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RESEARCH NOTE

A Dynamic Model for Laminated Piezoelectric Microbeam

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ABSTRACT

Piezoelectric beams are widely used in micro-electromechanical systems. At the microscale, the influence of the size effect on a piezoelectric beam cannot be ignored. In this paper, higher-order elasticity theories are considered to predict the behaviors of piezoelectric micro-structures and a size-dependent dynamic model of a laminated piezoelectric microbeam is established. The governing equations for the laminated piezoelectric microbeam are derived using the variational principle. The natural frequencies of piezoelectric microbeams are obtained by size-dependent dynamic models. The results reveal that the size effect can enhance the structural stiffness at the microscale. The natural frequency obtained by using the classical model is smaller than that obtained using the size-dependent model. Compared with the modified couple stress model, the modified couple stress model underestimates the size-dependent response. Thus, the modified couple stress model is a simplification of the modified strain gradient model. The influence of beam thickness on the natural frequency is also discussed. With increasing the thickness, the natural frequency of the size-dependent models gradually approaches the result of the classical model. If the value of ***h***/***l*** is greater than 15, the influence of the size effect can be neglected. Additionally, the relative thickness can influence the natural frequency, and if the relative thickness is greater than 5 or less than −5, the bilayer beam can be simplified to a single-layer beam.

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NOMENCLATURE

L	Beam length (m)	D_i	Vector of electrical displacement
h	Thickness(mm)	$\boldsymbol{e_{31}}$	Piezoelectric coefficient N/ (V m)
M	Bending moment (N·m)	l_n	Material length-scale parameters
b	Width (m)	**Greek Symbols**	
w	Deflection(m)	$\boldsymbol{\rho}$	Density (kg/m^3)
$q(x)$	Uniformly distributed load(N/mm)	ω	Natural frequency (Hz)
F	Shear force (N)	$\boldsymbol{\kappa}$	Dielectric constant
G	Material elastic modulus (GPa)	$\boldsymbol{\sigma}$	Stress tensor (Pa)
$A=bh$	Cross-sectional area of the beam(m^2)	ε	Strain tensor
$S=bh^2/2$	Static moment(m^3)	$\boldsymbol{\gamma}$	Dilatation gradient tensor
$I=bh^3/3$	Second moment of cross-sectional area(m^4)	χ	Rotation gradient tensor
$w(x,t)$	Amplitude (m)	η	Strain gradient tensor
E_i	Electric field	τ	Higher stress
$\boldsymbol{t}$	Relative thickness	λ	Lame constants
$\boldsymbol{H}$	Electric enthalpy density(J)	μ	Lame constants
$\boldsymbol{U}$	Strain energy (J)	ε_0	The permittivity of vacuum
$\boldsymbol{P}$	The polarization vector (C/m^2)	φ	Electric potential(V)
$\boldsymbol{W}$	Work done by the external forces(J)	$\boldsymbol{\theta}$	Rotation vectors
$\boldsymbol{T}$	Kinetic energy(J)	**Subscripts**	
u_i	Displacement vector	1	Piezoelectric layer
d	Distance from the neutral layer to the bottom layer(mm)	2	Elastic layer

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1. INTRODUCTION

Several piezoelectric structures are used in MEMS, such as micro-actuators [1] and micro-sensors [2]. Recently, people have been concerned about the energy crisis [3]. Thus, micro-energy harvesters [4], which are related to energy recovery, have attracted great research attention. The performance of the MEMS depends on the electromechanical control system [5] and the piezoelectric component's mechanical properties [6]. However, on the microscale, the performance of piezoelectric structures is size-dependent. Multani et al. [7] experimentally found that the piezoelectric effect of piezoelectric structures is proportional to the thickness. Bühlmann et al. [8] observed that the bending deflection of the PZT thin film decreases with decreasing thin film thickness at the microscale. Additionally, Shaw et al. [9] reported that the responses of a piezoelectric beam decrease when the thickness approaches to nanometer.

According to experiment reports, the size effect can influence the mechanical properties of piezoelectric microstructures. However, the classical piezoelectric theory [10] neglects the size effect phenomenon. Thus, the theoretical results obtained by this theory will have deviation from the experimental results. Because of the economic downturn caused by the COVID-19 pandemic [11], scholars want to establish more accurate theoretical calculation models to reduce the cost of trial and error. At micro scale, the constitutive relation of a microbeam model not only contains the classical material parameters, but also contains the material scale parameters $\boldsymbol{l_n}$ [1, 2]. However, the simple beam model doesn't consider the material scale parameters $\boldsymbol{l_n}$, it ignores the influence of size effect. Therefore, higher-order elasticity effects have been introduced into the classical piezoelectric theory to establish a modified piezoelectric theory. Higher-order elasticity effects comprising two branches have been proposed. They are the couple stress effects [12] and the strain gradient effects [13].

Yang et al. [14] used the modified couple stress (MCS) theory [15] to establish the laminated piezoelectric microcantilever model. Subsequently, Noori and Jomehzadeh [16] derived a functionally graded plate model. Bakhshi Khaniki and Hosseini Hashemi [17] found that the size effect can increase the natural frequency of a microbeam under free vibration. Additionally, based on the strain gradient elasticity theory [18], Nikpourian et al. [19] derived a size-dependent model for a piezoelectric microbeam. They found that considering the size effect, the amplitude of the microbeam is reduced, but the natural frequency has increased. Radgolchin and Moeenfard [20] applied the modified strain gradient (MSG) theory [21] to derive a mathematical model for a micro-bridge energy harvester.

To provide a theoretical basis for the researching of a micro beam piezoelectric energy harvester. This paper establisded a size-dependent dynamic model of a laminated piezoelectric microbeam. This piezoelectric microbeam model is able to reflect size effects more appropriately. And it has higher precision and wider universality than other types of microbeams. Furthermore, this piezoelectric microbeam model can provide the theoretical basis for designing of a micro beam piezoelectric energy harvester.

The structure of this paper is as follows: In section 2, higher-order elasticity effects are discussed to establish the modified piezoelectric theory. In section 3, a size-dependent dynamic model of a laminated piezoelectric microbeam is derived. Further, the natural frequency of piezoelectric microbeams is analyzed in section 4. Finally, section 5 gives the conclusion of this article.

2. THE MODIFIED PIEZOELECTRIC THEORY

Based on the MSG effect [21], the modified piezoelectric theory can be given as:

$$H = \frac{1}{2}\left(\sigma_{ij}\varepsilon_{ij} + p_i\gamma_i + \tau^{(1)}_{ijk}\eta^{(1)}_{ijk} + m^s_{ij}\chi^s_{ij} - D_iE_i\right) \tag{1}$$

where, D_i denotes the vector of electrical displacement, it is shown in Equation (2). E_i denotes the electric field, as shown in Equation (3).

$$D_i = -\varepsilon_0 E_i + P_i \tag{2}$$

$$E_i = -\varphi_{,i} \tag{3}$$

ε_{ij} is strain tensor, $\boldsymbol{\gamma_i}$ is dilatation gradient tensor, χ^s_{ij} is rotation gradient tensor, η_{ijk} represent strain gradient tensor, as shown:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{4}$$

$$\gamma_i = \varepsilon_{mm,i} \tag{5}$$

$$\begin{aligned}\eta^{(1)}_{ijk} = {} & \frac{1}{3}\left(\varepsilon_{jk,i} + \varepsilon_{ki,j} + \varepsilon_{ij,k}\right) - \frac{1}{15}\left[\delta_{ij}\left(\varepsilon_{mm,k} + 2\varepsilon_{mk,m}\right)\right. \\ & \left. + \delta_{jk}\left(\varepsilon_{mm,i} + 2\varepsilon_{mi,m}\right) + \delta_{ki}\left(\varepsilon_{mm,j} + 2\varepsilon_{mj,m}\right)\right]\end{aligned} \tag{6}$$

$$\chi^s_{ij} = \frac{1}{2}\left(\theta_{i,j} + \theta_{j,i}\right) \tag{7}$$

where, $\boldsymbol{\theta}$ is the rotation vector defined as:

$$\theta_i = \frac{1}{2}\left(curl(u)\right)_{,i} \tag{8}$$

The Cauchy stress σ_{ij}, higher stress p_i, $\tau^{(1)}_{ijk}$ and m^s_{ij} can be written as:

$$\begin{aligned} \sigma_{ij} &= \frac{\partial H}{\partial \varepsilon_{ij}} = c_{ijkl}\varepsilon_{kl} - e_{ijk}E_k \\ p_i &= \frac{\partial H}{\partial \gamma_i} = 2\mu l_0^2 \\ \tau^{(1)}_{ijk} &= \frac{\partial H}{\partial \eta^{(1)}_{ijk}} = 2\mu l_1^2 \eta^{(1)}_{lmn} \\ m^s_{ij} &= \frac{\partial H}{\partial \chi^s_{ij}} = 2\mu l_2^2 \chi^s_{ij} \\ D_i &= \frac{\partial H}{\partial E_i} = e_{ikl}\varepsilon_{kl} + \kappa_i E_k \end{aligned} \tag{9}$$

where λ and μ are the Lame constants, $\boldsymbol{l_n}$ (n= 0, 1, 2) are three material length-scale parameters.

3. MODELLING OF A PIEZOELECTRIC MICROBEAM

The laminated piezoelectric beam is shown in Figure 1.

If a beam length is much larger than beam width i.e., L >> h, the Euler-Bernoulli beam hypothesis model can be taken. The hypothesis model neglects the shear deformation of the beam. The electric field direction is assumed in the normal direction Z-axis. The displacement components are defined as follows [22]:

$$u = -(d+z)\frac{dw(x)}{dx} \qquad v = 0 \qquad w = w(x) \tag{10}$$

Substituting the Equation (10) into Equations (4) - (7), and ignored the strain gradient $\boldsymbol{\eta_{xxx}}$ [23]. the non-zero terms can be written as follows:

$$\varepsilon_{xx} = -(d+z)\frac{d^2w}{dx^2} \tag{11}$$

$$\gamma_z = -\frac{d^2w}{dx^2} \tag{12}$$

$$\begin{aligned} \eta^{(1)}_{xxz} &= \eta^{(1)}_{xzx} = \eta^{(1)}_{zxx} = -\frac{4}{15}\frac{d^2w}{dx^2} \\ \eta^{(1)}_{yyz} &= \eta^{(1)}_{yzy} = \eta^{(1)}_{zyy} = -\frac{1}{15}\frac{d^2w}{dx^2} \\ \eta^{(1)}_{zzz} &= \frac{1}{5}\frac{d^2w}{dx^2} \end{aligned} \tag{13}$$

Substituting Equations (11-13) into Equation (1), the electric enthalpy density is given as follows:

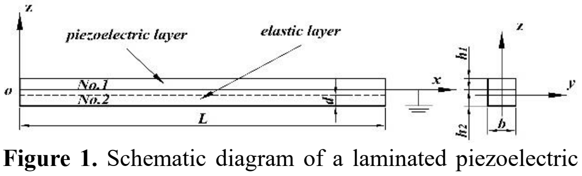

Figure 1. Schematic diagram of a laminated piezoelectric beam

$$\begin{aligned} H_1 = \frac{1}{2}\Big[& E_1\varepsilon_{xx}^2 + 2e_{31}\varphi_{,z}\varepsilon_{xx} + 2\mu_1 l_{0(1)}^2\gamma_z^2 + 6\mu_1 l_{1(1)}^2\left(\eta^{(1)}_{xxz}\right)^2 \\ & + 6\mu_1 l_{1(1)}^2\left(\eta^{(1)}_{yyz}\right)^2 + 2\mu_1 l_{1(1)}^2\left(\eta^{(1)}_{zzz}\right)^2 + 4\mu_1 l_{2(1)}^2\left(\chi^s_{xy}\right)^2 - \kappa_2\varphi_{,z}^2 \Big] \end{aligned} \tag{14}$$

The strain energy density of the elastic layer is given as follows:

$$\begin{aligned} U_2 = \frac{1}{2}\Big[& E_2\varepsilon_{xx}^2 + 2\mu_2 l_{0(2)}^2\gamma_z^2 + 6\mu_2 l_{1(2)}^2\left(\eta^{(1)}_{xxz}\right)^2 \\ & + 6\mu_2 l_{1(2)}^2\left(\eta^{(1)}_{yyz}\right)^2 + 2\mu_2 l_{1(2)}^2\left(\eta^{(1)}_{zzz}\right)^2 + 4\mu_2 l_{2(2)}^2\left(\chi^s_{xy}\right)^2 \Big] \end{aligned} \tag{15}$$

The summation of the electric enthalpy is shown as follows:

$$\Theta = b\int_0^L\int_{h_2}^{h_1} H_{total}\,dzdx = b\int_0^L\int_0^{h_1} H_1\,dzdx + b\int_0^L\int_{-h_2}^{0} U_2\,dzdx \tag{16}$$

The virtual work done can be defined as follows:

$$W_{ext} = \int_0^L q(x)w\,dx + \left[Fw\right]_0^L + \left[Mw'\right]_0^L + \left[M^h w''\right]_0^L \tag{17}$$

3. 1. Modelling for a Microbeam

The schematic diagram of the piezoelectric cantilever is shown in Figure 2.

The Hamilton's principle is shown as follows:

$$\delta\int_{t_1}^{t_2}\left(T - \Theta + W_{ext}\right)dt = 0 \tag{18}$$

Under the free vibration, $\boldsymbol{W_{ext}}$=0. The kinetic energy $\boldsymbol{T}$ is defined as follows:

$$T = \frac{1}{2}\int_0^L \rho_e A_e\left(\frac{\partial w}{\partial t}\right)^2 dx \tag{19}$$

Inserting Equations (16-17) and Equation (19) into t Equation (18), yields:

$$\begin{aligned} & \delta\int_{t_1}^{t_2}\left(T - \Theta + W_{ext}\right)dt \\ & = \int_0^L\left[-(a_1+a_3)w^{(4)} + e_{31}a_2\varphi_{,zxx} + q(x)\right]\delta w\,dx \\ & - \int_0^L\int_0^{h_1} b\left(e_{31}w'' + \kappa_2\varphi_{,zz}\right)\delta\varphi\,dzdx \\ & - \left[-(a_1+a_3)w^{(3)} + e_{31}a_2\varphi_{,zx} - F\right]\delta w\,|_0^L \\ & - \left[-b\int_0^L e_{31}(d+z)w'' + \kappa_2\varphi_{,z}dx\delta\varphi\,|_0^{h_1}\right] \\ & - \left[(a_1+a_3)w'' - e_{31}a_2\varphi_{,z} - M\right]\delta w'\,|_0^L = 0 \end{aligned} \tag{20}$$

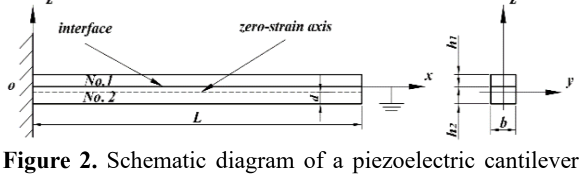

Figure 2. Schematic diagram of a piezoelectric cantilever microbeam under free vibration

where,

$$\begin{aligned} a_1 &= E_1\left(A_1 d^2 + 2dS_1 + I_1\right) + E_2\left(A_2 d^2 + 2dS_2 + I_2\right) \\ a_2 &= A_1 d + S_1 \\ a_3 &= A_1\left(2\mu_1 l_{0(1)}^2 + \frac{8}{15}\mu_1 l_{1(1)}^2 + \mu_1 l_{2(1)}^2\right) \\ &+ A_2\left(2\mu_2 l_{0(2)}^2 + \frac{8}{15}\mu_2 l_{1(2)}^2 + \mu_2 l_{2(1)}^2\right) \end{aligned} \tag{21}$$

Solving Equation (20), the mechanical and electrical governing equations can be expressed as Equations (22) and (23), respectively.

$$\left(\rho_1 A_1 + \rho_2 A_2\right)\frac{\partial^2 w(x,t)}{\partial t^2} + \left(a_1 + a_3\right)w^{(4)} - a_2 e_{31}\varphi_{,zxx} = 0 \tag{22}$$

$$\kappa_2 \varphi_{,zz} + e_{31} w''(x,t) = 0 \tag{23}$$

Inserting Equation (23) into electrical boundary conditions Equation (24), the electrical potential φ can be derived as Equation (25).

$$(d+z)e_{31}w'' + \kappa_2 \varphi_{,z}\,|_{z=h_1} = 0 \qquad \varphi|_{z=0} = 0 \tag{24}$$

$$\varphi = -\frac{e_{31}}{2\kappa_2} z^2 w'' - \frac{e_{31} d}{\kappa_2} z w''\,,\ \varphi_{,zxx} = -\frac{e_{31}(z+d)}{\kappa_2} w^{(4)} \tag{25}$$

Taking Equation (25) into Equation (22), leads to:

$$\left(\rho_1 A_1 + \rho_2 A_2\right)\frac{\partial^2 w}{\partial t^2} + \left(a_1 + a_3 + a_2 \frac{e_{31}^2 (z+d)}{\kappa_2}\right) w^{(4)} = 0 \tag{26}$$

where the ***w*** (***x***, ***t***) can be assumed as:

$$w(x,t) = W_0(x) e^{i\omega t} \tag{27}$$

Then substituting Equation (27) into Equation (26) gives:

$$J W_0^{(4)} e^{i\omega t} - (\rho A)_e W_0 \omega^2 e^{i\omega t} = 0 \tag{28}$$

where,

$$J = a_1 + a_3 + a_2 \frac{e_{31}^2 (z+d)}{\kappa_2}\,,\ (\rho A)_e = \rho_1 A_1 + \rho_2 A_2 \tag{29}$$

Solving Equation (28) then gives amplitude equation:

$$\begin{aligned} W(x) &= C_1 \sin(\beta x) + C_2 \cos(\beta x) \\ &+ C_3 \sinh(\beta x) + C_4 \cosh(\beta x) \end{aligned} \tag{30}$$

where,

$$\beta = \left(\frac{(\rho A)_e \omega^2}{J}\right)^{(1/4)} \tag{31}$$

The first order natural frequency can be written as Equation (32):

$$\omega_1 = 1.875^2 \sqrt{\frac{J}{(\rho A)_e L^4}} \tag{32}$$

3. 2. Analytical Flowchart The size-dependent dynamic model is solved using MATLAB. The design of the analytical flowchart is presented in Figure 3. First, the modified piezoelectric theory is established, and a size-dependent model of the microbeam based on this theory is developed. Second, the governing Equation (22) is solved using Hamilton's principle. Taking the amplitude, Equation (30), into the boundary condition, the natural frequency, Equation (32), can be obtained.

4. RESULTS AND DISCUSSION

The natural frequencies obtained using different size-dependent models are discussed in this section. The material parameters are shown in Table 1 [24]. The length scale parameters of the material can be assumed as $l_{01} = l_{11} = l_{21} = l_1$ and $l_{02} = l_{12} = l_{22} = l_2$, where $l_2 = 2 \times l_1 = 2 \times l$.

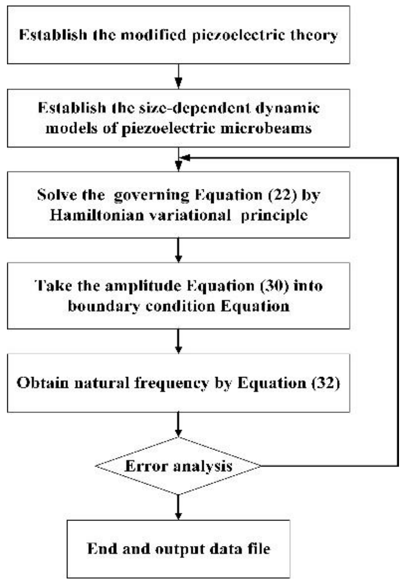

Figure 3. Analytical flowchart of the dynamic model

TABLE 1. Material parameters

Parameters	Value	Parameters	Value
G_1	126 GPa	h	1.2 μm
G_2	160 GPa	h_1	0.2μm
ρ_1	7.5×10^3 kg/m^3	h_2	1 μm
ρ_2	2.7×10^3 kg/m^3	L	20μm
ε_0	8.85×10^{-12} C/V·m	b	1 μm
κ_2	13×10^{-9} C/V·m	e_{31}	-6.5N/ (V m)

4. 1. Model Validation To validate the accuracy of the present model, the natural frequencies were compared with those reported data in previous research [25]; results are shown in Figure 4. The obtained results proved the accuracy of the model.

4. 2. Mechanical-electrical Coupling Responses under Free Vibration Figure 5 compares the dimensionless natural frequency of a piezoelectric microbeam predicted using the present and MCS models. ω is the natural frequency obtained using the size-dependent model, and ω_0 is the natural frequency that obtained using the classical model. ω is larger than ω_0, it means that the size effect can enhance the structural stiffness when h/l is smaller than 15. The results obtained using the MCS model are less than those obtained using the present model. This difference is due to the fact that the MCS model is a simplification of the present model. Furthermore, with an increase in the thickness, the influence of the size effect gradually decreases. When h/l is much greater than 15, the size effect can be neglected.

Figure 6 shows the effect of the relative thickness t on the natural frequency ω under free vibration. The t can be calculated using Equation (33). When t is greater than 5

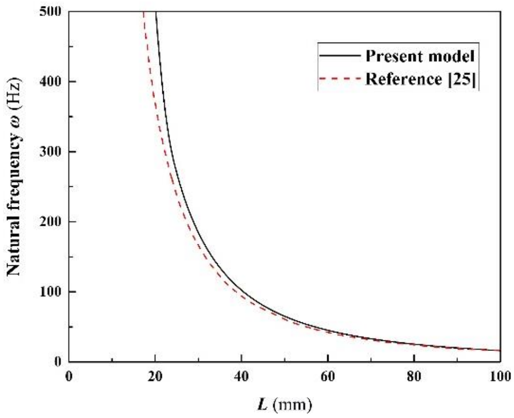

Figure 4. Model validation

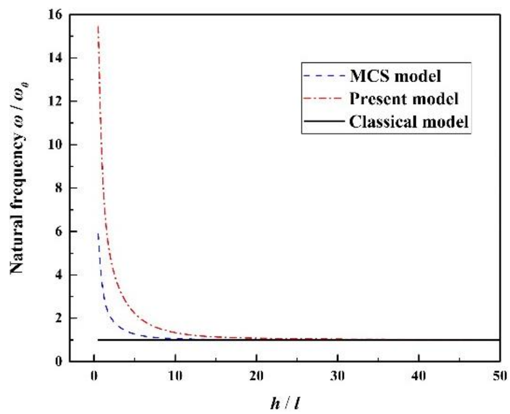

Figure 5. Influence of the h/l value on the natural frequency

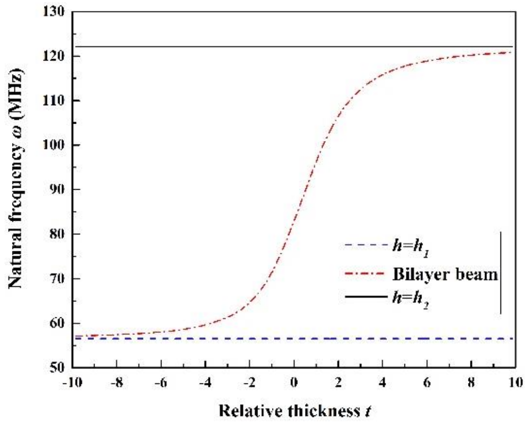

Figure 6. Influence of relative thickness t on the natural frequency

(the thickness of h_2 is 27 times greater than that of h_1), the ω is approximately equal to the ω of the single elastic layer no. 2 beam (122.06 MHz). When t is less than −5 (h_1 is 27 times greater than h_2), the ω of the piezoelectric microbeam is approximately equal to that of the single piezoelectric layer no. 1 beam (56.54 MHz).

$$t = \frac{h_2 - h_1}{\sqrt{h_1 h_2}} \tag{33}$$

5. CONCLUSIONS

In this article, a size-dependent dynamic model of a laminated piezoelectric microbeam is presented to predict the behaviour of a piezoelectric microbeam. The natural frequencies of microbeams are obtained under free vibration. Numerical results reveal that when the size effect is considered, the natural frequency of the piezoelectric microbeam increases, meaning that the size effect can strengthen the beam's structural stiffness. When h/l is larger than 15, the size effect can be neglected. Compared with the present model, the MCS model underestimates the size-dependent response. Thus, the MCS theory is a simplification of the MSG theory. In addition, the relative thickness can obviously influence the natural frequency. When the relative thickness is greater than 5 or less than −5, the laminated beam can be simplified to a single-layer beam. The influence of working conditions on the natural frequency will be considered in future research to study the design method of the MEMS.

6. ACKNOWLEDGEMENT

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Persian Abstract

چکیده

پرتوهای پیزوالکتریک به طور گسترده در سیستم های میکرو الکترومکانیک استفاده می شود. در مقیاس میکرو، تأثیر اثر اندازه بر یک پرتو پیزوالکتریک را نمی توان نادیده گرفت. در این مقاله، تئوری‌های الاستیسیته مرتبه بالاتر برای پیش‌بینی رفتار ریزساختارهای پیزوالکتریک در نظر گرفته می‌شوند و یک مدل دینامیکی وابسته به اندازه یک ریزپرتو پیزوالکتریک چند لایه ایجاد می‌شود. معادلات حاکم برای ریزپرتوهای پیزوالکتریک چند لایه با استفاده از اصل تغییرات به دست می‌آیند. فرکانس های طبیعی ریزپرتوهای پیزوالکتریک توسط مدل های دینامیکی وابسته به اندازه به دست می آیند. نتایج نشان می‌دهد که اثر اندازه می‌تواند سفتی ساختاری را در مقیاس میکرو افزایش دهد. فرکانس طبیعی به دست آمده با استفاده از مدل کلاسیک کوچکتر از فرکانس بدست آمده با استفاده از مدل وابسته به اندازه است. در مقایسه با مدل استرس زوج اصلاح شده، مدل استرس زوجی اصلاح شده پاسخ وابسته به اندازه را دست کم می گیرد. بنابراین، مدل تنش زوجی اصلاح‌شده، ساده‌سازی مدل گرادیان کرنش اصلاح‌شده است. تأثیر ضخامت پرتو بر فرکانس طبیعی نیز مورد بحث قرار گرفته است. با افزایش ضخامت، فرکانس طبیعی مدل های وابسته به اندازه به تدریج به نتیجه مدل کلاسیک نزدیک می شود. اگر مقدار h/l بیشتر از ۱۵ باشد، می‌توان از تأثیر اندازه چشم‌پوشی کرد. علاوه بر این، ضخامت نسبی می تواند فرکانس طبیعی را تحت تاثیر قرار دهد و اگر ضخامت نسبی بزرگتر از ۵ یا کمتر از ‎-۵ باشد، پرتو دولایه را می توان به یک پرتو تک لایه ساده کرد.