KINEMATICS AND DYNAMIC MODELLING OF THE HUMAN SHOULDER

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Abstract   This paper deals with kinematics and dynamics of the human shoulder in order to obtain the workspace related to the reachability of the wrist and applications to determine load for the wheelchair. The model contains six revolute degrees of freedom (D.O.F), two in *costoclavicular* and three in the *shoulder* joint and one in the elbow joint, while a six D.O.F model was developed according to given information of this study. A general computational procedure for the human shoulder given arbitrary trajectory is laid out in detail. Kinematics and dynamic behavior of the model were then considered. Finally, a numerical example involving a six-degree of freedom manipulator using the method is presented and the results are discussed for a specified trajectory.

چکیده   این مقاله مدلسازی سینماتیکی و دینامیکی شانه انسان را به منظور بدست آوردن فضای کاری ارائه می‌دهد. بدین منظور یک مدل شش درجه آزادی انتخاب و رفتار سینماتیکی و دینامیکی بررسی می‌شود. برای رفتار دینامیکی مدل، از روش لاگرانژ استفاده و یک الگوریتم عمومی ارائه می‌گردد. در انتها بعنوان ارائه کاربردهایی از معادلات بدست آمده برای سینماتیک مدل، فضای کاری و نیز شبیه سازی حرکت آن در فاز رانش ویلچر در یک مسیر فرضی و برای دینامیک مدل، محاسبه گشتاورهای مفاصل در فاز رانش ویلچر ارائه می‌شود.

INTRODUCTION

The shoulder mechanism includes a cons of bones (thorax, clavicle, scapula, and humerus) which connect the trunk and upper extremity. Bones of the shoulder griddle, i.e., scapula and clavicula, provide a movable but stable base for humeral motion. Motion of the humerus results from simultaneous motions of the sternoclavicular joint between the thorax and clavicula, of the acromioclavicular joint between clavicula and scapula, and of the glenohumeral joint between scapula and humerus [1-2]. In addition, to reach the utmost humeral elevation angle, rotations of the thorax are observed, with lateral rotations during one-sided elevation and backward rotations during two-sided elevation. The shoulder mechanism is a multifunctional joint. The number of functions of the upper extremity is almost infinite. It ranges from manipulating objects on a desktop to throwing a ball and lifting heavy objects. The existence of these tasks is feasible through the large range of motion and the number of degrees of freedom available in the shoulder mechanism (Figure 1).

The reachable workspace is defined as the volume within which all the points can be reached by a reference point of the mechanism [3]. The determination of the reachable workspace involves a considerable amount of numerical, time-consuming calculations. Therefore, because of its complexity, workspace calculation for mechanical manipulators is considered an area in which many

theoretical and numerical problems are still to be investigated.

Workspace properties can represent an important criterion in evaluation, programming, and design of mechanical manipulators, robots, and similar devices. A thorough analysis of the human arm workspace characteristics can therefore provide not only additional information for the design and control of artificial mechanisms but also can help us better understand the human arm proportions and motion properties. It can imply new directions in robotics research, especially in interaction with humans, such as telemanipulation, task-description, robot teaching and programming. There is a variety of applications that motivate the analytical study of workspace determination and evaluation. A comparison of the workspace properties, such as volume and shape, between the various types of manipulators can be used as an efficient tool for the design of future mechanisms [4]. On the other hand, for example, the study of the human arm mechanism and its workspace can help us to plan and control the rehabilitation procedure of a hemiplegic arm [5].

This paper presents a new method of kinematic model in order to obtain the workspace related to the reachability of the wrist and applications to determine load for the wheelchair. The model determines an approximated reachable workspace including the geometric properties, as well as some inside characteristics. The first part of the paper deals with the kinematic model of the human arm, while a suitable arrangement of joints is selected and other relevant parameters for workspace calculation are specified. In the second part, the procedure of workspace determination is described and the numerical example of a healthy left human arm is presented. Moreover, dynamic modeling for a six degree of freedom arm is dealt with for a given dynamic trajectory. Finally, to evaluate the performance of the proposed method simulation test is carried out.
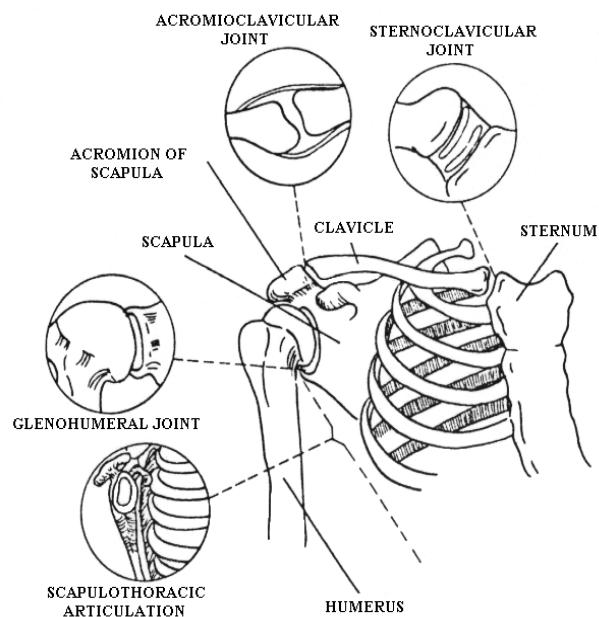

Figure 1. Schematic depiction of the bony structures of the shoulder complex and forearm.

BIOMECHANICS OF THE SHOULDER

The shoulder represents the first link in a mechanical chain of levers that extends from the shoulder to the fingertips. A compression of its components with those of other articulations reveals that the shoulder is the most intricate joint complex in the body. The combined and coordinated movements of four distinct articulations- *Glenohumeral, Acromioclavicular, Sternoclavicular* and *Scapulothoracic* allow the arm to be positioned in space for efficient function.

Range of Motion of the Shoulder Complex

Shoulder *Elevation* is defined as movement of the humerus away from the side of the thorax in any plane. Different types of shoulder elevation are possible depending on the chosen plane of motion. *Forward flexion* is shoulder elevation in the sagital plane, while *Abduction* is elevation in the frontal plane. Normal range of forward flexion and the range of abduction are both about 180 degrees.
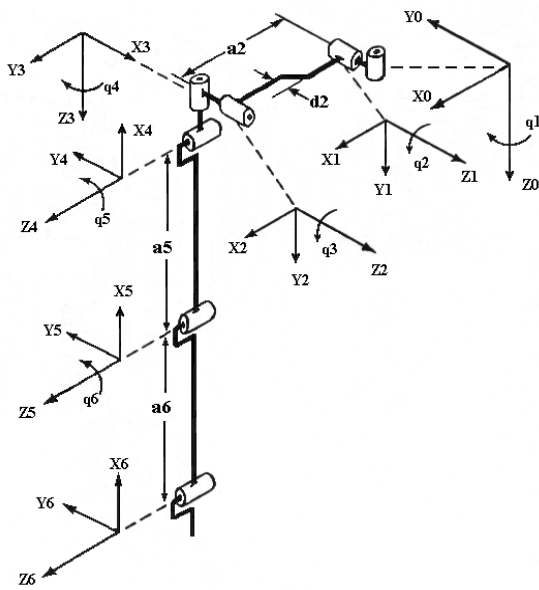
Figure 2. The six D.O.F simplified kinematic model of the shoulder mechanism.

Rotation about the long axis of the humerus is another functionally important shoulder motion. Both internal and external rotations can be performed with the humerus in varying degrees of elevation as shown in Figure 2. The range of internal and external rotation varies with the degree of arm elevation, but in general each may be accomplished to about 90 degrees and the maximum total range is about 180 degrees. Several other shoulder motions are, *Backward elevation* or *Extension* in the sagital plane, is possible to approximately 60 degrees. *Adduction* or *Depression* of the arm is the action of bringing the humerus closer to the side of the thorax and is normally limited by contact with the body.

Adduction with the arm moving in front of the body beyond the mid-line in an upward plane is possible to about 75 degrees. *Horizontal flexion* is defined as forward movement of the arm in a horizontal (transverse) plane, measured from a starting position of 90 degrees of *abduction*, the normal range of

horizontal flexion is approximately 135 degrees. Movement in the opposite direction from the same starting point, *horizontal extension*, has a normal range of approximately 45 degrees. Thus, the shoulder is capable of about 180 degrees of motion in the horizontal plane.

Range of Motion of the Elbow Elbow flexion and extension are accomplished through humeroulnar and *humeroradial* articulations. The range of flexion and extension can be predicted from the angular characteristics of the involved bony components. The angular value of the articulate surface of the *trochlea* of the humerus is 330 degrees, while that of the trochlear fossa of the *ulna* is 190 degrees. The difference is 140 degrees, which is the range of flexion-extension of the elbow. Similarly, 140 degrees is the difference between the angular value of the articulate surface of the *capitellum* (180 degrees) and that of the proximal *radial* head (40 degrees).

KINEMATIC MODELING OF THE SHOULDER MECHANISM

The kinematic structure of proposed model includes two joints corresponding to the shoulder motion. The first was named the inner shoulder joint, the second was named the outer shoulder joint. The elbow joint was replaced in the model by a simple rotation representing the elbow flexion-extension. The model thus includes three rigid links, the shoulder link representing the clavicula and scapula, the upper arm link representing the humerus, and the forearm link representing the radius and ulna. It must be stressed, however, that the model does not describe the motion of singular bones, but the spatial motion of the reference point on the wrist. The gross motions of the links can be roughly compared to the motion of some bones, although there is no direct reference.

TABLE 1. Link Parameters for a Kinematics of Shoulder.

Link Number	α_i	a_i	d_i	θ_i
1	90	0	0	θ_1
2	0	d_2	a_2	θ_2
3	-90	0	0	θ_3
4	-90	0	0	θ_4
5	0	0	$-a_5$	θ_5
6	0	0	$-a_6$	θ_6

Assume that the reference coordinate frame is fixed with its origin in the center of the inner shoulder joint (Figure 2). The x-axis lies in the medial direction, y in the anterior direction and z in the cranial direction. The complete kinematic model used for the workspace calculation contains six degrees of freedom. When all the joint angles are zero in the reference position, the shoulder link is parallel to x, the upper arm link to z, and the forearm link to y. There are two degrees of freedom in the inner shoulder joint, flexion-extension, and abduction-adduction: three rotations in the outer shoulder joint, flexion-extension, abduction-adduction (about y), and rotation (about z), and there is the elbow flexion-extension (about x).

Direct Kinematic Model of Shoulder   The Denavit-Hartenberg parameters for these coordinate frames according to the Figure 2 are listed in Table 1. So the 4×4 matrices $T_i^{i-1}(\theta_i)$ can be determined by:

$$T_1^0 = \begin{bmatrix} C1 & 0 & S1 & 0 \\ S1 & 0 & -C1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} C2 & -S2 & 0 & a2C2 \\ S2 & C2 & 0 & a2S2 \\ 0 & 0 & 1 & -d2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^2 = \begin{bmatrix} C3 & 0 & -S3 & 0 \\ S3 & 0 & C3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4^3 = \begin{bmatrix} C4 & 0 & -S4 & 0 \\ S4 & 0 & C4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^4 = \begin{bmatrix} C5 & -S5 & 0 & -a5C5 \\ S5 & C5 & 0 & -a5S5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_6^5 = \begin{bmatrix} C6 & -S6 & 0 & -a6C6 \\ S6 & C6 & 0 & -a6S6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3-1)

where Ci = $Cos(\theta_i)$ and Si = $Sin(\theta_i)$. The kinematics equation of this manipulator arm can be given by Equation 3-2 that is the functional relationship between the last link position and orientation and the displacement of all the joints [6-7].

$$T_6^0 = T_1^0(\theta_1).T_2^1(\theta_2).T_3^2(\theta_3).T_4^3(\theta_4).T_5^4(\theta_5).T_6^5(\theta_6)$$

(3-2)

Inverse Kinematic Solution   In this section, the problem of finding the joint angles are discussed. The hand-specified position and orientation are needed as follows:

$$T_6^0 = \begin{bmatrix} n_x & t_x & b_x & p_x \\ n_y & t_y & b_y & p_y \\ n_z & t_z & b_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(3-3)

The geometrical approach to the solution of the inverse kinematic was used. The geometrical inverse kinematic solution was based on the following equations:

$$\theta_6 = 2A\tan\left(\frac{L_2 \pm \sqrt{L_2^2 - L_3^2 + L_1^2}}{L_3 - L_1}\right) \quad (3\text{-}4)$$

$$\theta_1 = 2A\tan\left(\frac{-F \pm \sqrt{F^2 - d_2 + E^2}}{d_2 - E}\right) \quad (3\text{-}5)$$

$$\theta_2 = A\tan(S_2, C_2) \quad (3\text{-}6)$$

$$\theta_4 = A\tan(S_4, C_4) \quad (3\text{-}7)$$

$$\theta_5 = \text{Cos}^{-1}\left[\frac{a_6(C_1 n_y - S_1 n_y)}{a_6 S_4}\right] - \theta_6 \qquad (3\text{-}8)$$

$$\theta_3 = A\tan(S_3, C_3) \qquad (3\text{-}9)$$

where

$$L_1 = 2a_5(p_x n_x + p_y n_y + p_z n_z + a_6)$$
$$L_2 = 2a_5(p_x t_x + p_y t_y + p_z t_z)$$
$$L_3 = A^2 + B^2 + K^2 + a_5^2 - a_2^2 - d_2^2$$
$$A = p_x + a_6 n_x$$
$$B = p_y + n_y a_6$$
$$K = p_z + n_z a_6$$
$$E = -p_y - a_6 n_y - a_5 C_6 t_y + a_5 S_6 t_y$$
$$F = p_x + a_6 n_x + a_5 C_6 n_x - a_5 S_6 t_x$$
$$C_2 = \frac{p_y + n_y a_6 + a_5(C_6 n_y - S_6 t_y) - d_2 C_1}{a_6 S_1}$$
$$S_2 = \frac{p_z + n_z a_6 + a_5(C_6 n_z - S_6 t_z)}{a_2}$$
$$S_4 = \pm\sqrt{S_1^2(n_x^2 + t_x^2) + C_1^2(n_y^2 + t_y^2) - 2C_1 S_1(n_x n_y + t_y t_x)}$$

$$T_4^1(3,3) \Rightarrow -b_y C_1 + b_x S_1 = -C_4$$
$$T_4^2(1,3) \Rightarrow b_x C_1 C_2 + b_y C_2 S_1 + b_z S_2 = -C_3 S_4$$
$$\qquad (3\text{-}10)$$
$$T_4^2(2,3) \Rightarrow b_z C_2 - b_x C_1 S_2 - b_y S_1 S_2 = -S_3 S_4$$

$$\qquad (3\text{-}11)$$

The only unknown parameter in the Equation 3-10 is $Cos(\theta_3)$ and in the Equation 3-11 is $Sin(\theta_3)$.

WORKSPACE ANALYSIS AND MOVEMENT STIMULATION OF THE MODEL

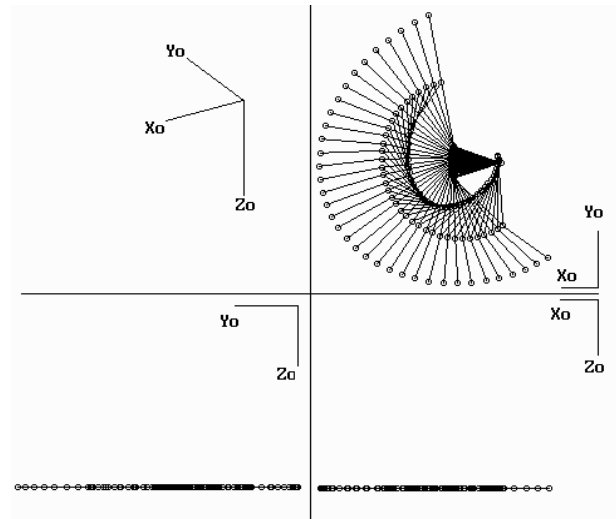In this part, the reachable workspace of the model is determined, using the kinematics equation and

Figure 3. The approximated human arm workspace (plane view.

also stimulating the movement of the model during an assumed motion in the wheelchair propulsion.

Workspace Analysis The reachable workspace is defined as the volume within which all the points can be reached by the center of the wrist. To represent the workspace of the model we need to determine the range of motion of each joint, according to the biomechanical properties of the human arm from a definite base point. The values of θ_1 through θ_6 related to this state were listed in Table 2. The ranges of motion of the $\theta_{1,...,6}$ according to biomechanical behavior of the human arm are given by:

$$\begin{bmatrix} -20 \\ -20 \\ -180 \\ -180 \\ 30 \\ 0 \end{bmatrix} \leq \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \leq \begin{bmatrix} 20 \\ 20 \\ 0 \\ 0 \\ 270 \\ 140 \end{bmatrix} \qquad (4\text{-}1)$$

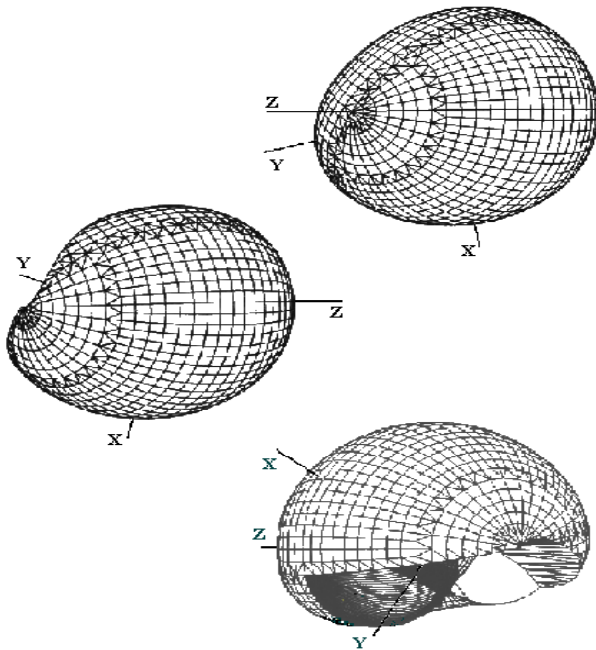The shoulder movement can be considered as a combination of movements in joints from both

Figure 4. The approximated human arm workspace related to the reachability of the selected reference point placed on the wrist.

sides of the clavicle bone; shoulder flexion-extension, and shoulder abduction-adduction. Based on the reference coordinate frame fixed on the sternoclavicular joint (Figure 2) and using the Equation 3-2, the workspace of the model is illustrated in the Figures 3 and 4.

Figure 3 shows an isometric view of workspace of the model in the horizontal plane. Moreover, Figure 4 shows a three dimensional workspace of the model, using a written program by the Microsoft *AutoCAD*.

These figures relate to a right arm, the body approximated by an elliptical cylinder, the arm links are approximated by lines, where the length of shoulder link is 0.11m, of the upper arm link is 0.27m, and forearm link is 0.23m.

DYNAMIC MODE

In this part the dynamic behavior of the model is described in terms of the time rate of change of the

TABLE 2. The Value of Joint Displacements in the Base Position.

θ_i	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
B.P	0	0	0	-90	90	0

arm configuration in relation to the joint torques exerted by the actuators. This relationship can be expressed by a set of differential equations using the *Lagrangian-formulation*. The closed-form dynamic equation of an n-D.O.F manipulator is given as follows [6-7]:

$$\sum_{j=1}^{n} H_{ij}\ddot{\theta}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} h_{ijk}\dot{\theta}_j\dot{\theta}_k + G_i = Q_i$$

i=1,…,n   (5-1)

where

$$H = \sum_{i=1}^{n}(m_i J_L^{(i)T} J_L^{(i)} + J_A^{(i)T} I_i J_A^{(i)})$$

$$h_{ijk} = \frac{\partial H_{ij}}{\partial \theta_k} - \frac{1}{2}\frac{\partial H_{jk}}{\partial \theta_i}$$

$$G_i = \sum_{j=1}^{n} m_j g^T J_{Li}^{(j)}$$

$$Q = \tau + J^T F_{ext}$$

$\tau_1(t),…,\tau_6(t)$ are the joint torques and $\theta_1(t),…,\theta_6(t)$ are the generalized coordinates.

The external forces and moments (F_{ext}) in the Equation 5-1, equated to zero and the inputs and initial condition listed in the Table 3. This work used MATHEMATICA for symbolic derivation, numerical solution [8-9]. It was chosen mainly because of its versatile symbolic manipulation capabilities, such as, symbolic simplification of polynomials and rational expressions, linearisation of trigonometric functions, automated evaluation of the relative significance of terms.
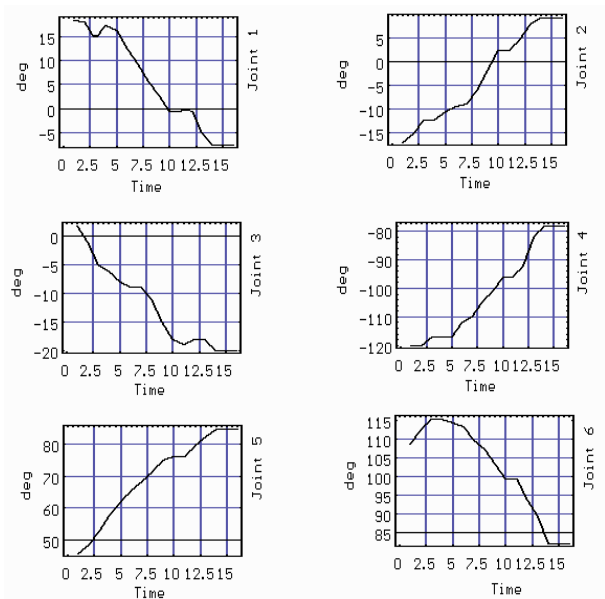
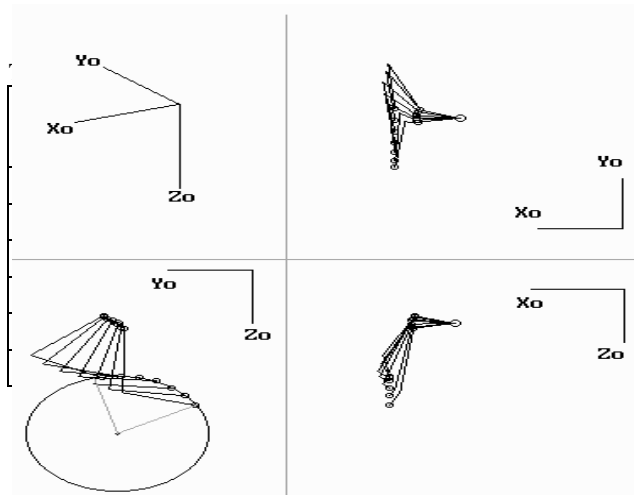**Figure 5.** Joint responses of kinematics model of shoulder.

Figure 6. Isometric view related to the right hand.

SIMULATION

A simulation study was carried out to further investigate the validity and effectiveness of the method presented above. Various desired trajectories were simulated and the following example related to a healthy left human arm is prescribed during an assumed wheelchair propulsion. Using obtained equations in the section (3), the joint displacements that lead the end-effector on desire points are found for given fourteen digitized trajectory points on a $75°$ arc (from $-15°$ through $60°$) according to the vertical axis on the ring of assumed wheelchair.

The length of the links approximated the same as the previous section (0.11m, 0.27m, 0.23m), the arc assumed on a circle with the diameter 0.50m and the center (0.14m, -0.04m, 0.45m) as compared with the reference coordinate frame.

Joint displacements ($\theta_{1,..,6}$) are obtained from the inverse kinematics for a desired trajectory. Figure 5 shows the variations of the joints displacements ($\theta_{1,..,6}$) in this course. Figure 6 shows its stimulation from isometric view, related to the right hand that provided using from a graphic program written by Microsoft *QBASIC*. The program was written using the Microsoft *Mathematica* and operated in a computer Pentium 266 MHz.

Furthermore, an example of inverse dynamic is also discussed. In this case, inputs are the desire trajectories, described as time functions $\theta_i(t)$ and the outputs are the joint torques to be applied at each instant by the actuators in order to follow the specified trajectories.

Using Equation 5-1, at each instant we computed joint velocities $\dot{\theta}_i(t)$ and joint acceleration $\ddot{\theta}_i(t)$ from the given time functions, and the substituted them to the left-hand side of Equation 5-1.

The endpoint forces F_{ext} in Equation 5-1 are chosen from results of the work of Moienzadeh [10], consist of forces and moments during wheelchair propulsion in fourteen points as shown in Figure 7. It shows the time varying forces required to execute the trajectory that depends on the joint velocities.

CONCLUSION

A mathematical model of the human arm kinematics is developed based on experimental evaluation of selective movements in the sagital, frontal, and horizontal plane. The developed model contains six revolute degrees of freedom for positioning the wrist and represents a redundant mechanism where each joint is dedicated to a specific task. The main goal of the paper is modeling the kinematic and dynamic behavior of the human arm, for this purpose in the first part of the paper biomechanical behavior of the human shoulder was studied. The consequence of this study is that a comparison of shoulder with other articulations reveals that it is the most intricate joint complex in the body. By simulating dynamic equations, further insights on the dynamic equations of the human arm were gained. The work also shows that there is a great benefit in using a symbolic derivation language when dealing with shoulder dynamics.

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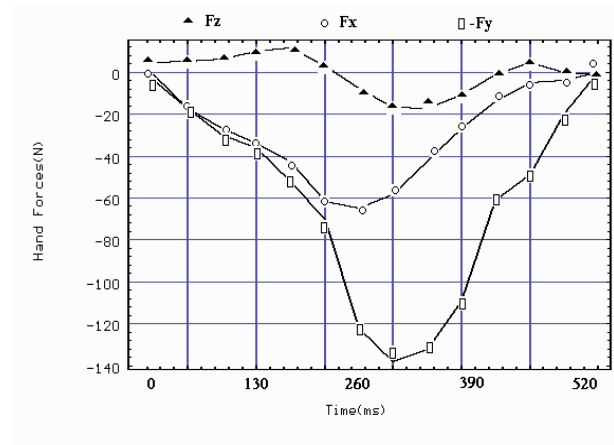
Figure 7. Hand forces against time subject to given trajectory.

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