A PARAMETRIC CATCHMENT EROSION MODEL

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Abstract The structural framework for a parametric catchment erosion model is proposed. A parameter optimization technique that provides a rational basis for the calibration of the model is developed. The adequacy of the model in representing a natural catchment is investigated.

چکیده یك مدل پارامتریك برای فرآیند فرسایش در حوزه آبریز طراحی و تدوین گردیده است. در طراحی این مدل فرآیند فرسایش به دو زیر فرآیند تولید و انتقال رسوب تفکیك و برای هر کدام فرمولی فیزیکی پیشنهاد گردیده است. در این مدل تصور بر این است که حوزه آبریزاز چند مخزن تشکیل گردیده و هر مخزن بعنوان یك سیستم تولید، ذخیره و انتقال رسوب عمل می‌کند. ضرائب و پارامترهای موجود در فرمولهای تولید و انتقال رسوب معرف خصوصیات فیزیکی حوزه و شرایط هیدرولیکی جریان آب و رسوب می‌باشند. برای تعیین مقادیر این پارامترها، مدل به یك روش بهینه سازی مجهز گردیده‌است. دقت و صحت مدل مورد تحقیق و کاربرد آن در برآورد میزان رسوب در حوزه آبریز مورد تأیید قرار گرفته‌است. علاوه بر برآورد میزان رسوب، این مدل‌را می‌توان به منظور تصحیح خصوصیات فیزیکی و هیدرولوژیکی حوزه و کاهش میزان رسوب بکار گرفت.

INTRODUCTION

From an engineering point of view, sedimentation problems which exist or may occur in response to man's activities in a region, and the methods of their solution by employing preventive measures can be identified by critical studies. In almost all circumstances the amount of sediment controls the relative seriousness of the problems. The higher the amount of sediment, the more serious the problems will be. The formulation of a catchment sediment model which can be used not only to estimate the sediment yield from a catchment but also to predict the effects of catchment treatment is, therefore, a valuable component in the development of sediment control systems. Such models can range from empirical formulae to sophisticated computer simulations. In this respect, the objective of this work can be briefly outlined as follows:

1. The development of a parametric catchment sediment model which can be used:
   a) to estimate the sediment yield from a natural catchment,and
   b) to predict the effects of catchment treatment and thereby expedite the development of sediment control systems
2. The development of a model that has rational parameter optimization techniques for calibration purposes.

HYPOTHESIS, PROCEDURE AND FORMULATION OF THE MODEL

To achieve the above objectives:

i) a general sediment transport function as an acceptable constitutive formula for the model is proposed.
ii) the processes of erosion which should be modelled as components of the model are defined.
iii) the structural framework of the model is introduced by incorporating the general sediment transport function proposed in (i) above, and the processes of erosion defined in (ii) above, and

iv) a parameter optimization algorithm to suit the model is formulated.

THE GENERAL SEDIMENT TRANSPORT FUNCTION

A vast number of sediment transport formulae has been developed by various authors since DuBoys[1] presented his tractive force theory, and reviews and detailed descriptions of various formulae have appeared in the literture.[4-7]

From a mathematical comparison of four well known and commonly used formulae, including:

Meyer- Peter and Muller formula [8]
Bagnold formula [9]
Yalin formula [10], and
Ackers and White formula [10]

the author (1982, 1989), inspired by the excellent work of Yalin [6], has shown that although various theories and approaches have been used in the development of these formulae, a general function for dimensionless sediment transport rate $\emptyset$ in terms of mobility number Υ can be reprented as:

$$\emptyset = \alpha (Y - Y_{cr})^{\gamma} \quad (1)$$

in which only the three dimensionless parameters α, Y_{cr} (threshold mobility), and γ are varied from one formula to another. For instance, in the Meyer-Peter and Muller formula $\alpha=8$, $\gamma=1.5$, and $Y_{cr} = 0.047$, and in the Ackers and White formula these parameters are functions of grain size D_{gr}.

Variations of parameters α, Y_{cr}, and γ are of great significance for model studies, since the range of values may be used in model calibration as the lower and upper limits of parameter values. These variations can also define the valid range of applicability of formulae in regard to flow and particle characteristics. For example, the Meyer-Peter and Muller formula with $Y_{cr} = 0.047$ is valid for large grain size, while the Ackers and White formula with $Y_{cr} = f(D_{gr})$ is reliable for a wider range of grain size (see Samani 1989 for others).

The unique characteristic of Equation 1 is its unlimited range of applicability. While all sediment transport formulae are applicable to a specific range of flow and sediment conditions, Equation 1 can be used for all flow and sediment regimes, by determining its optimum parameter values through optimization techniques (see the following sections).

Findings of Williams [12], Li et al.[13], Rendo-Herreno[14], Kumura[15], Sharma et al.[18] and Bhowmic[7] indicate the importance of surface runoff in influencing catchment sediment yield. These studies justify taking surface runoff as the main causative factor in production (detachment and supply) and transport of sediment.

By assuming that the mobility Y is directly porportional to the causative factor in transport of sediment, Equation 1 may therefore be written as:

$$q_s = \alpha (q_w - \beta)^{\gamma} \qquad q_w > \beta$$
$$q_s = 0 \qquad q_w \leq \beta \quad (2)$$

where q_s (L^3T^{-1}) and q_w (L^3T^{-1}) are sediment and runoff rates, respectively, β is some fraction of q_w indicating the threshold runoff for initial transport, and α & γ are system parameters. Parameter optimization techniques may be used to obtain values of parameters. α, β, and γ which, in turn, can be related to the catchment characteristics and flow-sediment conditions. Equation 2 may be used as a basis for more sophisticated computer sediment. systems in natural catchments.

THE SEDIMENT PRODUCTION MODEL

The development of the flow duration and sediment rating curves method [16, 17] was the first step to the estimation of the long term sediment yield of catchment. Since then various methods have been adopted in an attempt to estimate sediment yield from a catchment. These could be classified into four general approaches as follows:

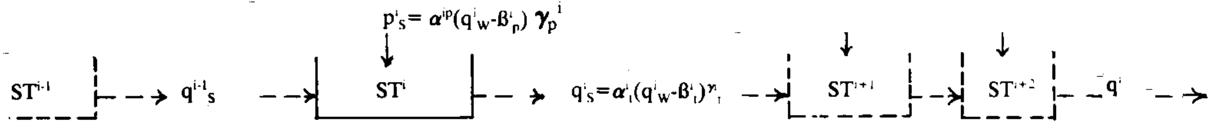


Figure 1. Illustrating multi-reservoir sediment model for one (ß = γ = 0), two (γ = 0) or three parameter processes. q_w^i is associated with sediment reservoir i. subscripts t and p denote transport and production respectively.

1) Statistical regression
2) Analytical
3) Unit sediment graph
4) Conceptual approach

Most exsiting sediment models that use the above approaches are black box or lumped and include transport, and are applied to the whole catchment. Our model is to be lumped but may be subdivided in three or four sub-catchment models each of which will have an independent production and transport model. Therefore, there is a requirement for a production model to be proposed. From the forgoing it apears that catchment runoff is a dominant driving force and, therefore, a model similar to that proposed for transport will be adopted for production, that is:

$$P_s = c\,(q_w - b)^n \qquad q_w > b$$
$$P_s = 0 \qquad q_w \leq b \qquad (3)$$

where q_w is runoff rate (L^3T^{-1}), P_s is sediment production rate (L^3T^{-1}), b, c, and n are parameters related to the catchment characteristics and flow and sediment conditions. Negev[18] and Fleming[19] used similar power functions to model the soil detachment (production) by runoff.

THE STRUCTURAL FRAMEWORK OF THE MODEL

The basic idea is to regard the catchment as being composed of several reservoirs in series. Each reservoir is a collection, storage and transmission system of water and sediment. In addition, each reservoir acts as a system for the production of sediment. It has been assumed that the main causative factor in generation and transportation processes is runoff rate. The erosion process is separated into subprocesses of production and transport each performing in accordance with a proposed physical rule (Equations 2 & 3). The two subprocesses of erosion are evaluated for each reservoir (Figure 1). The sediment available ST^i in any reservoir (i) is the material produced p_s^i by runoff q_w^i plus the materials carried q_s^i to it from the upper reservoir (i−1). This sum is compared with discharge capacity q_s of the reservoir. If the total produced sediment available for transport is less than the total discharge capacity, available sediment is the limiting factor in reservoir, and the sediment load carried to the next reservoir (i+1) or to the catchment outlet equals the amount of available material. However, if the total discharge capacity is greater than the sediment available for erosion, transportation is the limiting factor, and the surplus sediment load is stored in the reservoir. In brief, three equations form the basis of the model:

1)the trasport equation

$$q_s^{i,j} \quad (q_w^{i} \quad ß_t \quad \gamma_t^i \qquad q^{i} \quad ß_t^i$$
$$q_s^{i,j} = 0 \qquad q_w^{i,j} \leq ß_t^i \qquad (4)$$

which evaluates the sediment discharge capacity q_s (L^3T^{-1}) of reservofir (i) at the current time interval (j),

2) the production equation

$$p_s^{i,j} = (q_w \quad ß_p^{i})^{\gamma_p^i} \qquad q_w^{i,j} > ß_p^i$$
$$p_s^{i,j} = 0 \qquad q_w^{i,j} \leq ß_p^i \qquad (5)$$

which evaluates the sediment production

capacity P_s (L^3T^{-1}),
and
3) the continuity equation

$$ST^{i,j}=ST^{i,j-1}+p_s^{i,j}+q_s^{i-1,j}-q_s^{i,j} \quad (6)$$

which performs the reservoir storage balance.

Equation 6 finds the unknown reservoir storage $ST^{i,j}$ (at current time interval j) by summing the known initial reservoir storage $St^{i,j-1}$, the sediment yield $q^{i-1,j}$ transported from the upper reservoir (i−1), and the production yield $p_s^{i,j}$, then subtracting from it the sediment discharge $q_s^{i,j}$ released from storage (Figure 1 illustrates the above process schematically for a multi-reservoir system). Conceptually, each reservoir presents a sub-catchment which behaves as a homogeneous unit or as a particular source of sediment depending upon the physical characteristics of the land and flow condition. In other words, all properties of the catchment such as topography, soil erodibility, land-treatment, land use, channel fluvial characteristics etc., are reflected in a set of system parameters (the production and transport parameters).

ESSENTIAL STEPS IN OPTIMIZATION OF MODEL PARAMETERS

The calibration of model parameters through parameter optimization techniques encounters a number of problems and difficulties as the number of reservoirs increases. These problems include the presence of multi-minima, parameter redundancy, discontinuity of gradients, etc. To overcome these problems and to formulate the most complex model which can be optimized two steps are taken:

1) a synthesis procedure together with a series of numerical analyses are conducted, and
2) the following optimization algorithms* are employed and compared:

I. The Steeps descent algorithm
II. The Gill and Murray algorithm
III. The Pekham algorithm
IV. The Marquardt algorithm
V. The Nelder and Mead simplex algorithm

A SYNTHESIS PROCEDURE FOR MODEL FORMULATION

The so-called synthesis procedure is a progressive process which assembles the desired conceptual model by using simple models as parts. Simple models are those represented by a fewer number of reservoirs, processes and parameters (Figure 1, Equations 4 & 5). The procedure results in a large number of models, the difference among which lies in the complexity and conceptuality of their structures. The degree of complexity or number of dimensions of a model is expressed by:

$$NR \quad NP \quad NPP$$

where NR is the number of reservoir components
NP is the number of process per reservoir (transport and production, max. 2)
NPP is the number of parameters per process equation (max. 3 if parameters α, β and γ in eq. 2 are greater than zero).

Therefore, the degree of complexity can range from one dimension for a Single Reservoir-Single Process model (SR-SP), $\gamma=\beta=0$ in eq.2, to eighteen for a Triple Reservoir-Double Process model (3×2×3).

ERROR-FREE DATA AND OPTIMIZATION

Parameter optimization processes have been carried out using a set of artificialy generated data. This is to free the optimization studies from any error in real data. Such data has been generated by supplying the model under study (Reference model) with a given input and fixing its parameter values. The optimization process has been conducted for the model

* Standard routines from the NAG (Numerical Algorithm Group) library, University of Shiraz Computer Center.

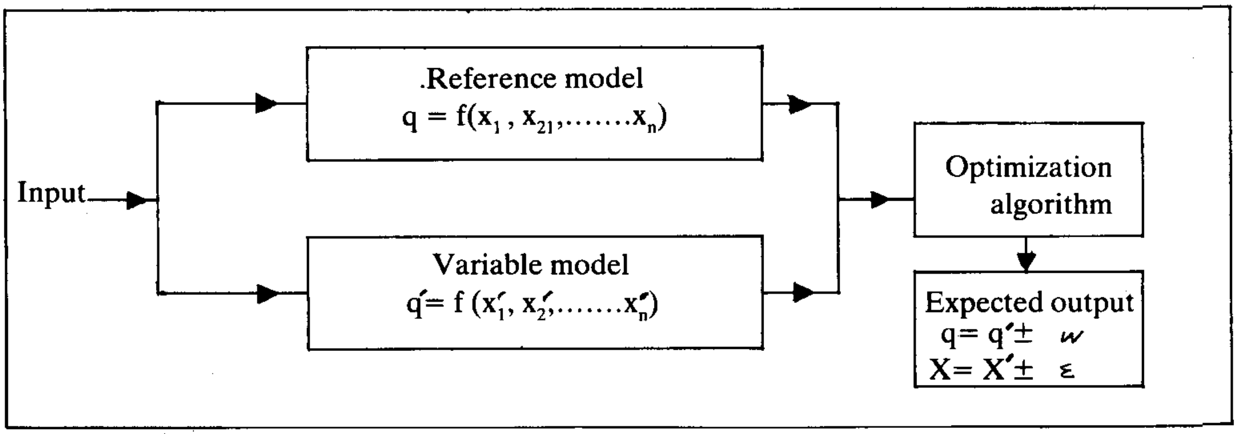


Figure 2. Generation of error-free data for model parameter optimization

(Variable model) by off-setting its parameter values away from those of the Reference model (see Figure 2). In fact the purpose of the Reference model is to define a target point for the optimization so that the deviation of end results of optimization from this target can be compared with the required tolerance.

To optimize the parameter of the Variable model (Figure 2), the objective function of form:

$$F=\sum^{m}\left(1-\frac{q_j'}{q_j}\right)^2 \qquad (7)$$

has been employed. Where q_j is the output from Reference model,

q_j' is the output from Variable model.

PARAMETER SENSITIVITY ANALYSIS

The term sensitivity is defined as a measure of the degree to which variation in a particular model parameter alters model output. To illustrate the parameter sensitivity analysis, the sensitivity curves have been plotted, (Figure 3). A sensitivity curve is a plot of percent change in value of a selected parameter against the sensitivity index SI:

$$SI=\sum_{j=}^{m}\left(1-\frac{q_j'}{q_j}\right)^2\times 100 \qquad (8)$$

where q_j' is the output from the variable model in which all parameters are kept unchanged except one.

Such curves bring to light the following:

a) The accuracy to which the individual parameter must be known in order to ensure successful model operation. That is, avoidance of parameters for which computation may fail or present difficulties.

b) The existance of insensitive parameters or a range of values for which a parameter acts redundantly.

c) The presence of multi-minima in the objective function.

d) The influence of parameter variation upon the model output.

These points have been fully demonstrated by Samani [20].

RESPONSE SURFACE ANALYSIS

A response surface is a three dimensional plot of sensitivity index with respect to two parameters of a model. The response surface is plotted in the same way as a sensitivity curve while perturbing two parameters of the model instead of one. The features of the model and its parameters detected by a sensitivity curve can also be detected by response surface. In addition, a response surface can detect the counter balancing effect of parameters caused by interdependence. One major cause for the

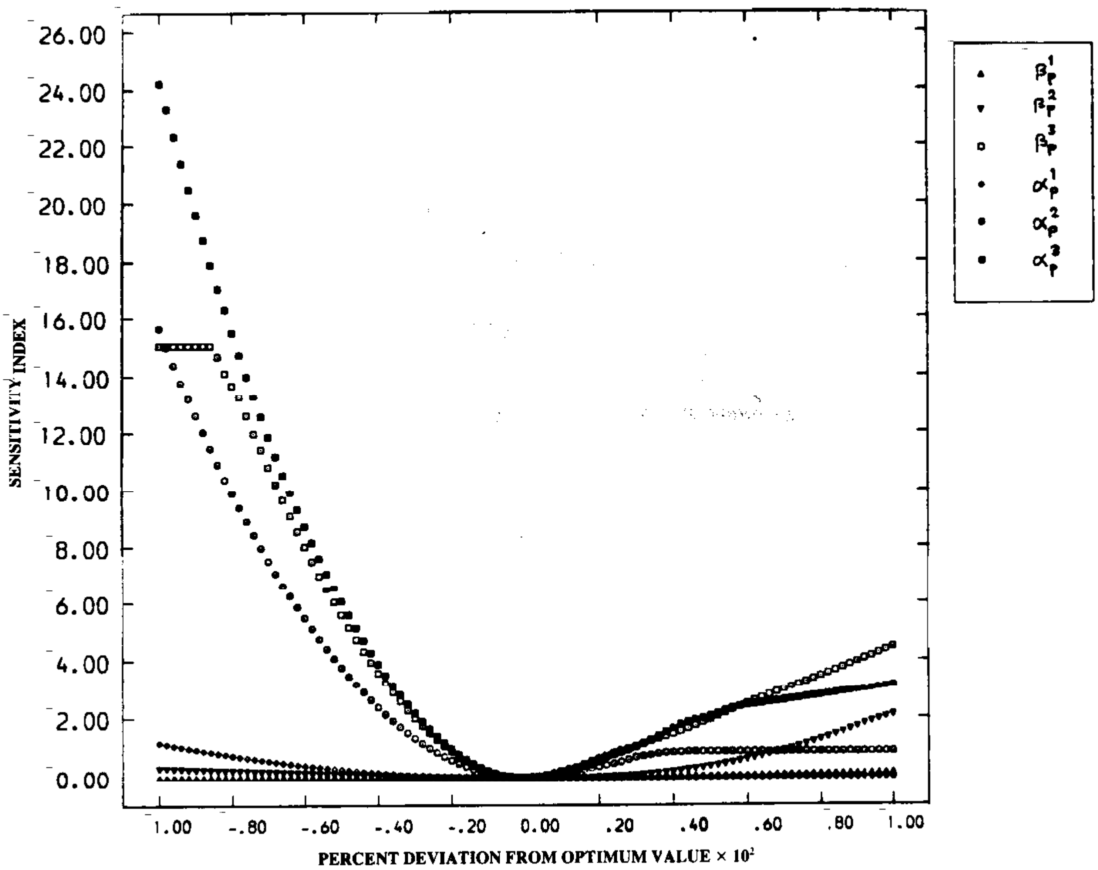


Figure 3. P.S. curves of TR-DP model with twelve parameters.

existance of two or more sets of roots for optimum solution is the counter balancing effect of parameters (Figure 4).

MODEL PARAMETER OPTIMIZATION

Prior to model parameter optimization the characteristics and structural features of each model formed by the synthesis procedure are investigated and identified by the conduction of the parameter sensitivity and response surface analyses. The analyses are essential in the development and calibration of the test models. Each test model is then optimized for several sets of starting parameter values using the above five optimization algorithms. Of these algorithms the steepest descent (Samani, [20] and Pekham[21]) algorithms were found more reliable. However, these two algorithms failed to optimize the objective function satisfactorily when a Quardruple Reservoir - Double Process (OR-DP) model with twenty four parameters was considered. Therefore, it seems that the TR-DP model with eighteen parameters is the most sophisticated model which can be handeled by the optimization algorithms that have been used. Figure 5 explains this failure. However, before accepting this fact some more assurance is needed. For example, because of an increase in the number of model parameters, the possibility that the number of input records may no longer be enough and may collapse into a subspace of less than n dimensions (n is number of model parameters) has to be investigated.

The number of runoff records has therefore been increased from the original number of 100, first to 200 and then to 500 and it was found that the problem still could not be sol-

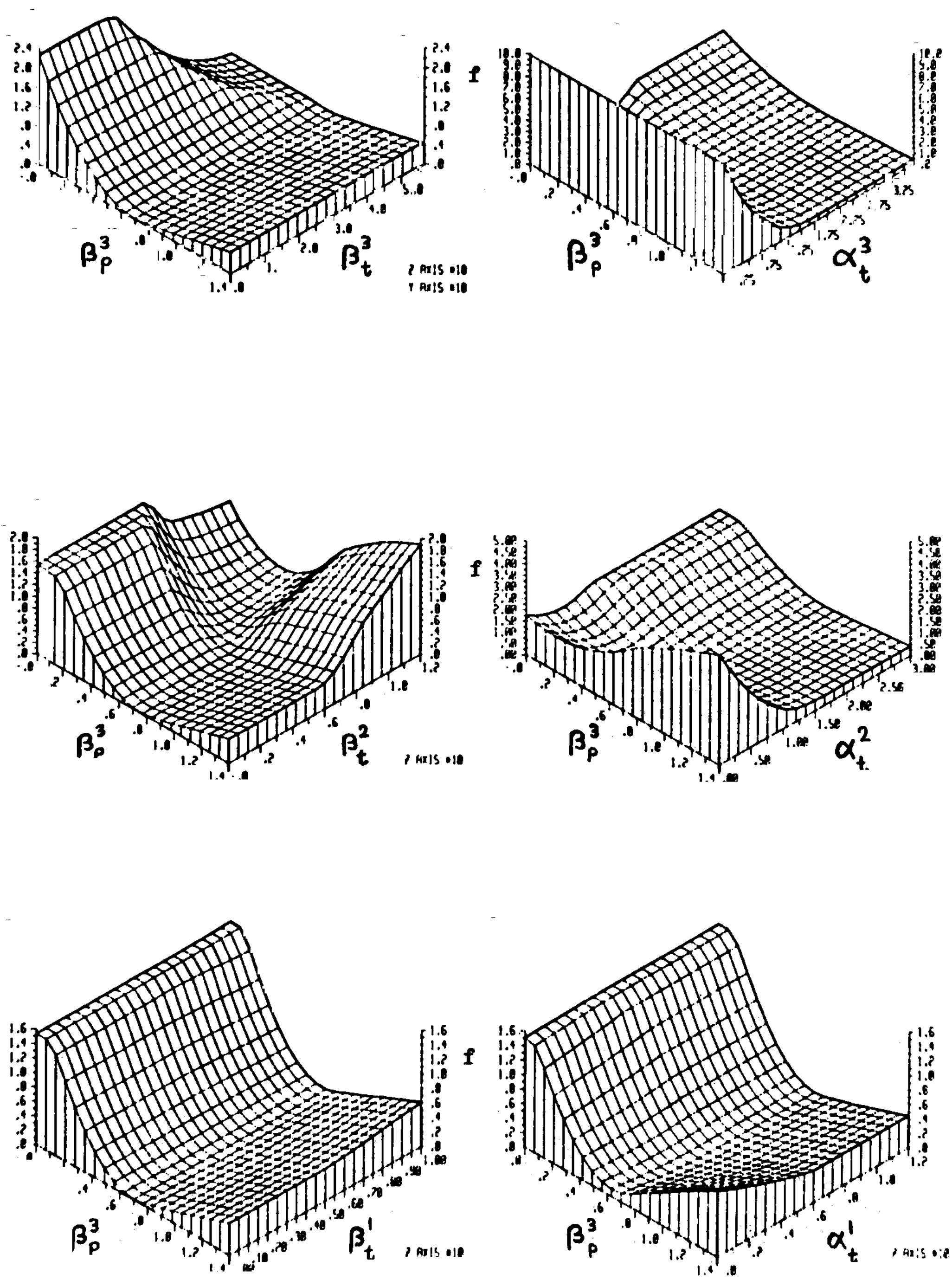
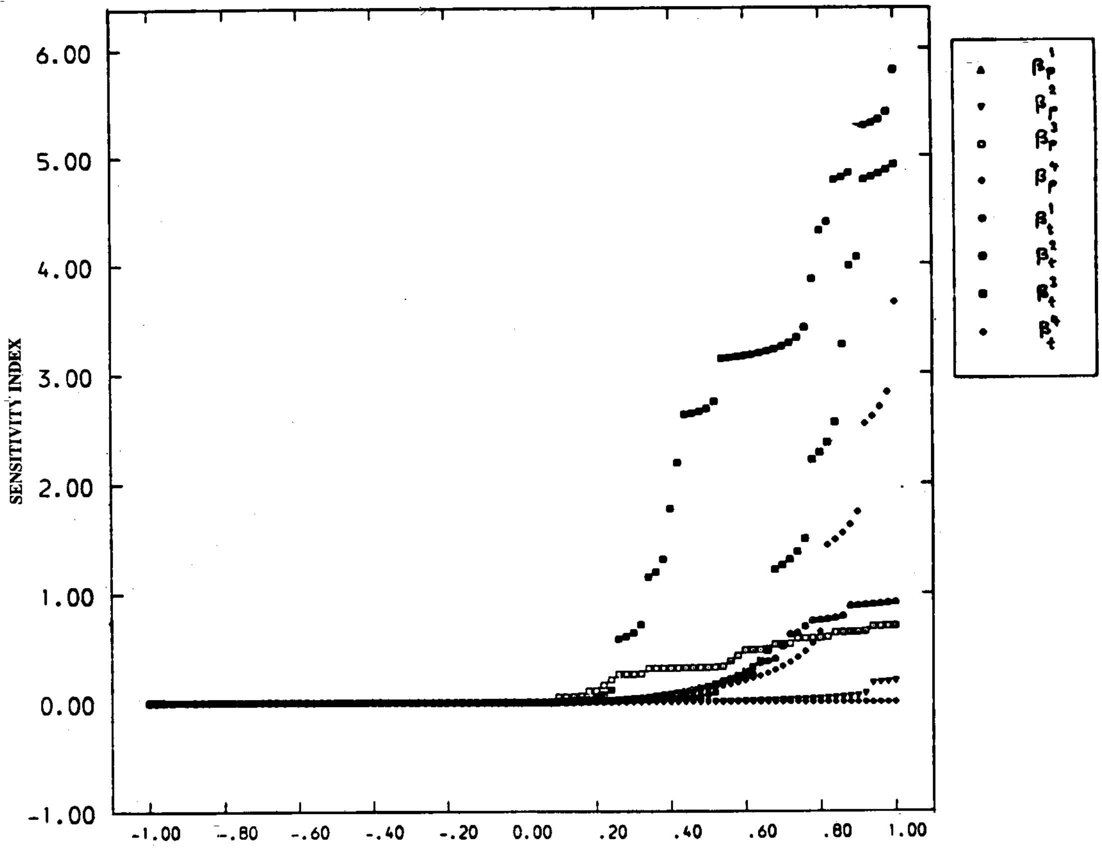


Figure 4. Response surfaces of TR-DP model with twelve parameters.



ved. On one or two occasions the values of the objective function were reduced further but compared to the extra cost and computer time, this was insignificant. Therefore, it was decided to accept that the TR-DP model with eighteen parameters was the limit for optimization.

For all test models the objective function is optimized to its minimum. However, depending on the starting parameter values, the optimum sets of parameter values are different, in other words, there exist several sets of roots for optimum solution. To overcome this the optimization algorithms have been complemented with a simple proposal which locates the optimum set of parameters from among all those sets which give a minimum value to the objective function by selecting the set which produces a minimum production and transport capacity. The value of this proposal is that it provides a rational basis for the parameter optimization of models of this sort[20].

VERIFICATION OF THE MODEL

The physical basis of the proposed model lends some confidence as to its suitability. However, no model can be fully accepted until it has been proved using several sets of real data. In the case of this model an area of concern is the complexity, i.e. the number of reservoir components that are required to form an adequate model. An attempt has been made to prove the adequacy of the model and its proposed optimization procedure but, in view of the lack of real data, this exercise has had to be limited to examining the sensitivity of the model to complexity by comparing the performance of a simple model with higher order models (Five and Ten Reservoir). For example, a set of reference sediment records was generated by a Five or Ten Reservoir model, and then the TR model was used to fit its response to that of Five or Ten Reservoir model.

From the above considerations it was found that a complex model which may be a more correct representation of a real system can be represented by the TR-DP model without losing too much accuracy. As a result a considerable advantage can be obtained in that parameter optimization becomes feasible.

The TR-DP model which was successfully fitted to the high order models is a rather complex model, simply because it involves eighteen parameters. In hydrological modelling there is no point in using more complex models if there is no great advantage offered in increased accuracy. To ascertain the simplest model which may be fitted equally to the high order models as the TR-DP model, the lower limit of model complexity has been examined by testing the accuracies of a number of low order models in representing Five and Ten Reservoir models. From this simulation it was found that among all models studied only the DR-DP model with twelve parameters shows an acceptable accuracy.

From the above points the following remarks can be made:

1) Although the model complexity is limited by the difficulties in optimizing model parameters, low order models, such as the Triple Reservoir - Double Process model with eighteen parameters, are as accurate as high order models in representing a natural catchment.
2) Lower order models which lend themselves to optimization do not represent the real system accurately.

EFFECTS OF RANDOM ERRORS IN DATA ON THE OPTIMUM PARAMETER VALUES

So far, this study has been based on error-free data. However, hydrological data inevitably contain errors. To study the effect of error-contaminated data on the optimum parameter values and on the process of optimization a number of models with different degrees of complexity was selected. Synthetic error-free sediment yield data were generated for each test model from a set of parameter values. Random errors were then introduced into all data records. By fitting the models to different combinations of error-free and corresponding error-contaminated records, the effects of errors on the overall response of models were studied.

Random errors (from a normal distribution whose mean and standard deviation are error-free) and 15% of error free values in data have no effect on the optimum parameter values and only increase the value of the objective function. Errors of higher magnitude change the parameter values insignificantly but have no effect on parameteric features (defined by the response surface plot) of the model. The proposed optimization method works successfully for data which involves random errors.

CONCLUSION

The catchment erosion model developed in this work can be used for the estimation of sediment yields from a catchment and could be applied to predict the effects of catchment treatment measures on those sediment yields which have an important role in the design of sediment control systems.

One important feature of the model is its proposed parameter optimization procedure. The value of this procedure is that it provides a rational basis for the calibration of the model.

Analyses and procedures used in the development and calibration of the model are useful and recommended for the formulation and calibration of hydrological models.

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