

**EARTHQUAKE INDUCED PERMANENT
DISPLACEMENT OF SLOPES:
A NUMERICAL STUDY**

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Abstract The conventional method to evaluate earthquake induced permanent deformations of slopes is the one proposed by Newmark. In this paper, permanent displacement of slopes is studied using a combination of distinct element and finite difference methods and the results of these numerical evaluations are compared with those of the Newmark's approach. Several parameters involved, including peak ground acceleration, period of excitation, friction and cohesion of the sliding surface, are considered in this study. The results show that the period of excitation is an important parameter in finding the displacements of slopes. In addition, it is shown that in resonance condition, the permanent displacement of a slope, derived by Newmark's method, can be twice the one predicted by the distinct element method. Finally, It is realized that, in frictional materials, permanent displacements of slopes obtained by the distinct element and Newmark's methods are in better agreement compared with the corresponding results in cohesive materials.

Key Words Slopes, Permanent Displacement, Distinct Element Method, Newmark's Method, Earthquake, Numerical Study

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INTRODUCTION

Newmark in 1965 [1] proposed the rigid block sliding model to evaluate the permanent displacement of slopes due to earthquakes. He rightly showed that the conventional safety factor of a slope is not a good measure of its safety during earthquake. He also showed that

the transient safety factor might be less than one for some time intervals. However, these situations may last for such a short time that the induced deformations will be very small and will not cause stability problems to the slope. Therefore, permanent displacement of a slope during earthquake is a logical measure of its safety. Newmark's idea was later on pursued

and improved by some researchers [2,3]. Goodman and Seed [4] showed that this method gives reasonable results for frictional dry slopes. Sarma [5], using the Skempton's parameters, modified the sliding block method to account for pore water pressure. A review of the Newmark's method and the techniques in finding permanent displacements of slopes has been reported in [6].

An important parameter in the sliding block method is the critical or yielding acceleration $k_c g$, i.e., the least acceleration causing instability of the sliding block. $K_c g$, with g being the gravitational acceleration, is a function of slope geometry and soil strength parameters. In rigorous evaluation of k_c , transient pore water pressure and slope geometry which change during earthquake, must be taken into account.

Another important parameter in the sliding block approach is the time history of ground acceleration, $a(t)$. In this method, by integrating the difference between $a(t)g$ and $k_c g$, the induced displacement of a slope can be calculated.

Using the sliding block approach, Ambraseys and Menu [3] calculated earthquake induced permanent displacements of slopes and proposed an envelope for finding the maximum ground deformation, provided that k_c to k_m ratio was known. k_m is the peak ground acceleration expressed as a fraction of the gravitational acceleration.

Gazetas and Uddin [7], by considering a prescribed sliding surface, conducted some numerical tests to find permanent deformation of slopes. Although, their studies are valuable in showing the power of numerical tools in finding earthquake induced displacements and the role of some parameters involved, there are some problems in their paper which have not been elaborated. Some of these problems follow:

1. The model used to simulate the interaction of

the sliding block and the slope body has not been clearly introduced.

2. It seems that the geometric non-linearity has not been taken into consideration.
3. The role of some important parameters, such as interface (joint) friction and cohesion has not been studied.

In this paper, permanent displacements of slopes are calculated using both the Newmark's and the distinct element methods and the results are compared. In the distinct element approach, the block is assumed to slide on a prescribed sliding surface.

CA2 computer program, developed by the first author [8], is used in the numerical analysis. CA2 is a two dimensional finite difference code which can solve static and dynamic problems. The material behavior can be linear or non-linear. Both elastic-perfectly plastic Mohr-Coulomb materials and elastic-plastic models with hardening or softening can be analyzed by the program. Large deformation problems can also be solved by this code. CA2 is also capable to analyze: soil-structure interaction problems, fluid flow in porous media, fluid-solid interaction in consolidation problem, cable-solid interaction (e.g reinforcement in rock engineering) and stability of slopes problems. In addition, CA2 can model the interaction of discrete bodies. This feature of CA2 is important in the distinct element analyses of slopes presented in this paper. The discrete blocks are internally discretized and, therefore, the deformability of discrete blocks is taken into consideration.

MATHEMATICAL MODEL

The typical soil slope considered in this study is discretized as shown in Figure 1. Since the analyses presented in this paper are for soil slopes with low modulus of elasticity, rigid block notion in the Newmark's method can not be

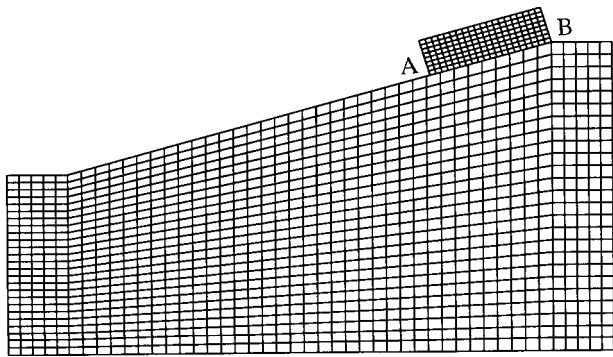


Figure 1. The discretization of the slope and the sliding block.

used directly. In fact due to the flexibility of the soil, the applied earthquake acceleration can not transfer to the sliding block without any change, as it is assumed in a rigid system. To account for soil flexibility, the slope is analyzed under earthquake excitation and the history of the induced acceleration in the soil block is calculated. The induced acceleration, in Newmark's method, is assumed to be the acceleration history of a finite difference node in the middle of line AB in Figure 1. The permanent displacement of the sliding block is then calculated by double integration of the difference between the above mentioned induced acceleration and $k_c g$.

In the distinct element approach, a sliding surface (AB) is assumed to exist within the soil mass and by dynamic analysis of this jointed system, permanent displacement of the sliding block is calculated.

The soil is assumed to be linear by elastic. The sliding surface behaves as an elastic-perfectly plastic Mohr-Coulomb model [9]. The material properties used for the soil and the sliding block are as follows:

E = Young modulus = 3 MPa

γ = Soil density = 1800 kg/m³

ν = Poisson's ratio = 0.2

$K_n = K_s$ = Normal and shear stiffnesses of the joint = 0.6 MPa/m

ϕ = Joint friction angle = 15° - 40°

C = Joint cohesion = 20-70 KPa

In the parametric study of the slope deformation, the joint friction angle and cohesion range from 15° to 40° (in 5° increments) and from 20 to 70 KPa (in 10 KPa increments) respectively.

For simplicity, the earthquake acceleration is assumed to be a cosine function of time with amplitudes ranging from 1 to 2 m/s². The shock is applied at the bottom boundary of the system to simulate a shear wave propagation. Its duration is 15 seconds and the period of excitation is taken to vary from 0.25 to 15 seconds. The boundary conditions in both the distinct element and Newmark analyses are the same. Initially, the lateral boundaries are fixed horizontally, while the bottom boundary is fixed vertically and a static analysis is performed under gravitational forces. To start the dynamic analysis, lateral boundaries are attached to special free field elements to absorb outgoing waves [10]. The bottom boundary is connected to vertically a fixed rigid foundation, whereas in the horizontal direction it is excited by a cosine acceleration shock. Since the deformation of the sliding block might be quite large, the analysis is performed by considering the geometric non-linearity of the problem. By performing several numerical experiments, a parametric study is conducted to clarify the importance of several parameters involved. These results are reported in the following sections.

NUMERICAL RESULTS

In this section, the results obtained from both the distinct element and the sliding block methods in finding the displacements of slopes are compared with.

In Figures 2 and 3, permanent displacements of the sliding block of a 15° slope as a function of earthquake period, for the Newmark's and

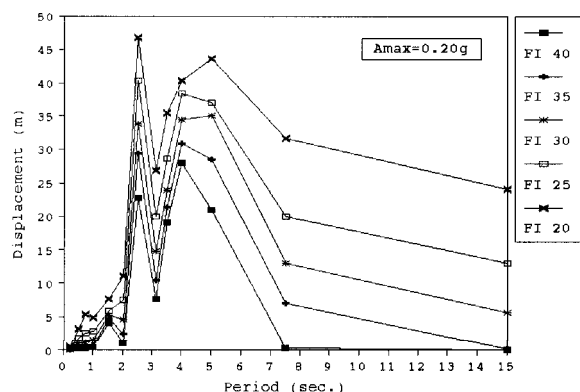


Figure 2. The permanent displacement of the block vs. the shock period (distinct element analysis).

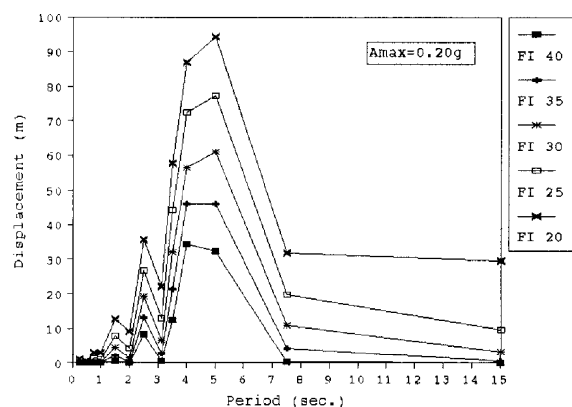


Figure 3. The permanent displacement of the block vs. the shock period (Newmark's analysis).

TABLE 1. First Three Modes Natural Frequencies and Periods of the Slope.

Natural Mode	Frequency	Period (s)
First	0.258	3.87
Second	0.433	2.31
Third	0.496	2.02

the distinct element analyses, are shown. The analyses are performed for different joint frictions. The joint cohesion is assumed to be zero. The amplitude of the earthquake shock is 0.2g. From these figures, it is realized that by increasing the interface friction, the corresponding permanent displacement is reduced. Comparison of Figures 2 and 3 shows that while the displacements derived by the two methods are in good qualitative agreement,

there are some quantitative differences between the results. Table 1 shows the natural periods and frequencies of the slope.

Referring to Table 1 and Figures 2 and 3, it can also be concluded that once the period of the applied shock approaches a natural period of the slope, the corresponding permanent displacement increases. This is expected due to the resonance phenomenon. It is interesting to note that in the distinct element analysis, resonance in the second mode is stronger than that of the corresponding mode in the Newmark's analysis. Similar analyses have also been conducted for slopes with cohesive interfaces [11] and the overall results are similar to the ones in the Figures 2 and 3.

The ratios of permanent displacements derived from the Newmark's method to the corresponding value derived from the distinct element method (R) as a function of the

normalized period ($\frac{T}{T_n}$), for different joint

frictions, are shown in Figure 4. The normalized period is defined as the ratio of applied period to the first mode natural period of the system. Figure 5 displays similar curves as those in Figure 4 for a slope with purely cohesive interface. From Figure 4, it might be realized that around the resonance period, the permanent displacement derived from the Newmark's method could be twice the one obtained by the distinct element method. This finding is consistent with the results reported in [7].

Comparison of Figures 4 and 5 demonstrates that for a cohesive soil, the difference between the permanent displacements derived by both methods is even higher compared to those of a frictional soil. An immediate practical conclusion of this finding is that the results of the Newmark's method are expected to be more reliable for frictional materials.

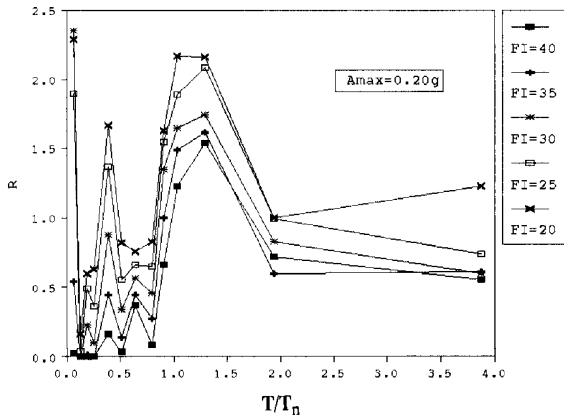


Figure 4. The ratios of the displacements derived from the Newmark's method to the corresponding values derived from the distinct element method (R) vs. period ratios (frictional interface).

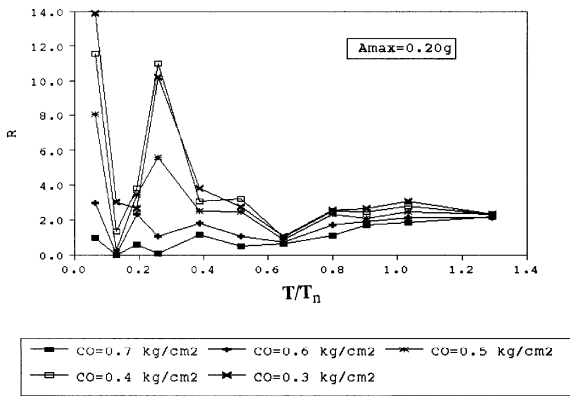


Figure 5. The ratios of the displacements derived from the Newmark's method to the corresponding values derived from the distinct element method (R) vs. period ratios (cohesive interface).

Figures 6, 7 and 8 display the computed permanent displacements of the sliding block, for a 15° slope, as functions of the interface frictions for different periods of excitations. Three different peak accelerations were used in these figures.

It should be noted that, as it is expected, the permanent displacement of the slope is a non-linear function of the interface friction. By comparing these figures, it is also realized that the displacement of the slope does not change linearly with the applied peak acceleration. Similar results have also been obtained for a

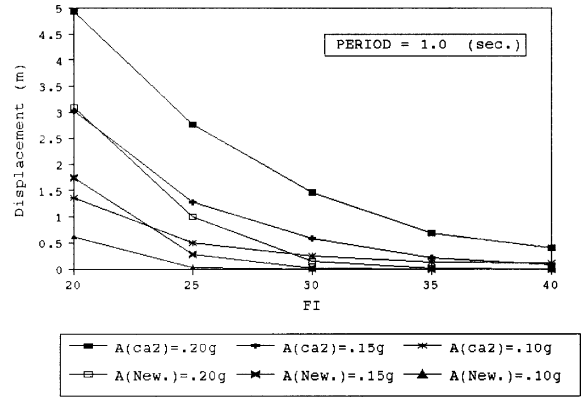


Figure 6. Permanent displacement of the slope vs. the interface friction angle. The excitation period is 1 second (below resonance).

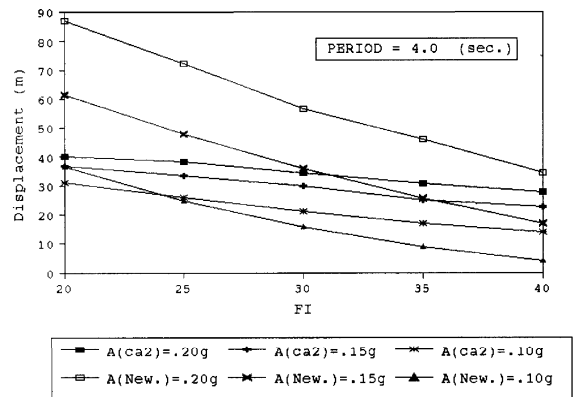


Figure 7. Permanent displacement of the slope vs. the interface friction angle. The excitation period is 4 seconds (resonance).

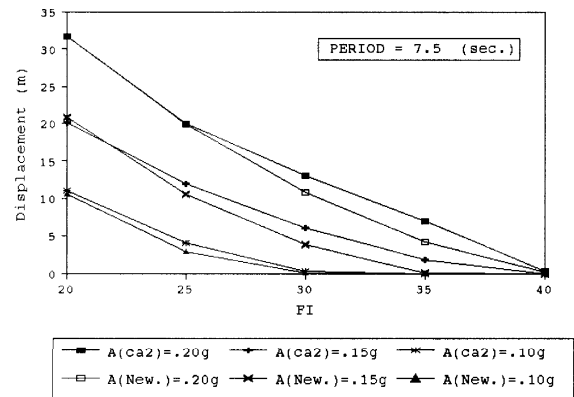


Figure 8. Permanent displacement of the slope vs. the interface friction angle. The excitation period is 7.5 seconds (above resonance).

cohesive soil which have been reported in [11].

To investigate the role of joint normal and

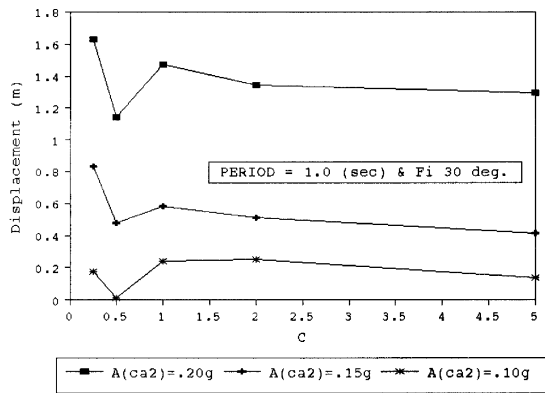


Figure 9. Permanent displacement of the slope vs. the joint stiffness parameter (C) for a 15 slope. Joint friction angle = 30 and the period of excitation is 1 second.

shear stiffnesses (K_n & K_s) on the permanent displacement of the slope, some numerical tests were conducted by multiplying the initial values of K_n and K_s (0.6 MPa/m) with a stiffness parameter (C). Figures 9 and 10 display the computed displacements as functions of the stiffness parameter. These figures have been plotted for different periods of excitations and different applied peak accelerations. It is observed that the interface normal and shear stiffnesses are not important parameters in the distinct element evaluation of permanent displacements of slopes. It is important to note that a very small joint normal stiffness can lead to numerical errors in the interpenetration of joint surfaces. This fact can be observed in the sharp changes of the curves in Figure 9, when the joint normal stiffness is a small number compared to the soil modulus.

To demonstrate the appropriate selection of the element sizes used in this paper, a sensitivity analysis was conducted using different grid sizes. Figure 11 shows the calculated results. The horizontal axis in this figure is the ratio of the tried grid size to that of the original one. This figure confirms that the domain of the analysis has been discretized appropriately and no mesh dependent error has occurred.

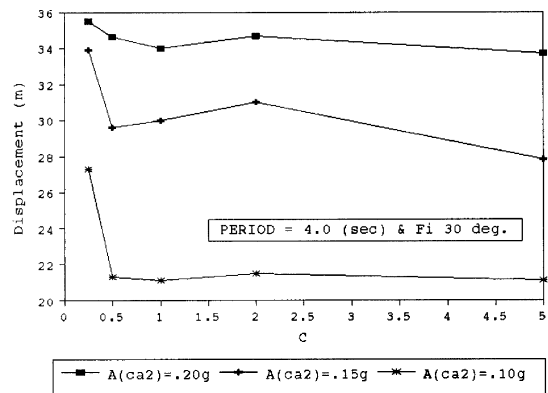


Figure 10. Permanent displacement of the slope vs. the joint stiffness parameter (C) for a 15 slope. Joint friction angle = 30 and the period of excitation is 4 seconds.

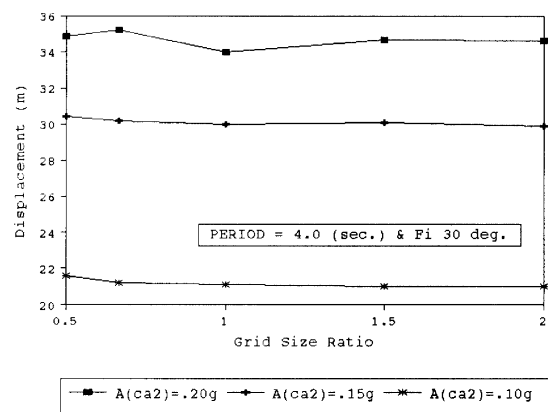


Figure 11. Permanent displacement of the slope vs. the grid size ratio for a 15 slope. The joint friction angle is 30 and the excitation period is 4 seconds.

CONCLUSION

In this paper, earthquake induced permanent displacements of slopes were studied and a comparison was made between the Newmark's method and the distinct element approach. In the latter method, a plane interface or joint was prescribed between the soil mass and the sliding block and parametric studies were conducted to investigate the important parameters involved in the slope deformation. The main findings are summarized as follows:

1. For a harmonic shock, with fixed duration and fixed amplitude, permanent displacement of a slope is a function of the applied frequency, i.e when the frequency of

the applied shock approaches the natural frequency of the slope, maximum displacements are induced.

2. During the first mode resonance, the displacement derived by the Newmark's method is greater than that obtained by the distinct element method.
3. The discrepancy between the slope displacements obtained by the Newmark's and the distinct element methods is greater for cohesive soils compared to the ones for frictional soils.
4. The displacement of the sliding block is almost independent of the interface normal and shear stiffnesses.

Some numerical runs for slopes with circular sliding surfaces were also conducted [11] which gave outputs similar to those presented in this paper.

An important parameter, which was not considered in this study, is the pore water pressure. In fact, pore pressure is not constant during earthquake excitations. This fact leads to a variable K_c in the Newmark's method and some complications in the distinct element approach.

Another important issue, which was not addressed in this paper, is the location of the prescribed interface or slip surface. A searching technique seems necessary to locate the critical slip surface of a slope. The critical slip surface is the one which gives the maximum permanent displacement. The location of this surface is a function of the slope geometry and the design earthquake. To find the location of this slip surface, the searching technique could be somehow similar to those used in the conventional limit equilibrium methods in finding slip surfaces with minimum safety

factors. The issues of pore water pressure and locating critical slip surface, during earthquake, need further investigations.

REFERENCES

1. Newmark N. M., "Effects of Earthquakes on Dams and Embankments", *Geotechnique*, Vol. 15, No. 2, (1965), 139-160.
2. Makdisi F. I. and Seed B., "Simplified Procedure for Estimating Dam and Embankment Earthquake-Induced Deformations", *Journal of the Geotechnical Engineering ASCE*, Vol. 104, No. GT7, (1978), 849-856.
3. Ambraseys N. N. and Menu J. M., "Earthquake-Induced Ground Displacements", *Earthquake Engineering and Structural Dynamics*, Vol. 16, (1988), 985-1006.
4. Goodman R. E. and Seed H. B., "Earthquake Induced Displacements in Sand Embankments", *Journal of Soil Mech. Fdn. Div.*, ASCE, SM2, (1966), 125-146.
5. Sarma S. K., "Seismic Stability of Earth Dams and Embankments", *Geotechnique*, Vol. 25, No. 4, (1975), 743-761.
6. Cai Z. and Bathurst R. J., "Deterministic Sliding Block Methods for Estimating Seismic Displacements of Earth Structures", *J. Soil Dynamics and Earthquake Engineering* (1996), 15.
7. Gazetas G. and Uddin N., "Permanent Deformation on Preexisting Sliding Surfaces in Dams", *J. Geotechnical Eng.*, ASCE, Vol. 120, No. 11, (1994), 2041-2061.
8. Fakhimi A. A., "Theory and User's Manual of CA2 Computer Program", Building & Housing Research Center, No. 262, Tehran, Iran, (1997).
9. Fakhimi A. A., "Numerical Modeling of Jointed Media", *Proceedings of the 4th International Conference on Civil Engineering* Sharif University of Technology, Tehran, Iran, Vol. II, (1997), 130-136.
10. Cundall P. A., Hansteen A. H., Lacasse S. and Selnes P. B., "NESSI, Soil-Structure Interaction Program for Dynamic and Static Problems", Norwegian Geotechnical Institute, Report 51508-9, (1980).
11. Moosavi M. J., "Numerical Evaluation of Earthquake Induced Displacement of Slopes", M.Sc. Thesis, School of Engineering, University of Tarbiat Modarres, Tehran, Iran, (1997).

Some notes about the referee No. (1) comments.

Paper title: "EarthquakeInduced Permanent ..."

Ref: 147-98

1- We have not explicitly claimed in the paper that the distinct element method is a more reliable approach, compared to that of Newmark, in finding permanent displacement of slopes due to earthquakes, but we generally believe that since distinct element method takes more details of the model into consideration, it should be a more appropriate approach.

We should remind you that distinct element method has been successfully used to analyze discrete systems and this fact also confirms the robustness of the method.

2- On page 4, we have informed the reader that analyses presented in the paper deal with soft soils. For this reason, a small Young modulus has been used. We agree that more numerical studies are needed with different soil rigidities to be able to compare Newmark and distinct element approaches more precisely, but, perhaps, all those results can not be presented in one paper. We might conduct such researches in future.

3- The referee believes that induced displacements are high compared to those in reality. Our comments in this regard are as follows:

- a) In any of our analyses, a single harmonic shock has been applied for the whole period of excitation. If the frequency of excitation approaches to one of the natural frequencies of the system, we expect great induced displacements. In reality, even if resonance happens, it might last for a short period of time. This is due to the complexity of real shocks compared to the simple cosine shock used in this study.

b) By using Ca2 program, we can analyze complicate systems under complex shocks. The reason for using a simple cosine function, is to study the influence of shock period (among others) on the system.

c) For realistic friction angles of granular materials (say $\phi=30$ to 35°) and when the system is far from the resonance situation, the induced permanent displacements approach to the ranges mentioned by the referee.

4- In this paper, we have not tried to compare our results with those in real situation. Our intention has been to compare two mathematical techniques which might be used to estimate earthquake induced displacements. In addition, we tried to show the role of important parameters involved. We believe that such detailed study which compares Newmark and distinct element methods have been performed for the first time in this paper.

Some notes about the referee No. (2) comments.

Paper title: "EarthquakeInduced Permanent ..."

Ref: 147-98

1- We did our best to correct grammatical mistakes and misprints.

2- We think that our formulation is correct. Ca2 computer program has been used for solving complex problems in some universities and it seems that Ca2 is a reliable code. For more clarification and more information about the mathematical formulation of the code, the referee is kindly requested to see reference No. 8.

3- The referee is referred to several papers, published in the literature, which use the elastic-perfectly plastic Mohr-Coulomb failure criterion for joints or interfaces. The joint model has been called "elastic perfectly plastic" for the following reason. If for a constant normal stress (S), the relative shear displacement is increased gradually, the shear resistance of the joint which increases linearly, becomes eventually a constant value, i.e $\tau = S \tan \phi$. This model has been extensively used by rock mechanics researchers (see e.g. reference No. 8&9 in the paper).

Some notes about the referee no. (3) comments.

Paper title: "EarthquakeInduced Permanent ..."

Ref: 147-98

1- Although, the title "Permanent displacement of slopes caused by horizontal harmonic base excitement", proposed by the referee sounds appealing, we believe that the current title of the paper is equally appropriate. In the current title, the "numerical study" has been emphasized, which is a key word in the paper.

2- Following the referee's comment, the paper of Gazetas and Uddin was cited in the introductory part of the paper.

In addition, the title: "Statement of the problem" has been replaced by: "Mathematical model".

3- A comparison has been made between our results and those in reference No. [10] (in the revised paper, the reference No. has been changed to 7) in page 9.

4- We think that Ambraseys and his coworkers have performed some researches regarding earthquake induced displacement of slopes. We do not claim that we are aware of all excellent works that have been done in this field.

5- The geometry of the slope has been shown in Figure 1. The boundary conditions during initial static and final dynamic situations have been fully explained in page 6.

Regarding to the free field boundaries, a reference has been made to reference No. 10.

The applied shock is a simple cosine function which does not seem to need more elaboration.

With respect to "determination of input accelerogram in Newmark's method" and "capabilities of Ca2 program", some descriptions were added to the paper (please see page 4 and 5).

6- The referee believes that induced displacements are high compared to those in reality. Our comments in this regard are as follows:

- a) In our analysis, a simple harmonic shock has been applied for the whole

period of excitation. If the frequency of excitation approaches to one of the natural frequencies of the system, we expect great induced displacements. In reality, even if resonance happens, it might last a time ~~This is due to the complexity of real shocks, compared to the simple cosine shock used in this study.~~

b) By using Ca2 program, we can analyze complicate systems under complex shocks. The reason for using a simple cosine function, is to study the influence of shock period (among others) on the system.

c) For realistic friction angles of granular materials (say $\bar{f} = 35^\circ$) and when the system is far from the resonance situation, the induced permanent displacements approach to those observed in reality (please see Figures 2 & 3).

7- We appreciate greatly the referee's comments regarding the mistakes in Figures 4 and 5. In these figures, the abscissa axis shows $\frac{T}{T_n}$ (not $\frac{F}{F_n}$), in which T is the period of excitation and T_n is the first natural period of the system. The necessary corrections, regarding these mistakes, were applied to the paper.

8,9- The statements, "The role of K_c in sliding block ..." and "In fact by linearly increasing" were removed from the paper.

10- "A planner interface", in the conclusion section, was replaced by "plane interface".

11- The word "transient" in Page 2, was replaced by "permanent".

12- Grammatical mistake and misprints were corrected.

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