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A Robust Control Chart for Monitoring Autocorrelated Multiple Linear Profiles in Phase I

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A B S T R A C T

Many problems do not have one or more variables that determine quality characteristics. In these situations, as a solution method, a profile is descibed by linking independent variables to the response variable. One of the common assumptions in most monitoring schemes is the assumption of independent residuals. Contravention of this assumption can lead to misleading results of the control chart. On the other hand, when the data are contaminated, the classical methods of estimating the parameters do not perform well. Such situations require robust estimation methods. Hence, this paper proposes a robust method to estimate the process parameters for Phase I monitoring autocorrelated multiple linear profiles. The developed control chart is appraised in the absence and presence of contaminated data through comprehensive simulation studies. The results showed that the robust estimator decreases the impact of contaminated data on the performance of the proposed control chart for all outlier percentages and shift magnitudes. Generally, in all three scenarios, including outliers in the model parameters and error variance, the robust approach performs better than the comparative method.

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1. INTRODUCTION

Control charts are an essential tool for quality practitioners to improve industrial and service processes. For example, Sogandi and Vakilian [1] used control chart to estimate a step change in Gamma regression profiles. Sometimes, the quality characteristic of a product or process can be described by a relationship between response and predictor variable(s) typically known as a profile. Profiles can be categorized based on their functional forms into polynomial profiles, simple linear profiles, multiple linear profiles, generalized linear model profiles and so on. As the first review papers on profile monitoring, Woodall et al. [2] and Woodall [3] provided a comprehensive introduction and research gaps on profile monitoring. In this respect, Saghaei et al. [4] surveyed different types of profiles, and introduced the definition and applications of profile monitoring. In real applications, John and Vaibha [5] also demonstrated the application of the control chart for monitoring the quality characteristics exhibiting a nonlinear profile during time. Sogandi and Vakilian [1] and Khedmati et al. [6] surveyed AR(1) autocorrelated structure to estimate a change point in simple linear and polynomial profile, respectively. Niaki et al. [7] also provided a control chart based on the generalized linear test to monitor coefficients of the simple linear profiles. More recently, Abbasi et al. [8] presented a new monitoring scheme for non-parametric profiles using an adaptive Exponentially Weighted Moving Average (EWMA) control chart. This control chart, EWMA is developed under a type II censoring life test by mohammadipour et al. [9]. For the sake of brevity, other related research about profile monitoring is referred to Maleki et al. [10], in which an overview is performed on research published during the period 2008–2018.

In the aforementioned studies, the profile parameters are often estimated by methods, which perform appropriately without outliers. However, in many real cases, there may exist some contamination on the

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samples due to many reasons, such as the worker's fault. Applying the classical methods of parameter estimation in the presence of outliers would lead to inaccurate estimations and as a result, the erroneous performance of the monitoring scheme. To deal with these challenges, robust estimations are better properties than classical estimations. As one of the pioneering robust works, Khoo [11] suggested two time-weighted robust monitoring schemes for the process variance that the interquartile sample range for the control limits. In the profile monitoring field, Xuemin et al. [12] suggested a robust distribution-free approach to monitor linear profiles using rank-based regression to monitor nonparametric profiles. For simple linear profiles, Ebadi and Shahriari [13] used two robust methods, including the M-estimator and Huber estimator, in Phase I data with contamination. Similarly, Shahriari et al. [14] applied two methods for robust estimation of complex profiles using a nonparametric method for Phase I monitoring. After that, Shahriari and Ahmadi [15] proposed a robust estimation of complicated profiles. Also, Hakimi et al. [16] employed robust approaches using the M-estimator and the redescending M-estimator for Phase I monitoring of the logistic regression profile to reduce the impact of contaminated data. Furthermore, Ahmadi et al. [17] proposed a robust wavelet-based profile monitoring in Phase II in a two-stage process. For a simple linear profile, Hassanvand et al. [18] used two robust M-estimators for the parameter estimation to eliminate the detrimental impact of outliers in Phase I monitoring. After that, Kordestani et al. [19] suggested a monitoring scheme for monitoring multivariate simple linear profiles based on a robust estimation method. Moheghi et al. [20] considered robust estimation to monitor model parameters in GLM-based profiles with contaminated data. In recent years, Khedmati and Niaki [21] considered simple linear profiles based on robust parameter estimation in multistage processes in Phase-I. They proposed two robust methods, namely the MM-estimator and Huber's M-estimator with outliers in historical data. Despite of the many studies in profile monitoring, there are few works for robust profile monitoring with autocorrelation within profile data.

The critical assumption in many profile monitoring procedures is that the observations within or between profiles are independent. However, there are many cases in the real world where this assumption is violated. So far, some work has shown correlations within or between profiles. In Phase I monitoring, Jensen et al. [22] suggested a mixed model to describe the autocorrelation structure within each profile. Moreover, Jensen and Birch [23] used nonlinear hybrid models to extend a monitoring scheme for autocorrelated nonlinear profiles. Afterward, Soleimani et al. [24] suggested a transformation to remove the autocorrelation structure between observations within simple linear profiles. In a similar method, Soleimani and Noorossana [25] studied the impact of autocorrelation in Phase II monitoring in multivariate simple linear profiles. Another research in this scope is Narvand et al. [26] in which they extended a Phase II monitoring scheme for auto-correlated linear profiles. In this paper, they used Hotelling's T^2, multivariate cumulative sum, and multivariate EWMA control charts to monitor the process. In Phase II monitoring, Soleimani and Noorossana [27] developed a control chart for the multivariate simple linear profiles considering autocorrelation between observations for each profile. In the same type of autocorrelation, Yang et al. [28] suggested two Shewhart multivariate control charts to monitor a linear profile as well. To eliminate the effects of autocorrelation, Soleimani et al. [29] proposed three methods based on time series models for monitoring multivariate simple linear profiles with autocorrelation between profiles. Also, they demonstrated that the presence of outliers has a deleterious effect on the control chart performance.

Among a few works concentrating on robust methods for profile monitoring, only Kamranrad and Amiri [30] developed a robust control chart for auto-correlated simple linear profiles. Ahmadi et al. [31] suggested a control chart for Phase II monitoring of multiple linear profiles in which two robust estimate methods, the M-estimator, and fast-τ-estimator, were used. They showed their robust control chart based on M-estimator performs better than the fast-τ-estimator under high contamination data. To the best of the authors' knowledge, there is no more research on robust estimation for autocorrelated profiles monitoring. Hence, in this research, we considereed the robust monitoring of autocorrelated multiple linear profiles in Phase I. On this subject, the robust estimation approach will be appraised using the control signal probabilities. Besides, we survey the benefits of using the proposed approach against the classical estimation method with and without outliers. The structure of this paper is as follows: The second section provides the statistical model and corresponding assumptions of the considered process. Then, the classical and robust estimators were reviewed for the model parameters of autocorrelated multiple linear profiles. Section 3 proposed robust control chart for monitoring autocorrelated multiple linear profiles. Section 4 related to the performance evaluation by some simulation results to validate the proposed robust control chart. Finally, our concluding remarks and future studies provided in section 5.

2. STATISTICAL MODEL AND ASSUMPTION

In this section, we model the problem and describe the corresponding assumptions. Let *m* samples of observations be available, and *n* fixed values of the predictor variable in each sample. We define the autocorrelated multiple linear profile model for the j^{th} sample profile in which (x_i, y_{ij}) is the observation vector $(j=1,2,...,m)$. Assume that the process is in a state of statistical control, the autocorrelation within the profile can be modeled using Equation (1):

$$\begin{aligned} y_{ij} &= \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_p x_{pi} + \varepsilon_{ij}, \\ \varepsilon_{ij} &= \varphi \varepsilon_{(i-1)j} + a_{ij}, \end{aligned} \tag{1}$$

where y_{ij} is i^{th} observation in j^{th} sample profile $(i=1,2,...,n)$. Let x_{ip} the p^{th} value of the independent variable for i^{th} observation which is fixed from sample to sample. Also, $\beta_k (k=0,1,2,..., p)$ are the parameters of the regression model in the autocorrelated multiple linear profile. ε_{ij} 's are the autocorrelated error terms and a_{ij} 's are independent identically normal distributed with mean zero and variance σ^2. We assume that there is autocorrelation within a multiple linear profile and the autocorrelation structure is a first-order autoregressive (AR(1)) model. In the following subsection, we will show how to eliminate the autocorrelation structure between observations within multiple linear profile.

2. 1. Autocorrelation Elimination Method

The autocorrelation structure among error terms leads to autocorrelation between data in each profile. Hence, a transformation method should be used to remove the impact of autocorrelation. In this regard, each observation is transformed using Equation (2):

$$y'_{ij} = y_{ij} - \varphi y_{(i-1)j} \tag{2}$$

According to Equations (1) and (2) can be easily written for $(i\text{-}1)^{th}$ observation in the j^{th} profile.

$$y_{(i-1)j} = \beta_0 + \beta_1 x_{(i-1)1} + \beta_2 x_{(i-1)2} + ... + \beta_p x_{(i-1)p} + \varepsilon_{(i-1)j}, \tag{3}$$

By replacing Equations (1) and (3) into Equation (2), and simplification it, for each observation, we will obtain Equation (4):

$$\begin{aligned} y'_{ij} &= \beta_0 (1-\varphi) + \beta_1 (x_{1i} - \varphi x_{1(i-1)}) + ... \\ &+ \beta_p (x_{pi} - \varphi x_{p(i-1)}) + (\varepsilon_{ij} - \varphi \varepsilon_{(i-1)j}), \end{aligned} \tag{4}$$

leading to Equation (5):

$$y'_{ij} = \beta'_0 + \beta'_1 x'_{1i} + \beta'_1 x'_{2i} + ... + \beta'_p x'_{pi} + a_{ij}, \tag{5}$$

In which $\beta'_0 = \beta_0 (1-\varphi)$, and a_{ij} 's are independent random variables with mean zero and variance σ^2. Moreover, $\beta'_k = \beta_k$, $x'_{ki} = x_{ki} - \varphi x_{k(i-1)}$, for each explanatory variable $(k=1,2,..., p)$. As it is clear Equation (5) is a multiple linear profile with independent error terms. In the next section, the proposed methods of parameter estimation are given.

2. 2. Robust Estimation of Model Parameters

Usually, for uncontaminated cases, the ordinary least-square estimation (LSE) method is utilized to estimate the model parameters. For each sample, the least-square estimator for $\boldsymbol{\beta} = (\beta_0, \beta_1, ..., \beta_p)$ is achieved using minimizing the sum of squared errors $\left|(\mathbf{y}-\mathbf{x}\boldsymbol{\beta})^{\mathrm{T}}(\mathbf{y}-\mathbf{x}\boldsymbol{\beta})\right|$, and it is given by Equation (6).

$$\hat{\boldsymbol{\beta}} = \left(\mathbf{x}^{\mathrm{T}}\mathbf{x}\right)^{-1} \mathbf{x}^{\mathrm{T}}\mathbf{y}. \tag{6}$$

Equation (6) should be derived using all samples, even out-of-control profiles. This effect is known as the masking effect and leads to changing the value of statistics. The signicant outliers intensify this impact. To deal with this challenge, a robust estimation method should be used. If few contaminated data exist in a random sample, there are two methods to cope with this sample, including eliminating it and keeping it in which some information may be eliminated, or inaccurate estimates may be achieved. Hence, applying robust estimations is rational because they give unbiased estimations, under both contaminated data and outlier free. On this subject, researchers have applied robust regression with slighter sensitivity to outliers using appropriate weighting method. Many robust estimators have been suggested so far. Among these methods, M-estimator is the most introduced method introduced by Huber [32] because it has higher efficiency. On the other hand, Ahmadi et al. [32] showed the M-estimator is better than the fast-τ-estimator in high contamination for Phase II monitoring of multiple linear profiles. Hence, we estimate the parameters of autocorrelated multiple linear profile using the M-estimator, which are a generalization of maximum likelihood estimation.

Usually, s is the median absolute deviation, a robust estimator, defined as follows by Abu-Shawiesh [33] according to Equation [7]:

$$s = \frac{med\left|e_i - med\left(e_i\right)\right|}{0.6745}. \tag{7}$$

Considering s as a robust scale estimate, M-estimator could be calculated using minimizing a function $\rho(.)$ of regression residuals according to Equation (8):

$$\min \sum_{i=1}^{n} \rho\left(\frac{e_i}{s}\right), \tag{8}$$

in which ρ is a function of Huber or bisquare weight function. The bisquare function, as one of the main weighting functions is used here. The $\psi(x)$ is the derivative of $\rho(.)$ and the other functions, from a family of bisquare function given by Shahriari et al. [34]

according to Equations (9) and (10):

$$\rho(x)=\begin{cases}1-\left(1-\left(\frac{x}{k}\right)^2\right)^3 & |x|\le k\\ 1 & |x|>k\end{cases}, \quad (9)$$

$$w(x)=\begin{cases}\left(1-\left(\frac{x}{k}\right)^2\right)^2 & |x|\le k\\ 0 & |x|>k\end{cases}, \quad (10)$$

where k value should be chosen so that the resultant estimate would have a suitable asymptotic σ^2. Shahriari et al. [34] proved that these functions apply well with k=4.68.

2. 3. Proposed Robust Monitoring Scheme for Autocorrelated Multiple Linear Profiles

We use T_I^2 which is based on intra-profile pooling and sample average in Phase I monitoring. Consider $\hat{\boldsymbol{\beta}}_j$ can be shown by $(\hat{\beta}_{0j},\hat{\beta}_{1j},\hat{\beta}_{2j},...,\hat{\beta}_{pj})$ vector for each profile. Note that estimation of $\text{var}(\hat{\boldsymbol{\beta}}_j)$ is equal to Equation (11). For more details see Yeh et al. [35].

$$\hat{\text{var}}(\hat{\boldsymbol{\beta}}_j)=\left(\mathbf{X}^{\mathbf{T}}\mathbf{W}_{\mathbf{j}}\mathbf{X}\right)^{-1}. \quad (11)$$

Hence, an estimate of variance-covariance matrix could be obtained by taking the average of values of $\hat{\text{var}}(\hat{\boldsymbol{\beta}}_j)$ according to the $\mathbf{S}_I=\frac{1}{m}\sum_{j=1}^{m}\hat{\text{var}}(\hat{\boldsymbol{\beta}}_j)$. In a similar way, the estimation of average parameters is equal to the $\bar{\hat{\boldsymbol{\beta}}}=\frac{\sum_{i=1}^{m}\hat{\boldsymbol{\beta}}_j}{m}$ across all m samples. Therefore, T_I^2 control chart is obtained by Equation (12) to monitor the regression model parameters in autocorrelated multiple linear profiles.

$$T_{I,j}^2=\left(\hat{\boldsymbol{\beta}}_j-\bar{\hat{\boldsymbol{\beta}}}\right)^{\mathbf{T}}\mathbf{S}_I^{-1}\left(\hat{\boldsymbol{\beta}}_j-\bar{\hat{\boldsymbol{\beta}}}\right). \quad (12)$$

The proposed T_I^2 control chart trigger a statistical alarm when $T_{I,j}^2>UCL$ in which Upper Control Limit (UCL) is obtained by $UCL=(p+1)F_{p+1,m(n-p-1),\alpha}$. In this regard, Figure 1 depicts a general graphical scheme about the robust control chart for Phase I monitoring of autocorrelated multiple linear profiles.

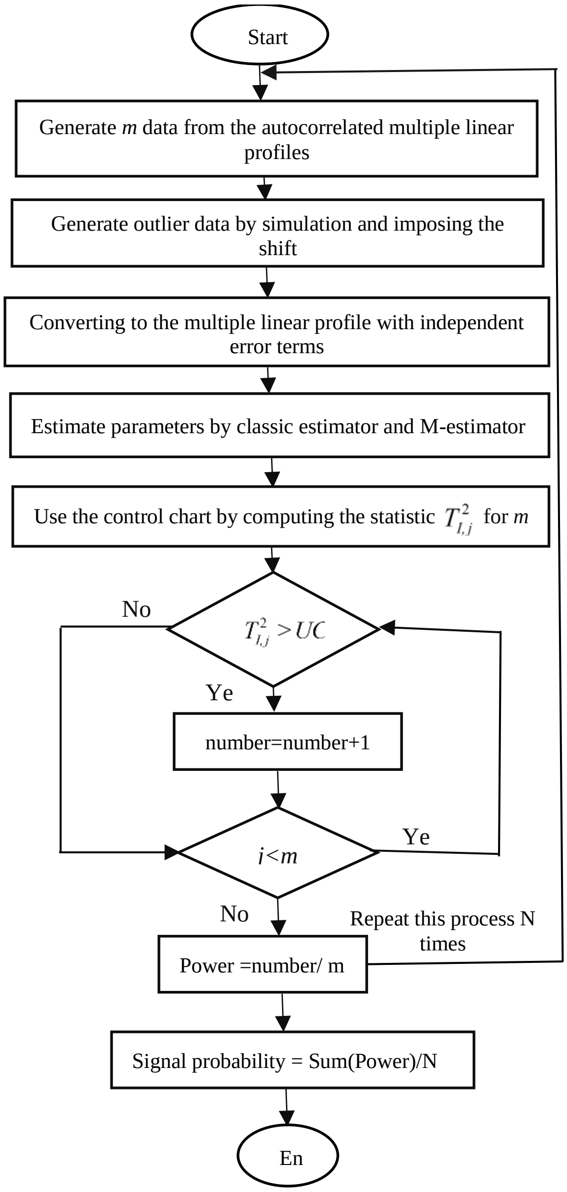

Figure 1. Flowchart of proposed robust monitoring scheme

3. SIMULATION STUDY AND PERFORMANCE EVALUATION

In this section, taking into account contaminated data, some simulation studies are provided to evaluate the performance of the proposed monitoring scheme in Phase I. The number of runs in Monte-Carlo simulation is 10000 in R software. On this subject, to apply classical and robust estimators, a simulation example of an autocorrelated multiple linear regression model is utilized to generate the data by Equation (13):

$$y_{ij}=3+1.2x_{1i}+1.3x_{2i}+\varepsilon_{ij},$$
$$\varepsilon_{ij}=0.8\varepsilon_{(i-1)j}+a_{ij}, \quad (13)$$

In which a_{ij} is the independent random variable, and follow a Normal distribution with mean 0 and $\sigma^2=1$. Let explanatory variables equal to $\mathbf{x}_0=(1,1,...,1)$, $\mathbf{x}_1=(0.2,0.4,0.6,...,4)$ and $\mathbf{x}_2=(0.1,0.3,0.5,...,2)$. Consider that 10 observations are generated for each level of explanatory variables. Hence, the total number of observations in each profile is 200. To appraise the estimation of the model parameters, 30 random samples are generated under different shifts and given contamination percentages. After that, a percentage of the simulated data is contaminated by shifting the model parameters of the autocorrelated multiple linear profile as $\beta_0 + \lambda,\ \beta_1 + \lambda, \beta_2 + \lambda$.

In this regard, different contamination percentages are considered using global outliers to evaluate the robust and classical estimates. Then, the mean and standard deviation of estimates in autocorrelated multiple linear profile are calculated under global outlying conditions. In the global contamination, a given percent of observations in all profiles should be replaced with contaminated data. For this aim, (*c*) percent of the data of each profile include outliers, and (100-*c*) percent of them are simulated by the pre-specified autocorrelated multiple linear profile. In other words, 10 levels were randomly selected from all the profiles, and in this regard, even levels are considered. According to the conducted simulation study, Table 1 shows the accuracy and standard deviation of both estimators in the presence of outliers in which λ ($\lambda = 0.3, 0.6, 0.9, 1.2, 1.5$) is the shift magnitude in the intercept.

Based on the simulation results provided in Table 1, in the absence of contamination, robust and classical estimators are almost identical. Also, it can be inferred that the proposed robust estimation method outperforms the classical estimation method in the presence of contamination. That is, the robust estimator gives more accurate estimates of β_0 compared to the estimator obtained by the LSE method regardless of outlier percentages and shift magnitudes. The conventional criterion used in Phase I monitoring for performance comparison of control charts is the probability of signal. Hence, we calculate signal probability of T^2 control chart after estimation of the regression coefficients. When there are no outliers in the process, the upper control limit of the T^2 control chart is set equal to 10.83 considering α=0.005. In this regard, Table 2 gives simulation results for different shifts with contamination in the intercept parameter.

TABLE 1. Performance evaluation of classical and robust estimations under contamination in β_0

Method		Classic		Robust	
c	**Shift (λ)**	**Parameter estimation**	**Standard deviation**	**Parameter estimation**	**Standard deviation**
5	**0**	2.911	1.142	2.999	0.940
	0.3	3.104	1.147	3.007	0.941
	0.6	3.155	1.175	3.025	0.943
	0.9	3.221	1.189	3.031	0.944
	1.2	3.284	1.200	3.080	0.955
	1.5	3.419	1.208	3.137	0.966
10	**0**	3.006	1.164	2.974	0.939
	0.3	3.247	1.170	3.006	0.940
	0.6	3.303	1.178	3.038	0.945
	0.9	3.382	1.183	3.127	0.952
	1.2	3.558	1.193	3.206	0.972
	1.5	3.576	1.231	3.267	0.988
15	**0**	3.040	1.142	2.962	0.936
	0.3	3.255	1.156	3.054	0.946
	0.6	3.370	1.163	3.172	0.954
	0.9	3.414	1.181	3.231	0.982
	1.2	3.777	1.191	3.279	0.984
	1.5	3.945	1.284	3.428	1.002
20	**0**	3.054	1.181	2.976	0.949
	0.3	3.243	1.182	3.031	0.956
	0.6	3.472	1.191	3.193	0.960
	0.9	3.738	1.201	3.330	0.976
	1.2	3.834	1.206	3.578	0.981
	1.5	4.125	1.246	3.761	0.986
25	**0**	3.077	1.281	3.072	0.959
	0.3	3.343	1.290	3.111	0.966
	0.6	3.482	1.131	3.273	0.970
	0.9	3.838	1.322	3.360	0.986
	1.2	3.935	1.336	3.777	0.991
	1.5	4.204	1.346	3.851	0.992

TABLE 2. Performance of T^2 control chart for shifts of various magnitudes in the presence of contamination in β_0

Method		Classic	Robust
c	**Shift (λ)**	**Signal probability**	**Signal probability**
5	**0**	0.005	0.005
	0.3	0.328	0.616
	0.6	0.461	0.679
	0.9	0.563	0.741
	1.2	0.667	0.771
	1.5	0.754	0.809
10	**0**	0.005	0.005
	0.3	0.444	0.733
	0.6	0.643	0.834
	0.9	0.820	0.916
	1.2	0.889	0.961
	1.5	0.960	0.987

15	**0**	0.005	0.005
	0.3	0.566	0.787
	0.6	0.802	0.925
	0.9	0.931	0.981
	1.2	0.982	0.977
	1.5	0.992	0.998
20	**0**	0.005	0.005
	0.3	0.629	0.856
	0.6	0.871	0.957
	0.9	0.969	0.996
	1.2	0.985	0.999
	1.5	0.999	0.999
25	**0**	0.005	0.005
	0.3	0.658	0.876
	0.6	0.882	0.966
	0.9	0.973	0.998
	1.2	0.986	0.999
	1.5	0.999	0.999

According to Table 2, when there is contamination in the intercept parameter, the robust control chart will show considerably better performance than classical control chart. Moreover, it shows that the presence of outliers in the clean observations causes to increase the signal probabilities. Also, whatever the magnitude of shifts increases, the signal probability values will be larger in both estimators. Similar to the previous tables, when there is contamination in β_1, Tables 3 and 4 summarize the estimators and signal probability of T^2 control chart, respectively.

TABLE 3. Performance evaluation of classical and robust estimations under contamination in β_1

Method		**Classic**		**Robust**	
c	**Shift (λ)**	**Parameter estimation**	**Standard deviation**	**Parameter estimation**	**Standard deviation**
5	**0**	1.055	1.124	1.154	0.923
	0.3	1.244	1.165	1.186	0.941
	0.6	1.487	1.171	1.234	0.950
	0.9	1.625	1.185	1.493	0.957
	1.2	1.989	1.158	1.711	0.929
	1.5	2.226	1.164	2.024	0.969
10	**0**	1.045	1.170	1.182	0.964
	0.3	1.300	1.208	1.259	0.967
	0.6	1.519	1.167	1.381	0.951
	0.9	1.736	1.180	1.464	0.952
	1.2	2.044	1.185	1.988	0.973
	1.5	2.299	1.197	2.074	0.988
15	**0**	1.002	1.148	1.173	0.948
	0.3	1.359	1.152	1.229	0.944
	0.6	1.617	1.159	1.267	0.957
	0.9	1.805	1.167	1.427	0.974
	1.2	2.010	1.160	1.551	0.979
	1.5	2.358	1.186	2.035	0.982
20	**0**	1.013	1.168	1.208	0.933
	0.3	1.367	1.204	1.276	0.935
	0.6	1.606	1.152	1.426	0.949
	0.9	1.853	1.156	1.848	0.950
	1.2	2.002	1.166	2.015	0.974
	1.5	2.393	1.226	2.054	0.991

TABLE 4. Performance of T^2 control chart for shifts of various magnitudes in the presence of contamination in β_1

Method		**Classic**	**Robust**
c	**Shift (λ)**	**Signal probability**	**Signal probability**
5	**0**	0.005	0.005
	0.3	0.262	0.569
	0.6	0.274	0.591
	0.9	0.292	0.638
	1.2	0.314	0.655
	1.5	0.349	0.752
10	**0**	0.005	0.005
	0.3	0.285	0.605
	0.6	0.382	0.668
	0.9	0.427	0.724
	1.2	0.508	0.764
	1.5	0.566	0.815
15	**0**	0.005	0.005
	0.3	0.336	0.634
	0.6	0.466	0.761
	0.9	0.577	0.837
	1.2	0.675	0.895
	1.5	0.799	0.943
20	**0**	0.005	0.005
	0.3	0.405	0.742
	0.6	0.593	0.833
	0.9	0.749	0.940
	1.2	0.837	0.966
	1.5	0.922	0.994

The obtained results from simulation runs show both classical and robust estimation methods are almost similar under the clean data. However, robust estimator decreases the effect of outliers on the mean of estimated parameters with outliers. In other words, robust estimator values are closer to the in-control β_1 than the classical estimator under different shifts and outlier observations. Moreover, comparing the standard deviation of them demonstrates that the robust estimation method is better than the least-square estimation method in the presence of outliers. Note that the classical estimation method of standard deviation performs roughly better than the robust estimation method without contamination. Afterward, outliers are generated with shift in β_2 and the corresponding mean and standard deviation values are given in Table 5. Also, simulation results of signal probability of the proposed control chart are given in Table 6.

TABLE 5. Performance evaluation of classical and robust estimations under contamination in β_2

Method		Classic		Robust	
c	**Shift (λ)**	**Parameter estimation**	**Standard deviation**	**Parameter estimation**	**Standard deviation**
5	**0**	1.024	1.147	1.301	0.936
	0.3	1.382	1.167	1.342	0.941
	0.6	1.482	1.170	1.404	0.953
	0.9	1.715	1.181	1.615	0.961
	1.2	2.001	1.185	1.935	0.979
	1.5	2.206	1.198	2.055	0.999
10	**0**	1.052	1.129	1.328	0.914
	0.3	1.452	1.114	1.378	0.933
	0.6	1.771	1.114	1.562	0.934
	0.9	1.839	1.117	1.781	0.937
	1.2	2.116	1.149	2.027	0.935
	1.5	2.220	1.157	2.129	0.936
15	**0**	1.041	1.140	1.306	0.916
	0.3	1.440	1.145	1.310	0.925
	0.6	1.592	1.147	1.415	0.932
	0.9	1.877	1.149	1.762	0.961
	1.2	1.994	1.163	1.896	0.968
	1.5	2.307	1.185	2.220	0.980
20	**0**	1.051	1.177	1.321	0.901
	0.3	1.176	1.197	1.354	0.913
	0.6	1.527	1.148	1.452	0.947
	0.9	1.740	1.176	1.671	0.955
	1.2	1.992	1.180	1.987	0.942
	1.5	2.136	1.197	2.096	0.960

TABLE 6. Performance of T^2 control chart for shifts of various magnitudes in the presence of contamination in β_2

Estimation method		Classic	Robust
c	**Shift (λ)**	**Signal probability**	**Signal probability**
5	**0**	0.005	0.005
	0.3	0.246	0.575
	0.6	0.252	0.576
	0.9	0.257	0.579
	1.2	0.273	0.586
	1.5	0.372	0.599
10	**0**	0.005	0.005
	0.3	0.286	0.608
	0.6	0.299	0.644
	0.9	0.342	0.666
	1.2	0.407	0.677
	1.5	0.443	0.743
15	**0**	0.005	0.005
	0.3	0.325	0.654
	0.6	0.428	0.731
	0.9	0.510	0.788
	1.2	0.593	0.828
	1.5	0.671	0.887
20	**0**	0.005	0.005
	0.3	0.371	0.686
	0.6	0.525	0.824
	0.9	0.623	0.906
	1.2	0.734	0.955
	1.5	0.844	0.988

Similarly, Table 5 demonstrates satisfactory performance for robust estimator under global outliers, as the proposed robust estimator reduces their impact. The robust estimator with no outliers has 0.936 standard deviation, which is lower than the classical estimator (1.147). A close match between the robust estimator and the corresponding actual value is shown in Table 5. Also, Table 6 shows that the developed T^2 chart by a robust estimator is a more efficient scheme than the T^2 chart based on the classical estimator in Phase I monitoring.

To take account into the impact of contminations on the variance of error terms, let (1-*c*) percent of ε_{ij}'s independently follow a Normal distribution as with N(0, σ^2). Besides, let *c* percent of the residuals generate an another Normal distribution. In other words, a model (say uncontaminated case) in which all observation are from N(0,1). A model for symmetric variance disturbances in which each observation has (1-*c*)% probability of being drawn from N(0,1) distribution and a *c*% probability of being drawn from N(0,9).The Mean Squared Error (MSE) criterion is applied to appraise the capability of error terms variance estimators. A smaller MSE value indicates a more accurate estimation of the parameter. Figure 2 shows the MSE of σ^2 estimations if there is contamination in the variance of the error terms.

Figure 2 illustrates when shift magnitudes and outlier percentages increase, robust approach performs better than classical approach. Furthermore, low contamination in variance of the ε_{ij}'s does not significantly affect classical estimation of parameters. While, by increasing the contaminated error terms variance, the classical estimator of ε_{ij}'s variance becomes significantly different from the actual value. Despite the satisfactory performance of the classical estimator for some low values of σ and *c*, with moderate and large contamination rates, it has worse performance than the robust estimation method. In these simulation studies, the maximum estimates for variance of the ε_{ij}'s based on the classical estimator was 11.32. However, this value is 1.58 for the robust estimator. Moreover, as shown in Figure 3, the robust scheme increases the contamination percent. Besides, we taken into account other simulations with different σ and *c* values. For the brevity, these simulation studies, not given here, support the results shown in these figures.

4. A REAL CASE

To show the practicality and effectiveness of the proposed robust control chart, we present a real case derived from the automotive industry given by Amiri et al. [36]. Specifically, when evaluating an automobile

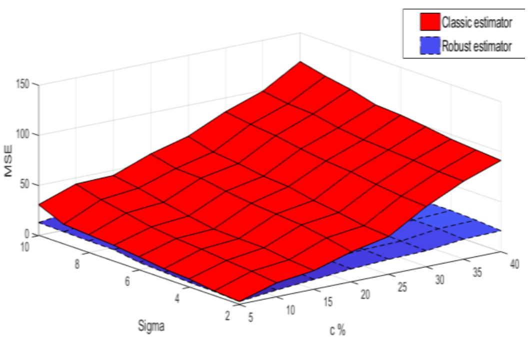

Figure 2. The comparison of MSE of variance estimations for *c*% contamination in error terms distribution and different shifts

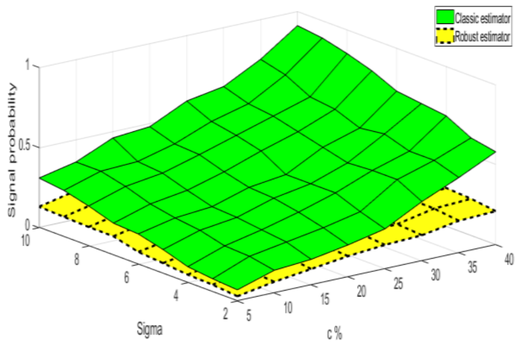

Figure 3. Performance of T^2 control chart for *c*% contamination in error terms distribution and different shifts

engine's performance, a crucial quality characteristic is how torque production relates to engine speed stated in revolutions per minute. We have 26 engines available for the initial phase of analysis pertaining to the engine data. Within each engine, we establish a set of speed values, including 1500, 2000, 2500, 2660, 2800, 2940, 3500, 4000, 4500, 5000, 5225, 5500, 5775, and 6000 RPM, and collect corresponding torque measurements. Consequently, we obtain a profile of interest consisting of 14 data points per engine. They showed for this data set that a quadratic polynomial works well according to Equation (14). In is worth mentioning that polynomial profiles is a special case of multiple linear profiles.

$$y_j = \beta_0 + \beta_1 x_j + \beta_2 x_j^2 + \varepsilon_j, \tag{14}$$

In which y_j denotes torque values, and x_j show RPM values in which $x_j = x$. The model parameters are estimated for each profile by high values of the adjusted coefficient of determination. Figure 4 depicts a scatterplot showcasing the data for one specific engine, identified as Engine number 1791. The figure serves as an example to demonstrate that the speed values have been adjusted for mean correction in order to mitigate the impact of multicollinearity. The variance inflation factors showed that there is no multi-collinearity between explanatory variables. They used a run chart to check independence of residuals over time assumption and showed that the clustering and trend hypothesis tests are significant and as a result it can be concluded that the residuals are correlated. In other words, the process of evaluating model adequacy revealed that we are dealing with a situation where there is a correlation between the residuals and therefore between the observations in each profile. Hence, the data set can be modeled by autocorrelated multiple linear profiles.

Amiri et al. [36] showed an AR(1) error structure using Draftman's display for the data set. Besides, the estimate of the mean vector and the covariance matrix under the classical and robust methods are reported in Table 7.

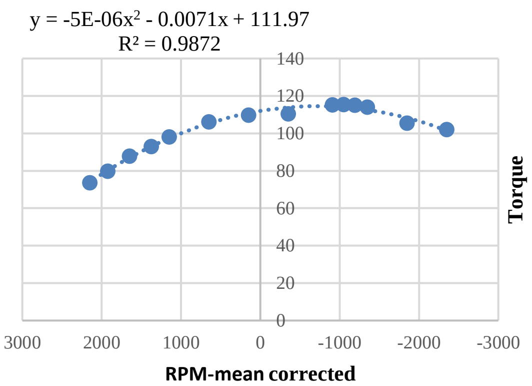

Figure 4. A second-order multiple linear profile fit for engine number 1791

TABLE 7. The parameter estimates of the real case obtained by the classical and the robust estimator

Estimates		Sample mean		Sample standard deviation	
Outlier percentages		Classic	Robust	Classic	Robust
0	β_0	111.2589	111.2586	1.5299	1.529876
	β_1	−0.005985	−0.005775	0.000496	0.000489
	β_2	−0.0000049	−0.0000051	0.0000031	0.00000309
20	β_0	114.577	111.4313	1.69988	1.52969
	β_1	−0.007798	−0.005784	0.005096	0.000477
	β_2	−0.0000041	−0.00000508	0.00004	0.0000032

The classical and robust estimates indicate that the process mean as well as the process standard deviation are influenced by outliers. To examine the stability of the process and identify any unusual profiles among the 26, we can establish UCL equal to 0.7089 at 95% confidence level for Phase II of the control chart. Then, we can employ $T^2_{I,j}$ based on classic and robust approach. Hence, we use the T^2 control chart to monitor the process mean. The control chart statistics are also determined for the sample data points using Equation (12) with the classic and robust estimators and plotted Figure 5. From Figure 5, it is clear that robust control charts gave a quick out of control signal by last two data points whereas classical estimator-based control chart did not signal any alarm.

5. CONCLUSION AND FURTHER STUDIES

An assumption commonly used in profile monitoring schemes is that residuals will be independent. The control chart can be misled if this assumption is violated. From another standpoint, when the data are contaminated, the

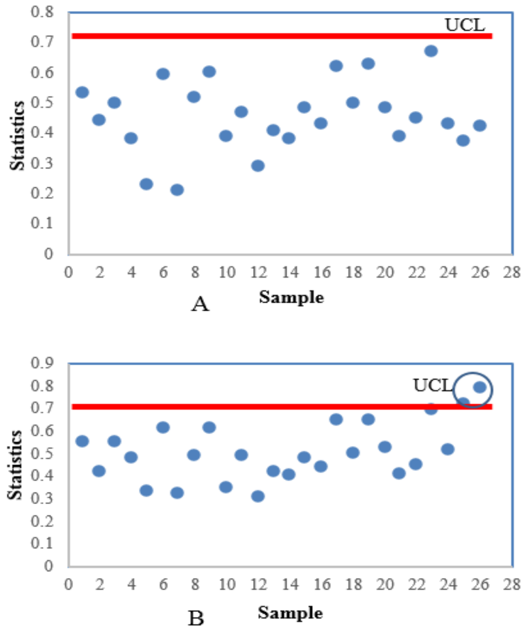

Figure 5. Plots of sample data for T^2 control chart with classical (A), and robust (B), estimators

classical estimation methods do not perform well. Hence,this paper proposed a robust approach for Phase I monitoring of autocorrelated multiple linear profiles.

The proposed control chart was appraised in the absence and presence of contaminated data through extensive simulation studies. The simulation results showed that without outliers, the classical and robust method performed partly the same in parameter estimation of the model. Considering these results, when the outlier magnitude increased, the estimations achieved by the classical estimator deviated considerably from their actual values. While, the estimates computed based on the robust estimator were close to the reference values. We showed the LSE method was affected by the outlier data, however, the M-estimator decreased their effect. Generally, in all scenarios including contaminations in the model coefficients and error variance, the robust approach performed better than the classical method. Besides, the performance of the T^2 control chart with the classical and the robust estimates was appraised by extensive comparison under different shift magnitudes with and without outliers. When the regression parameters were estimated using the robust method, the capability of the proposed T^2 control chart enhanced under different shifts in the parameters of regression model.

Considering the proposed robust approaches for multistage processes can be a fruitful subject for future research. Also, investigating the performance of the proposed robust estimator under autocorrelation between profiles and contamination data can be considered as a further research.

6. REFERENCES

1. Sogandi, F. and Vakilian, F., "Isotonic change point estimation in the ar (1) autocorrelated simple linear profiles", ***International Journal of Engineering, Transactions A: Basics***, Vol. 28, No. 7, (2015), 1059-1067. doi: 10.5829/idosi.ije.2015.28.07a.12.
2. Woodall, W.H., Spitzner, D.J., Montgomery, D.C. and Gupta, S., "Using control charts to monitor process and product quality profiles", ***Journal of Quality Technology***, Vol. 36, No. 3, (2004), 309-320. doi: 10.1080/00224065.2004.11980276.
3. Woodall, W.H., "Current research on profile monitoring", ***Production***, Vol. 17, (2007), 420-425. doi: 10.1080/07408170600998769.
4. Saghaei, A., Noorossana, R. and Amiri, A., "Statistical analysis of profile monitoring, Wiley Online Library, (2013).
5. John, B. and Agarwal, V., "A regression spline control chart for monitoring characteristics exhibiting nonlinear profile over time", ***The TQM Journal***, Vol. 31, No. 3, (2019), 507-522. doi: 10.1108/TQM-11-2018-0183.
6. Khedmati, M., Soleymanian, M.E., Keramatpour, M. and Niaki, S., "Monitoring and change point estimation of ar (1) autocorrelated polynomial profiles", ***International Journal of Engineering, Transactions A: Basics***, Vol. 26, No. 9, (2013), 933-942. doi: 10.5829/idosi.ije.2013.26.09a.11.
7. Niaki S.T.A., Abbasi, B. and Arkat, J., "A generalized linear statistical model approach to monitor profiles", ***International Journal of Engineering, Transactions A: Basics,***, Vol. 20, No. 3, (2007). doi: 10.5829/idosi.ije.2007.20.03.006.
8. Abbasi, S.A., Yeganeh, A. and Shongwe, S.C., "Monitoring non-parametric profiles using adaptive ewma control chart", ***Scientific Reports***, Vol. 12, No. 1, (2022), 14336. doi: 10.1038/s41598-022-39011-0.
9. Mohammadipour, P., Farughi, H., Rasay, H. and Arkat, J., "Designing exponentially weighted moving average control charts under failure censoring reliability tests", ***International Journal of Engineering,Transactions B: Applications***, Vol. 34, No. 11, (2021), 2398-2407. doi: 10.5829/ije.2021.34.11b.03.
10. Maleki, M.R., Amiri, A. and Castagliola, P., "An overview on recent profile monitoring papers (2008–2018) based on conceptual classification scheme", ***Computers & Industrial Engineering***, Vol. 126, (2018), 705-728. doi: 10.1016/j.cie.2018.08.030.
11. Khoo, M.B., "Robust time weighted control charts for the process variance", ***International Journal of Reliability, Quality and Safety Engineering***, Vol. 12, No. 05, (2005), 439-458. doi: 10.1142/S0218539305001977.
12. Zi, X., Zou, C. and Tsung, F., "A distribution-free robust method for monitoring linear profiles using rank-based regression", ***IIE Transactions***, Vol. 44, No. 11, (2012), 949-963. doi: 10.1080/0740817X.2011.605559.
13. Ebadi, M. and Shahriari, H., "Robust estimation of parameters in simple linear profiles using m-estimators", ***Communications in Statistics-Theory and Methods***, Vol. 43, No. 20, (2014), 4308-4323. https://doi.org/10.1080/03610926.2012.721914
14. Shahriari, H., Ahmadi, O. and Samimi, Y., "Estimation of complicated profiles in phase i, clustering and s-estimation approaches", ***Quality and Reliability Engineering International***, Vol. 32, No. 7, (2016), 2455-2469. doi: 10.1002/qre.2049.
15. Shahriari, H. and Ahmadi, O., "Robust estimation of complicated profiles using wavelets", ***Communications in Statistics-Theory and Methods***, Vol. 46, No. 4, (2017), 1573-1593. doi: 10.1080/03610926.2015.1114893.
16. Hakimi, A., Amiri, A. and Kamranrad, R., "Robust approaches for monitoring logistic regression profiles under outliers", ***International Journal of Quality & Reliability Management***, (2017). doi: 10.1108/IJQRM-02-2015-0024.
17. Ahmadi, O., Shahriari, H. and Samimi, Y., "A robust wavelet based profile monitoring and change point detection using s-estimator and clustering", ***Journal of Industrial and Systems Engineering***, Vol. 11, No. 3, (2018), 167-189. doi: 10.1080/24725854.2018.1565149.
18. Hassanvand, F., Samimi, Y. and Shahriari, H., "A robust control chart for simple linear profiles in two-stage processes", ***Quality and Reliability Engineering International***, Vol. 35, No. 8, (2019), 2749-2773. doi: 10.1002/qre.2543.
19. Kordestani, M., Hassanvand, F., Samimi, Y. and Shahriari, H., "Monitoring multivariate simple linear profiles using robust estimators", ***Communications in Statistics-Theory and Methods***, Vol. 49, No. 12, (2020), 2964-2989. doi: 10.1080/03610926.2018.1568453.
20. Moheghi, H.R., Noorossana, R. and Ahmadi, O., "Glm profile monitoring using robust estimators", ***Quality and Reliability Engineering International***, Vol. 37, No. 2, (2021), 664-680. doi: 10.1002/qre.2689.
21. Khedmati, M. and Niaki, S.T.A., "Phase-i robust parameter estimation of simple linear profiles in multistage processes", ***Communications in Statistics-Simulation and Computation***, Vol. 51, No. 2, (2022), 460-485. https://doi.org/10.1080/03610918.2019.1653916
22. Jensen, W.A., Birch, J.B. and Woodall, W.H., "Monitoring correlation within linear profiles using mixed models", ***Journal of Quality Technology***, Vol. 40, No. 2, (2008), 167-183. doi: 10.1080/00224065.2008.11918540.
23. Jensen, W.A. and Birch, J.B., "Profile monitoring via nonlinear mixed models", ***Journal of Quality Technology***, Vol. 41, No. 1, (2009), 18-34. doi: 10.1080/00224065.2009.11917898.
24. Soleimani, P., Noorossana, R. and Amiri, A., "Simple linear profiles monitoring in the presence of within profile autocorrelation", ***Computers & Industrial Engineering***, Vol. 57, No. 3, (2009), 1015-1021. doi: 10.1016/j.cie.2009.06.019.
25. Soleimani, P. and Noorossana, R., "Investigating effect of autocorrelation on monitoring multivariate linear profiles", ***International Journal of Industrial Engineering & Production Research,*** , Vol. 23, No. 3, (2012), 187-193. doi: 10.22059/ijiepr.2012.30681.
26. Narvand, A., Soleimani, P. and Raissi, S., "Phase ii monitoring of auto-correlated linear profiles using linear mixed model", ***Journal of Industrial Engineering International***, Vol. 9, (2013), 1-9. doi: 10.1186/2251-712X-9-22.
27. Soleimani, P. and Noorossana, R., "Monitoring multivariate simple linear profiles in the presence of between profile autocorrelation", ***Communications in Statistics-Theory and Methods***, Vol. 43, No. 3, (2014), 530-546. https://doi.org/10.1080/03610926.2012.665554
28. Zhang, Y., He, Z., Zhang, C. and Woodall, W.H., "Control charts for monitoring linear profiles with within-profile correlation using gaussian process models", ***Quality and Reliability Engineering International***, Vol. 30, No. 4, (2014), 487-501. doi: 10.1002/qre.1589.
29. Soleimani, P., Noorossana, R. and Niaki, S., "Monitoring autocorrelated multivariate simple linear profiles", ***The International Journal of Advanced Manufacturing Technology***, Vol. 67, No. 5-8, (2013), 1857-1865. doi: 10.1007/s00170-012-4648-4.
30. Kamranrad, R. and Amiri, A., "Robust holt-winter based control chart for monitoring autocorrelated simple linear profiles with contaminated data", ***Scientia Iranica***, Vol. 23, No. 3, (2016), 1345-1354. doi: 10.24200/sci.2016.2142.

31. Ahmadi, M.M., Shahriari, H. and Samimi, Y., "A novel robust control chart for monitoring multiple linear profiles in phase ii", ***Communications in Statistics-Simulation and Computation***, Vol. 51, No. 11, (2022), 6257-6268. https://doi.org/10.1080/03610918.2020.1799228

32. Huber, P.J., "Robust regression: Asymptotics, conjectures and monte carlo", ***The Annals of Statistics***, (1973), 799-821. doi: 10.1214/aos/1176342503.

33. Abu-Shawiesh, M.O., "A simple robust control chart based on mad", ***Journal of Mathematics and Statistics***, Vol. 4, No. 2, (2008), 102. doi: 10.3844/jmssp.2008.102.107.

34. Shahriari, H., Ahmadi, O. and Shokouhi, A.H., "A two-step robust estimation of the process mean using m-estimator", ***Journal of Applied Statistics***, Vol. 38, No. 6, (2011), 1289-1301. https://doi.org/10.1080/02664763.2010.498502

35. Yeh, A.B., Huwang, L. and Li, Y.-M., "Profile monitoring for a binary response", ***IIE Transactions***, Vol. 41, No. 11, (2009), 931-941. https://doi.org/10.1080/07408170902735400

36. Amiri, A., Jensen, W.A. and Kazemzadeh, R.B., "A case study on monitoring polynomial profiles in the automotive industry", ***Quality and Reliability Engineering International***, Vol. 26, No. 5, (2010), 509-520. doi: 10.1002/qre.1127.

Persian Abstract

چکیده

بسیاری از مسائل صرفاً دارای یک یا چند متغیر نیستند که بتوانند مشخصه های کیفیت را تعیین کنند. در این مواقع، به عنوان یک راه حل، یک پروفایل با پیوند دادن متغیرهای مستقل به متغیر پاسخ معرفی می‌شود. یکی از مفروضات رایج در اکثر رویه های کنترلی، فرض مستقل بودن باقیمانده‌ها است. نقض این فرض می تواند منجر به نتایج گمراه کننده در نمودار کنترل شود. از سوی دیگر، زمانی که داده ها آلوده هستند، روش های کلاسیک تخمین پارامترهای عملکرد خوبی ندارند. چنین شرایطی نیازمند روش های برآورد قوی است. از این رو، این مقاله یک روش قوی برای تخمین پارامترهای فرآیند برای نظارت بر نمپ روفایل‌های خطی چندگانه خود همبسته برای فاز I پیشنهاد می‌کند. نمودار کنترل توسعه‌یافته در غیاب و حضور داده‌های آلوده از طریق مطالعات شبیه‌سازی جامع ارزیابی می‌شود. نتایج شبیه سازی‌های گسترده، نشان داد که برآوردگر قوی تأثیر داده‌های آلوده را بر عملکرد نمودار کنترل پیشنهادی برای همه درصدهای دورافتاده و مقادیر مختلف تغییرات کاهش می‌دهد. به طور کلی، در هر سه سناریو، از جمله نقاط پرت در پارامترهای مدل و واریانس خطا، رویکرد قوی بهتر از روش کلاسیک عمل می کند.