PRESSURE CALCULATION IN THE FLOW BETWEEN TWO ROTATING ECCENTRIC CYLINDERS AT HIGH REYNOLDS NUMBERS

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Abstract This paper reports the result of an analytical investigation of a steady, incompressible and viscous flow between two eccentric, rotating cylinders at high Reynolds number. A one dimensional case is far from reality because the gap between the cylinders is very small. Further, when their axes are displaced by a small distance, usually caused by bearing loads, two dimensional effects become obvious. Here the equations of motion are perturbed and made linear. Then utilizing the boundary layer theory, these equations are solved for the case of large Reynolds number. The results are compared to those of other works.

Key Words Pressure Calculation, Rotating Cylinders, Perturbation Method

چکیده این مقاله نتیجه یک بررسی تحلیلی از جریان دائمی و غیر قابل تراکم و لزج بین دوسیلندر خارج از مرکز و درحال چرخش در اعداد رینولدز بالا می باشد. مورد یک بعدی این مسأله خیلی از واقعیت به دور است زیرا فاصله بین سیلندرها خیلی کوچک بوده و وقتی که محور آن ها به فاصله کوچکی جابجا شوند (که معمولاً در اثر بار وارد برآن ها به آسانی این امر انجام می گیرد)، اثرات دو بعدی بودن جریان واضح می شوند. دراین جا ما با اختلال بهٔ وجود آوردن در معادلات حرکت، آن ها را خطی کرده و سپس با کمک تئوری لایه مرزی این معادلات را برای مورد عدد رینولدز بزرگ حل می کنیم. در انتها نتایج بدست آمده با دیگر نتایج مقایسه می شوند.

INTRODUCTION

This study is concerned with the viscous incompressible steady flow between two rotating eccentric cylinders. The analysis of the dynamics of a bearing is incomplete without the knowledge of the pressure distribution inside the lubricating fluid. In order to find the pressure distribution, the governing equations of the flow motion are to be solved. This is not an easy task even if the flow is considered to be two-dimensional by assuming that the cylinders are infinitely long. In the case when the cylinders are concentric, the flow is one dimensional and an exact solution for the pressure distribution is easily found. One of the early approximate equations in this case is known as the Reynolds equation. Wannier [1] established that this equation arises as a first approximation of all quantities appearing in the Stokes equations when expanded in powers of the film thickness, usually small. This equation was solved by Sommerfeld [2] as early as 1904 and Kamal [3] has presented a brief summary of Sommerfeld's solution. The most recent study of this analysis using perturbation methods has been done by Rahimi and Ajam [4]. However, these cases are far from reality because in the bearings the gap between the cylinders in very small and when their axes are displaced by a small distance, usually caused by bearing loads, two dimensional effects become obvious. Many scholars have investigated this problem. Kulinski and Ostrach [5] and Diprima and Stuart [6] have expanded the flow quantities in powers of small modified Reynolds number and small eccentricity. Their solution are only good for relatively small values of Reynolds number. Only Wood [7] has considered solutions that are valid for any value of Reynolds number. He expanded the flow quantities in powers of an eccentricity measure in a modified bipolar coordinate system. His solutions are given in terms of Bessel functions of complex order and complex argument and also secular terms are found in the straightforward expansions. Moreover, Wood presents only the first order corrections, but stability analysis shows that the square of the entricity is needed. Following Wood, Selmi [8] has also looked at this problem. However, their studies are more directed toward flow stability and lack pressure calculations. Some other recent works are the one by Szeri [9] which is numerical and the work of Luis San Andres [10] which considers the effects of journal misalignment on the operation of a hydrostatic bearing. We will compare our result to that of Szeri.

In this work the second-order corrections are presented along with the first-order results in terms of elementary functions. This is done by taking the distance between the axes of the cylinders, known as eccentricity, very small and assuming large Reynolds numbers.

Therefore, the study deals with two small perturbation parameters, the eccentricity and inverse of the Reynolds number. The latter makes the problem interesting because it requires a singular perturbation theory treatment. The flow quantities are expanded in powers of the inverse of Reynolds number, which is different from other works. When seeking this expansion the problem becomes a singular perturbation problem because this parameter is multiplied by the highest derivative term in the existing equations. While it is comparatively easy to find a uniform expansion for a regular perturbation problem, it is not so easy to find one for a singular one. At the end by finding the flow quantities, composite pressure is obtained. This pressure is compared with the results of other works.

MATHEMATICAL ANALYSIS OF THE PROBLEM

In this section the equations governing an incompressible, viscous flow between two rotating cylinders are considered in polar coordinates. These cylinders are infinitely long and of radii a and b with axes displaced by a small distance e, as shown in Figure 1. The angular velocity of the inner and outer cylinders are Ω_1 and Ω_2 respectively. The flow is assumed to be two-dimensional, the fluid has constant properties, and body forces are assumed to be negligible. The governing non-dimensional equations in polar coordinates are the continuity and the r and θ components of the momentum equations:

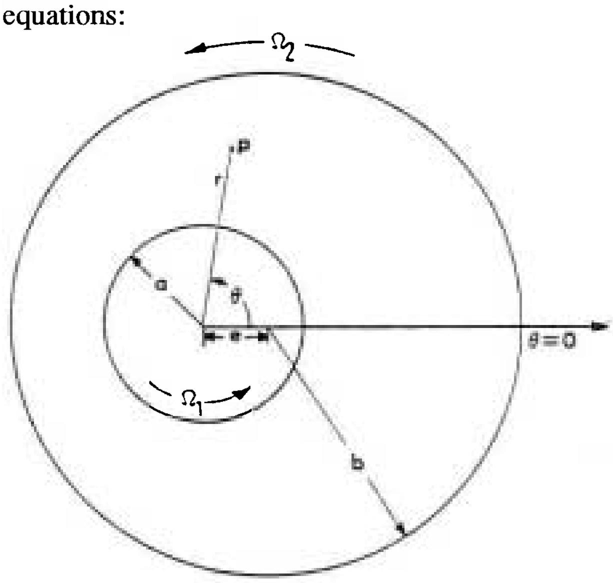

Figure 1. Description of geometry in polar coordinates.

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(V_\theta)=0 \tag{1}$$

$$V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{V_\theta^2}{r}=-\frac{\partial P}{\partial r}+\frac{1}{R}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_r)\right)+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial r}\right] \tag{2}$$

$$V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}=-\frac{1}{r}\frac{\partial P}{\partial r}+\frac{1}{R}\left[\frac{\partial}{\partial r}+\left(\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}\right] \tag{3}$$

with the following nondimensional boundary conditions:

$$\begin{aligned}
&V_r(1,\theta)=0\\
&V_\theta(1,\theta)=1\\
&V_r(r_0,\theta)=-(\Omega_2/\Omega_1)\,\varepsilon\,\mathrm{Sin}\,\theta\\
&V_\theta(r_0,\theta)=(\Omega_2/\Omega_1)\,(b^2+r_0^2-e^2)/2ar_0
\end{aligned} \tag{4}$$

In these equations and with the help of Figure 1,

$$R=\frac{\Omega_1 a^2}{\nu} \tag{5}$$

$$r_0=\varepsilon\,\mathrm{Cos}\,\theta+\sqrt{\frac{b^2}{a^2}-\varepsilon^2\,\mathrm{Sin}^2\,\theta} \tag{6}$$

$$\varepsilon=\frac{e}{a} \tag{7}$$

Note here ν is kinematic viscosity. These equations can be simplified by introducing the vorticity (ω) and stream function (ψ) such that

$$V_r=\frac{1}{r}\frac{\partial \psi}{\partial \theta} \tag{8}$$

$$V_\theta=-\frac{\partial \psi}{\partial r} \tag{9}$$

By taking the Curl of the vector form of the momentum equation, the vorticity transport equation results:

$$V_r\frac{\partial \omega}{\partial r}+\frac{V_\theta}{r}\frac{\partial \omega}{\partial \theta}=\frac{1}{R}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \omega}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2}\right] \tag{10}$$

where

$$\omega=\frac{1}{r}\frac{\partial}{\partial r}(rV_\theta)-\frac{1}{r}\frac{\partial}{\partial \theta}(V_r) \tag{11}$$

If Equations 8 and 9 are substituted into Equation 11, the so-called Poisson's equation results:

$$\omega=-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right)-\frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} \tag{12}$$

The boundary conditions on ψ are:

$$\begin{aligned}
&\frac{\partial \psi}{\partial r}(1,\theta)=0\quad,\quad \frac{\partial \psi}{\partial r}(1,\theta)=-1\\
&\frac{\partial \psi}{\partial r}(r_0,\theta)=-(\Omega_2/\Omega_1)\,(b^2/a^2+r_0^2-\varepsilon^2)/2r_0\\
&\frac{\partial \Omega}{\partial \theta}(r_0,\theta)=-(\Omega_2/\Omega_1)\,\varepsilon\, r_0\,\mathrm{Sin}\,\theta
\end{aligned} \tag{13}$$

Equations 10 and 12 along with the boundary conditions 13 represent the mathematical model for the flow motion between two rotating eccentric cylinders in polar coordinates. Note that, as of Equation 6 the outer cylinder (r_0) is a function of ε which causes problems when seeking an approximate solution to the flow equations by expanding the flow quantities in terms of ε. For this reason we follow Wood and introduce a modified bi-polar coordinate system in which the description of the boundaries is not a function of ρ and can be described with a single coordinate variable.

We use the conformal transformation:

$$Z=\frac{\zeta+\gamma}{1+\gamma\zeta} \tag{14}$$

with $Z = re^{i\theta}$ and $\zeta = \rho e^{i\phi}$ and

$$\gamma = -2\varepsilon\left[\left(\frac{b}{a}\right)^2 - 1 - \varepsilon^2 + \sqrt{\left(\frac{b^2}{a^2} - 1 - \varepsilon^2\right)^2 - 4\varepsilon^2}\,\right]^{-1} \qquad (15)$$

This transforms the two eccentric cylinders in the Z(r, θ) plane into two concentric cylinders in the $\zeta(\rho, \phi)$ plane. As a result, the circular sections of the inner and outer cylinders are transformed to the coordinate lines $\rho = 1$ and $\rho = \beta$ in the ρ, ϕ coordinate system, where

$$\beta = \frac{(b/a) + \varepsilon - \gamma}{1 - (b/a)\gamma - \varepsilon\gamma} \qquad (16)$$

Figure 2 shows a superposition of the polar and modified bi-polar coordinate systems. The coordinate variables ρ and ϕ have been modified so as to generate two polar variables r and θ when the cylinders are coaxial.

The stream function in the new coordinate system is defined by:

$$u_\rho = \frac{\sqrt{J}}{\rho}\frac{\partial\psi}{\partial\phi} \qquad (17)$$

$$u_\phi = -\sqrt{J}\,\frac{\partial\psi}{\partial\phi} \qquad (18)$$

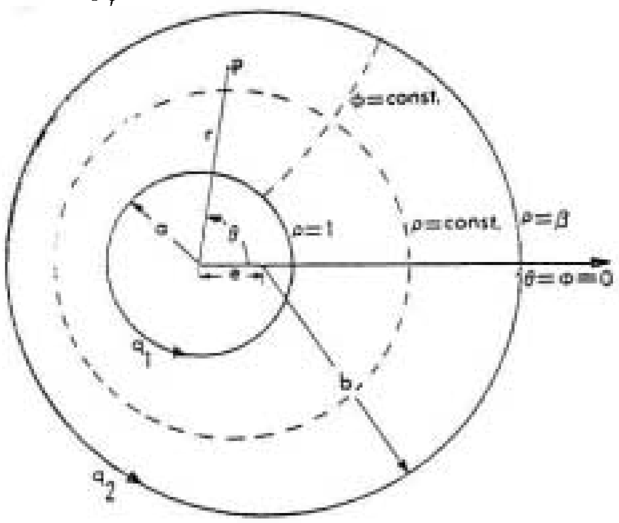

Figure 2. Description of geometry in the modified bi-polar coordinates.

where u_ρ and u_ϕ are the components of the velocity field in the ρ and ϕ coordinates, respectively, and J, the Jacobian of the transformation, is given by

$$J = \frac{(1 + 2\gamma \cos\phi + \gamma^2\rho^2)^2}{(1 - \gamma^2)^2} \qquad (19)$$

The vorticity-stream function equations in the new coordinate system become

$$\frac{1}{\rho}\left[\frac{\partial\psi}{\partial\phi}\frac{\partial\omega}{\partial\rho} - \frac{\partial\psi}{\partial\rho}\frac{\partial\omega}{\partial\phi}\right] = \frac{1}{R}\nabla^2\omega \qquad (20)$$

$$\omega = -J\,\nabla^2\psi \quad (21)$$

where $\nabla^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho}\frac{\partial^2}{\partial\phi^2}$

and the boundary conditions are transformed to

$$\frac{\partial\psi}{\partial\varphi}(1, \phi) = 0 \qquad (22)$$

$$\frac{\partial\psi}{\partial\varphi}(1, \phi) = -\frac{1}{\sqrt{J(1, \phi)}} \qquad (23)$$

$$\frac{\partial\psi}{\partial\varphi}(\beta, \phi) = 0 \qquad (24)$$

$$\frac{\partial\psi}{\partial\varphi}(\beta, \phi) = -\frac{q_2/q_1}{\sqrt{J(\beta, \phi)}} \qquad (25)$$

where $q_1 = \Omega_1 a$, $q_2 = \Omega_2 b$, and ψ is bounded at t= 0. Our aim is to find an approximate analytical solution to those equations.

It is clear from Equation 15 that γ is a measure of the eccentricity (ε). It is, therefore, a small parameter since ε is small, and it reduces to zero when ε reduces to zero. This parameter (γ) is used here as our first perturbation parameter to linearize the equations of motion. This is done by assuming the following asymptotic expansions for the stream function and vorticity:

$$\psi = \psi_0 + \gamma\, \psi_1 + \gamma^2\, \psi_2 + \ldots \tag{26}$$

$$\omega = \omega_0 + \gamma\, \omega_1 + \gamma^2\, \omega_2 + \ldots \tag{27}$$

and J as

$$J = 1 + 4\gamma \operatorname{Cos}\phi + 2\gamma^2 (1 + \rho^2 + 2\rho^2 \operatorname{Cos}^2\phi) + \ldots \tag{28}$$

If J is written as

$$J = J_0 + \gamma\, J_1 + \gamma^2 J_2 \tag{29}$$

then by comparison with Equation 28

$$J_0 = 1 \tag{30}$$

$$J_1 = 4\gamma \operatorname{Cos}\phi \tag{31}$$

$$J_1 = 2\,(1 + 2\,\rho^2 + \rho^2 \operatorname{Cos} 2\,\phi) \tag{32}$$

This analysis will be confined to second-order accuracy in γ. Upon the substitution of Equations 6, 7, 8, and 9 in Equations 20 and 21 and equating the coefficients of different powers of γ to zero, three sets of equations are obtained. Each set solves for $\omega_0, \psi_0, \omega_1, \psi_1$, and ω_2, ψ_2, respectively. The algebra is very tedious and is not presented here. We only mention that in the process of solving these equations, the factor of inverse of the Reynolds number, taken here as a very small number, appears in front of the highest derivatives. Therefore, we have to do an inner and outer expansion taking the inverse of the Reynolds number as a second perturbation parameter. Solutions so far are called outer solutions, see Nayfeh [11]. Notice here that as γ goes to zero the cylinders become coaxial and the flow is one-dimensional. Therefore the flow quantities at the zeroth level of (ω_0 and ψ_0) are independent of ϕ. Since the highest derivatives of the governing equations are multiplied by the small parameter 1/R, then the problem described here is referred to as a boundary layer problem. It is also known as a singular perturbation problem. Thus these equations are solved by the method of matched asymptotic expansions, see Reference 11. The solutions obtained above are only valid outside the boundary layer region. Therefore, solutions that are valid inside the boundary layer are developed by means of introducting stretching transformations to account for the rapid change of the dependent variable inside the boundary layer. These are called inner solutions. Again the algebra is very tedious here and we only mention that the stretching transformations that characterize these boundary layers are

$$\xi = \frac{\rho - 1}{\lambda} \qquad \text{near } \rho = 1$$

and $\qquad \lambda = \sqrt{R}$

$$\eta = \frac{\beta - \rho}{\lambda} \qquad \text{near } \rho = \beta$$

The next task is to match the outer solutions and inner solutions to yield a uniform expansion that is valid throughout the domain of interest. This expansion is referred to as composite expansion. This is done by using Van Dyke's matching principles, see Reference 11. Here we only present the composite solution for u_ϕ component of velocity. The solution for the function ψ up to the second-order is

$$\psi = \psi_0 + \gamma\psi_{10} + \gamma R^{-1/2}\psi_{11} + \gamma^2\psi_{20} + O(\gamma R^{-1}, \gamma^2 R^{-1/2}, \gamma^3) \tag{33}$$

In this expansion $O(\gamma R^{-1}, \gamma^2 R^{-1/2}, \gamma^3)$ indicates that the accuracy involved in this expansion is up to the maximum of γ^3, $\gamma^2 R^{-1/2}$, γR^{-1}.

$$\psi_0 = -\frac{1}{2} A\,\rho^2 - B \ln \rho \tag{34}$$

where

$$A = \frac{\beta q^2/q_1 - 1}{\beta^2 - 1} \tag{35}$$

$$B = \frac{\beta - \beta q^2/q_1}{\beta^2 - 1} \tag{36}$$

$$\psi_{10} = [A\beta^2\rho^{-1} - A(1+\beta^2)\rho + A\rho^3] \operatorname{Cos}\phi \tag{37}$$

$$\psi_{11} = F_{11}(\rho, \phi) + P_{11}(\zeta, \phi) + K_{11}(\eta, \phi) \tag{38}$$

and

$$F_{11} = \{[\frac{2}{\delta}\frac{\beta^2}{\beta^2-1}(\delta + \beta\frac{q_2}{q_1}) + \frac{2}{\delta}A\beta^2(\delta - 1)]\rho^{-1} + [\frac{2}{\delta}\frac{\beta^2}{\beta^2-1}(\frac{\delta}{\beta^2} + \beta\frac{q_2}{q_1}) + \frac{2}{\delta}A(\beta^2 - \delta)]\rho\} \operatorname{Cos}(\phi - \frac{\pi}{4}) \tag{39}$$

$$P_{11} = 2[A(1+\beta^2) - 1]_e^{-\sqrt{2/2}\zeta} \operatorname{Cos}\frac{\sqrt{2}}{2}\zeta \operatorname{Cos}(\phi - \frac{\pi}{4}) + 2[A(1+\beta^2) - 1]_e^{-\sqrt{2/2}\zeta} \operatorname{Sin}\frac{\sqrt{2}}{2}\zeta \operatorname{Sin}(\phi - \frac{\pi}{4}) \tag{40}$$

$$R_{11} = 2\frac{\beta}{\delta}[A(1-\beta^2) + \beta\frac{q_2}{q_1}]e^{(-\sqrt{2/2})(\delta/\beta)\eta} \operatorname{Cos}\frac{\sqrt{2}}{2}\frac{\delta}{\beta}\eta \operatorname{Cos}(\phi - \frac{\pi}{4}) + 2\frac{\beta}{\delta}[A(1-\beta^2) + \beta\frac{q_2}{q_1}]e^{(-\sqrt{2/2})(\delta/\beta)\eta} \operatorname{Sin}\frac{\sqrt{2}}{2}\frac{\delta}{\beta}\eta \operatorname{Sin}(\phi - \frac{\pi}{4}) \tag{41}$$

in which

$$\zeta = R^{1/2}(\rho - 1) \tag{42}$$

$$\eta = R^{1/2}(\beta - \rho) \tag{43}$$

and

$$\delta = \sqrt{\beta q_2} - /q_1 \tag{44}$$

and finally we have.

$$\psi_{20} = -\frac{1}{2}A\rho^4 + [A(1+\beta^2) - \frac{1}{2}\beta\frac{q_2}{q_1}]\rho^2 + (\beta\frac{q_2}{q_1} - 2A\beta_2)\ln\rho + [-A\beta^4\frac{1-\beta^2}{1-\beta^4}\rho^{-2} + A\frac{1-\beta^6}{1-\beta^4}\rho^2 - A\rho^4] \operatorname{Cos}2\phi \tag{45}$$

Now from Equations 17 and 18 the components of velocity can be found. Next we will find the second-order pressure.

First by substituting Relations 8 and 9 into Equation 2, we write this equation in terms of ψ function. Then, by transferring the terms into the (ρ, ϕ) coordinate system, Equation 2 is written in terms of $\partial\psi/\partial p$ and $\partial\psi/\partial\phi$. Finally by substituting the solution 33 into this equation, we can integrate for pressure. Notice that since the quantities used in the process of calculating pressure are composite, then the obtained relation for pressure is also composite solution. This result is shown in the next section.

NUMERICAL RESULTS AND DISCUSSION

Here we present some numerical results for velocity component u_ϕ and pressure, along with a general discussion. Figures 3 and 4 show velocity profiles in

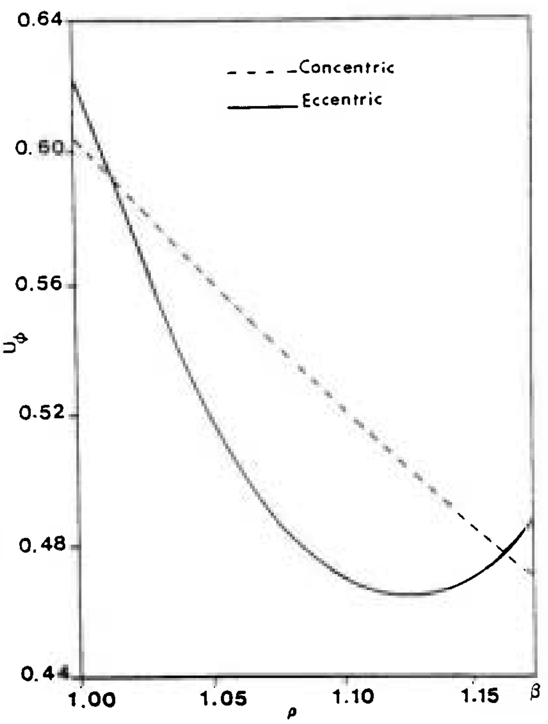

Figure 3. Variation of velocity component u_ϕ with ρ at maximum gap ($\phi = 0$) for b/a= 1.2, $\varepsilon = 0.1$ ($\gamma = -0.2467$), R= 200, and $q_2/q_1 = 0.75$.

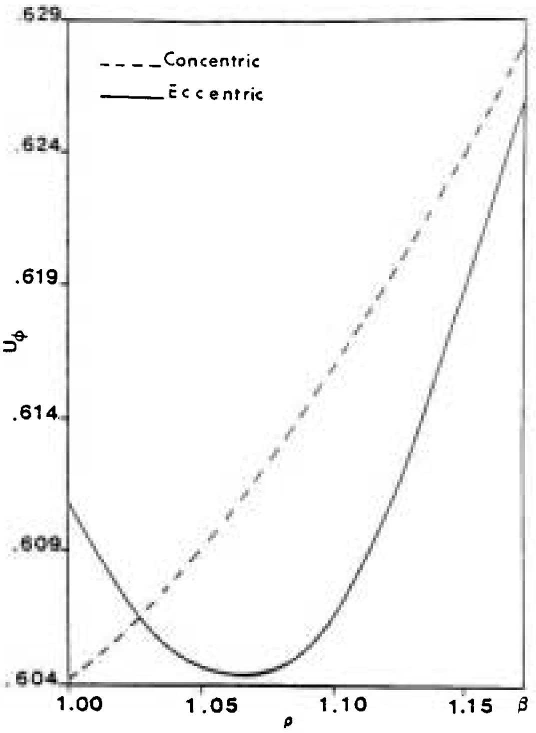

Figure 4. Variation of velocity component u_ϕ with ρ at maximum gap ($\phi = 0$) for b/a= 1.2, ε= 0.1 (γ = -0.2467), R= 200, and q_2/q_1= 1.

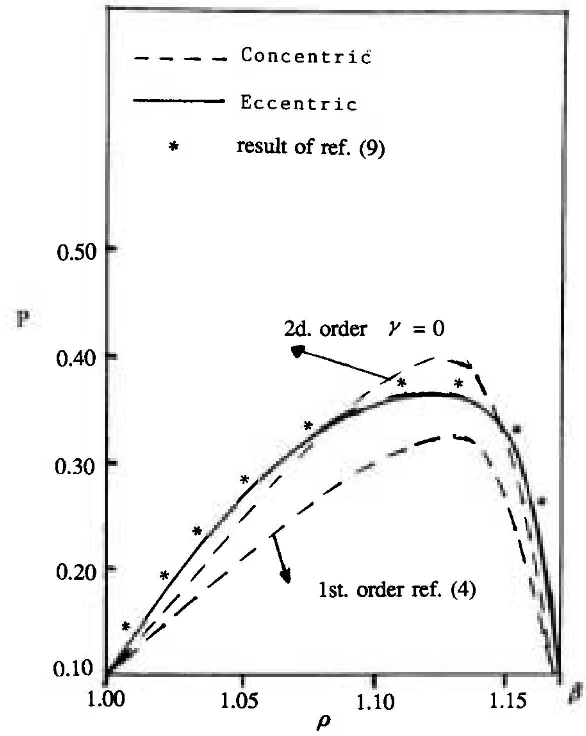

Figure 5. Variation of composite pressure with ρ at maximum gap ($\phi = 0$) for b/a= 1.2, ε= 0.1 (γ = -0.2467), R= 200, and q_2/q_1= 0.75.

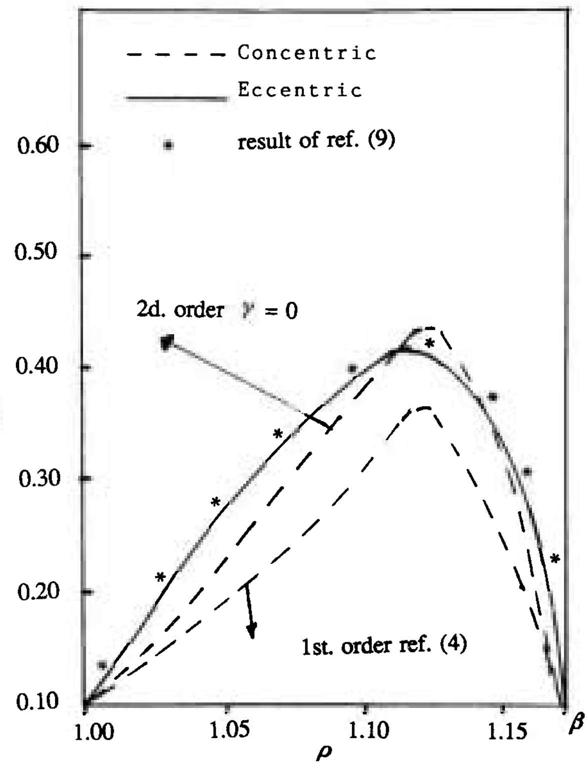

Figure 6. Variation of composite pressure with ρ at maximum gap ($\phi = 0$) for b/a= 1.2, ε= 0.1 (γ = -0.2467), R= 200, and q_2/q_1= 1.0.

the ϕ direction at the maximum gap (ϕ = 0) for different speed ratios (q_2/q_1) of the two cylinders. Note that the speeds of the inner and outer cylinders are $q_1\sqrt{J(1,\phi)}$ and $q_2\sqrt{J(\beta,\phi)}$, respectively. A comparison of the profiles presented in each of these figures emphasizes the importance of two-dimensional effects when the cylinders are eccentric. Figures 5 and 6 show the composite pressure for the maximum gap (ϕ = 0) for different speed ratios of both cylinders, along with comparison of other solutions. As these figures show, the trend of the composite pressure is as the intersection of an outer and inner solution which is expected. This trend can be seen directly in the result of Reference 4, but only for the first-order accuracy. This result is also compared with the second-order result here. Work of Szeri, et. al [9] in the area of approximations in hydrodynamic lubrication is one of the most recent studies which calculates the pressure in flow between rotating cylinders using a numerical approach. As Figures 5 and 6 show, the results of Reference 9 have been compared with our analytical results for pressure which is very much agreeable. At the same time, a comparison of the second-order and first-order results reveals the effect of two-dimensionality while proves its superiority over the first-order results.

An important issue to be noted is that as discussed in Reference 7, the method of expanding the stream function as a series in γ cannot be expected to hold for large Reynolds numbers if the outer cylinder is at rest. The reason for this may be seen by referring to the boundary layer approximation. If the series were valid and a boundary layer existed on the outer cylinder, the speed of the inviscid flow at the outer cylinder would be expected to be $O(\gamma\ q_1)$. The thickness of the boundary layer there would then be $O(\gamma^{-1/2} R^{-1/2})$. Thus for a given value of γ and a Reynolds number much greater than $1/\gamma$ (a necessary condition for the boundary layer to exist), the flow quantities of the boundary layer would not be analytic functions of γ. A similar objection would apply if the inner cylinder were at rest. That is why we as of Reference 7 have assumed that neither cylinder is at rest. We have also supposed that the cylinders rotate in the same sense. With this provision, the speed of the unperturbed flow does not vanish at any point of the fluid and some simplification thereby results in the behaviour of large overall Reynolds numbers.

CONCLUSION

An analytical investigation of a steady, incompressible and viscous flow between two eccentric, rotating cylinders at high Reynolds number using singular perturbation method up to second-order accuracy has been presented. Our main goal has been to calculate second-order pressure in terms of elementary functions to compare with already obtained numerical results as well as first-order results. We have acheived this goal by following the work of Wood [7] and introducing a modified bi-polar coordinate system. We have dealt with two small perturbation parameters, the eccentricity and inverse of the Reynolds number, causing a singular perturbation theory treatment. The flow quantities have been expanded in powers of the inverse of Reynolds number which is different from other works and handled a singular perturbation problem in the form of inner and outer expansions, Reference 11. Comparison of our results with first-order results reveals the effect of two-dimensionality and at the same time proves its superiority over them. It is important to note that as of Reference 7, the problem of Taylor vortices would not appear here since both cylinders are rotating.

It is also important to note that in this problem we did not have to go through doing asymptotic matching formally, as of Reference 4. Although the trend of our composite solution was the same as that of this reference, the composite solution in our work was obtained by intersecting the outer solution with the inner solution graphically. This is done to circumvent going through the cumbersome procedure of Van Dykes asymptotic matching discussed in Reference 11.

LIST OF SYMBOLS

inner cylinder radius	a	tangential velocity	q
constant	A	polar coordinate	r
outer cylinder radius	b	Reynolds number	R
constant	B	vel. in bi-polar coord.	u_ϕ
eccentricity	e	vel. in bi-polar coord.	u_ρ
function	F_{11}, P_{11}, R_{11}	vel. in polar coord.	v_r
complex number	$i=\sqrt{-1}$	vel. in polar coord.	v_θ
Jacobian	J	conformal transformation	z
Pressure	P		

Greek Letters

constant	β	vorticity function	ω
perturbation	γ	variable	η
polar coord.	θ	new variable	ζ
constant	δ	bi-polar coord.	ρ
e/a	ε	bi-polar coord.	ϕ
kinematic coeff.	ν	angular velocity	Ω
flow function	ψ		

Subscripts

1st. term of per. expansion	0	1 st. order approx.	10
2 nd. term of per. expansion	1	1 st. order approx.	11
3 rd. term of per. expansion	2	2 nd. order approx.	20

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