# BASIC ISSUES IN IDENTIFICATION SCHEME OF A SELF-TUNING POWER SYSTEM STABILIZER

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Abstract Power system stabilizers have been widely used and successfully implemented for the improvement of power system damping. However, a fixed parameter power system stabilizer tends to be sensitive to variations in generator dynamics so that, for operating conditions away from those used for design, the effectiveness of the stabilizer can be greatly impaired. With the advent of microprocessor technology an adaptive controller, a controller which adapts itself to the changes in system dynamic characteristics, offers an attractive proposition in power system control. The heart of the so-called an adaptive self-tuning power system stabilizer is its identification scheme by which unknown system/controller parameters are estimated. This paper addresses some of the basic issues in implementation of a recursive least square estimator, when applied to an unknown power system. Digital simulation results are presented.

چکیده امروز پایدار کننده های سیستم قدرت نقشی حساس را در پایداری شبکه های بزرگ بهم پیوسته قدرت ایفاً می نمایند. از آنجا که پایدار کننده های سنتی بعلت تغییر پیوسته شرایط دینامیکی سیستم، عملکرد بهینه خود را ممکن است از دست بدهند پایدار کننده وفقی خود تنظیم، میتواند با شناسائی سیستم بصورت بلادرنگ و تغییر پارامترهای کنترل کننده بکمک یک کامپیوتر، این مشکل را تاحد زیادی حل نماید. دراین مقاله نکات کلیدی در بکارگیری یک الگوریتم شناسائی سیستم نمونه ارائه میگردد.

#### INTRODUCTION

Synchronous generators are equipped with Automatic Voltage Regulators (AVR) to maintain the voltage close to the desired value as operating conditions and system load change. The general trend towards constructing larger unit sizes and longer transmission lines, mainly due for economic reasons, has significantly reduced transient stability margins [1] so that the use of high gain, low time constant voltage regulators in synchronous machines employing thyristor excitation is the common industrial practice [2]. Nevertheless, the approach has a detrimental effect on system damping [3] so that to compensate for its effect and to improve the system damping behavior during transient conditions, an additional control signal via the so called power system stabilizer (PSS), may be superimposed on the normal voltage error signal of an automatic voltage regular [4]. Since the generator dynamics change significantly with the operating conditions and system configurations, it is very difficult or even impossible via a priori design to determine a single set of controller (PSS) parameters which provides satisfactory performance for all conditions [5]. It is for this reason that an adaptive self-tuning power system stabilizer, i.e., a stabilizer which adapts itself to changing dynamic characteristics has so much potential to improve power system performance.

A self-tuning controller is capable of adjusting its controller parameters. A computer, as the heart of the controller, may be used to evaluate the system dynamic characteristics on-line, and adjust the regulator parameters according to a prespecified strategy (see Figure 1). Self tuning algorithms may be divided into two main categories [6]:

Implicit types: where the controller parameters are identified directly.

**Explicit types:** where the system parameters are identified on-line and used to calculate the controller parameters.

In either of the aforementioned types, a self tuner contains a recursive parameter estimator. Many different

estimation schemes have been used, for example stochastic approximation, least squares, instrumental variables, extended Kalman filtering and maximum likelihood methods. These are covered in some surveys [7,8] and books [9,10,11].

For high signal/noise ratio (say > 10), which is typical of the systems studied in this paper, the recursive least squares algorithm is suggested. In the section which follows, this algorithm is described in some detail. Thereafter, basic issues in implementation of the algorithm, namely persistently of excitation, sampling period selection and model order selection and validation are discussed and digital simulation results of a typical power system are presented and the capabilities of the algorithm are appreciated.

# RECURSIVE LEAST SQUARES IDENTIFICATION

When a digital computer is used for control, it is convenient to design and analyze the system in terms of a discrete model. A single-input/ single-output randomly disturbed system can be described as:

A 
$$(q^{-1})$$
  $y(t) = q^{-k} B(q^{-1}) u(t) + V(q^{-1}) \xi(t)$  (1)

where

$$A(q^{-1}) = 1 + a_1q^{-1} + ... + a_{n_1}q^{-n_2}$$
 (2)

$$B(q^{-1}) = b_0 + b_1 q^{-1} + ... + b_{nb} q^{-nb}$$
(3)

$$C(q^{-1}) = 1 + c_1 q^{-1} + ... + c_{nc} q^{-nc}$$
 (4)

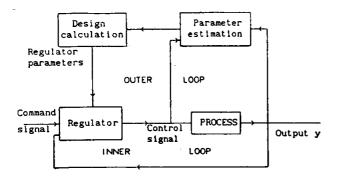


Figure 1. Block diagram of a self-tuning regulator (design calculation block is eliminated for implicit types)

k: system time delay in sample instants

 $q^{-1}$ : backward shift operator, ex.,  $q^{-k}u(t)=u(t-k)$ 

y(t), u(t): system output and input, respectively, corresponding to the sampling instant

 $\xi(t)$ : an uncorrelated random sequence of zero mean which disturbs the system

As no pure delay is involved, for the deterministic (or near deterministic) system studied in this paper, the governing equation is:

$$A(q^{-1}) y(t) = q^{-1} B(q^{-1}) u(t)$$
 (5)

If the system order is known, then the knowledge of the input disturbance and the output response can be used to determine the parameters of the system model via least squares identification as follows:

Equation (5) can be written in difference form as:

$$y(t) = -a_1 y (t-1) - a_2 y(t-2) ... - 2_{na} y(t-na) +$$

$$b_0 u(t-1) + ... + b_{nb} u(t-nb)$$
(6)

or in vector / matrix form as:

$$\mathbf{v}(\mathbf{t}) = \mathbf{x} \ (\mathbf{t}) \ \mathbf{\theta} \tag{7}$$

where

$$\theta^{T} = [a_1, \dots, a_m, b_0, \dots, b_m]$$
 (8)

$$\mathbf{x}(t) = [-y(t-1), -y(t-na), u(t-1), ..., u(t-nb)]$$
 (9)

As time progresses and additional samples are available, the following set of relationships can be established:

$$y(t) = x(t)$$

$$y(t+1) = x(t+1)$$

$$y(t+N) = x(t+N)$$

or

$$\mathbf{Y} = \mathbf{X}\mathbf{\Theta} \tag{11}$$

where Y and X are left hand side and right hand side

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vectors, respectively.

Consider now, how an estimate of the parameter vector  $\hat{\theta}$ , can be obtained by a least squares approach. Using estimate  $\hat{\theta}$ , the prediction of the system output  $\hat{Y}$  is given by:

$$\hat{\mathbf{Y}} = \mathbf{X}\hat{\boldsymbol{\theta}}$$
 (12)

and the prediction error  $\varepsilon$  by:

$$\mathbf{\varepsilon} = \mathbf{Y} - \widehat{\mathbf{Y}} = \mathbf{Y} - \mathbf{X}\widehat{\boldsymbol{\theta}} \tag{13}$$

The value of  $\hat{\theta}$  which minimizes the squares of the error can be found as follows:

$$\mathbf{s} = \boldsymbol{\varepsilon}^{\mathsf{T}} \, \boldsymbol{\varepsilon} = (\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\theta}})^{\mathsf{T}} \, (\mathbf{Y} - \mathbf{X} \hat{\boldsymbol{\theta}}) \tag{14}$$

$$\frac{\hat{\mathbf{d}}\mathbf{s}}{\hat{\mathbf{d}}\hat{\mathbf{\theta}}} = -2\mathbf{X}^{\mathsf{T}} (\mathbf{Y} - \mathbf{X}\hat{\mathbf{\theta}}) = 0 \tag{15}$$

hence

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^{\mathsf{T}}\mathbf{X})^{-1} \mathbf{X}^{\mathsf{T}}\mathbf{Y} \tag{16}$$

So by providing the observation matrices X and Y to the estimator, an estimate of  $\theta$  can be obtained provided ( $X^TX$ ) is non-singular. If steady state values are used for the data record, rows of X become equal, making ( $X^TX$ ) singular. To avoid this occurrence, the system variables should be changing sufficiently. This can be ensured by employing a persistently exciting input.

The previous formulation is suitable only for off-line estimation. It is possible, however, to convert it into a recursive form which enables old estimates (denoted by  $\hat{\theta}(t)$  to be updated as  $\hat{\theta}(t+1)$  as new data becomes available giving an on-line facility. Appropriate rearrangements and manipulations result in the following recursive scheme:

### i) Form x (t+1) using the new data

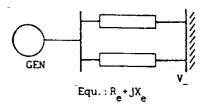


Figure 2. Single machine infinite busbar system under study

ii) Form ε (t+1) using

$$\varepsilon$$
 (t+1)= y(t+1) - x(t+1)  $\hat{\theta}$ (t)

iii) Form P (t+1) using:

$$\mathbf{P}(t+1) = \rho \mathbf{P}(t) \left\{ \mathbf{I} - \frac{\mathbf{x}^{T}(t+1) \mathbf{x}(t+1) \mathbf{P}(t)}{1 + \mathbf{x}(t+1) \mathbf{P}(t) \mathbf{x}^{T}(t+1)} \right\}$$

(where  $P(t) = (X^TX)^{-1}$  is introduced) iv) Update  $\theta(t)$  obtain  $\theta(t+1)$  as:

$$\hat{\boldsymbol{\theta}}(t+1) = \hat{\boldsymbol{\theta}}(t) + \mathbf{P}(t+1) \mathbf{X}^{\mathsf{T}}(t+1) \boldsymbol{\varepsilon}(t+1)$$
 (19)

Initially  $\hat{\theta}(t)$  may be set to zero and P made a large diagonal matrix (say  $10^6$  I). As time increases, the elements of the covariance matrix, P, will decrease in size as the estimates become more accurate. If, therefore, the system parameters change, the ability of the recursive estimator to respond will be limited by the small size of the P(t) matrix elements. To overcome the difficulty, the forgetting factor,  $\rho$  (.97-1.0) is introduced which serves to progressively phase out the effects of the past data. The covariance matrix does not reduce to zero now and the tracking of time varying parameters can be accommodated.

# BASIC ISSUES IN IMPLEMENTATION OF A RECURSIVE LEAST SQUARES ESTIMATOR

## **Continuous and Discrete System Representation**

The continous system under study is a single machine

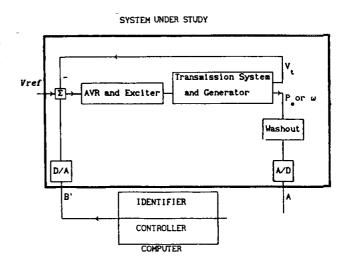


Figure 3. System representation with a self-tuning PSS

connected to a large power system (represented by an infinite busbar) via two parallel transmission lines as depicted in Figure 2. All data and information are available from Reference 12. The system representation with the adaptive PSS is shown schematically in Figure 3.

The discrete system prediction model representing the system behavior between points A, and B, in Figure 3 is given by:

A 
$$(q^{-1})$$
 y(t)=  $q^{-1}$  B  $(q^{-1})$ u (t) (20)

where y(t) and u(t) are the values of the output and input at each sampling time instant and:

$$A (q^{-1}) = 1 + a_1 q^{-1} + + a_{na} q^{-na}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + + b_{nb} q^{-nb}$$
(22)

#### **Basic Issues**

System identification is carried out through the standard recursive least squares method. In estimating system parameters, the persistence of the disturbance, the sampling period and model order selections are of great concern to the analyst.

#### **Persistent Excitation**

As described above an estimate of the parameter vector can be obtained provided the system variables are changing sufficiently. For the deterministic or near deterministic system considered in this paper, this can be ensured by employing a persistently exciting input. Two signals are of interest here:

White noise
Pseudo - Random Binary Sequence (PRBS)

Although both of them have been effectively used, white noise suffers from the disadvantage that its generation is more difficult. Moreover, it may result in a longer converge time. The PRBS can be more easily generated in a digital computer. The signal can take on only two states, say + a and -a; the state can change only at discrete intervals of time  $\Delta t$ ; the change occurs in a deterministic pseudo random manner and the sequence is periodic with period  $N\Delta t$  where N is an integer. The effective frequency band

covered by a PRBS is [13]:

from 
$$1/N\Delta t$$
 to  $1/3\Delta$  Hertz (23)

The power of the signal can thus be controlled for the frequency range of interest. For the studies conducted in this work, either a white noise signal generated by a computer system library or a PRBS of 0.1 sec bit width ( $\Delta t = 0.1 \text{ sec}$ ), and 6.5 sec. sequence (N= 63) duration is used so that the frequency covered is from 0.16 to 3.3 Hertz which is the normal frequency range of interest [14]. The amplitude is normally chosen such that the system output (input to the computer) deviation from the operating condition is within  $\pm 1\%$ .

#### Sampling Period Selection

The choice of the sampling period is of great importance. If is too large, the sampled-data representation may be entirely erroneous. If it is too low, the process computer may be unable to perform the required task in the given period. A study conducted by Lee [15] shows that a sampling period of up to 100 msec may be considered acceptable for a digital power system stabilizer operating at a natural frequency of around 1 Hz. This sampling period is used throughout this work.

#### **Model Order Selection and Validation**

To sufficiently represent the system dynamics, the

TABLE 1. Various System Parameters/Operating Conditions Under Study.

Condition	Active power	Reative power	Ext. Reactance	V∞
	p.u.	p.u.	p.u	
	.5	એ 1 <b>:3</b> 539	.7	.94
. 2	1.0	.5	0.8	1.0
3	<b>5</b>	.5	.1	1.0
4	.6	5	(2)	1.05
5	1.0	.0	s <b>4</b> 5	1.0
6	<sup>19</sup> .5	೯೩: 0. ್ಮ ಕ್ರ	1.0	1.1
7	1.0	7	.15	1.05
8 -31	ar 3 mies	isa 6 <b>2</b> - ⇔di		.95

prediction model order (Equation 20) should be chosen with great care. The order should be chosen as low as possible to save computational time but not so low as to sacrifice the accuracy of the system dynamic representation.

As an aid in choosing the model order, eigenvalue analysis may be employed. Digital simulation studies are carried out by using *Advanced Continuous Simulation Language* (ACSL) [16]. Extensive eigenvalue analysis is performed by linking ACSL with a linear analysis package. Eight different operating conditions / system parameters are considered (see Table 1). The general pattern of all eigenvalues shifts is shown Figure 4 whenever the gain of a conventional PSS of the form given by:

G(s) = 
$$K\left(\frac{1+0.15s}{1+0.09s}\right)\left(\frac{10s}{1+10s}\right)$$
 (24)

varies over a wide range (for details see Appendix A). The first mode observed is associated with rotor oscillations while the second one is primarily associated with the field voltage. The results clearly demonstrate that the latter (normally called *exciter mode*) is highly damped in open loop conditions (notice where the open loop eigenvalues are located). This suggests that the system dynamic, being mainly dictated by the local mode, may be estimated by a second order prediction model.

To actually confirm the validity of the assumption, the open-loop identification results for a typical system condition (P=1.0 p.u., Q=0.0 p.u. and Xe=0.1 p.u.), with the electrical power as the input are shown in Figure 5, when a Gaussian noise or a PRBS signal perturbs the system. With faster convergence for the PRBS case, all parameters converge to their final values with an associated large reduction in the trace of the covariance matrix. Another test shown in Figure 6 further confirms that the estimated system output closely follows the sampled data output when a PRBS is applied to the voltage reference junction. Tests were also carried out with the speed as the input to the stablizer with similar conclusions. For all eight conditions, the estimation results are summarized in Table 2.

Comparing the natural frequency and the damping ratio (dictated by the system pole locations) as calculated from the estimated model with those of the simulated system will further demonstrate how the system dynamics are closely predicted by the estimated model. The validity of the estimated zero locations, is, however, checked by comparing the system phase lag (between output, namely, electrical power or speed, and input, i.e., the V<sub>ref</sub> junction) at the frequency of the most interest (i.e. natural frequency) as predicted from the discrete model with that of the

TABLE 2. System Parameters for All Conditions

	P A	CONDITIONS							
INPUT	R A	1	2	-3	4	5	6	7	-8
SPEED	$\mathbf{a}_{_{\mathbf{l}}}$	-1.61	-1.83	-1.12	-1.40	-1.18	-1.68	-1.46	-1,60
	a,	0.98	11.11	10.75	0.90	0.81	0.98	0.98	0.95
	$b_0$	0.11	0.20	0.21	0.43	0.46	0.123	0.53	0.08
1	b,	0.35	0.65	0.55	1.24	1.25	0.40	1.63	0.22
ELEC.	a,	-1.67	-1.83	-1.07	-1.34	-1.14	-1.63	-1.45	-1.55
	$a_2$	0.98	1.10	0.83	0.90	0.8	0.94	0.98	0.92
	b,	0.29	0.38	0.48	0.97	1.01	0.34	1.26	0.19
	b,	-0.30	- <del>-</del> 0.48	I -0.38	-0.62	-40.71	-0.29	18.0- K	-0.17

simulated system. the following procedure is followed:

\*For the estimated model, the undamped natural frequency  $(\omega_n)$  and the damping ratio  $(\xi)$  are calculated using the standard procedure [17]. The phase lag produced is calculated from the prediction model with q replaced by exp  $(j \omega_n T')$ , where T' is the sampling period.

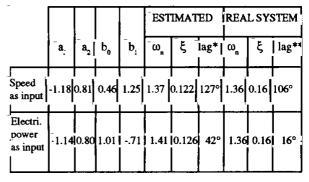
\*For the simulated (continuous) system, the eigenvalue analysis yields the natural frequency and the damping factor. An analytical approach presented in Reference [3] is then employed to calculated the phase lag at the natural frequency.

The results are summarized in Table 3 with the following conclusions:

\*The natural frequencies, damping factors and the phase lags are in close agreement. While the former two are dependent upon the a parameters, the phase lag is sensitive to all system parameters (i.e. a's and b's). Being more dependent upon the parameters, the discrepancies observed

TABLE 3. Comparison Chart of Simulated (Continuous)
System and Estimated Model.

SYSTEM CONDITION:  $P = 1.0 \ p.u., Q = 0.0 \ p.u. \ Xe = 0.1 \ p.u., V = 1.p.u.$ 



<sup>\*</sup>Phase Lag of the Estimated Model at Natural Frequency

\*\* Phase Lag of the Continuous System at the Natural
Frequency

(Between Speed or Power and the V, Junction)

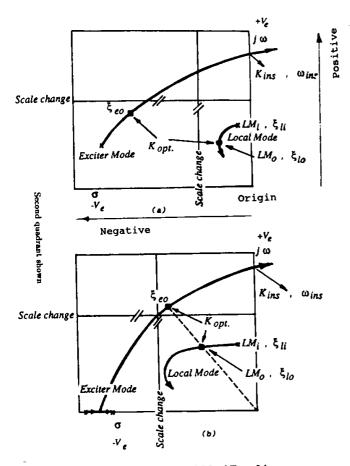


Figure 4. Eigenvalue analysis for PSS of Eqn. 24
a) general pattern for conditions 1,2,6 and 8
b) general pattern for conditions 3,4,5 and 7
(For definitions, see Table A, Appendix A)

in the phase lags are mainly due to the inaccuracies involved in the identification process. However, as suggested by de Mello andConcordia, as long as the difference between the machine phase lag and the phase lead produced by the PSS is within 30°, acceptable performances should be achieved. Reference [18] serves to show that for the self-tuning stablizer tuned based on the aforementioned estimated model, this criterion is well satisfied.

\*The 90° shift between the speed and electrical power suggests the system phase lag between either of these outputs to the system input ( $V_{\rm ref}$  junction) as predicted by the estimated model should be 90° out of phase. The results in Table 3 demonstrate that in the natural frequency of interest, this is closely satisfied.

### DISCUSSION

Simulation results of the typical power system studied show how effectively an identification routine may be employed to predict system dynamic characteristics. The

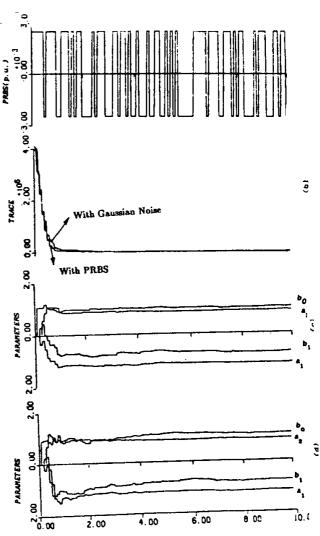


Figure 5. Identification test at P = 1.0 p.u. and Q = 0.0 p.u. (electrical power as the input).

- a) PRBS signal input
- b) trace of the covariance matrix
- c) parameters with PRBS peturbation signal
- d) parameters with gaussian noise peturbation signal

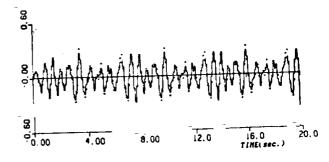


Figure 6. Comparison of the actual and estimated system output Solid: estimated output,

Dashed: actual output (sampled data)

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scheme has been effectively employed in a self-tuning power system stabilizer as described in References [19-24]. However, the capabilities discussed are not limited only to self-tuning applications, but to predict power system dynamic characteristics when system data are not available or models are not accurate.

It should be mentioned, however, that if a greater accuracy and as a result a better matching between estimated model and real system are required, a third order model may be employed. These results in an estimation of an additional pole and extra burden on the process computer and if can be tolerated, is justified [25]. In multimachine cases, higher model order should be definitely employed to predict interarea as well as local modes [26].

#### CONCLUSION

The paper described basic issues in implementation of an identification routine of a self-tuning power system stabilizer. The scheme is quite fast and well suited for real time applications. On-line results together with some practical issues in real time conditions will be published soon. The author is in the process of analyzing identification routines in multimachine situations.

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#### APPEDNIX A

With electrical power as the input to the PSS, the required phase

TABLE A. Summary of Eigenvalue analysis.

	con.1	con.2	con.3	con.4	con.5	con.6	con. 7	con8
K <sup>1</sup> <sub>opi</sub>	.5	1.2	5	.28	26	4	28	4
K <sub>ins</sub>	9.0	5.8	7.5	3.3	3.3	7.5	2.7	13.0
- Wins	41.0	40.0	50.0.	45.0	47.0	40.0	45.0	40.0
LM <sub>i</sub> <sup>4</sup>	-4.±j 5.6	+.5±j 5.1	-2.1±j 8.1	-1.0±j 7.3	-1.4±j 8.4	-0.2±j 4.9	-0.2±j 7.3	-0.5±j . 5.3
LM <sub>0</sub>	7±j 4.5	-0.1±j 4.3	-3.8±j 4.3	-3.6±j 4.2	-4.3±j 5.16	.42±j 4.0	-3.4±j 4.2	67±j 4.6
ξ <sub>li</sub> 6	-07	094	25	<sup>-</sup> 14	<sup>-</sup> 16	-04	.026	.t
ξ <sub>10</sub> <sup>7</sup>	.15	.033	65	- .65	65	103	62	14
ξ <sub>e0</sub>	62	340	65	.65	65	.62	62	.70

lead is normally low so that a single stage lead PSS is generally sufficient. The gain, K (in eqn. 24) is varied over a wide range. For the general pattern shown in Figure. 4, the detailed results are shown in Table A (see also ref. [5]).

- 1. Optimum value of K
- 2. The value of K corresponding to instability

- 3. The frequency corresponding to instability gain in rad/sec.
- 4. Local mode poles for K = 0
- 5. Local mode poles for the optimum value of K
- 6. Damping factor for local mode at K = 0
- 7. Damping factor for local mode at optimum gain
- 8. Damping factor for exciter mode at optimum gain