RESEARCH NOTE

STOCHASTIC SYNTHESIS OF DROUTHS FOR RESERVOIR STORAGE DESIGN

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Abstract Time series techniques are applied to Ghara-Aghaj flow records, in order to generate forecast values of the mean monthly river flows. The study of data and its correlogram shows the effect of seasonality and provide no evidence of trend. The autoregressive models of order one and two (AR1, AR2), moving average model of order one and ARMA (1,1) model are fitted to the stationary series, where the AR2 model yields results which are statistically compatible with the past records. For a better understanding of the behavior the spectral analyses of the data and exhaustive statistical tests are also provided. Synthetic data generated by AR2 model together with historical data are then used to obtain reservoir storage for varying design periods, and draft rates. For this, the Stall's method which is a more sophisticated form of mass curve is employed. Results are given as two design curves.

Key Words Stochastic Models, Reservoir, Storage, Design

چکیده بمنظورتسهیل طراحی یک سد مخزنی برروی رودخانه قره آغاج در استان فارس آمار دبی متوسط ماهانه این رودخانه با استفاده از تکنیکهای تجزیه و تحلیل سریهای زمانی بسط داده شده است. تجزیه و تحلیل داده ها نشان می دهد که آمار دبی این رودخانه فاقد مولفهٔ لنگر بوده و نمود فصلی درآن کاملاً مشهود است. پس از تفکیک نمود فصلی و ایستا شدن آمار، مدلهای استوکستیک AR1 ، AR2، MA و ARMA (1,1) درسری ایستا برازانده شده که درمیان این مدلها، مدل AR2 بعنوان بهترین مدل انتخاب شده است. آمار سنتز شده توسط مدل AR2 را با آمار واقعی ترکیب نموده و با استفاده از روش Stall حجم مخزن سد را برای خشکسالیهای مختلف با دوره های برگشت متفاوت و همچنین برای میزان آب مصرفی مختلف محاسبه و نتیجه بصورت دو منحنی طراحی ارائه گردیده است.

INTRODUCTION

The length of records of rivers in the Fars province is inadequate in comparison with the design period of a dam. Therefore, for a realistic design it is necessary to utilize not only the historical data but synthesis sequences which could possibly occur in the future.

To generate forecast values of the mean monthly river flows, time series techniques which preserve the relevant statistical parameters of the past records are applied to the longest records of Fars rivers; that is, those of the Ghara-Aghaj at Band-e-Bahman. The statistical methods for analysing time series are well established. The traditional methods are mainly concerned with isolating the deterministic components (trend and periodicities) and the stochastic components. The latter may then be explained in terms of stochastic models, such as moving-average or autoregressive models. If time series techniques result in a model truly representative, the synthetic data should provide records some of which can be more critical than those observed in the past and these can be combined with historical records for design purposes.

THE GHARA-AGHAJ RIVER FLOW ANALYSIS

The longest record of the Ghara-Aghaj stretches back for 15 years; this is at Band-e-Bahman where the catchment intercepted is 2410 square kilometers. The first step in time series analysis showed that no significant trends or long term fluctuations were present in the series of recorded Ghara-Aghaj river flows. This was easily ascertained by inspection of the series of annual means against the sample mean. Another procedure, the weighted moving-average method, has also failed to reveal trends. Obviously, in the absence of trends, the removal of periodicities (annual and seasonal variation) is a prerequisite to the analysis of the stochastic components. In order to achieve this serial correlograms and spectrum have been used.

Serial correlogram is a plot of the autocorrelation coefficient against time lag [2] and is a graphical representation of the storage processes in the catchment. The correlogram of the Ghara-Aghaj river flows is shown in Figure 1(a). The presence of annual cycle is obvious from this figure.

Another well-known means for the exploration of the internal structure of a time series is a spectrum. In contrast to the correlogram which considers the evaluation of a process through time, a spectrum considers the frequency properties of a time series. The spectrum is a plot of the spectral density against frequency [7]. The spectrum of the Ghara-Aghaj river flows is shown in Figure 2, which confirms the presence of an annual cycle with a sharp peak at 0.52 cycle per month.

Removal of Periodicities

Periodicities of the analyzed data (depicted by Figures 1(a) & 2) were removed in two simple steps:

a) each individual observation was subtracted from the corresponding observation in the previous water year;

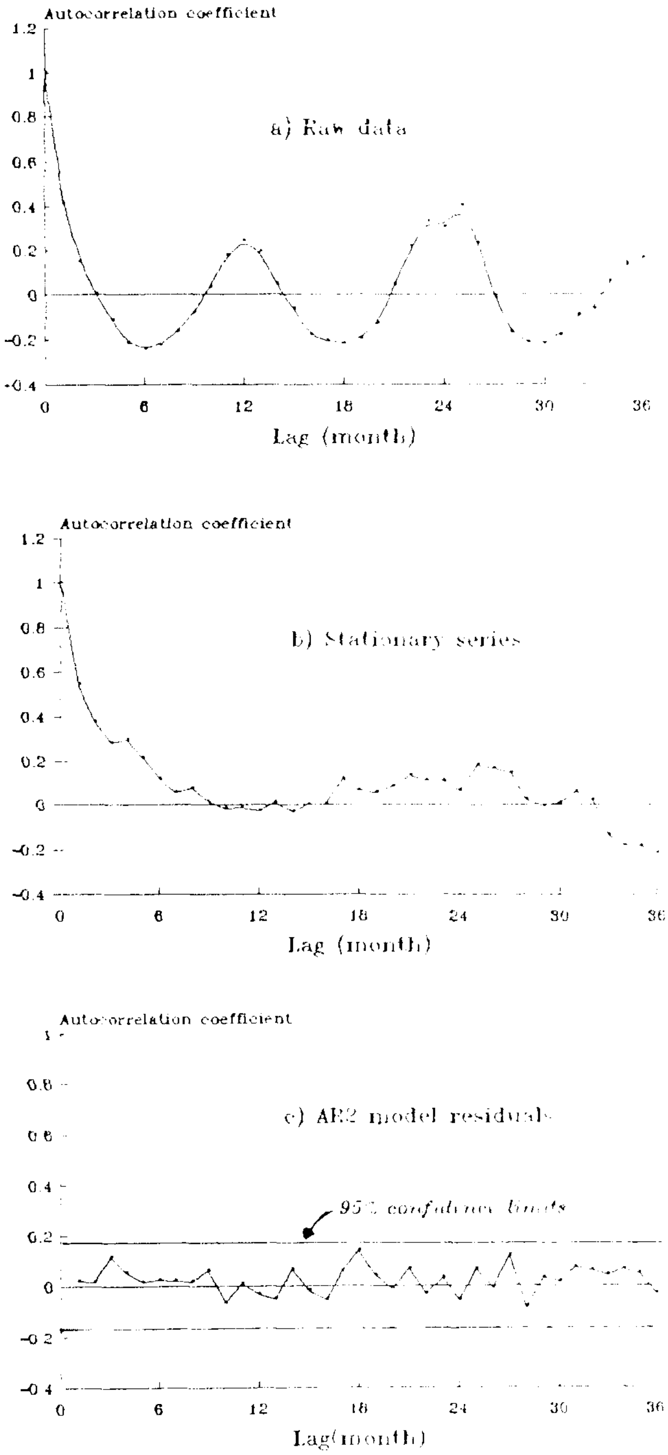

Figure 1. Serial correlogram of the monthly mean flow of Ghara Aghaj river

this may be considered as the removal of annual cycle.

b) the first step was followed by the simple procedure of calculating the twelve monthly averages and subtracting the appropriate one from each individual record and then dividing the result by the standard deviation; this may be considered as the removal of seasonal effects.

The correlograms of the stationary series (Figure 1(b)), shows that the first four coefficients are significantly different from zero. In other words, the coefficients die exponentially and the coefficients beyond lag one are not significantly different from zero. The spectrum of the stationary series (Figure 2), demonstrates the efficiency of the simple procedure described above in removing periodicities.

A STOCHASTIC MODEL FOR THE GHARA-AGHAJ RIVER

Three well-known stochastic models are autoregressive, moving average and autoregressive moving average models. The correlogram of a given time series provides some ideas as to which stochastic model will be appropriate to describe a time series. The correlogram of the stationary series (Figure 1(b)) shows that the coefficients die exponentially and that the coefficients beyond lag one are not significantly different from zero. This is an adequate criterion for an AR model of a short memory [2]. Therefore, an AR1 (Markov 1) model or an AR2 model seem

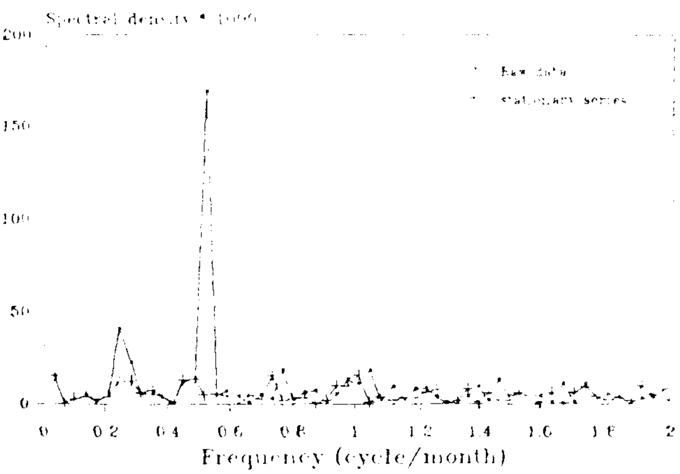

Figure 2. Spectrum of monthly mean flow

appropriate for the Ghara-Aghaj, that is:

$$q_t = a_1 q_{t-1} + a_2 q_{t-2} + Z_t \qquad (1)$$

where q_t, q_{t-1} and q_{t-2} are flows in monthts, t, t-1 and t-2, Z_t is the random component, and $a_1 = r_1$ and $a_2 = 0$ for an AR1 model; and

$$a_1 = \frac{r_1(1 - r_2)}{1 - r_1^2} \qquad a_2 = \frac{r_2 - r_1^2}{1 - r_1^2}$$

for an AR2 model.

For the Ghara-Aghaj the AR2 model was found more appropriate. The correlogram of Z_t is shown in Figure 1(c). Note that the 95% confidence limit is equal to $\pm 2/\sqrt{N}$. Fitting MA and ARMA to the stationary series of Ghara-Aghaj records was found to be unsatisfactory.

Hurst's Coefficient

To Justifiy the apropriateness of the AR2 model for the Ghara-Aghaj, another criterion known to hydrologists as the Hurst's coefficient was calculated. According to Hurst *et al*. [4] and Mandelbort and Wallis [8], the ratio of range R to the standard deviation of flow s in a sequence of N values is $R/s = k.N^H$. The exponential constant H should not be significantly different to 0.5 if an AR model based on a short memory is to be adopted. For the Ghara-Aghaj the mean value of Hurst's coefficient was found to be equal to 0.52. This is not considered to be significantly different to 0.5.

Synthesized Data

Data generation based on an AR2 model is done using one half of the record at a time. The synthetic sequences are tested against the other half by plotting the cumulative distribution functions of Figure 3. This figure reveals the close match between the historical and synthesized data. Further statistical tests including Kolmogorov Smirnov two-sample tests, confirm that the synthesized and historical data come from the same population. In the following synthetic stream, flows generated by the AR2 model are combined with historical flows to obtain the reservoir storage capacity for varying design periods and draft rates. Utilizing AR generated stream flows, for the prediction of reservoir storage capacity is also recommended by Kendall and Dracup [6] and Masahiko *et al.* [9].

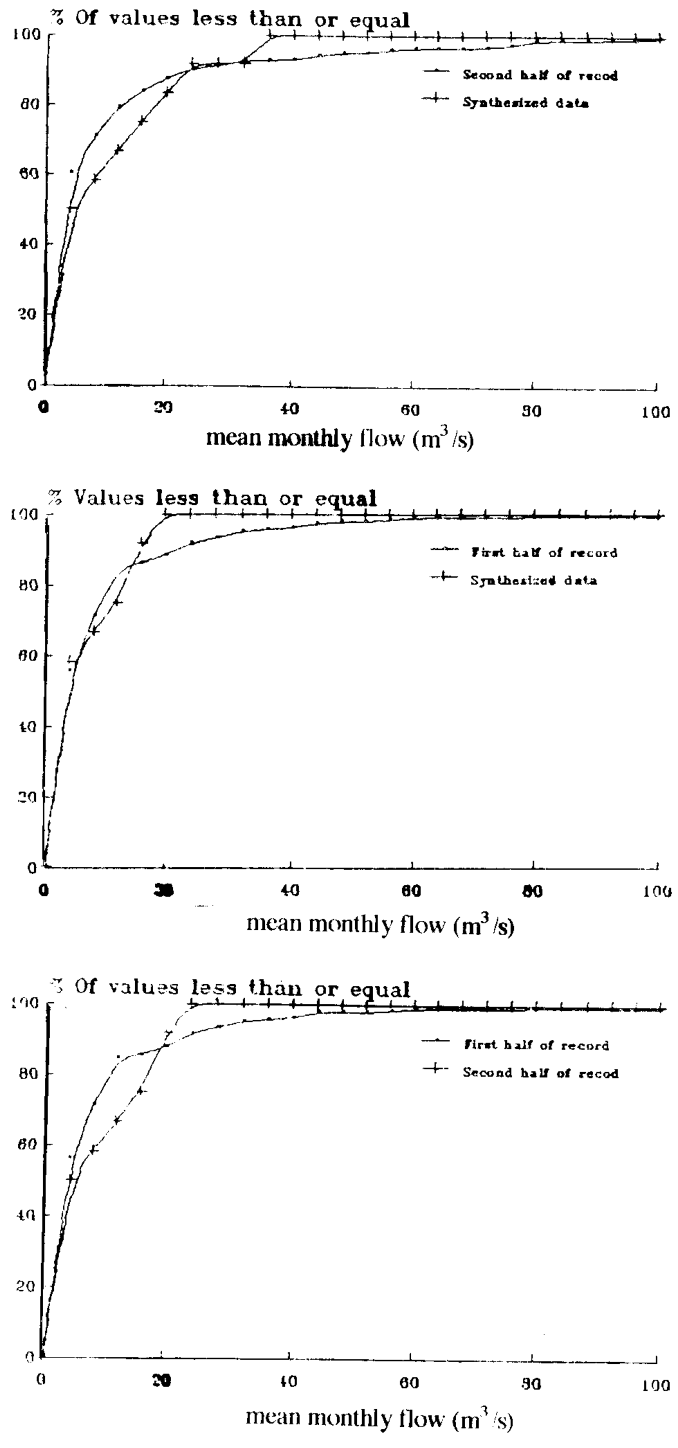

Figure 3. Cumulative distribution functions

RESERVOIR STORAGE DESIGN

In order to determine reservoir capacities for varying rates of outflow and design periods, the Stall's method [10] has been computerized. Basically, the approach is to find nonoverlapping critical low flow (drouth) periods of m month and rank them. If N is the number of years of data, the lowest sequence is assumed to have a mean return period of N. The K^{th} lowest sequence will have a mean return period of N/K. In fact the plotting position of form P=K/N is used. The application of Stall's method to the data for several low flow series is presented in Figures 4 and 5. Figure 4 can help the designer to obtain the gross

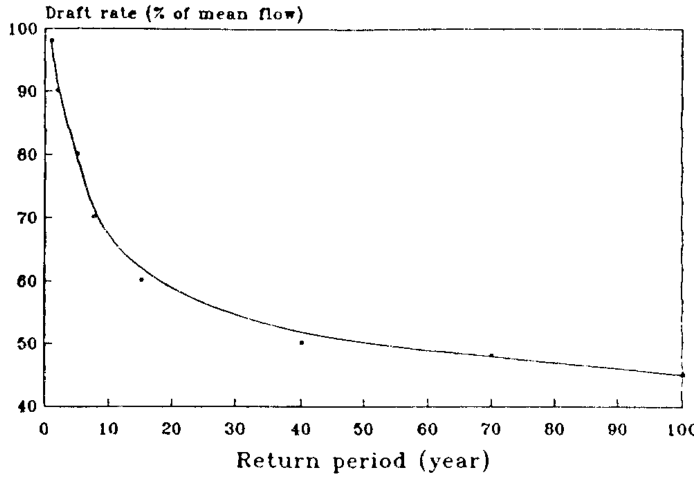

Figure 4. Draft rate available during a 30-year drouth from various reservoir capacities

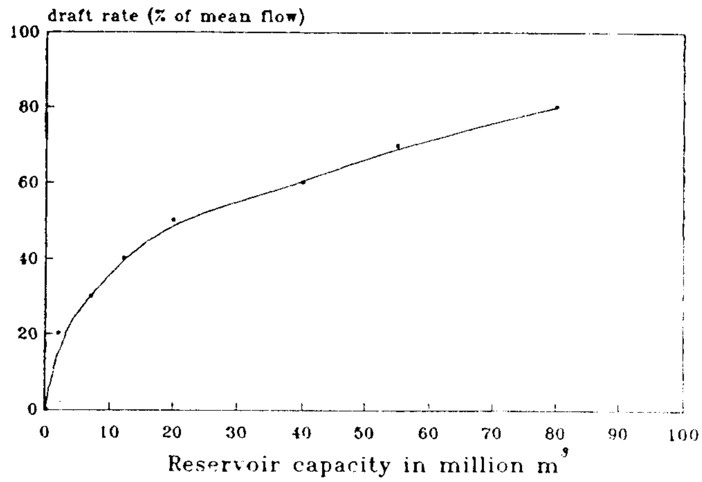

Figure 5. Draft rate available during selected recurrence intervals from a theoretical reservoir on Ghara-Aghaj with a capacity of $14 \times 10^7 m^3$

draft rate from reservoirs of various capacities during a drouth of selected recurrence intervals. Figure 5 represents the performance of a particular reservoir of a known capacity during drouths which have various recurrence intervals.

The end portion of the first curve tends to increase its slopes. In other words, an increment of reservoir capacity provides lesser increment of available draft rate. In contrast, the end portion of the second curve tends to decrease its slope, so that an increase in the draft rate results in a water shortage during a drouth which has lower return periods. These two points allow the designer to choose an optimum design.

CONCLUSION

The methodology described in this paper constitutes a rational approach to the solution of the problem of determining the gross yield of a reservoir, and represents an efficient overall use of basic stream flow data to aid the hydrologist's judgment in the selection of an optimum design. Difficulties of design due to lack of long sequences of reliable river flow data may be overcome by the stochastic models as data generating systems. Generated stream flows by AR models are more reliable than those generated by conventional methods. As a result the Stall's method complemented by AR models gives a better and more accurate estimate of reservoir storage capacity, and provides a more reliable program for controlling the draft rate and hence preventing drouths.

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