RESEARCH NOTE

MACHINE REPAIR QUEUEING SYSTEM WITH NON-RELIABLE SERVICE STATIONS AND HETEROGENEOUS SERVICE DISCIPLINE

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Abstract This investigation deals with M/M/R/N machine repair problem with R non-reliable service stations which are subjected to unpredictable breakdown. There is provision of an additional server to reduce backlog in the case of heavy load of failed machines. The permanent service stations repair the failed machines at an identical rate μ and switch to faster repair rate μ_f when all service stations are busy. By using matrix geometric theory, the formulae for obtaining the optimal number of service stations is derived.

Key Words Machine-repair, Markov Queue, Non-reliable, Server Breakdown, Additional Server, Matrix Geometric

چکیده این پژوهش به مسألهٔ تعمیر ماشین M/M/R/N با R ایستگاه خدمت‌دهی غیرقابل اعتماد - که در معرض از کار افتادگی غیرقابل پیش‌بینی هستند - می‌پردازد. درصورتیکه برای ماشینهای ازکارافتاده بار کاری سنگینی وجود داشته باشد، امکان تدارک خدمت دهندهٔ اضافی وجود دارد تا سفارشات عقب افتاده کاهش یابد. وقتی تمام ایستگاههای خدمت‌دهی مشغولند، ایستگاههای خدمت دهی دائمی، ماشینهای ازکارافتاده را با نرخ μ تعمیر کرده و به نرخ تعمیر سریعتر μ_f انتقال می‌یابند. در این مقاله، با استفاده از نظریهٔ هندسه ماتریسی، فرمولی برای کسب تعداد بهینه ایستگاههای خدمت‌دهی ارائه می‌شود.

INTRODUCTION

The machine repair problem has been studied in many frame works over the past three decades. Feller [1] introduced M/M/R/N machine-repair problems and obtained analytic steady-state solution. Many researchers from have occasionally studied the server breakdown machine-repair systems [cf. 2,3,4] . Jain [5] investigated diffusion approximation for $G^*/G/r$ machine interference problem with spares. Sztrik and Bunday [6] analyzed the asymptotic behavior of the machine interference problem with machines and a single operator. Jain and Premlata [7] investigated M/M/R machine repair problem with reneging and obtained some measures of effectiveness.

In some queueing systems it is feasible to introduce the additional servers in order to reduce the waiting time of customers as well as system cost. The queueing systems with additional servers have been rarely studied. Murari [8] developed bulk service finite queueing system with a special additional channel. Mukaddis and Zaki [9] studied M/M/I queueing system with additional servers for a long queue. Varshney et al. [10] investigated M/M/m/K queueing system with additional servers. Recently Jain and Ghimire [11] developed nopassing M/M/m/K queue having additional servers for a longer queue.

In many real life situations service stations to which failed machines as taken to be repaired may be subject to unpredictable breakdown. Such a service station is termed as "non-reliable" service station. Several researchers have investigated machine-repair problems with non-reliable service stations. Avi-itzhak and Naor [12] first introduced the M/M/I queueing system with non-reliable service station. Neuts and Lucantoni [13] studied infinite source Markovian M/M/N queueing system with N-non-reliable service stations. Recently Wang and Hsu [14] developed the cost model to obtain the optimal number of non-reliable service stations.

In many day-to-day advanced technology of machining systems, from the optimization standpoint, the service rate may be speeded up or slowed down in order to reduce the cost associated with service. In this investigation we study M/M/R/N machine-repair system with R non-reliable service stations where the server switches to faster service rate when all service stations are busy. The model under study has its wide spread applications in real life situations such as in production systems in textile windings, for mathematical description of terminal computer systems, pilotless aircraft systems etc.

MATHEMATICAL MODEL AND ANALYSIS

We study the M/M/R/N machine-repair problem with N identical operating machines and R non-reliable service stations. The failure time of each machine is assumed to be exponential distributed with mean rate λ . The service stations repair the failed machines exponentially with rate μ and switch to fast rate μ_f $(>\mu)$ when all the service stations are busy in providing service to failed machines. The service stations may breakdown in Poisson fashion with rate α and are repaired exponentially with service rate β . An additional server having service rate μ_a is provided when number of failed machines equal to N_i . The rate dependent failure and repair rates are respectively given by

$$\lambda_n = \begin{cases} (N-n)\lambda, & o \le n < N \\ O & otherwise \end{cases}$$

and

$$\mu_n = \begin{cases} n\mu, & 1 \le n \le R-(i+1) \\ (R-i)\mu_f, & (R-i) \le n < N_1 - 1 \\ (R-i)\mu_f + \mu_a, & N_1 \le n \le N \end{cases}$$

We denote P, (n) as the probability that there are i broken service stations and n failed machines.

The steady-state equations for P_i (n) are:

(I) i=0

$$(N\lambda + R\alpha)p_0(O) = \mu p_0(1) + \beta p_1(0), \qquad n=0 \qquad (1)$$

$$[(N-n)a+Ra+n\mu]p_0(n) = [N-n+1]\lambda p_0(n-1) + \beta p_1(n) + [n+1]\mu p_0(n+1)$$

$$1 \le n \le R-1 \qquad (2)$$

$$[(N-n)a+Ra+R\mu_f]p_0(n) = [N-n-1]\alpha p_0(n-1) + \beta p_1(n) + R\mu_f p_0(n+1)$$

$$R \le n \le N_1 - 1 \qquad (3)$$

$$[Ra+R\mu_f+\mu_a]\, p_0\,[N_1] = \lambda p_0\,[N_1-1] + \beta p_1\,[N_1]$$

n=N_1 (4)

$$[[N-n]\,\lambda+R\mu+\mu_a]\, p_0(n) = [N-n+1]\,\lambda p_0\,[n-1] + \beta p_1\,(n) + [R\mu+\mu_a]\, p_0\,[n+1]$$

$N_1+1 \leq n \leq N-1$ (5)

$$[Ra+R\mu_f+\mu_a]\, p_0\,[N] = \lambda p_0\,[N-1] + \beta p_1\,[N],$$

n=N (6)

$[ii]$ $1 \leq i. \leq R-1$

$$[N\lambda - [R-i]\, a+i\beta]\, p_i\,[0] = [R-i+1]\, ap_{i-1}\,[0] + \mu p_i\,[1] + [i+1]\,\beta p_{i+1}\,[0] \qquad n=0 \quad (7)$$

$$[[N-n]\,\lambda + [R-i]\, a+A+n\mu+i\beta]\, p_i\,(n) = [N-n+1]\,\lambda p_i\,[n-1] + [n+1]\,\mu p_i\,[n+1]$$

$$+ [R-i+1]\, ap_{i+1}\,(n) + [i+1]\,\beta p_{i+1}\,(n)$$

$1 \leq n \leq R-[i+1]$ (8)

$$[[N-n]\,\lambda + [R-i]\, a + [R-i]\,\mu_f+i\beta]\, p_i\,(n) = [N-n+1]\,\lambda p_i\,[n-1] + [R+i\ \mu_f p_i\,[n+1]$$

$$+ [R-i+1]\, ap_{i+1}\,(n) + [i+1]\,\beta p_{i+1}\,(n)$$

$R-i \leq n \leq N_1-1$ (9)

$$[(N-N_1)\lambda + [R-i]\, a + [R-i]\,\mu_f+\mu_a+i\beta]$$

$$p_i(N_1) = (N-N_1+1)\lambda p_i(N_1-1) + (R-i+1)$$

$$ap_{i-1}(N_2) + [i+1]\,\beta p_{i+1}(N_1) \qquad n=N_1 \quad (10)$$

$$[[N-n]\,\lambda + [R-i]\, a + [R-i]\,\mu + \mu_a + i\beta]\, p_i\,(n) = [N-n+1]\,\lambda p_i\,[n-1]$$

$$+ [R-i+1]\, ap_{i-1}\,(n) + [i+1]\,\beta p_{i+1}\,(n)$$

$N_1+1 \leq n \leq N-1$ (11)

$$[[R-i]\, a + [R-i]\,\mu_f+\mu_a+i\beta]\, p_i\,[N] = \lambda p_i\,[N-1] + [R-i+1]\, ap_{i-1}\,[N] + [i+1]\,\beta p_{i+1}\,[N] \qquad n=N$$

(12)

$[iii]$ $i = R$

$$[N\lambda+R\beta]\, p_R\,[0] = ap_{R-1}\,[0], \qquad n=0 \quad (13)$$

$$[[N-n]\,\lambda+R\beta]\, p_R\,(n) = [N-n+1]\,\lambda p_R\,[n-1] + ap_{R-1}\,(n), \qquad 1 \leq n \leq N_1-1 \quad (14)$$

$$[[N-N_1]\,\lambda + [\mu+R\beta]]\, p_R\,[N_1] = [N-N_1+1]\,\lambda + p_R\,[N_1-1] + ap_{R-1}\,[N_1], \qquad n = N_1$$

(15)

$$[[N-n]\,\lambda + [\mu_a+R\beta]]\, p_R\,(n) = [N-n+1]\,\lambda p_R\,[n-1]\,] + ap_{R-1}\,(n), \qquad N_1+1 \leq n \leq N-1 \ldots$$

(16)

$$[\mu_a+R\beta]\, P_R\,[N] = \lambda p_R\,[N-1] + ap_{R-1}\,[N],$$

n=N (17)

Solving equation (13) to (17) recursively for p***R*** (n) in terms of p***R-1***(n) we have

$$p_R\,[0] = \frac{a}{[N\lambda+R\beta]}\, p_{R-1}\,[0] \quad (18a)$$

$$p(n) = \frac{a \prod_{i=0}^{n-N_1} [N-N_1-i+1]\lambda}{\prod_{i=0}^{n-n_1} \{[N-N_1-i]\lambda+\mu_a+R\beta\}} \gamma + \frac{a \prod_{i=0}^{n-N_1} [N-N_1-i-1]\lambda}{\{[N-N_1+1]\lambda+R\beta\} \prod_{i=0}^{n-n_1} \{[N-N_1-i]\lambda+\mu_a+R\beta\}} P_{R-1}(N_1-1)$$

$$+a \sum_{s=1}^{n-N_1} \frac{\prod_{i=s}^{n-N_1} [N-N_1-i+1]\lambda}{\prod_{i=s}^{n=N_2+1} \{[N-N_1-i+1]\lambda+\mu_a+R\beta\}} P_{R-1}(s+N_1-1)$$

$$+\frac{a}{\{[N-n]\lambda+\mu_a+R\beta\}} P_{R-1}(n)$$

$1 \leq n \leq N$ (18b)

where $Y = \sum_{j=1}^{N_2-1} \frac{\prod_{r=j}^{N_2-1} [N-r+1]\lambda}{\prod_{r=j}^{N_1} \{[N-r+1]\lambda+R\beta\}} P_{R-1}(j-1)$

$1 \leq n \leq N_1-1$

MATRIX GEOMETRIC SOLUTIONS

Using matrix-geometric theory (See Neuts[13]) to find the steady-state probabilities, we have transition rate matrix A in tridiagonal form:

$$A = \begin{bmatrix} Y_0 & Z_0 & & & & & \\ X_1 & Y_1 & Z_1 & & & & \\ & X_2 & Y_2 & Z_2 & & & \\ & \vdots & \vdots & \vdots & & & \\ & & & X_{R-2} & Y_{R-2} & Z_{R-2} & \\ & & & & X_{R-1} & Y_{R-1} & Z_{R-1} \\ & & & & & X_R & Y_R \end{bmatrix} \quad (19)$$

where each element of this matrix is a square matrix of order N+1 and given as follows.

$$X_i = -[R-i+1]\alpha I, \qquad 1 \leq i \leq R \quad (20)$$

$$Z_i = -[i+1]\beta I, \qquad 1 \leq i \leq R \quad (21)$$

I being a square unit matrix of order N+1 and

$$Y_i = \begin{bmatrix} Y_{0,i} & Z_{0,i} & & & & & \\ X_{1,i} & Y_{1,i} & Z_{1,i} & & & & \\ & X_{2,i} & Y_{2,i} & Z_{2,i} & & & \\ & \vdots & \vdots & \vdots & & & \\ & & & X_{N-2,i} & Y_{N-2,i} & Z_{N-2,i} & \\ & & & & X_{N-1,i} & Y_{N-1,i} & Z_{N-1,i} \\ & & & & & X_{N,i} & Y_{N,i} \end{bmatrix} \quad (22)$$

The elements of Y*i* for $0 \leq i \leq R$ are

$x_{n,i} = -[N-n+1]\lambda, \qquad 0 \leq n \leq N$

$$Y_{n,i} = \begin{cases} [N-n]\lambda + [R-i]a + i\beta + n\mu & : 1 \leq n \leq R-[i+1] \\ [N-n]\lambda + [R-i]a + i\beta + +[R-i]\mu_f & : R-i \leq n \leq N_1-1 \\ [N-n]\lambda + [R-i]a + i\beta + [R-i]\mu_a & : N_1 \leq n \leq N \end{cases}$$

$$Z_{n,i} = \begin{cases} -(n+1)\mu; & 1 \leq n \leq R-(i+1) \\ (R-i)\mu_f; & (R-i) \leq n < N_1-1 \\ (R-i)\mu_f + \mu_a; & N_1 \leq n \leq N \end{cases}$$

steady-state probability vector of A and $P_i = \{P_i(0), P_i(1), \ldots P_i(N)\}$ *for* $0 \leq i \leq R$ the steady-state equation PA=0 can be expressed as

$$P_0 Y_0 - R\alpha p_1 = 0 \quad (23)$$

$$i\beta p_{i-1} - p_i Y_i + [R-i]\alpha p_{i+1} = 0, \quad (24)$$

$$R\beta p_{R-1} - p_R Y_R = 0 \quad (25)$$

Performing successive substitutions, we have

$$p_0 = R a p_1 Y_0^{-1} \quad (26)$$

and

$$p_1 = [R-1]\alpha p_2 [Y_1 - Y_{0.0}]^{-1} \quad (27)$$

$$p_i = [R-i]\,\alpha p_{i+1}\{Y_i - Y_{0,i-1}\}^{-1}\ ,\ 2 \le i \le R-i \quad (28)$$

$$P_R\left[Y_R - R\alpha\beta\{Y_{R-1} - Y_{0,R-2}\}^{-1}\right] = 0 \quad (29)$$

where

$$Y_{0,0} = R\alpha\beta Y_0^{-1} \quad (30)$$

and

$$Y_{0,i-1} = [R-i-1]\,a\,[i\beta]\,\{Y_{i-1} - Y_{0,i-2}\}^{-1} \quad (31)$$

P_R obtained from equation (29) and P_0 , P_1 , ... , P_{R-1} obtained from equations (26)-(28) can also be determined by normalizing condition $\sum_{i=0}^{R} P_i B = 1$, where B is the unit column vector.

STEADY-STATE CHARACTERISTICS OF THE SYSTEM

System characteristics and their expressions are obtained as follows:

L_O = The expected number of failed machines when all the service stations are operative.

$$= \sum_{n=1}^{N} np_i(n)$$

L_i = The expected number of failed machines in the system when i service stations are broken down

$$= \sum_{n=1}^{N} np_O(n) \quad , \quad 1 \le i \le R$$

L = The expected total number of failed machines in the system

$$= \sum_{i=0}^{R} \sum_{n=1}^{N} np_i(n)$$

$$= \sum_{i=0}^{R} L_i$$

L_q = The expected total number of failed machines in the queue

$$= \sum_{i=0}^{R} \sum_{n=R-1}^{N} \left[n-(R-i)p_i(n)\right]$$

E(O) = The expected number of operative machines in the system

$$= N - \sum_{i=0}^{R} L_i$$

$$= N\text{-}L$$

E(D) = The expected number of defective (broken) service stations

$$= \sum_{i=1}^{R} \sum_{n=0}^{N} ip_i(n)$$

E(I) = The expected number of idle service stations in the system

$$= \sum_{i=0}^{R-1} \sum_{n=0}^{R-i} \left[(R-i)-n\right]p_i(n)$$

E(B) = The expected number of busy service stations

$$= R\text{-}E(D)\text{-}E(I)$$

M.A. = Machine availability

$$= 1 - \frac{\sum_{i=0}^{R} L_i}{N} = \frac{E(O)}{N}$$

O.U. = Operative utilization (the fraction of busy service stations)

$$= \frac{E(B)}{R}$$

DISCUSSION

Machines repair problem under study is of immense importance due to its specific applications in textile production system, in mathematical description of terminal computer system and in pilotless aircraft systems for the military use. The provision of additional server and heterogeneous service discipline are common in various machining system of day-to-day increasing complex technology world. Several performance characteristics obtained may be helpful to determine optimal operating policies for system designers.

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