A NON-SINUSOIDAL REFERENCE WAVE FOR PWM AC DRIVES

S.H. Hosseini, A. Servatian and A. Seifi

Department of Electrical Engineering
University of Tabriz
Tabriz, Iran

Abstract In this paper we propose a suitable reference wave for Pulse Width Modulation (PWM) AC Drives. Staircase reference waves whose levels are calculated to eliminate certain harmonics are studied and a certain staircase reference waveform with L levels is constructed. When L is made very large in limit, this staircase waveform approaches a continuous one which is called Quasine (Quasi + Sine). This approach compared to the classical harmonic elimination method has the advantage that the magnitude of non-triplen harmonics is reduced. A computer program by TURBO PASCAL 6 was developed to enable us to have a comparison between the harmonic contents of output voltage for various reference waveforms.

Key Words Staircase Reference Waves, P.W.M. AC Drives, Harmonic Elimination, T.H.D. Minimization

چکیده امروزه هدفی که در وارونگرهای منبع ولتاژ (Inverters) به عنوان راه انداز (Drive) ماشین های AC تعقیب می شود در هرچه نزدیک تر نمودن شکل موج خروجی وارونگر به یک شکل موج سینوسی می باشد. در واقع با حذف هارمونیک های مرتبه پائین تلفات کل در سیستم کمینه شده و با پائین آوردن ضریب کل اعوجاج «THD» شکل موج خروجی، گشتاور نامطلوب نوسانی موتور کاهش می یابد. دراین مقاله شکل موجهای مرجع پله ای که ارتفاع آنها برای حذف هارمونیک های خاصی محاسبه می شود مورد مطالعه قرار گرفته و اشتقاق چندین نوع شکل موج به عنوان شکل موج مرجع در فرآیند هماهنگ دهی (مدولاسیون «PWM» عرضه می شود. به کمک یک برنامه کامپیوتری بوسیله «TURBO PASCAL6» مقایسه ای بین نتایج بدست آمده در خروجی وارونگر با شکل موج های مرجع مختلف انجام داده و ازبین آنها قابلیت شکل موج مرجع پیشنهادی «Quasine» را نشان داده ایم. دراین روش با کاهش هارمونیک های غیر مضرب ۳ می توان انتظار داشت که سروصدای موتور متصل به این نوع وارونگر کاهش یابد. درضمن ۳۳ درصد صرفه جوئی درساخت رقمی (دیجیتالی) شکل موج خواهیم داشت که از مزایای مضاعف آن می باشد.

INTRODUCTION

The objective which is followed in AC machine drives is to make the output of drive similar to a sine wave. There have been various presumptions about the word "similarity" or "optimality". In some papers (as in [1] and [2]) elimination of lower order harmonics is believed to be an optimality criterion. Here, switching instants are determined so that in PWM technique, for a given fundamental, certain harmonics are eliminated. Some authors have tried to optimize the output by minimizing the overall system losses [3], and some others have attempted to minimize the unwanted pulsating torque of motor or Total Harmonic Distortion (THD) of the stator current [4]. Unfortunately, optimized PWM has to date no identifiable modulation process [4,5]. This is due to the nonlinear nature of the problem and its complexity. Hence, optimization calculations, in optimized PWM implementation, are carried out off line to achieve switching instants, so that a certain criteria is met. Switching instants are saved in an EPROM and used in digital or microprocessor implementation of this method. The difficulty encountered in this method is that of excessive data, especially if the inverter is to work over a large scale of voltage and frequency [6]. Thus, we are led to *sampling strategy*. Historically, *natural sampling strategy* has had precedence and has been used more than other methods. In this method, swithing instants are determined by the analogue intersection of reference and carrier waveforms.

Emerging microprocessors and rapid progresses made in digital processing technology, has meant that consideration has been given to the possibility of implementation of drives with higher accuracy and efficiency. Unfortunately, programming microprocessors for natural sampling intended to real time control is rather intricate, because finding the intersections leads to transcen-dental equations which consume too much time. Fortunately, *regular sampling strategy* helps us in this and makes the microprocessorized implementation of inverters more practical [6,7,8]. In this method the reference waveform is sampled regularly, using minimal storage to hold its samples, and the widths of output pulses are calculated by a simple linear relationship using these samples. Since for applications in which we need lower frequencies, optimized PWM requires a huge memory, the regular sampling strategy is justified. Cases are reported in which the output of inverter is divided into different frequency regions and at lower frequency regions regular sampling strategy is used but at higher ones optimized PWM is realized [8]. In some papers in order to take advantage of the optimality of optimized PWM, as well as the practicability of sampling strategy, effort is made to compromise between the two. To this end non-sinusoidal references are considered [1,4,9]. In [1], for natural sampling, a staircase reference is found so that for unity modulation index (M), 5th and 7th harmonics are eliminated from output, and various levels of voltages are generated by multiplying the found reference by M. In [4], for regular sampling strategy, the spectrum of reference waveform giving minimum THD under various Ms and Ps is analysed and approximated by a third harmonic-added sine (*sine + 3rd*).

In this paper, staircase references are studied the levels of which are calculated to eliminiate certain harmonics from them this is done according to the idea that for rather large Ps the output of inverter follows the reference waveform. Then, a staircase reference waveform is constructed so that n first odd-numbered non-triplen harmonics are eliminated from its spectrum. If n is made very large in limit, this staircase waveform approaches a continuous one which is called Quasine [after Thorborg]. This approach in relation to the classical harmonic elimination method has the advantage that the magnitudes of non-triplen harmonics are reduced, because the triplen harmonics here contain an amount of the energy of the wave. In addition, the existing harmonics are distributed broader and resemble white noise more than the harmonics of classical method. So, in this case it is expected to have lower audible noise.

We developed a computer program to have a comparison among several waveforms as a reference. This user-friendly program is capable of calculating and drawing various waveforms of inverter and the spectrum of its output. This program simulates natural sampling to find the switching pattern of output. For staircase references its results are the same as regular sampling, so the achieved results can be applied to microprocessorized implementation.

EXISTING REFERENCE WAVEFORMS CONSIDERED

Figure 1 shows the basic structure of inverter and Figure 2

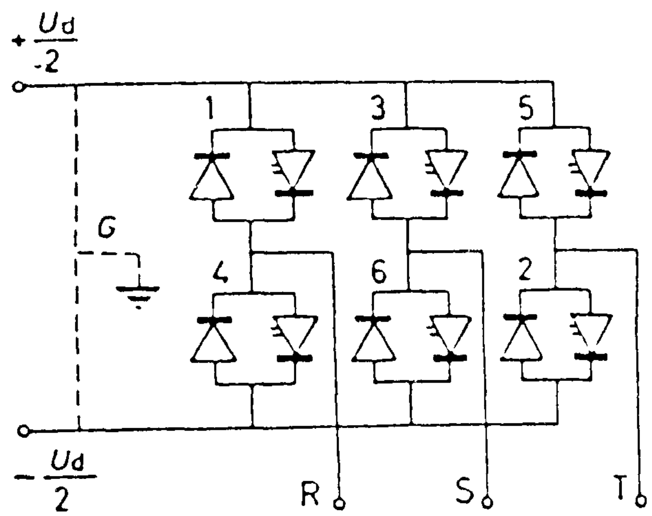


Figure 1. Inverter structure

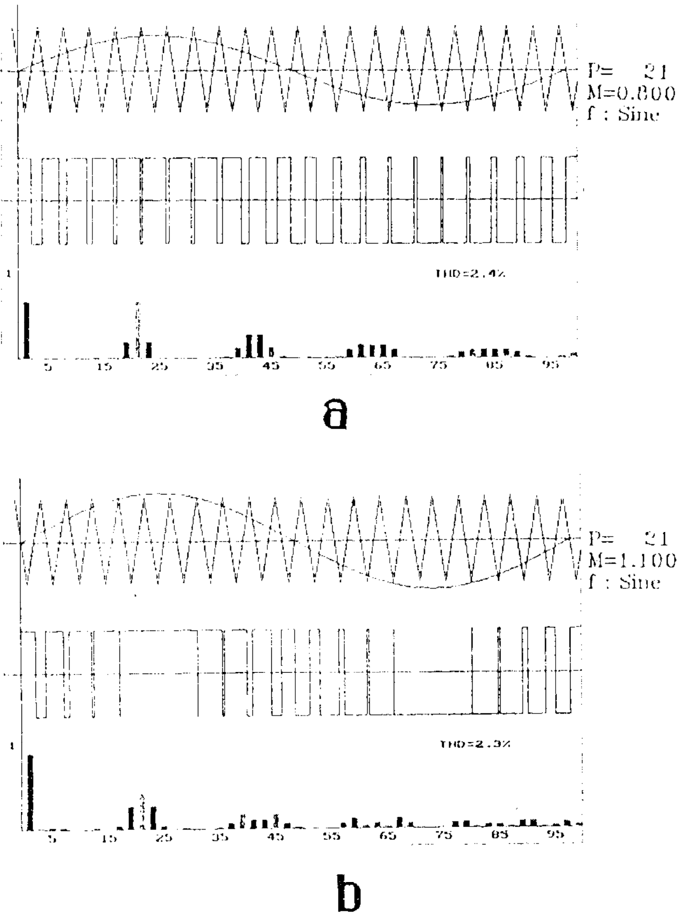


Figure 2. Sinusoidal PWM; (a), (b) samples of SPWM

shows the algorithm of producing output phase voltage in Sine PWM technique with natural sampling.

In this technique the final objective is the selection of the best reference waveform so that the quantity of output voltage THD becomes the best approximation to a Sine shape.

It is seen that if P is a relatively large number, the harmonic content of inverter output voltage will be nearer to the harmonic content of reference waveform and follow it. Inverter output voltage harmonics originate from two sources: one, reference waveform harmonics and the other, harmonics caused from changing pulse amplitude to pulse duration.

Since edges originate from switching, even if reference waveform is exactly sinusoidal, the output voltage will contain harmonics. These harmonics contain narrow bands around the carrier frequency and its multiples. If carrier and reference waveforms posses odd symmetry around 0° phase and even symmetry around 90° phase, the Fourier series of output voltage will be constructed from sinusoidal terms with odd harmonics. If we apply three-phase symmetry to turn on and off the switches, triplen harmonics will not be present in the phase-to-phase voltage and will not transfer to the load. Thus it is better to choose P as a multiple of 3 because harmonic components of it and all its multiples eliminate from line voltage. The elimination of triplen harmonics gives us one degree of freedom and this permits the reference waveform to contain these harmonics because harmonics will not be transferred to the load. The added harmonics can be useful from two aspects: first, that the amplitude of harmonics around the carrier frequency and its multiples is reduced and output voltage frequency spectrum becomes more monotone; the other, that if harmonics are added logically to reference waveform we can have more amplitude for fundamental component of output voltage than when we use sinusoidal reference waveform for a constant modulation index [1].

We will have maximum amplitude for the fundamental component of output phase voltage when phase switches turn on and off with reference wave frequency ($M>>1$).

The fundamental of a square wave which is considered here as a 1-level staircase, with amplitude 1, has the rms value:

$$U_{(1)0}=\frac{4}{\pi}\cdot\frac{1}{\sqrt{2}}=\frac{2\sqrt{2}}{\pi} \quad (1)$$

This is the highest obtainable value and is used as a benchmark, for other references or output voltage all harmonic components are expressed in per unit (pu) or % of that value.

As referred to earlier, in this paper we use natural sampling technique for calculating switching times in order to have a uniform judgment among different reference waveforms. The total harmonic distortion, which is a measure of closeness in shape between a waveform and its fundamental component is defined as:

$$THD=100\,.\,\frac{1}{U_{(1)}}\cdot\left[\sum_{n=5}^{\infty}\left(\frac{U_{(n)}}{n}\right)^{2}\right]^{1/2}\% \quad (2)$$

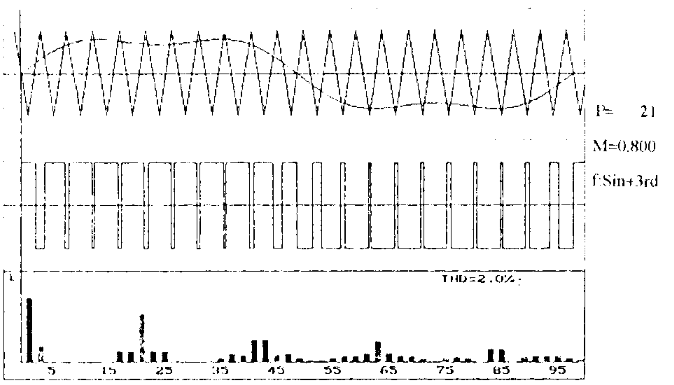
where n= 6i ±1 (i= 1,2,...) and $U_{(n)}$ nth harmonic amplitude of voltage in pu. If we assume that load is pure inductive this value (THD) will be equal to rms harmonics current that flows through it.

Sinusoidal PWM

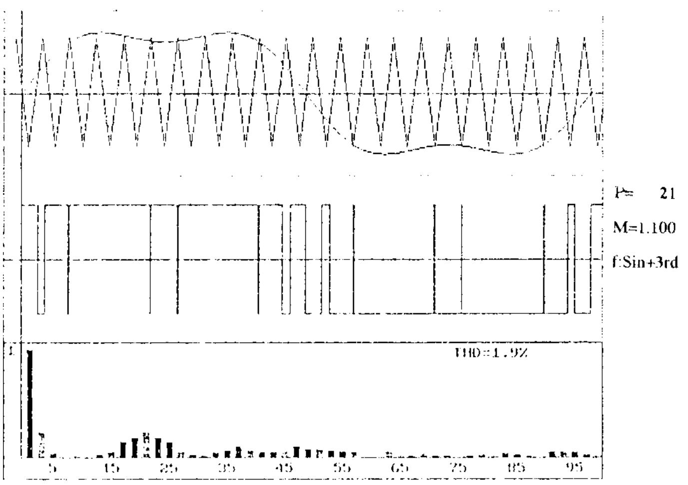
This waveform is the most elementary waveform used as a reference wave. For $M \leq 1$ we are in linear region and output voltage $U_{(1)}$ is proportional to M. Maximum value of $U_{(1)}$ in linear region (M=1) is equal to 0.785 p.u. for M>1 linear proportionality decays and notches gradually disappear. Figures 2(a) and 2(b) show samples of SPWM.

Third Harmonic-Added Sine

In [4] it has been shown that if we choose waveform:



a



b

Figure 3. Sine + 3rd PWM; **(a)**, **(b)**. Waveforms

$$R(\omega t) = M \left(\sin(\omega t) + \frac{1}{4}\sin(3\omega t)\right) \qquad (3)$$

as a reference wave for regular sampling strategy we will have a state near the optimum one. Since in this paper we assume linear region for output voltage in interval 0<M<1, we multiply a coefficient to this function so that when M=1 the maximum of function equals one, for comparison between different waveforms.

$$R(\omega t) = M \cdot \frac{12}{7}\sqrt{\frac{3}{7}} \cdot \left(\sin(\omega t) + 1/4 \sin(3\omega t)\right) \qquad (4)$$

Figures 3(a) and 3(b) show a combination of carrier, reference and output voltage wave shapes and the spectrum of output for M=0.8 and M=1.1.

In Figures 3(c) and 3(d) the variation of ouput voltage fundamental component and THD versus M is shown. It is seen that $U_{(1)}$ is more than SPWM and THD especially near M=1.

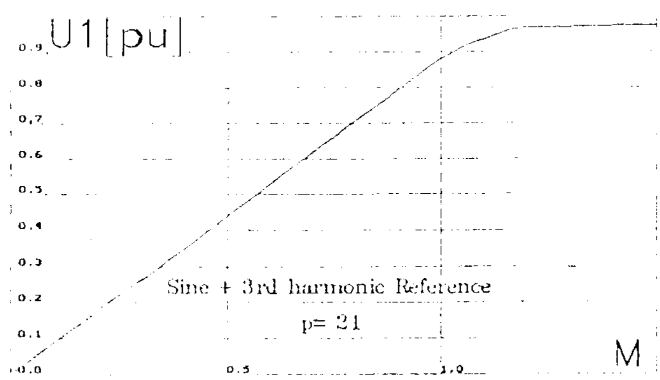


Figure 3(c). Fundamental component versus M

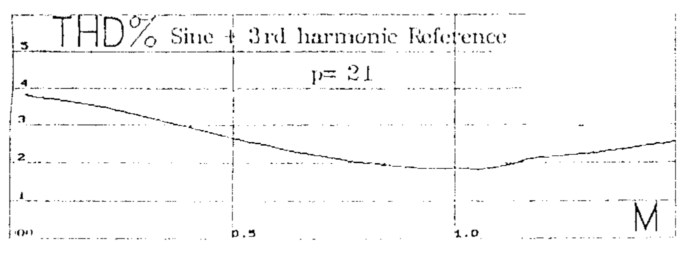


Figure 3(d). THD versus M

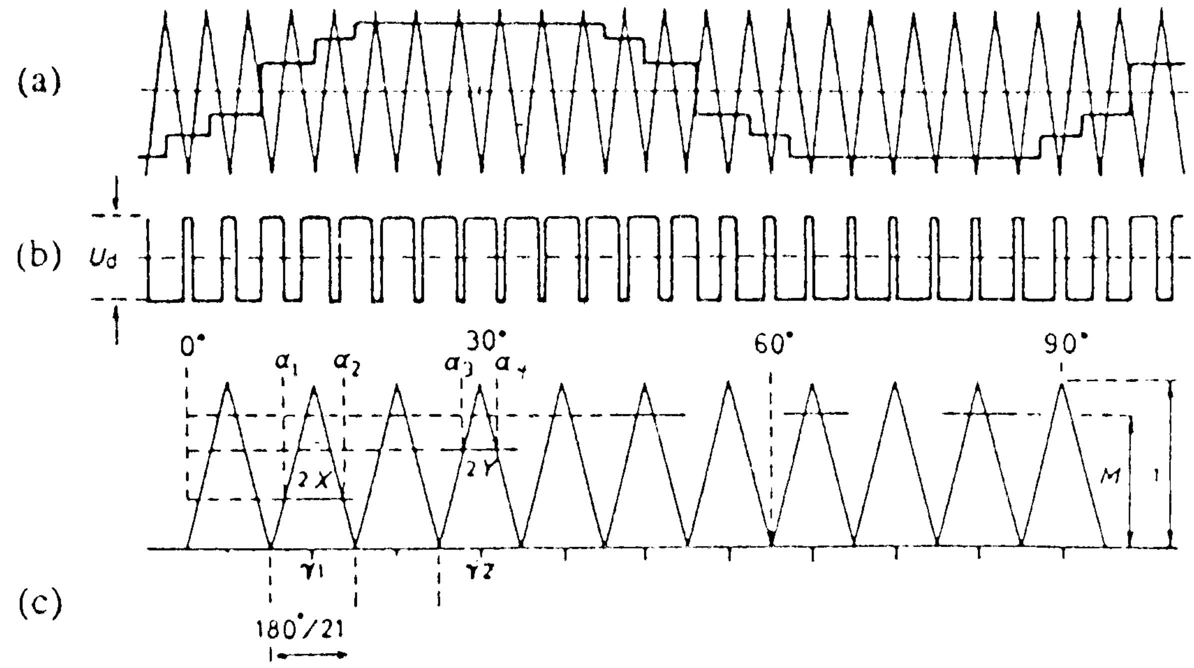


Figure 4. Staircase PWM
(a) Three-level staircase voltage and triangular waveform
(b) Phase potential waveform
(c) Rectified carrier waveform and levels

Optimized PWM

Here, the object of optimization is harmonic elimination. For a given P we calculate a staircase reference waveform with L levels such that when $M=1$, first to $(L-1)$ th harmonics eliminate from line voltage.

For $0<M<1$ it can be predicted (conclusions also confirm) that the amplitude of unwanted harmonics is low. We assume that each stair of reference waveform from the first to the $(L-1)$ th stair intersects Q (is an integer) tooth of triangular waveform teeth (Figure 4).

If we assume that $\alpha_1, \alpha_2, \alpha_3, \ldots$ are edges of output phase voltage waveform, the nth harmonic will be calculated in the follwing manner.

$$U_{(n)}(p.u) = \frac{1}{n}\left|\int_{0}^{\alpha_1} \sin(n\alpha)\,dn\alpha - \int_{\alpha_1}^{\alpha_2} \sin(n\alpha)\,dn\alpha + \ldots + \int_{\alpha_{end}}^{\pi/2} \sin(n\alpha)\,dn\alpha\right|$$

$$= 2/n\left|\cos n\alpha_1 - \cos n\alpha_2 + \cos n\alpha_3 - \ldots - 0.5\right|$$

$$= 4/n\left|\sin(n\gamma_1)\sin(nx_1) + \sin(n\gamma_2)\sin(nx_2) + \ldots - 0.25\right| \quad (5)$$

where $x_1, x_2, \ldots$ are half widths of notches and $\gamma_1, \gamma_2, \ldots$ are central angles of notches. Figure 5 shows these variables.

$$\gamma_i = \frac{\pi}{2P}(4i - 1) \;;\quad i = 1,2,\ldots,\left[\frac{P}{4}\right] \quad (6)$$

If we let relation (5) equal zero, the nth harmonic will be eliminated. If we name the harmonics that must be eliminated by $b_1, b_2, \ldots, b_{L-1}$, eventually, system equations

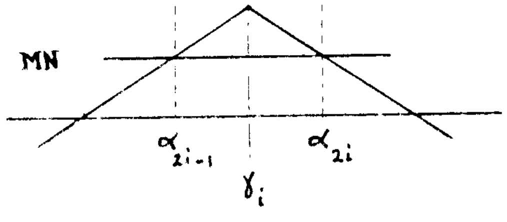


Figure 5. Definition of γ_i and x_i

for Q pulses in each step become as the following simple form:

$$f_i(x_1, x_2, \ldots, x_{L-1}) = \sum_{j=1}^{L-1} \sin(b_i x_j) . \sum_{k=1}^{Q} \sin(b_i . \gamma_{k+(j-1)Q}) - 0.25 = 0 \quad (7)$$

Values of x_i (half widths of notches) are calculated by solving these nonlinear equations with the Newton-Raphson method. Values of N_i can be easily calculated using the equation

$$N_i = 1 - \frac{2P}{\pi} x_i \quad (8)$$

In general form α_i can be calculated using

$$\alpha_i = \pi / 2P \; [2i - (-1)^i M N_j] \quad (9)$$

$j = 1 + (i-1) \text{ div } (2Q)$ (div means integer division)

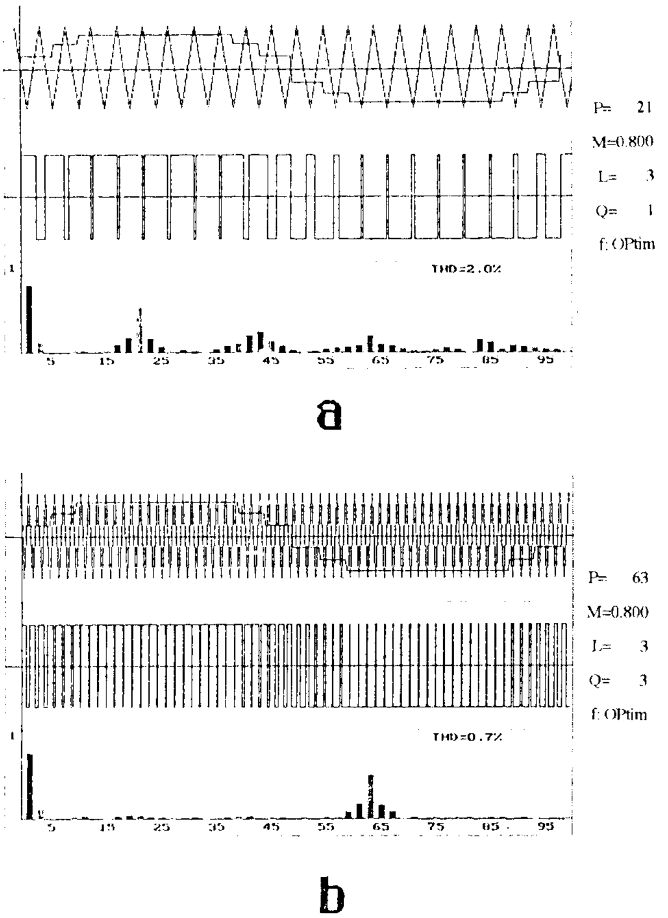


Figure 6. Optimised PWM; **(a)**, **(b)**. Waveforms

All $U_{(n)}$ can be calculated using all α_i in relation (5). THD can be calculated by Equation 2. Figures 6(a) and 6(b) show two kinds of optimized waveforms. Variation of $U_{(1)}$ and THD versus M have been drawn in Figures 6(c) and 6(d). It is seen that fundamental component of output is very high (in M=1, $U_{(1)}$= 0.9225 against $U_{(1)}$= 0.8806 for sine + 3rd) but THD is lower than SPWM, and a little amount more than PWM with Sine + 3rd reference wave.

The major difficulty in optimized PWM is the dependece of reference wave upon P and if we want to vary P, amplitude of levels will change. Also nonlinear system (7) does not have a solution in all states and for some combinations of P, Q, L nonlinear equations diverge.

DERIVATION OF THE NEW WAVEFORMS

Sine-Stair PWM

In this section we choose a staircase waveform as reference and try to calculate angles of edges and step levels in such a way that some of low order odd harmonics are elimi-

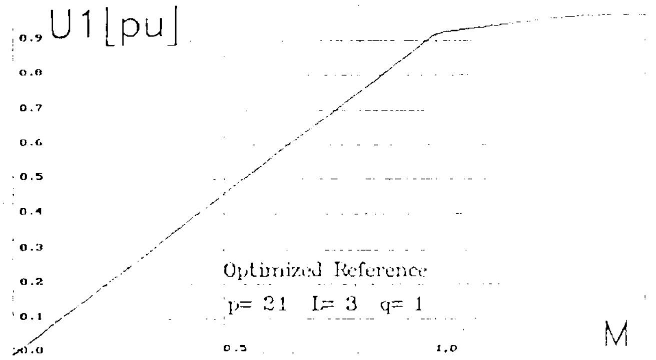


Figure 6(c). Fundamental component versus M

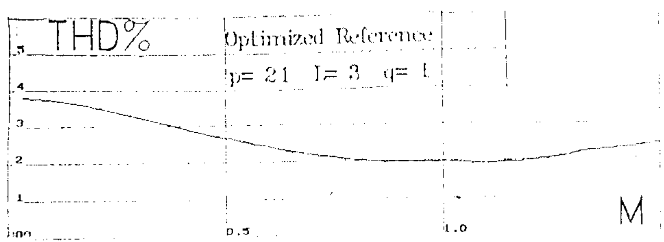


Figure 6(d). THD versus M

nated. As before it is assumed that an unknown staircase waveform has odd symmetry around 0° phase and even symmetry around 90° phase.

We show step levels in interval 0° to 90° with N_1, N_2, ... N_L and angles of step edges with $a_1, \alpha_2, \ldots, \alpha_{L-1}$, L number of steps and $N_L = 1$. The nth harmonic coefficient of reference voltage Fourier series in p.u. is equal to:

$$R_{(n)} = \frac{1}{U_{(1)0}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\pi} \cdot 4 \left| \int_0^{\pi/2} U \sin n\alpha \, d\alpha \right|$$

$$= \frac{1}{n} \left| \int_0^{\alpha_1} N_1 \sin n\alpha \, dn\alpha + \int_{\alpha_1}^{\alpha_2} N_2 \sin n\alpha \, dn\alpha + \ldots \right.$$

$$\left. + \int_{\alpha_{L-1}}^{\pi/2} N_L \sin n\alpha \, d\, n\alpha \right|$$

$$= \frac{1}{n} \left| N_1 + (N_2 - N_1) \cos n\alpha_1 + (N_3 - N_2) \cos n\alpha_2 + \ldots \right.$$

$$\left. + (1 - N_{(L-1)}) \cos n\alpha_{L-1} \right| \qquad (10)$$

If $R_{(n)}$ is equated to zero, the nth harmonic of stair reference voltage will be eliminated. For a given L we have $2(L-1)$ free parameters: $N_1, N_2, \ldots N_{L-1}$ and $\alpha_1, \alpha_2, \ldots, \alpha_{L-1}$.

In this section we will try to calculate these parameters in such a way that the number of $2(L-1)$ low-order odd harmonics are eliminated from the reference wave. In this manner using (10) the following system equations are achieved:

$$N_1 + (N_2 - N_1) \cos 3\alpha_1 + (N_3 - N_2) \cos 3\alpha_2 + \ldots$$
$$+ (1 - N_{L-1}) \cos 3a_{L-1} = 0$$
$$N_1 + (N_2 - N_1) \cos 5\alpha_1 + (N_3 - N_2) \cos 5\alpha_2 + \ldots$$
$$+ (1 - N_{L-1}) \cos 5a_{L-1} = 0$$
$$\vdots$$
$$N_1 + (N_2 - N_1) \cos((4L-3)\alpha_1) + \ldots$$
$$+ (1 - N_{L-1}) \cos((4L-3)\alpha_{L-1}) = 0 \qquad (11)$$

$(2L - 2$ equations$)$

Calculation of $\alpha_1, \alpha_2, \ldots, \alpha_{L-1}$ from (11) can be simplified by equating the relation of nth harmonic elimination with the relation of $(4L$-n) th harmonic elimination term by term.

With this assumption the volume of system equations will be half and nth harmonic elimination will be equal to $(4L$-n) th harmonic elimination.

For example, if $L = 4$ the 3rd - 13th, 5th-11th, 7th-9th harmonics will be eliminated together.

$$\cos(4L - n)\alpha_i \equiv \cos n\alpha_i; \qquad i = 1, 2, \ldots, L-1$$
$$(4L - n)\alpha_i \equiv 2k\pi \pm n\alpha_i; \qquad k = 0, \pm 1, \pm 2, \ldots \qquad (12)$$

By selecting k=i, α_i is calculated to be:

$$\alpha_i = i \frac{\pi}{2L} \qquad (13)$$

This result is interesting since waveform is constructed from equal width steps. The width of each step is a[illegible] $\pi/2L$.

This property makes it possible to use this waveform in microprocessorized sampling strategy. Amplitudes of levels are calculated from the following system equations:

$$\sum_{j=1}^{L-1} \sin\left[(2i+1)(2j-1)\frac{\pi}{4L}\right] . N_j = -\frac{\cos[(2i+1)(L-1)\pi/2L]}{2\sin\frac{(2i+1)\pi}{4L}}$$

$$i = 1, 2, 3, \ldots L-1 \qquad (14)$$

These equations are linear with respect to unknowns N_1, $N_2, N_3, \ldots, N_{L-1}$ and can be solved by ordinary methods such as Gauss elimination.

$$R_{(1)} = N_1 + (N_2 - N_1) \cos\alpha + (N_3 - N_2) \cos 2\alpha + \ldots$$
$$+ (1 - N_{L-1}) \cos(L-1)\alpha \qquad (15)$$

Using (14) and (15) it can be shown that

$$U_{(1)} = LN_1 \qquad (16)$$

Calculations can be checked by (16).

Figure 7(a) shows a sample of this reference wave for $P=21$ and $L=3$. Amplitude of 2 to $(L-1)$th low-order odd harmonics (3,5) are not eliminated but they have small amplitudes such as we have predicted before. The comparison between this reference waveform and sinusoidal reference waveform due to $U_{(1)}$ (fundamental component of inverter output phase voltage) and output THD shows that in this state $U_{(1)}$ is better but THD is worse than SPWM. For example, at $M=1$, $U_{(1)}=0.82$ (against 0.786 for SPWM) but THD= 2.49% (against 2.31% for SPWM).

If we are not sensitive to the small increase of THD this waveform will be prior to sine wave because it has higher output voltage.

If $L \to \infty$, staircase waveform that is calculated from (14) will approach a pure Sine wave with unit amplitude. The name of Sine-Stair is used for this reason.

Quasine Stair PWM

It was mentioned that in a three-phase symmetric system, triplen harmonics are not present in phase-to-phase voltages. Therefore, there is no need to eliminate these harmonics, but instead we can eliminate more non-triplen harmonics. Thus in system (11) triplen harmonic equations will not appear. Using a similar method it can be shown that for this waveform we have

$$\alpha_i = \frac{i\pi}{3L} \qquad (17)$$

Thus in this waveform all the steps from the first up to the $(L-1)$th have the same width as:

$$\alpha = \frac{\pi}{3L} \qquad (18)$$

and the last step starts from $\pi/3$. Also, in the list of harmonics to be eliminated, both the harmonics having the same order from the beginning of the list and its ending will be eliminated together. Elimination of one necessitates elimination of the other. For example, for $L=4$ the following harmonics must be eliminated: 5th, 7th, 11th, 13th, 17th, 19th. They are classified in this manner: 5th-19th, 7th-17th, 11th-13th.

In Figure 7(b) output voltage spectrum for $L=3$ is shown. Comparing this figure with 7(a) (Sine-Stair) we will see that in this state fundamental component amplitude is high and THD value has been decreased too because there is third harmonics in Quasine stair reference wave.

Figures 7(c) and 7(d) show variation of THD, $U_{(1)}$ versus M for $L=3$, $P=21$. It is seen that at $M=1$ we have achieved a very good voltage ($U_{(1)}=0.9176$ p.u.) and in this state THD has relatively low value (THD= 2.18%).

Quasine PWM

If L approaches infinity in Quasine stair waveform we will have the following continuous waveform:

$$R(\omega t)=\begin{cases} 2\sin(\omega t+\pi/6)-1 & 0\le \omega t\le \frac{\pi}{3} \\ 1 & \frac{\pi}{3}\le \omega t\le \frac{2\pi}{3} \\ 2\sin(\omega t-\pi/6)=-1 & \frac{2\pi}{3}\le \omega t\le \pi \end{cases}$$

$$\text{and } R(\omega t+\pi)=-R(\omega t) \qquad (19)$$

Figures 8(a), 8(b) and 8(c) show samples of this waveform used as a reference wave. As (19) shows, waveform has formed from sine and line pieces, hence the name "Quasine" (Quasi + Sine). From Fourier analysis, the rms value of the fundamental of Quasine is $U_{(1)}=\sqrt{2}/\sqrt{3}/0.9=0.907$ pu. and the rms value of the nth harmonic in pu is:

$$R_{(n)}(\text{p.u})=\frac{1-2\cos\left(\frac{n\pi}{3}\right)}{n^3-n}=\begin{cases} \frac{3}{n^3-n} & \text{for } n=3,9,15\ldots \\ 0 & \text{else} \end{cases}$$

(in pu with respect to (1)) (20)

Fourier series of this waveform contains fundamental component and triplen odd harmonics. If we write three elements of the function (19) we will have: (not in p.u)

$$R(\omega t)=\frac{2}{\sqrt{3}}\sin(\omega t)+\frac{1}{2\pi}\sin(3\omega t)+\frac{1}{60\pi}\sin(9\omega t)+\ldots \qquad (21)$$

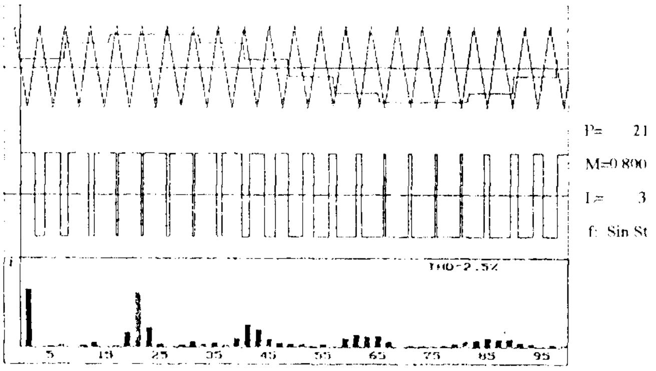


Figure 7 (a). Sine-Stair PWM

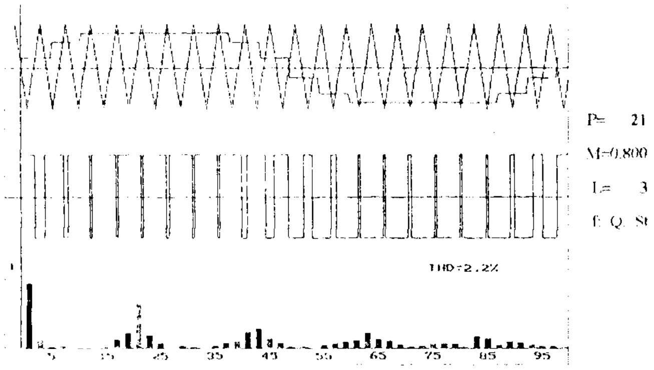


Figure 7(b). Quasine-Stair PWM

It is seen that function has some third and ninth harmonics. If we compare this relation with (3) at $M=1$ we will see that: firstly, the Quasine fundamental component is high, and this will cause more voltage to be obtained from inverter in linear zone of variation of $U_{(1)}$; secondly, third-harmonic amplitude of Quasine is less; thirdly, Quasine has some ninth harmonic which (3) hasn't. Because of the ninth harmonic, Quasine can be constant in interval [π/3, 2π/3]. Figures 8(a) and 8(b) show a sample of inverter output phase voltage spectrum. Figures 8(c) and 8(d) show variation $U_{(1)}$, THD versus M.

COMPARISON

We developed a computer program by TURBO PASCAL 6. Using this program we compared various reference waveforms; Sine, Optimized PWM in sense [1], Sine + 3rd (Suboptimal) in sense [4], Sine-Stair, Quasine-Stair and

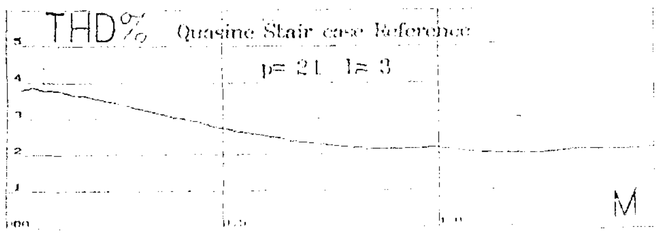


Figure 7(c). THD versus M for Quasine - Stair

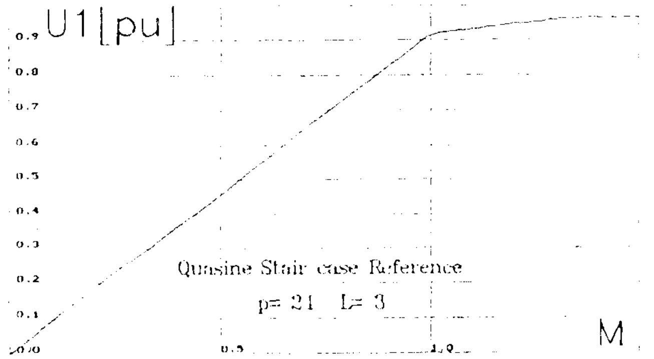


Figure 7(d). Fundamental component versus M for Quasine - Stair

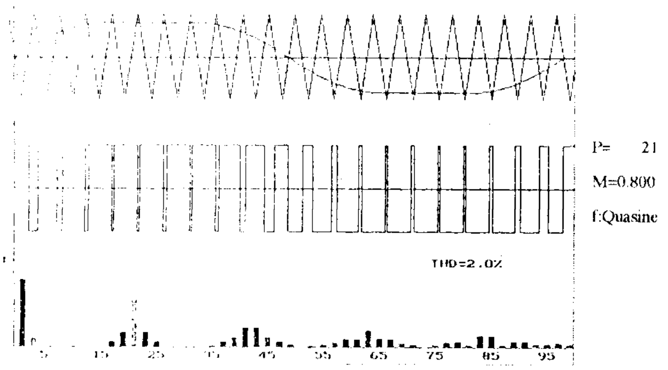
Quasine.

In all the comparisons $P=21$, $L=3$, $Q=1$ are assumed. The results of these comparisons show that: Sine and Sine-Stair are not suitable because their fundamental components are small and their THDs are high. The less amplitude of output fifth harmonic for $M \geq 1.2$ is the only advantage that can be mentioned for Sine waveform. From the point of view of amplitude of $U_{(1)}$, we see that Optimized method is the best and gets first order. Quasine-Stair, Quasine and Sine + 3rd get second, third and fourth orders repectively. If we compare in THD base it is seen that in different Ms we have different states. Table 1 shows three states at three Ms.

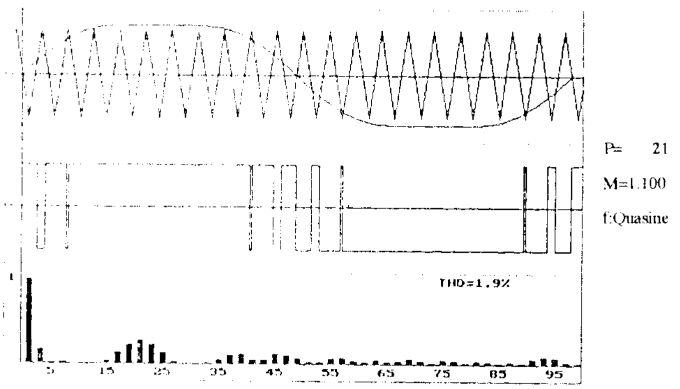
As is seen, for $M=1$, Sine + 3rd has better (less) THD. But for Ms greater than 1 its state is degraded. This problem can be caused by fifth harmonic that is produced from this waveform.

For $M<0.65$ Quasine's THD is better than Sine + 3rd's. Taking into account that difference between THDs is not very much, it can be seen that Quasine has more smooth variation of THD than the other waveforms.

The other disadvantage of Optimized method is the



a



b

Figure 8. Quasine PWM; **(a)**, **(b)**. Waveforms

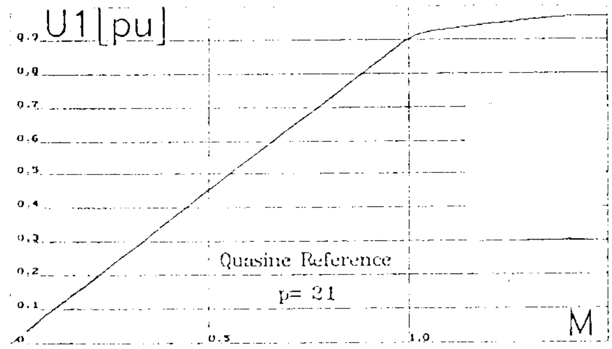


Figure 8(c). Fundamental component versus *M*

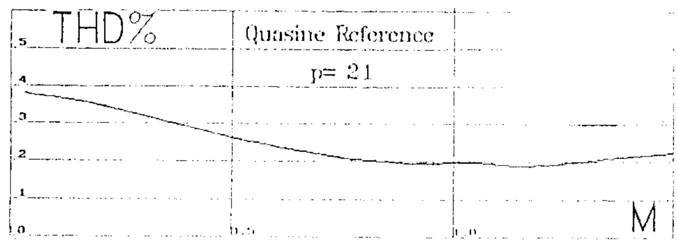


Figure 8(d). THD versus *M*

dependence of waveform on *P*.

CONCLUSION

Derivation of several waveforms as a reference waveform were explained. Sine waveform was laid aside because it has low fundamental component of output and high THD. If we want *P* to be constant, Optimized method will give more fundamental voltage than the other considered waveforms. But its THD is more than Quasine's THD. In *M*s less than 1 and those near it, Sine + 3rd gives good THD, but its amplitude of fundamental component is small and in addition at *M*>1, its THD increases quickly (because fifth harmonic exists in this waveform). From the point of view of THD (to be low) and fundamental component of output (to be high) Quasine is a compromise among all of these waveforms.

TABLE 1.

M= 0.4		M= 1		M=1.2	
	THD%		THD%		THD%
Quasine	2.92	Sine + 3rd	1.86	Quasine	1.91
Optimized	2.94	Quasin	1.98	Optimized	1.99
Sine + 3rd	2.95	Optimized	1.99	Quasine - Stair	2.00
Quasine - Stair	3.01	Quasine - Stair	2.18	Sine + 3rd	2.16

Quasine has no dependence on P, its fundamental component is high and its THD is low. Variation of its THD is less too. Since the nature of Quasine is so that its magnitude is constant during one third of the period, if this waveform is to be used in digital implementation, there will be a 33% saving in memory. Therefore, considering the above reasons, Quasine is recommended for use in inverters.

NOMENCLATURE

M: Modulation index, ratio of reference waveform amplitude to carrier waveform amplitude

P: Sampling rate, ratio of the frequency of carrierwaveform to reference waveform

L: Number of levels per a quarter of staircase reference waveform

Q: The number of pulses per level for optimise PWM.

THD: Total harmonic distortion

$U_{(1)}$: Fundamental component of inventer output phase voltage in per unit.

$U_{(1)b}$: The maximum obtainable fundamental component phase voltage from output, the base of per unitizing of output voltage

$U_{(n)}$: nth harmonic of inverter output phase voltage in per unit.

N_i: The level of ith step

REFERENCES

1. K. Thorborg and A. Nyström: Staircase PWM: an uncomplicated and efficient modulation technique for AC motor drives. *IEEE Trans. Power Electronics*, Vol. 3, No. 4, pp 391-398, Oct. (1988).
2. S.R. Bowes and P.R Clark: Transputer-based harmonic elimination PWM control of inverter drives, *IEEE trans. on Industry Applications*, Vol 28, No. 1, pp 72-80, Jan/Feb. (1992).
3. F.C. ZACH: Comparison of voltage and current spectra using optimized forms of inverter-pulse width modulation, *IEEE trans. on Power Electronic*, Vol. 4, No. 1, pp 64-72, Jan. (1989).
4. S.R. Bowes and A. Midoun: Suboptimal switching. Strategies for microprocessor-controlled PWM drives, *IEEE proceeding*, Vol. 132, Pt B, No. 3, pp 133-148, May (1985).
5. S. Halasz: Correspondence to 'Suboptimal switching strategies for microprocessor-controlled PWM inverter drives', *IEEE proceeding*, Vol. 132, Pt. B, No. 6, p 344, Nov (1985).
6. S.R. Bowes and P.R. Clark: Simple microprocessor implementation of new regular-sampled harmonic elimination PWM techniques, *IEEE Trans. on Industry Appl.*, Vol. 28, No. 1, Jan/ Feb. (1992).
7. J. Richardson and O. Kuker: Implementation of a PWM regular sampling strategy for AC drives, *IEEE Trans. on Power Electronics*, Vol. 6, No. 4, pp 645-655, Oct. (1991).
8. B.K. Bose and H.A. Sutherland: A high performance pulse width modulator for inverter-fed drive system using a micro-computer, *IEEE Trans. Ind. Appl.*, Vol 19, pp 235-243, Mar/ Apr. (1983).
9. A.M. Tzynodlowski: Non-sinu soidal modulating functions for three-phase inverters, *IEEE Trans. On Power Electronics*, Vol. 4, No. 3, pp. 331-338, July (1989).
10. K. Thorborg: *"Power Electronics"*, Prentice Hall Int. (UK) (1988).