A NEW APPROACH VERSUS BENDERS' DECOMPOSITION IN AGGREGATE PRODUCTION PLANNING

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Abstract This paper proposes a simplified solution procedure to the model presented by Akinc and Roodman. The Benders' decomposition procedure for analyzing this model has been developed, and its shortcomings have been highlighted. Here, the special nature of the problem is exploited which allowed us to develop a new algorithm through surrogating method. The two methods are compared by several numerical examples. Computational experience with these data shows the superiority of the new approach. In addition, the required computer programs have been prepared by the authors using TURBO PASCAL 7.0 to execute the algorithm.

Key Words Aggregate Production Planning, Benders' Decomposition, Surrogating Method

چکیده در ایـن مقاله یک روش حل ساده برای حل مسئله اکین و رودمن در برنامه ریزی تولید ادغامی ارائه مـی شود. روش تجزیه بندرز برای تحلیل این مدل مورد بررسی مجدد قرار گرفته است و نواقص و کمبودهای آن مدنظـر واقـع شـده اسـت. طبیعـت خـاص این مسئله باعث شد که الگوریتم جدیدی تحت عنوان الگوریتم جانشـینی ارائـه شـود. هـر دو روش توسـط حـل مسائل عددی مختلف مورد بررسی قرار گرفته اند. نتایج این محاسـبات برتری الگوریتم جدید را نشان می دهد. بعلاوه در این تحقیق به برنامه های کامپیوتری متعددی نیاز بود که توسط نویسندگان مقاله با استفاده از توربوپاسکال (روایت هفتم) نوشته شده اند.

INTRODUCTION

Aggregate Production Planning (APP) is an intermediate range production problem (the planning horizon is generally three months to two years), in which one attempts to achieve a (cost–effective) balance between productive capacity, on one hand, and forecasts of fluctuating demand, on the other. The production manager typically has at his disposal a set of production options with which to achieve the balance. Despite the strong interest that has been shown in APP models reported examples of successful implementation have been rare [1]. Akinc and Roodman [1,2], first suggest several reasons for this failure, and then introduce a mixed integer-programming model for aggregate production planning that attempts to address the limitations they have listed. The reformulation of APP by Akinc and Roodman may be seen in the modeling framework of this paper.

Bender's decomposition [3] is a well-known constraint generation approach for problems where certain variables make the problem more difficult, for example Mixed Integer Linear Programming (MILP) problems that is the nature of the problem here. A disadvantage of this approach is that the Benders' master problem is often very hard to formulate which can make the method inefficient. Possibilities of solving the Benders master problem by speeding up the procedure have been dealt with by, for example, Cote and Laughton [4].

Akinc and Roodman [1] proposed a specialized version of Benders' decomposition for analyzing this model. The salient features of this complicated procedure are listed in [1]. For complete description, the reader is referred to Akinc and Roodman [2].

Various papers, e.g. Paula jr. and Maculan [5] and Rana and Vickson [6] use Benders' decomposition with Lagrangean relaxation applied to the Benders' cuts [3] in the master problem.

Diaby [7] presented an implicit enumeration procedure for solving pure integer 0/1 minimax problems, which arise in the context of Benders' decomposition for mixed integer 0/1 linear programming problems.

However, Holmberg [8] gave some small counter examples of MILP problems, which this approximations (of Benders' master problem), fails to solve, i.e. yields bounds worse than the LP relaxation or even fails to find any feasible solution.

Aryanezhad [9] proposed a new algorithm for solving a special version of this aggregate production planning with zero-one variables. In this procedure the master problem of the APP was not an optimization problem, but the subproblems were basically transportation models. The transportation structure of subproblems, and the non-optimal nature of the master problem, made the new approach quite superior to the previous mixed integer programming routines.

A branch and bound method based on a dual ascent and adjustment procedure is developed by Holmberg [10] and compared to application of a modified Benders' decomposition method.

In this paper the full model of APP has been reconsidered and a new algorithm is proposed. The procedure we propose for analyzing this MILP model is a modified version of Benders' decomposition that exploits the intricacies of both the restricted master problem and the related subproblems. Here, the restricted master problem will be developed by surrogating method (Glover [11]) that is a binary variable problem (BVP). We will show that the optimal solution of this BVP is straightforward. This result and the proposed incumbent solution to initiate the new approach will speed up the procedures. In order to show the superiority of the new algorithm the authors developed all the required programming codes through TURBO PASCAL 7.0 for Gomory's all integer dual method [12], Balas zero-one additive algorithm [13], strongest surrogate constraint [11,14], Benders' decomposition [3] and so on. Several numerical examples have been generated and solved by the new algorithm and Benders' decomposition. The results are tabulated and the execution time and number of iterations have been compared.

THE MODELING FRAMEWORK

To state the model, it is useful to distinguish between two types of production strategies, which we shall call production modes (PM), and capacity adjustments (CA). Production modes will refer to the alternative methods that can be used to produce units of output. They will be distinguished from one another by their cost structures and/or the technology that they employ. Typical production modes would include regular time scheduling, overtime scheduling, use of a more labor-intensive production method, subcontracting, substitution of lower quality raw materials, etc. Capacity adjustments, in contrast, will refer to the choices that determine the levels at which the production modes operate. Capacity adjustments might include changing the size of the workforce, activating idle equipment, re-scheduling preventive maintenance, etc.

In the model that follows, a binary variable, y_{ikt} will be used to denote whether the k^{th} CA is used with mode "i" in period "t". In order to avoid introducing additional variables, we will denote the use of a mode "i" in period "t" by y_{i1t}, (the first CA). The function of properly counting any mode-related fixed costs will be performed by y_{i1t}. To illustrate the relationship between y_{i1t}, and y_{ikt}, $k \neq 1$, consider overtime production as mode "i" which can be scheduled in two increments with known capacities 6-8 P.M. weekday program or a weekend program. One would model this situation by representing the existence of any overtime production by y_{i1t} and the two CA's by y_{i2t} and y_{i3t}. Constraints:

$$y_{i1t} \geq y_{ikt} \text{ and } k = 2,3$$

would force $y_{i1t} = 1$ and properly incur any fixed costs associated with any overtime before the two CA's may be employed. In this case, y_{i1t} itself has a capacity adjustment of zero but it represents a precondition for the use of CA's.

In some cases, a mode may be associated with a primary CA. For example, in regular time production, the capacity due to the existing work force has priority over any CA's due to hiring programs; likewise, in subcontracting there may be a preferred supplier who is resorted to first before any other supplier (CA's) is activated. In such cases, the capacities of these priority adjustments can simply be associated with the variable y_{i1t}. This scheme of representing modes and CA's gives a certain degree of modeling flexibility, whereby the overall capacity level can be represented by a y-vector:

$$y = \langle y_{ikt} \rangle$$

The model can be stated as follows:

Minimize:

$$\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{\ell=1}^{T} C_{it\ell}X_{it\ell} + \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{k=1}^{K_{it}}(a_{ikt}Z^{+}_{ikt} + f_{ikt}y_{ikt} + b_{ikt}Z^{-}_{ikt})$$

Subject to:

$$\sum_{\ell=1}^{T} X_{it\ell} \le R_{it}(y) \qquad i = 1,\ldots,M \quad t = 1,\ldots,T \qquad (1)$$

$$\sum_{\ell=1}^{T} X_{it\ell} \ge L_{it}(y) \qquad i = 1,\ldots,M \quad t = 1,\ldots,T \qquad (2)$$

$$\sum_{i=1}^{M}\sum_{t=1}^{T} X_{it\ell} \ge F_{\ell} \qquad \ell = 1,\ldots,T+1 \qquad (3)$$

$$y_{ikt} = y_{ik0} + \sum_{r=1}^{T}(Z^{+}_{ikr} - Z^{-}_{ikr}) \qquad i = 1,\ldots,M \quad k = 1,\ldots,K_{it} \quad t = 1,\ldots,T \qquad (4)$$

$$y_{ikt} \le y_{i1t} \qquad i = 1,\ldots,M \quad k = 2,\ldots,K_{it} \quad t = 1,\ldots,T \qquad (5)$$

Linear Configuration Constraints on y including those representing desired end conditions.

$$X_{it\ell} \ge 0 \qquad (6)$$

$$y_{ikt}, Z^{+}_{ikt}, Z^{-}_{ikt} = 0,1$$

where

- M = the number of production modes,
- T = the number of time periods,
- K_{it} = the number of capacity adjustments for production mode "i" in period "t",
- F_{ℓ} = forecast demand in period "ℓ" (F_{T+1} desired ending inventory level),
- $C_{it\ell}$ = the variable cost to produce one unit of output using production mode "i" (regardless of its capacity level) in period "t" to meet demand in period "ℓ".
- $X_{it\ell}$ = the number of units produced using production mode "i" in period "t" to meet demand in period "ℓ",
- a_{ikt} = the fixed cost to first establish capacity adjustment "k" for production mode "i" at the beginning of period "t",
- b_{ikt} = the fixed cost to remove capacity adjustment "k" for production mode "i" at the beginning of period "t",
- f_{ikt} = the fixed cost to operate capacity adjustment "k" for production mode "i" in period "t",
- Z^{+}_{ikt} = 1, if capacity adjustment "k" for production mode "i" is started at the beginning of period "t", 0, otherwise,
- y_{ikt} = 1, if capacity adjustment "k" for production mode "i" is in use during period "t", (y_{ik0} initial condition) 0, otherwise,
- Z^{-}_{ikt} = 1, if capacity adjustment "k" for production mode "i" is removed at the beginning of period "t", 0, otherwise,
- $R_{it}(y)$ = the capacity of production mode "i" in period "t" (a linear function of y), and
- $L_{it}(y)$ = a lower bound on output of production mode "i" in period "t".

In general, additional production modes will lead to additional constraints in the framework, where as additional capacity adjustment will change the form of the R_{it} (y) and L_{it} (y).

Constraints 1 specify that total production cannot exceed available capacity for each mode in each period, while Constraints 2 establish minimum quotas on total production for each mode in each period. Constraints 3 require that demand be met in each period (although they do not necessarily preclude back-ordering). Then (4) specifies the relationship that must exist among "start-up", "shut-down", and "operating" variables for each production mode and/or capacity adjustment (Z^{+}_{ikt}, Z^{-}_{ikt} and y_{ikt}, respectively). Finally, constraints (5) as discussed earlier, require that the mode "i" is in place before any further capacity increment can be employed.

The constraints in (6) are those which might be required, depending on the particular problem characteristics, to establish logical relationships among PMs and, CAs as well as desired end-of the period conditions for PMs and CAs.

MODEL ANALYSIS

Surrogating Method Here, we show that the main constraints of the master problem will be replaced by the strongest surrogate constraint. The surrogate constraints are the non-negative linear convex combinations of the main constraints and the objective function [11,14]. The generated problem with the strongest surrogate constraints is a valid relaxation of the main problem with zero-one variables.

DEFINITION 1: Suppose $Ay \le b$ is the constraints of a special problem. Suppose $V \ge 0$ is given. Then the surrogate constraint for this problem is $V^T Ay \le V^T b$, [11,14].

DEFINITION 2: Suppose the objective function of a given problem is a minimization problem. Suppose there exists at least two surrogate constraints the strongest surrogate constraint is the one, which implies less value for the objective function.

Consider a linear mixed integer programming with zero-one variables:

$$\left.\begin{array}{ll} \left.\begin{array}{ll} \text{Min} & dx + cy \\ \text{S.t.} & Dx + Ay \ge b \\ & 1 \ge y \ge 0 \end{array}\right\} (MIP') \\ \quad\quad\; x \ge 0 \\ \quad\quad\; y \text{ integer} \end{array}\right\} (MIP)$$

The dual of (MIP) for a given $y = \bar{y}$ is

$$\begin{array}{ll} \text{Max} & U_0 = U(b - A\bar{y}) \\ \text{S.t.} & UD \le d \\ & U \ge 0 \end{array}$$

Let us define:

$$U_K = (U_{k_1}, U_{k_2}, \dots, U_{k_{|K|}}),$$

$$k_1, k_2, \dots, k_{|K|} \in K, K \subset P, P = \{1,2,\dots,p\}$$

Suppose U_{k_i} is an extreme point of the following constraint (1) for all i=1,2,…,|K|.

$$\{UD \le d, U \ge 0\} \quad (1)$$

Now, the integer problem of Benders' decomposition (1962) is as follows:

$$\begin{array}{lll} \text{Min} & Z & \\ \text{S.t.} & Z \ge cy + U_k^T (b - Ay),\ k=1,2,\dots,p & (IP) \\ & 1 \ge y \ge 0 \text{ and integer} & \end{array}$$

Then in terms of this notation, any Benders' integer subproblem like (IP) can be formulated as follows:

$$\left.\begin{array}{ll} \left.\begin{array}{ll} \text{Min} & Z \\ \text{S.t.} & Z1 \ge (1c)\, y + U_K^T (b - Ay) \\ & 1 \ge y \ge 0 \end{array}\right\} (BIP'_K) \\ \quad\quad\; y \text{ integer} \end{array}\right\} (BIP_K)$$

Suppose $V_K \ge 0$ is a given vector, then the surrogate constraint for (BIP_K) can be written as:

$$V_K^T (1Z) \ge V_K^T (1c)\, y + V_K^T U_K^T (b - Ay) \quad (2)$$

Then, the strongest surrogate constraint for zero-one variables y, which maximizes $V_K \ge 0$ will be:

$$\max_{V_K \geq 0}\{\min_{y=0,1}\{[V_K^T 1]^{-1}[V_K^T(1c)\, y + V_K^T U_K^T (b-y)]\}\} \quad (3)$$

Since V_K's are the convex combination vectors, therefore $V_K^T 1$ should be equal to one, or $V_K^T 1=1$. In other words (3) is equivalent to (4).

$$\max_{V_K \geq 0, V_K^T 1=1}\{\min_{y=0,1}\{[c - V_K^T U_K^T A]\, y + b^T(U_K V_K)\}\} \quad (4)$$

Since y is a binary variable, then optimization of the relationship inside bracket in (4) is straightforward. In the other words:

$$y_i = \begin{cases} 1, \text{ if } (c - V_K^T U_K^T A)_i < 0, \\ 0, \text{ otherwise.} \end{cases}$$

Now, let us define a new vector W such as $W_j = \max\{0, -(c - V_K^T U_K^T A)_j\}$.

Then, (4) may be changed to a linear programming model with the help of new vector W. That is,

$$\begin{aligned} \text{Max} \quad & b^T U_K V_K - 1^T W \\ \text{S.t.} \quad & A^T U_K V_K - W \leq c \\ & 1^T V_K = 1 \qquad\qquad (BSP_K) \\ & V_K, W \geq 0 \end{aligned}$$

The optimal solution of (BSP_K) would imply ($\overline{V}_K$), to be the strongest surrogate constraint for Benders integer programming (BIP).

Now, let us consider (BIP'_K), and write its dual problem:

$$\begin{aligned} \text{Max} \quad & b^T U_K V_K - 1^T W \\ \text{S.t.} \quad & 1^T V_K = 1 & (5) \\ & A^T U_K V_K - c^T V_K - W \leq 0 & (6) \\ & V_K, W \geq 0 \end{aligned}$$

Since $1^T V_K=1$, Then, $c^T V_K = 1(c^T V_K) = (1c^T) V_K = (c1^T) V_K = c\,(1^T V_K) = c$, the Constraint 6 will be replaced by (7):

$$A^T U_K V_K - W \leq c \quad (7)$$

Therefore, the dual of (BIP'_K) will be as follows:

$$\begin{aligned} \text{Max} \quad & b^T U_K V_K - 1^T W \\ \text{S.t.} \quad & A^T U_K V_K - W \leq c \\ & 1^T V_K = 1 \qquad\qquad (DBIP'_K) \\ & V_K, W \geq 0 \end{aligned}$$

Here, we see that (BSP_K) and $(DBIP'_K)$ are the same. In other words, the optimal solution of the dual problem of (BIP'_K) would imply $\overline{V}_K$ which is the strongest surrogate constraint for (BIP_K). Therefore, the strongest surrogate constraint would be as follows:

$$Z \geq cy + \overline{V}_K U_K^T (b - Ay) \quad (8)$$

Trivial Solution of (BIP) The biggest obstacle associated with Benders' decomposition is the solution of master problem. However, the special structure of the APP problem in conduction with the strongest surrogate constraint method, make the master problem to have one constraint with zero-one variable. So, in each iteration, we have the following optimization problem:

$$\begin{aligned} \text{Min} \quad & Z \\ \text{S.t.} \quad & Z \geq (c - \square A)\, y + \square\, b \\ & y = 0,1 \end{aligned}$$

The optimal solution of this problem is straightforward. Hence,

$$y_i = \begin{cases} 1, \text{ if } (c - \square A)_i < 0, \\ 0, \text{ otherwise.} \end{cases}$$

The Initial Solution Let us consider Benders' final integer programming when all of the constraints are generated:

$$\left.\begin{aligned} \text{Min} \quad & Z \\ \text{S.t.} \quad & \left.\begin{aligned} & Z \geq cy + U_P^T(b - Ay) \\ & 1 \geq y \geq 0 \end{aligned}\right\} (BIP'_P) \\ & y \text{ integer} \end{aligned}\right\} (BIP_P)$$

The strongest surrogate constraint could be derived through the solution of (BSP_P), which is the dual problem of (BIP'_P).

$$\begin{array}{ll} \text{Max} & b^T U_P V_P - 1^T W \\ \text{S.t.} & A^T U_P V_P - W \le c \\ & W \ge 0 \\ & 1^T V_P = 1 \\ & V_P \ge 0 \end{array} \qquad (BSP_P)$$

The strongest surrogate constraint would be:

$$Z \ge cy + (U_P V_P^*)^T (b-Ay) \qquad (9)$$

Let us write the dual problem of MIP$'$, which is:

$$\begin{array}{ll} \text{Max} & b^T\pi - 1^T\alpha \\ \text{S.t.} & A^T\pi - \alpha \le c \\ & \alpha \ge 0 \\ & D^T\pi \le d \\ & \pi \ge 0 \end{array} \qquad (DMIP')$$

Now, let us prove that (DIMP$'$) and (BSP_P) are equivalent.

Theorem. (V_P^*, W^*) is an optimal solution for (BSP_P), if and only if $\pi^* = U_P V_P^*$ & $\alpha^* = W_P^*$ is an optimal solution for (DMIP$'$).

Proof. The objective function and the first two constraints will be the same if we do the desired transformation. The remaining constraints will be the same if we show that the two sets of (10) and (11) are equivalent.

$$\begin{cases} D^T\pi \le d \\ \pi \ge 0 \end{cases} \qquad (10)$$

$$\begin{cases} 1^T V_P = 1 \\ V_P \ge 0 \end{cases} \qquad (11)$$

The set (10) is always feasible and bounded. So, if we suppose that $\pi = U_P V_P$ is its vertices then the linear convex combination of its vertices is also its solution. Therefore, (11) is satisfied. Conversely, suppose (11) is true. Then by reconsidering (11) we have,

$$\begin{cases} D^T U_P \le d \\ U_P \ge 0 \end{cases} \qquad (12)$$

Multiplying (12) by $V_P \ge 0$ and summing it up, we will have $D^T U_P V_P \le d\,(1V_P)$.
Since $(1V_P) = 1$, then

$$D^T U_P V_P \le d \qquad (13)$$

By considering the transformation $\pi = U_P V_P$ in (13) we have $D^T\pi \le d$.
Since $V_P \ge 0$ and $U_P \ge 0$ then $\pi = U_P V_P \ge 0$ and the proof is complete.

If (π^*, α^*) solve (DMIP$'$), then the strongest surrogate constraint for (BIP_P) will be as follows:

$$Z \ge cy + \pi^* (b-Ay).$$

Then the trivial solution of the following surrogate integer problem (SIP) will yield $\bar{y}$. This $\bar{y}$ will be an initial value for starting the new algorithm. The objective value of Z will be a lower bound for the objective value of MIP.

$$\begin{array}{ll} \text{Min} & Z \\ \text{S.t.} & Z \ge cy + \pi^* (b-Ay) \\ & y = 0,1 \end{array} \qquad (SIP)$$

THE NEW ALGORITHM

In this section we are in a position to introduce a new algorithm based on the above analysis. We will also mention some properties, which reduce the computational efforts.

Step 1 (Initialization): Solve the linear program (DMIP$'$) and derive π^*. Then derive the straightforward binary solution of (SIP) and call it $\bar{y}$.
Put $Z = Z^\ell$, $Z^u = +\infty$, and k=1. GO TO STEP 2.

Step 2: Solve the following linear program:

$$\begin{array}{ll} \text{Max} & U_0 = U(b-A\bar{y}) \\ \text{S.t.} & UD \le d \\ & U \ge 0 \end{array}$$

Put $U_k \leftarrow U$. Then update $Z^u = \min\{Z^u, U_0 + c\bar{y}\}$. GO TO STEP 3.

Step 3: Construct the main constraint $Z \geq cy + U_k(b-Ay)$ with the help of U_k in step 2. If this inequality is a redundant constraint, then GO TO STEP 6. Otherwise, GO TO STEP 4.

Step 4: Suppose $K=\{1,2,...,k\}$. If k=1 then $\overline{V}_K=1$ and GO TO STEP 5.
Otherwise solve ($DBIP'_K$). Find the optimal V_K. Then put $\overline{V}_K \leftarrow V_K$, and GO TO STEP 5.

Step 5: Construct the strongest surrogate constraint as the following:

$$Z \geq cy + \overline{V}_K U_K^T (b-Ay)$$

For k=1 this constraint is unique. Develop the surrogate integer problem (SIP) as follows:

$$\begin{aligned} \text{Min} \quad & Z \\ \text{S.t.} \quad & Z \geq cy + \overline{V}_K U_K^T (b-Ay) \qquad \text{(SIP)} \\ & y = 0,1 \end{aligned}$$

The optimal solution of (SIP) is straightforward:

$$y_i = \begin{cases} 1, \text{if} \, (c - \overline{V}_K{}^T U_K{}^T A)_i < 0, \\ 0, \text{otherwise.} \end{cases}$$

Put $\overline{y}$ equal to the optimal y. If $\overline{y}$ is a repeating value, then GOTO STEP 6. Otherwise, find $Z^{\ell} = c\overline{y} + \overline{V}_K{}^T U_K^T (b-A\overline{y})$ and GO TO STEP 7.

Step 6: Solve the following integer program:

$$\begin{aligned} \text{Min} \quad & Z \\ \text{S.t.} \quad & Z \geq cy + U_i (b-Ay), \; i = 1,2,\ldots,k \\ & y = 0,1 \end{aligned}$$

This is the only integer programming problem that should be solved only once in this new approach. Put $\overline{y}$ equal to the optimal value of y, and Z^{ℓ} = min Z. Then GO TO STEP 7.

Step 7: If $Z^{\ell} < Z^u$, then $k \leftarrow k+1$ and GO TO STEP 2.

Otherwise, $Z^{\ell} = Z^u$ and $\overline{y}$ is the final optimal y. For finding the final optimal x, solve the following linear programming problem.

$$\begin{aligned} \text{Min} \quad & dx \\ \text{S.t.} \quad & Dx \geq b - A\overline{y} \\ & x \geq 0 \end{aligned}$$

Then $x = \overline{x}$, $y = \overline{y}$, $Z^{\ell} = Z^u = d\overline{x} + c\overline{y}$ are the final optimal solution of the original MILP.

NUMERICAL EXAMPLE

For purposes of illustration, we have solved several examples based on the example given by Akinc and Roodman [1]. One of the illustration problems has the following parameters:

There is a three period planning horizon with forecast demands of 800, 950, 1250 units of output. Production data is summarized below:

1. Regular Time Workforce. Initial regular work force is 20 workers where each worker contributes 35 units to output. An additional increment of 10 workers is available. Hiring/firing and incremental payroll costs associated with this option are \$400, \$250 and \$700 per worker, respectively.
2. Overtime. Actual overtime production is restricted to no more than 20% of regular time production. With this constraint, the company can employ overtime increment at capacity of 70 units/period. The variable overtime penalty is \$6 per unit for the overtime increment. In addition, there is a \$30/period incremental fixed overhead expense.
3. Inventory-Backlog. Inventory carrying cost is \$5/unit/period while demand can be backlogged at a cost of \$20/ unit/period.

In order to represent this problem within the framework given earlier, we may define production modes of (1) regular time, (2) overtime, and hiring-firing, as capacity adjustment to regular time production. The following zero-one variables will be required to model these:

Mode 1. Regular Time Production
Adjustment:
Incremental Labor (Hire/Fire) y_{11t}

Mode 2. Overtime Production Adjustment:

Overtime Increment (Use/Not Use) y_{21t}

The cost parameters for the problem will be as follows:

Inventory/ Backlog/ Operating

$$C_{1t\ell} = \begin{cases} 5(\ell - t) & \ell \geq t \\ 20(t - \ell) & \ell < t \end{cases}$$

$$C_{2t\ell} = \begin{cases} 6 + 5(\ell - t) & \ell \geq t \\ 6 + 20(t - \ell) & \ell < t \end{cases}$$

Hiring/Firing

$a_{11t} = 400$, all t (hiring)
$f_{11t} = 700$, all t (additional payroll, etc.)
$b_{11t} = 250$, all t (firing)

Overtime

$a_{21t} = 0$, all t
$f_{21t} = 30$, all t
$b_{21t} = 0$, all t

The Model. The model is then

Minimize:

$$\sum_{i,t,\ell}^{2,3,3} C_{it\ell} X_{it\ell} + \sum_{t=1}^{3} (400 Z_{11t}^{+} + 700 y_{11t} + 250 Z_{11t}^{-}) + \sum_{t=1}^{3} y_{21t}$$

Subject to:

(Labor Hrs.) $\sum_{\ell=1}^{3} X_{1t\ell} \leq 700 + 350 y_{11t}$, all t

(O.T. Cap) $\sum_{\ell=1}^{3} X_{2t\ell} \leq 70 y_{21t}$, all t

(O.T. Limit) $70 y_{21t} \leq 0.20(700+350 y_{11t})$, all t

$$y_{11t} = \sum_{r=1}^{t} (Z_{11r}^{+} - Z_{11r}^{-}), \text{ all t}$$

$$\sum_{i,t}^{2,3} X_{it\ell} \geq F_{\ell}, \text{ all } \ell$$

$X_{ikt} \geq 0$, y_{ikt}, Z^{+}_{ikt}, $Z^{-}_{ikt} = 0,1$

Optimal Solution. The optimal solution of this problem in summary form is to schedule overtime production mode in period 3, and to hire 10 workers in period 1 and let them work in periods 1-3. The cost of this solution is \$3750 consisting of \$1220 of variable (inventory, backordering, overtime penalty) and \$2530 of fixed costs (hiring, firing, and fixed wages of 10 employees, and incremental overhead due to overtime).

This problem was solved by Benders' decomposition technique, through solving four linear programming problems and four integer-programming problems. However, the new approach solved this problem by solving ten linear programming problems, but one integer programming.

COMPUTER PROGRAMMING CODES

To compare, the new approach and Benders' decomposition technique, the authors prepared the following computer programming codes through TURBO PASCAL 7.0.

1. Revised Simplex Method.
2. Dual Simplex Method.
3. Gomory All integer Dual Method.
4. Balas Zero-one Additive Algorithm.
5. Strongest Surrogate Constraint Method.

COMPUTATIONAL EXPERIENCE

In order to test the solution algorithm and the new approach, a large number of problems were generated based on the numerical example of Akinc and Roodman [1].

TABLE 1. Comparison of Time reduction in the New Approach with Respect to Benders' Decomposition.

Dimension of the problems			Benders' Decomposition				New Approach					Percent of reduction time
			Number of Iterations	Computation Time			Number of Iterations		Computation Time			
Real variables	Integer variables	Constraints		Total	LP	IP	LP	IP	Total	LP	IP	Total
2	2	4	2	1.60	0.60	1.00	3	1	1.20	0.90	0.30	37.50
4	2	4	3	1.85	0.60	1.25	2	1	1.40	1.00	0.40	24.32
8	4	8	3	2.60	1.00	1.60	5	1	1.90	1.10	0.80	26.92
8	6	6	6	2.45	0.90	1.55	5	1	1.70	1.10	0.60	30.61
12	8	12	8	2.80	1.00	1.80	8	1	2.00	1.25	0.75	28.57
12	24	12	5	4.50	1.30	3.20	11	1	3.10	2.20	0.90	31.11
18	12	12	10	6.15	2.10	4.10	10	1	3.90	2.60	1.30	36.59
18	12	18	4	6.00	2.10	3.90	9	1	3.80	2.70	1.10	36.67
20	14	12	12	7.65	2.45	5.20	12	1	5.40	3.30	2.10	29.41
26	16	16	16	12.10	4.25	7.45	16	1	8.65	4.75	3.90	28.51
30	22	20	20	16.45	4.40	12.05	20	1	11.25	6.60	4.65	31.61
36	24	14	10	21.10	6.10	15.00	22	1	14.25	6.85	7.40	32.46

These problems were solved by Benders' decomposition and by our New Approach (NA) on a 486 DXII computer with 8MB Ram. The dimensions of the problem, number of iterations, time of computation, number of LP and IP, and percent of computational time reduction of the NA with respect to the Benders' decomposition are tabulated in Table 1. In this table we can see that computation time of the new approach is smaller than Benders' decomposition algorithm. The number of integer program problems in the new approach was always only one problem, while in the Benders' procedures it varies with the number of iterations i.e., twenty integer program problems in the largest example.

CONCLUSION

In this research a new approach was developed for solving more flexible kind of aggregate production problems as formulated by Akinc and Roodman [1]. Some of the advantages of the new approach compared with Benders' decomposition procedure are listed below:

- Replacing the integer subproblems by linear problems.
- Reduction of the number of integer programming problem, to one.
- Generation of a good initial solution.
- Finding a good lower bound for the objective function in the first step of the NA.
- Reduction of the computational time.

The new approach was applied to the new formulation of aggregate production planning problem with zero-one variables. However, the application of the new algorithm for general mixed integer programming is an open question.

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