EVALUATION OF SCATTERING PARAMETERS

M. Hosseini
Department of Physics
Ferdowsi University
Mashhad, Iran

Abstract A simple algorithm is presented for evaluating scattering parameters of a two-port network. This technique uses the indefinite admittance matrix to calculate scattering or any other desired set of two-port parameters.

چکیده یک الگوریتم ساده برای تعیین پارامترهای تفرق یک شبکه دوقطبی ارائه می شود. دراین روش از ماتریس ادمیتانس نامعین جهت محاسبه پارامترهای تفرق و یا هرنوع پارامترهای دوقطبی دلخواه استفاده شده است.

INTRODUCTION

In any combined circuit analysis/optimization computer program, the analysis subroutine plays a dominate role. The associated algorithm invariably occupies the bulk of the computer time, being called upon to re-compute the overall network response each time any parameter is varied [1, 2]. It is therefore essential that this algorithm be made efficient, which in turn influences the choice of network matrix. Of the many types which may be employed, we use the indefinite admittance matrix (IAM) as eminently suitable, both on account of its high efficiency and by being most appropriate for combinations of two and three-terminal networks [3].

Sometimes it is advantageous to use a combination of matrix techniques, rather than a single procedure. The former approach may be particularly useful where different types of network are involved which have a common node or ground connection. This paper describes how, with the aid of a simple algorithm, a relatively inexperienced computer programmer can apply the relevant procedures to typical multiterminal circuit analysis, halving the number of iterations required in a typical two-port s-parameters optimization.

COMPUTATION OF S-PARAMETERS

In a vast number of network problems, however complicated the network, the relationship between electrical properties at an output port and an input port is the item of predominant interest. We will therefore seek to compute the two-port parameters for such a network in terms of s-parameters.

Consider a network in which there are n independent node-pairs, labelled sequentially, 1, 2, ... i, ... o, ... (n-1), n, where i and o are those terminal pairs acting as input and output ports, respectively. We need consider only these node-pairs and may ignore all others, so we can use one of the standard forms of two-port parameters to represent the network between the input and output ports. If the representation is to be in the form of Z-parameters, for example, we require to find the elements of the Z-matrix of Equation 1.

$$\begin{bmatrix} V_i \\ V_o \end{bmatrix} = \begin{bmatrix} Z_{ii} & Z_{io} \\ Z_{oi} & Z_{oo} \end{bmatrix} \begin{bmatrix} I_i \\ I_o \end{bmatrix} \qquad (1)$$

The IAM for the n-independent node-pairs would be of the form

$$[I] = [Y]\,[V] \qquad (2)$$

where all the elements $[I]$ are zero except for I_i and I_o. If we take node-pair n as the reference we may obtain a non-singular matrix $[Y_{n-1}]$ by removing from $[Y]$ the nth row and column. Provided that the determinant of this matrix is non-zero we may then obtain the inverse of the admittance matrix $[Y_{n-1}]$, hence re-write Equation 1 as

$$[V] = [Z]\,[I] \tag{3}$$

The matrix Equation 3 represents (n - 1) simultaneous equations. Since, however, there are only four unknowns, we may reduce these to two equations only, i.e. the ones involving the independent variables, thus relating to the input and output terminals. We now factorize the $[Y_{n-1}]$ matrix into the lower triangular matrix $[L]$ and upper triangular matrix $[U]$ with diagonal elements of unity. Hence we calculate the jth column via the following algorithm [4]:

$$u_{ij} = (y_{ij} - \sum_{k=1}^{i-1} \ell_{ik} u_{kj}) / \ell_{ii} \qquad i<j$$

$$\ell_{ij} = y_{ij} \sum_{k=1}^{j-1} \ell_{ik} u_{kj} \qquad i\geq j \tag{4}$$

where u_{ij} and ℓ_{ij} are elements of $[U]$ and $[L]$, respectively, utilizing

$$[Y_{n-1}]^{-1} = [U]^{-1}\,[L]^{-1} \tag{5}$$

Our problem is to invert the triangular matrices $[L]$ and $[U]$. The inverse of a lower (upper) triangular matrix is also a lower (upper) triangular matrix [5]. Now let $[L]^{-1}=[\ell'_{ij}]$ with $\ell'_{ij}=Q$, i<j and let L_i and L_j^{-1} be, respectively, the ith column of $[L]$ and the jth of $[L]^{-1}$. Then for $1 \leq k \leq n$:

$$L_k^{-1} L_k = l'_{kk} l_{kk} = 1$$

$$L_k^{-1} L_j = \sum_{i=j}^{k} \ell'_{ki} \ell_{ij} = 0 \qquad j= k-1, k-2, ..., 1 \tag{6}$$

from which we may calculate ℓ'_{ki}, i=k, k-1, ... , 1 for any k, thereby obtaining all the elements of $[L]^{-1}$. The algorithm is as follows:

$$\ell'_{kk} = 1/\ell_{kk} \qquad k = 1, \ldots, n$$

$$\ell'_{kj} = (-\sum_{i=j+1}^{k} \ell'_{ki} \ell_{ij}) / \ell_{jj} \qquad j = k-1, \ldots, 1 \tag{7}$$

Similarly we may calculate $[U]^{-1} = [u'_{i,j}]$ the only difference being the fact that the diagonal elements of $[U]$ are unity. Calculation of $[L]^{-1}$ and $[U]^{-1}$ followed by the matrix multiplication $[U]^{-1}[L]^{-1}$, finally arrives at the following algorithm:

$$Z_{ij} = \sum_{k=i}^{n} u'_{ik}\, \ell'_{kj} \tag{8}$$

It is, of course, a matter of routine transformation to obtain any other desired set of two-port parameters from these, e.g. scattering or transmission parameters.

CONCLUSION

In summary, evaluation of s-parameters by conventional means, requires two separate runs by a nodal analysis program. This means that with a generator connected to port i, only s_{ii} and s_{oi} can be evaluated. Similarly by removing the generator from port i and connecting to port o, s_{io} and s_{oo} can be evaluated. We have shown that this algorithm may be used to evaluate the s-parameters in one run, and this halves the number of iterations required in a typical analysis/optimization program.

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