vertical displacements versus depth due to vertical excitation (a_0 = 1.0).

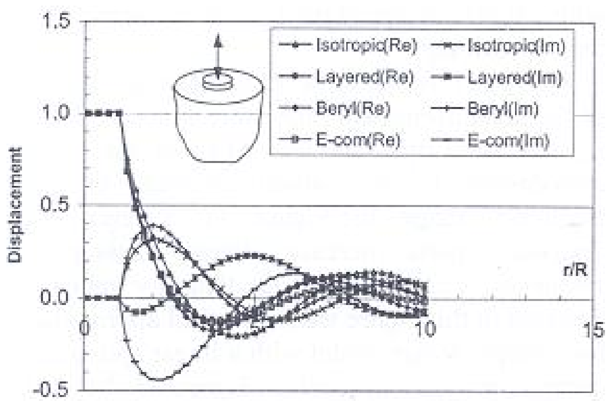

Figure 4. Vertical displacements versus radial distance due to vertical excitation (a_0 = 3.0).

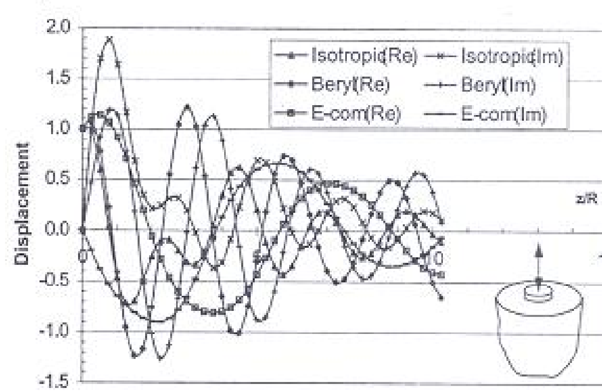

Figure 7. Vertical displacements versus depth due to vertical excitation (a_0 = 3.0).

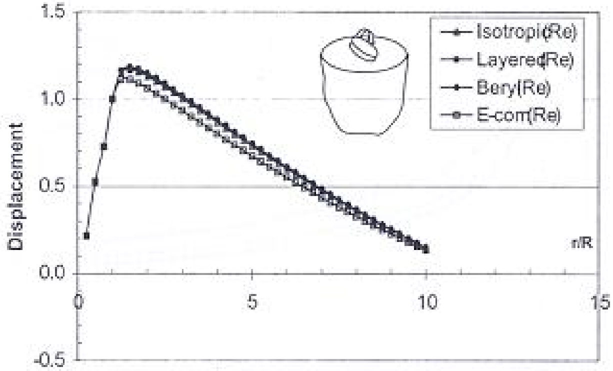

Figure 8. Vertical displacements versus radial distance due to rocking excitation (a_0=0.1). The imaginary parts are equal to zero.

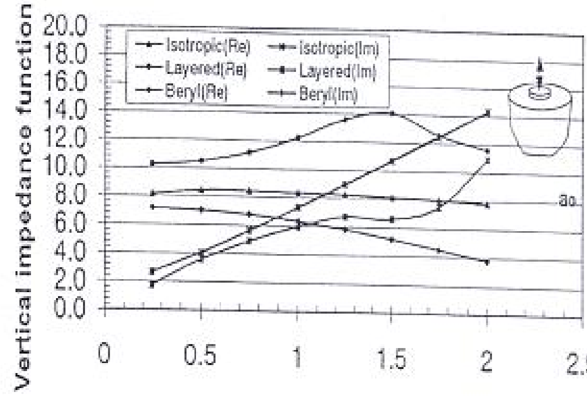

Figure 11. Real and imaginary parts of vertical impedance function versus a_0.

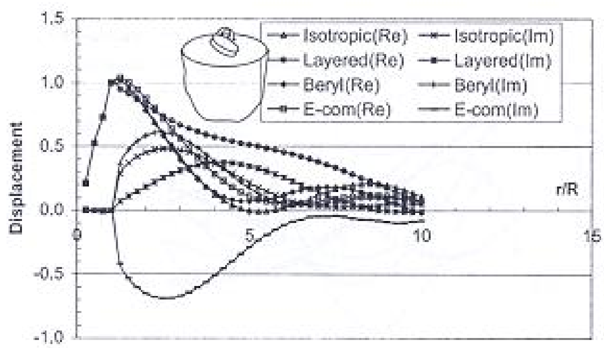

Figure 9. Vertical displacements versus radial distance due to rocking excitation (a_0 =1.0).

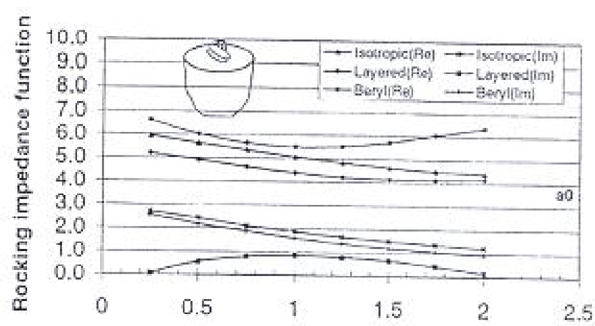

Figure 12. Real and imaginary parts of rocking impedance function versus a_0.

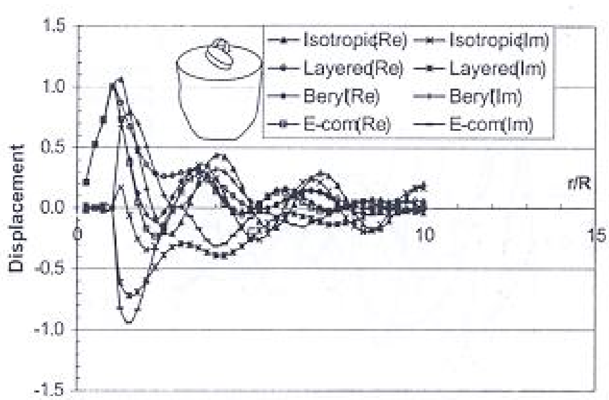

Figure 10. Vertical displacements versus radial distance due to rocking excitation (a_0 = 3.0).

disk. In these figures, displacements are plotted with r/R whose orientation is taken normal to the axis of rocking.

Figures 11 and 12 show the variation on frequency of the non-dimensional impedance functions k_{zz} and $k_{\varphi\varphi}$, respectively. The real parts of the vertical impedances k_{zz} are about constant over the frequency range in Figure 11, whereas their imaginary parts increase almost linearly with frequency; the facts indicates that any impedance function in this figure would be well approximated by a simple Voigt model with a linear spring and a dashpot arranged in parallel. Figure 12 shows that the imaginary parts are noticeably smaller than those in Figure 11, indicating that the material-disk systems are less damped for their rocking motions.

5. CONCLUSION

The vertical and rocking impedances of a rigid foundation resting on a semi-infinite transversely isotropic medium were obtained in the frequency domain. In the present approach, the pressure distribution on the soil-foundation interface was approximated by a linear combination of known pressure patterns. In order to provide a proper perspective on the accuracy of the present method, the impedance of the disk on an isotropic half space was first obtained, and compared with the rigorous solution of previous authors. The solution obtained was in good agreements with the rigorous solution, demonstrating the accuracy of the solution by the present approach. Real parts of the vertical impedances k_{zz} of disk on different transversely isotropic media are all about constant over a wide non-dimensional frequency range, whereas their imaginary parts increase almost linearly with frequency. The fact indicates that any impedance function would be well approximated by a simple Voigt model with a linear spring and a dashpot arranged in parallel. On the other hand, the imaginary parts of the rocking impedances $k_{\varphi\varphi}$ for these materials are noticeably small when compared with those of k_{zz}, indicating that the rocking motions of the disks are less damped than their vertical motions.

REFERENCES

1. Reissner, E. and Sagoci, H. F., "Forced Torsional Oscilations of an Elastic Half-Space, I", ***J. Appl. Phys.***, Vol. 15, (1944), 652-654.
2. Arnold, R. N., Bycroft, G. N. and Warburton, G. B., "Forced Vibration of a Body on an Infinite Elastic Solid", ***J. Appl. Mech.***, Vol. 22, (1955), 391-400.
3. Bycroft, G. N., "Forced Vibration of a Rigid Circular Plate on a Semi-Infinite Elastic Space and on an Elastic Stratum", ***Phil. Trans. R. Soc. Series A***, Vol. 248, (1956), 327-368.
4. Awoboji, A. O. and Grootenhuis, P., "Vibration of Rigid Bodies on Semi-Infinite Elastic Media", ***Proc. R. Soc. Series A***, Vol. 287, (1965), 27-63.
5. Gladwell, G. M. L., (1968),"Forced Tangential and Rotatory Vibration of a Rigid Circular Disk on a Semi-Infinite Solids", ***Int. J. Engng Sci.***, Vol. 6, pp 591-607.
6. Noble, B., "Dual Bessel Function Integral Equations", ***Proc. Camb. Phil. Soc. Math. Phys.***, Vol. 59, (1963), 351-371.
7. Fabrikant, V. I., "Elastic field around a circular Punch", ***ASME J. Appl. Mech.***, (1988), Vol. 55, 604-610.
8. Hanson, M. T., "The Elastic field for an Upright or Tilted Sliding Circular Flat Punch on a Transversely Isotropic Half-Space", ***Int. J. Solids Structures***, Vol. 31, (1994), 567-586.
9. Noorzad, A. and Kongai, K., "Effect of Seepage on Impedance of Circular Rigid Foundation", ***Proc. Of 9th JSCEE***, (1995).
10. Noorzad, A., "Dynamic Interaction between a Rigid Body and the Surrounding Semi-Infinite Poroelastic", PhD Thesis Submitted to Tokyo University, (1994).
11. Mita, A. and Luco, J. E., "Dynamic Response of Embedded Foundations: a Hybrid Approach", Comput. Methods Appl., Mech. Eng., 63, (1987), 233-259.