IJE TRANSACTIONS C: Aspects Vol. 36 No. 06, (June 2023) 1150-1165

International Journal of Engineering

Journal Homepage: www.ije.ir

The Equipment Scheduling and Assignment Problem in the Overhaul Industry

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PAPER INFO

Paper history:
Received 06 March 2023
Received in revised form 28 March 2023
Accepted 30 March 2023

Keywords:
Genetic Algorithm
Flow Shop
Shortest Processing Time
Particle Swarm Optimization

ABSTRACT

In this article, equipment overhaul is considered in a multi-stage flow shop scheduling problem. In this problem, the equipments are disassembled in the first stage, overhaul and repairs are done on the equipment in parallel workshops in the second stage, and the assembly operation is done in parallel workshops in the third stage. Considering a three-stage overhaul with parallel machines in the second and third stages is new in the overhaul industry. The sequence of equipment processing is determined in the first stage, as well as the allocation and sequence of equipment in the second and third stages should be done in such a way that the total completion time of jobs is minimized. Unlike most articles, the sequence of processing jobs is not the same in all stages and changes with the use of decoding. For the next innovation: in order to solve the problem, a new mathematical model is presented. Two new improved algorithms, Genetic Algorithms (GA) and Particle Swarm Optimization (PSO) are presented to solve the problem in large dimensions. By using the shortest processing time (SPT) heuristic, these two algorithm have been improved and Hybrid GA (HGA) and Hybrid PSO (HPSO) algorithms have been presented. In order to achieve better results with the current conditions, the parameters setting is done by one-way analysis of variance (ANOVA). Finally, it is possible to improve the performance of the equipment by applying the discussed issues.

doi: *10.5829/ije.2023.36.06c.14*

1. INTRODUCTION

The equipment and physical assets of the organization undergo depreciation during their life cycle, and this puts the organizations in intermittent periods of time in front of the decision to decommission or refurbish the equipment. In many cases, the renovation and overhaul of industrial equipment, if implemented correctly, is a decision that can help the organization in creating the most value from physical assets.

Luh et al. [1] investigated the effect of overhaul, they considered three cases. Based on the results, after using the equipment overhaul, the average cost of delay is significantly reduced from 53.81 in case 1 to 13.66 in case 2, which leads to the reduction of the average possible cost from 59.84 to 27.38. The standard deviation of the feasible cost is also significantly reduced from 27.53 to 3.03, implying a more predictable asset delivery. As a result, asset delivery becomes more reliable and the standard deviation of the feasible cost is further reduced from 3.03 to 0.88.

As another example, an Maintenance, Repair, and Overhaul (MRO) activity in the aviation industry is typically a scheduled periodic check to determine the condition of an aircraft or its components, including service, repair, modification, overhaul, and inspection. The MRO industry can be described as a strong supporter in the aviation industry for the provision of aircraft parts and services. For airlines, MRO costs cover 12% of total annual operating costs, the third hidden cost after fuel and operations. As the number and age of aircraft worldwide increases, the annual growth rate of the MRO market is now 4.2%, and the MRO market is forecast to reach $87.8 billion by 2024. With incumbent MRO companies competing for markets and new entrants entering, intense competition puts them under great pressure to increase

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Please cite this article as: M. Torkashvand, N. Shamami, H. Bigdeli, The Equipment Scheduling and Assignment Problem in the Overhaul Industry, ***International Journal of Engineering, Transactions C: Aspects***, Vol. 36, No. 06, (2023), 1150-1165

profit margins and optimize business operations. Therefore, effective management technologies and tools are needed for MRO companies to improve operation efficiency and reduce operation cost [2].

According to a general definition, it can be said that overhaul is the comprehensive and complete restoration of an asset to bring it to an acceptable condition through reconstruction or replacement of its parts and subassemblies. The purpose of the overhaul is actually to restore the optimal performance of the equipment while ensuring its reliability. The overhaul process of industrial equipment generally includes the following steps [3]:

- Segregation
- Disassembly
- Cleaning
- Inspection
- Repair (or replace)
- Assembly and testing

In “segregation”, the equipment is separated for overhaul. In “disassembly”, the components of each equipment are separated. In “cleaning”, the operation of cleaning separated parts is performed. After cleaning, each part is inspected to determine the need for repair (“inspection”). In the “repair (or replace)” stage, the repair or replacement operation is performed on the parts that have been specified from the previous stage. Finally, after the repair, the repaired or replaced or cleaned parts are put together and then tested and the “assembly and testing” operation is performed. In this article, all the above steps are done in three steps, so that the steps of segregation, disassembly, cleaning and inspection are known as disassembly, and two steps with the same title are mentioned in this article.

The overhaul word is usually used for mechanical equipment and this process is generally implemented in the middle of the useful life of the equipment. Sometimes, instead of the word "overhaul", the term "major repairs" is used. In order to overhaul some equipment, it is necessary to remove them from the site and move them to a suitable place for the implementation of the reconstruction process, although many equipment must also be overhauled at their location. In the overhaul process of industrial equipment, sometimes there is a need to rebuild some parts and sub-category equipment. For the parts that cannot be repaired, they must be replaced.

Overhaul operation can be done as an emergency and after an unexpected failure or form of a predetermined program and when the equipment is out of operation. In general, the implementation of the overhaul process imposes two types of costs on the organization, one is the direct costs of the overhaul process and the other is the costs associated with the equipment being unavailable during the overhaul. Correct planning and based on the health status of the equipment during its life cycle can adjust the scheduling of the overhaul operations in such a way that both types of costs are minimized.

In this century, organizations are operating in a very unstable and dynamic environment, and they are always facing risks and threats, which will undoubtedly face very important challenges if they are not responded to in a timely and quick manner. Considering that the use duration of military equipment is longer than that of commercial equipment, and changes in it usually take longer, one of these key factors in using equipment is overhaul and optimization, which has been the focus of most managers and organizations in recent years. according to these conditions, the scheduling of overhaul and repairs of devices and equipment can show a significant role in the field of defense in military industries, whenever any of the devices and equipment needs major repairs and optimization, sent to the overhaul and optimization centers, the readiness and combat power of that unit will decrease in the same proportion, and with this description, any speed and agility will help to strengthen the power and readiness of that unit. The level of readiness can be a standard for allocating funds, purchasing equipment and support items, as well as carrying out military actions in peacetime or wartime. The readiness of equipment is one of the basic components in the operational readiness of organizations, which is the main goal of the maintenance and repair system and its existential philosophy in the process of supporting organizations.

In this plan, a flow shop scheduling problem is proposed in order to reduce the cost and time of equipment overhaul. Also, in order to prevent additional repairs, it is used to diagnose and predict defects before failure and reduce waste. One of the most important defense factors of any country is its equipment. One of the necessities of presenting this research is keeping the equipment ready and having the maximum ability when needed. Its importance is related to the time that the protection of the country and people's lives are involved. It is used in times of peace to maintain security and in times of war to speed up the preparation of equipment. In order to increase the defense power, the preparation and availability of equipment for use in emergency situations, repairs and overhaul of the equipment are carried out. In case of possible threats, it will be possible to use the equipment when needed. Also, proper, accurate, high-reliability and high-quality overhaul and optimization scheduling can increase the trust of employees and the organization in the internal overhaul and repair system, and as a result, increase the morale and self-confidence of technical personnel, which causes a kind of lack of dependence on external factors. Also, the scheduling of repairs, overhaul and optimization with high quality and reliability will prevent rework and return of refurbished equipment and tools to the centers, which can be effective in reducing the cost and time of the organization's personnel.

The following sections are presented below. In the second part, a literature review is presented, which examines the articles presented in this field. The third part of the mathematical model is defined along with the limitations. In the fourth section, classical solution methods for large-scale problems are presented. In the fifth section, improved problem solving methods for large sizes are shown. Calculation results for small and large problem sizes as well as sensitivity analysis are analyzed in the sixth section. Finally, the conclusion is made in the seventh section.

2. LITERATURE REVIEW

Reményi and Staudacher. [3] investigated the maintenance, repair and overhaul operations of aircraft engines. Since it is difficult to control maintenance planning and maintenance operations, the focus of research has been on improving maintenance operations by finding appropriate scheduling for job shop operations in maintenance operations. They emphasized that scheduling can improve maintenance operations. According to the needs of different researches, they presented a simulation model. The maintenance and repair process is defined in 3 separate stages: disassembly, repair and assembly. Qin et al. [4] proposed an integrated aircraft maintenance scheduling and hangar layout planning problem. They developed a mathematical model incorporating the variation of parking capacity and blocking during aircraft movements. To obtain good quality feasible solutions for large scale instances, presented a rolling horizon approach incorporating the enhanced mathematical model. Pinhão et al. [5] studied integration of line balancing and scheduling problems in aircraft engines assembly and presented integer programming models define a standard work for multi-skilled operators. Models were implemented in the system and hierarchically solved by the CPLEX on data of two different assembly lines from a Brazilian MRO company.

Dinis et al. [6] proposesed a framework for the qualitative and quantitative characterization of maintenance work to support MRO organizations in performing capacity planning and scheduling. They proposed a framework, entitled Framework for Aircraft Maintenance Estimation (FRAME), is intended to allow MROs in managing this uncertainty throughout the maintenance planning process and comprises for that end a set of requirements for data treatment and a method for data analysis.

Sharma and Rai [7] proposed arithmetic reduction of age based virtual age model to estimate reliability parameters by considering the complete MRO as imperfect and provides the likelihood and log-likelihood functions for parameter estimation of the proposed model and also presents the various extensions of the proposed model.

Huang et al. [8] presented a simulation optimization for maintenance, repair and overhaul scheduling problem based on multi credit models. A simulation model with a high degree of credibility, which is probable and time-consuming. The solution space was divided into several parts. They also divided the process into three parts: disassembly workshop, repair workshop and assembly workshop. The repair workshop consists of one or more multifunctional machines with one or more parallel machines. Mayto et al. [9] presented a research with the aim of improving the scheduling of machines so that the maximum completion time of job is minimized in the scheduling of job shop using genetic algorithm. They created a new plan to carry out the repair process of the CT7 engine cold section module at PT XYZ. Chang and Abdullah [10] presented an operations management model that creates the concept of soft production and sustainable development in the maintenance process and the management process for factory maintenance, repairs and overhaul. Aircraft maintenance, repairs and overhaul are very important for air operations. MRO company should focus on delivery, quality, cost and flexibility in order to increase customer satisfaction. Luh et al. [1] presented a new formulation for repair and overhaul services so that key characteristics such as asset arrival and non-deterministic processing times of parts are presented in the model including an overhaul center and several repair shop centers. They developed a solution method including a combination of the Lagrange method and probabilistic dynamic planning in order to schedule operations so that the total tardiness and inventory maintenance costs are minimized. Liu et al. [2] presented a scheduling problem in the class of maintenance, repair and overhaul systems. Considering all the key characteristics such as disassembly, non-deterministic material recovery, material matching requirements, possible routing and possible processing times, they formulated a scheduling problem as a simulation problem. In order to solve the problem, they presented two combined algorithms. For a set of jobs, the problem is to determine the sequence of jobs for disassembly in the disassembly shop, the sequence of components for repair in the repair shop, and the sequence of jobs for assembly operations in the assembly stage, so that the weighted sum of tardiness times is minimized. Li et al. [11] presented a scheduling optimization method for maintenance, repairs, and operation of service resources in complex products, which improves customer satisfaction, increases product value, and increases competitive advantage. First, they analyzed the scheduling problem in service resources, presented a mathematical model for the service scheduling problem, and presented three objective functions including minimizing the waiting time of customers, reducing excess resources, and maximizing the resource performance cost index. Then each of the three objective functions have been analyzed based on three methods including improved genetics, combined weight coefficient optimization method and non-dominant sorted genetic method. Tran et al. [12] proposed Swarm Intelligence Algorithm, Ant Colony Optimization in order to solve the scheduling problem of MRO processes with two objective functions of minimizing the maximum completion time and the total tardiness times of all jobs. They considered three stages for the MRO process: all components are inspected in the inspection stage (first stage), the repair operations are performed based on the defined sequence (scheduling is done in this stage) in the second stage, the release operation is done in the third stage.

Yuan et al. [13] proposes a capacitated fuzzy disassembly scheduling model with cycle time and environmental cost as parameters, which has broad applications in remanufacturing and many other production systems. A mixed-integer mathematical programming model proposed to minimize the cycle time and environmental cost, whilst a metaheuristic approach based on a fruit fly optimization algorithm (FOA) is developed to find a fuzzy disassembly scheduling scheme.

Guo et al. [14] due to the heavy problems of accompanying support jobs, high timely requirements and the mismatch between the types and number of repair units and support activities in wartime, together with the scheduling of equipment maintenance activities, they considered the problem of flexible flow shop scheduling and presented a scheduling model for equipment maintenance activities based on the accompanying maintenance group. Based on the limited ability of repairs and the repair time of the repair team, the proposed model was considered to schedule the activities of the repair team with the aim of minimizing the time spent on repairs. Rahman et al. [15] believe that the MRO process is an action to control, identify and ensure the life of a defense equipment. Indonesia should have MRO concept in defense industry. The relevant issue is how to prioritize the weighting and develop strategic planning for the maintenance and overhaul of warships implemented by Fasharkan Surabaya to support the readiness of naval operations. The aim of the presented research is to create a strategic planning in the development of MRO of Fasharkan Surabaya warships in support of maritime operations readiness. Considering that the problem investigated in this article is a special type of flow shop, some articles have been presented in this field. Fathollahi-Fard et al. [16] proposed a sustainable Distributed Permutation Flow Shop Scheduling Problem (DPFSP). The study total energy consumption related to production, and the social factors linked to job opportunities and lost working days. In order to solve the problem they presented a novel multi-objective learning-based heuristic is established, as an extension of the Social Engineering Optimizer (SEO). Gholizadeh et al.[17] addressed this challenge more broadly, this paper presents an optimization model for the problem of flexible flowshop scheduling in a series-parallel waste-to-energy (WTE) system. A preventive maintenance (PM) policy is proposed to find an optimal sequence for processing tasks and minimize the delays.

Amirian and Sahraeian [18] proposed an effective multi-objective differential evolution algorithm (MDES) to solve a permutation flow shop scheduling problem (PFSSP) with the modified Dejong's learning effect. The proposed algorithm combines the basic differential evolution (DE) with local search and borrows the selection operator from NSGA-II to improve the general performance. Naseri et al. [19] consider the production environment of no-wait reentrant flow shop with the objective of minimizing makespan of the jobs. They constructed simulated annealing (SA), genetic algorithm (GA) and a bottleneck based heuristic (BB) algorithms to solve the problem. A summary of the subject literature is shown in Table 1. According to Table 1, two of the articles are job shop. In four articles, the second stage is the job shop, and in three articles, a three-stage flow shop is presented. In order to complete the presented problems and fill the gaps in the literature, in this article, parallel machines are considered in the second and third stages of the flow shop.

According to the reviewed literature, in this research, a combined flow shop scheduling problem will be presented in order to maintain, repair and overhaul military equipment in the army. In this problem, the equipment considered for the scheduling of repairs and overhaul are placed in a multi-stage flow shop where the first stage is equipment disassembly, the second stage is repairs and overhaul, and the third stage is assembly. The innovations of the problem are:

TABLE 1. Summary of the subject literature

Row	Author	Problem
1	Tran et al. [19]	Job shop
2	Guo et al. [14]	Flexible job shop
3	Reményi and Staudacher [18]	Job shop in the second stage
4	Huang et al. [8]	Job shop in the second stage
5	Mayto et al. [9]	Job shop in the second stage
6	Liu et al. [2]	Job shop in the second stage
7	Chang and Abdullah [10]	Three-stage flow shop
8	Luh et al. [1]	Three-stage flow shop
9	Rahman et al. [15]	Three-stage flow shop

- Providing improved meta-heuristic algorithms to solve large-scale problems
- Considering parallel machines in the second and third stages
- Regarding the three-stage flow shop for equipment overhaul
- Presenting a new model to accurately solve the problem in small dimensions

3. MATHEMATICAL MODEL

The problem investigated in this article is a three-stage flow shop problem for equipment repairs and overhaul. In this problem, there are n number of machines to perform overhaul operations in a three-stage flow shop, where the first stage has a single machine, the second stage has m_2 machines, and the third stage has m_3 parallel machines. If the equipment is considered as job and each stage is considered as a machine, the problem under investigation is a three-stage flow shop problem where there is one machine in the first stage and identical parallel machines in the second and third stages. The problem model is presented by considering the following assumptions [20]:

- The number of steps is fixed and equal to 3.
- Each machine processes only one job at a time in each step.
- Storage space between stages is considered free.
- Each job is processed on only one machine at a time.
- The operation of a job in each step starts when its operation in the previous step is completed.

Parameters and Indices	
n	The number of jobs
m_2	The number of machines in the second stage
m_3	The number of machines in the third stage
s	Position indexes, {1,2,... , n}
i, j, r	The jobs indexes {1,2,... , n}
e	The machines index in the second stage {1,2, … , m_2}
l	The machines index in the third stage {1,2, … , m_3}
tf_j	the processing time of job j at the first stage
ts_j	the processing time of job j at the second stage
tt_j	the processing time of job j at the third stage
A	A large positive number

Decision varibales	
$Q_{i,s}$	1 If job i is processed in position s in the disassembly stage otherwise 0.
$Z_{i,s,e}$	1 If job i is processed in position s on machine e in the overhaul stage otherwise 0.
$W_{i,s,l}$	1 If job i is processed in position s on machine l in the assembly stage otherwise 0.
COF_s	Completion time of job in position s at disassembly stage
$COS_{s,e}$	Completion time of the job in position s on the machine e in the overhaul stage
$COT_{s,e}$	Completion time of the job in position s on the machine l in the third stage

$$\text{Minimize } \sum_{s=1}^{n}\sum_{l=1}^{m_3} COT_{s,l} \quad (1)$$

$$\sum_{s=1}^{n} Q_{i,s} = 1 \quad \forall\, i = 1,2,\dots,n \quad (2)$$

$$\sum_{i=1}^{n} Q_{i,s} = 1 \quad \forall\, s = 1,\dots,n \quad (3)$$

$$\sum_{i=1}^{n} Q_{i,s-1} \geq \sum_{j=1}^{n} Q_{j,s} \quad \forall\, s = 2,\dots,n \quad (4)$$

$$\sum_{s=1}^{n}\sum_{e=1}^{m_2} Z_{i,s,e} = 1 \quad \forall\, i = 1,2,\dots,n \quad (5)$$

$$\sum_{i=1}^{n} Z_{i,s,e} \leq 1 \quad \forall\, s = 1,\dots,n;\ e = 1,2,\dots,m_2 \quad (6)$$

$$\sum_{i=1}^{n} Z_{i,s-1,e} \geq \sum_{j=1}^{n} Z_{j,s,e} \quad \forall\, s = 2,\dots,n;\ e = 1,2,\dots,m_2 \quad (7)$$

$$\sum_{s=1}^{n}\sum_{l=1}^{m_3} W_{i,s,l} = 1 \quad \forall\, i = 1,2,\dots,n \quad (8)$$

$$\sum_{i=1}^{n} W_{i,s,l} \leq 1 \quad \forall\, s = 1,\dots,n;\ l = 1,2,\dots,m_3 \quad (9)$$

$$\sum_{i=1}^{n} W_{i,s-1,l} \geq \sum_{j=1}^{n} W_{j,s,l} \quad \forall\, s = 2,\dots,n;\ l = 1,2,\dots,m_3 \quad (10)$$

$$COF_s \geq COF_{s-1} + \sum_{i=1}^{n} tf_i * Q_{i,s} \quad \forall\, s = 2,\dots,n, \quad (11)$$

$$COF_1 \geq \sum_{i=1}^{n} tf_i * Q_{i,1} \quad (12)$$

$$COS_{s,e} \geq COS_{s-1,e} + \sum_{i=1}^{n} tt * Z_{i,s,e} - A * \left(1 - \sum_{i=1}^{n} Z_{i,s,e}\right)$$
$$\forall\, e = 1,2,\dots,m_2\,;s = 2,\dots,n \quad (13)$$

$$COS_{s,e} \geq COF_r + ts_i * Z_{i,s,e} - A * \left(2 - Z_{i,s,e} - Q_{i,r}\right)$$
$$\forall\, i = 1,2,\dots,n; s,r = 1,2,\dots,n;\ e = 1,2,\dots,m_2 \quad (14)$$

$$\forall\, l = 1,2,\dots,m_3\,;s = 2,\dots,n \quad (15)$$

$$COT_{s,l} \geq COS_{r,e} + tt_i * W_{i,s,l} - A * \left(2 - W_{i,s,l} - Z_{i,r,e}\right)$$
$$\forall\, i = 1,2,\dots,n; s,r = 1,2,\dots,n;;\ e = 1,2,\dots,m_2\,;\ l = 1,2,\dots,m_3 \quad (16)$$

$$Q_{i,s} \in \{0,1\}\,,\ Z_{i,s,e}, W_{i,s,l} \in \{0,1\}$$
$$\forall\, i = 1 \,.\dots.\, n\,; e = 1,2,\dots,m_2\,;\ s = 1,.\dots.\,n\,; l = 1,\dots,m_3 \quad (17)$$

Equation (1) shows the objective function of the problem, which is to minimize the total time to complete the jobs. Constraint 2 indicates that each job is assigned to only one position in the first stage. Constraint 3 specifies that each position must be assigned a job in the first stage. Constraint 4 determines that a position in the first stage is completed when its previous position is occupied by a job. Constraint 5 shows that every job in the second stage must be assigned to a position of one of the machines. Constraint 6 specifies that a position of a machine is occupied by at most one job. Constraint 7 shows that a position of a machine is occupied in the second stage when its previous position is occupied. Constraint 8 specifies that every job in the third stage must be processed by one machine and it is not possible to stop processing and move between machines. Constraint 9 shows that at most one job is assigned to each position of each machine in the third stage. Constraint 10 specifies that in the third stage, a position of a machine is completed when its previous position is occupied by a job. Constraint 11 specifies the time to complete the job in the first stage. Constraint 12 specifies the completion time of the job that is in the first position of the first stage. Constraints 13 and 14 specify the time to complete the jobs in the second stage. Constraints 15 and 16 specify the time to complete the job in the third stage. Constraint 17 specifies binary variables. Constraint 18 determines the range of continuous variables.

4. SOLUTION METHOD

It is possible to solve the problem in small dimensions using the model. For the two-stage scheduling problem with parallel machines in one stage and the objective function of maximum completion time, Chen [21] showed that the problem is NP-hard. Considering that the problem investigated in this article is a three-stage flow shop with parallel machines in the third stage, it is more complicated than the problem addressed in the literature [21], so it is NP-hard.

Considering that the problem is NP-hard, it is possible to solve it in small dimensions in reasonable time using the exact solution approach, and the solution time is long for larger dimensions, Therefore, two meta-heuristic algorithms GA and PSO have been used to solve the problem in large dimensions. In the following, the common parts of two algorithms and each of the algorithms are presented separately.

4. 1. Genetic Algorithm Genetic algorithm is one of the most well-known metaheuristic algorithms that was first presented by Holland [22]. This algorithm is population based and inspired by nature. Each solution to the problem is known as a chromosome, which is created from the combination of genes. By making changes in each chromosome, a new solution is created. Genetic algorithm includes mutation and crossover operators to create a new population. Genetic algorithm in many articles including Yazdani et al. [23], Tavakkoli-Moghaddam [24], Ghafari [25] has been used.

The presented algorithm consists of several parts, which are: 1. Presenting the solution representation in order the problem coding 2. Decoding the problem from the way it is displayed in the second and third stages 3. Creating the initial population in the amount of N_{POP} 4. Randomly selecting the solution from the existing population in order to performing crossover operations with the probability of P_{cross} 5. Random selection of solution in order to perform mutation operations with the probability of P_{mut} 6. Performing local search in order to improve the solutions. In this paper, an improved genetic algorithm is presented. According to the objective function of the problem, the algorithm presented in this problem is a combination of the genetic algorithm with the shortest processing time (SPT) heuristic method [26], which is called HGA. In the following, the steps of the algorithm are described until generating a new solution.

4. 1. 1. Solution Representation Each chromosome in the genetic algorithm is specified by using a representation in coding. Assuming that there are 10 jobs, the solution representation in the genetic algorithm presented in this article is according to Figure 1 and based on the sequence of job processing [27]. In this representation, the solution to the sequence of jobs processing in the first stage is determined. In the second and third stages, considering that there are parallel machines, the processing sequence on each machine is determined by decoding.

4. 1. 2. Decoding After determining the sequence of jobs in the first stage, considering that the second and third stages include parallel machines, the sequence of processing jobs will be different. For this purpose, after finishing the processing of a job in the first stage, in the second stage, the job is processed on a machine that is free or free earlier than other machines. It is the same for the third stage.

4. 1. 3. Initial Population In order to implement the genetic algorithm, an initial population of solution with the size of N_{POP} should be created and by applying changes to each solution, a new population is generated. According to the representation of the solution that presented in the previous section, the initial population is randomly generated and after making the necessary changes, it is updated in each iteration.

4	7	8	2	5	10	6	3	9	1

Figure 1. Solution representation

4. 1. 4. Crossover Operator In order to create a new population and improve the solutions, the crossover operator in the genetic algorithm is applied to two randomly selected solution. In order to perform the crossover in this section, the partially mapped crossover (PMX) crossover operator has been used [28]. In this operator, two points are randomly selected on the parents. The numbers between these two points are transferred from the first parent to the second child and from the second parent to the first child. We used the first parent to fill the rest of the positions in the first child.

If the selected number is not between two points, the position is filled and if there is, the opposite number is selected in the opposite child. An example of this type of crossover is shown in Figure 2. The crossover operator in the presented algorithm is performed with the probability of P_{Cross} on a solution.

Parent 1	1	2	3	4	5	6	7	8	9	10
Parent 2	2	7	8	5	4	9	6	3	10	1
Child 1	*	7	8	5	*	*	*	*	*	*
Child 2	*	2	3	4	*	*	*	*	*	*
Child 1	1	7	8	5	*	6	*	*	9	10
Child 2	*	2	3	4	*	9	6	*	10	1
Child 1	1	7	8	5	4	6	2	3	9	10
Child 2	7	2	3	4	5	9	6	8	10	1

Figure 2. Crossover operation

Parent	4	7	8	2	5	10	6	3	9	1
child	4	7	8	2	3	5	10	6	9	1

Figure 3. Mutation operation

4. 1. 5. Mutation Operator In order to make small changes in each solution, the genetic algorithm uses the mutation operator. This operator in the genetic algorithm produces a new solution by applying changes to each solution. If the new solution is better than the previous solution, it will be replaced, otherwise the previous solution will remain unchanged. Different modes for performing mutation are presented by different authors. In this article, insert mutation is used for mutation [20], which is shown in Figure 3.

In this type of mutation, two genes are randomly selected. The second gene is transferred after the first gene. Numbers between two genes of a cell move to the right. For example, according to Figure 3, the genes in cells 4 and 8 are randomly selected. The 8th gene has been moved to the side of the 4th gene, and the numbers between the two genes have been moved one cell to the right. As a result, a new solution has been obtained. The mutation operator in the presented algorithm is performed with the probability P_{mut} on a solution.

4. 1. 6. Updating The Solutions After performing crossover and mutation operations, the value of the objective function is calculated. If the obtained solutions are better than the current solution, the new solution will replace the current solution, otherwise the solution will remain unchanged.

4. 2. Particle Swarm Optimization The particle swarm optimization algorithm was proposed by Kennedy, and Eberhart [29] and has been successfully used in many fields of science. The PSO algorithm has been used by Daliri et al. [30] and Rabbani et al. [31].

This algorithm is one of evolutionary computing techniques and was invented by imitating the flight of birds or the movements of fishes and the exchange of information between them. In this algorithm, each solution is only one particle in the search space. All particles have a fitness value that is evaluated by the fitness function that must be optimized. In addition, each particle i has a position in the d-dimensional space of the problem, which is represented by the vector (19) in the t iteration.

$$X_i^t = (X_{i1}^t, X_{i2}^t, \dots, X_{id}^t) \tag{19}$$

Also, this particle has a speed that guides its movement and is represented by the vector (20) in the tth repetition:

$$V_i^t = (V_{i1}^t, V_{i2}^t, \dots, V_{id}^t) \tag{20}$$

In each iteration of the search, each particle is updated considering the two best values. The first value is related to the best solution that the particle has experienced so far. This value is called the best P. The second best followed by the particle swarm optimization algorithm is the best position obtained so far in the population. This value is the general optimum, which is called the best g.

$$V_i(t+1) = WV_i(t) + C_1\, r_{1,i}(t)\big(p_i(t) - X_i(t)\big) + C_2\, r_{2,i}(t)\left(p_g(t) - X_i(t)\right) \tag{21}$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \tag{22}$$

After these two best values are found, the position and velocity of each particle is updated by relations (21) and (22). In the above relationships, t represents the repetition number, and variables C_1 and C_2 are learning factors. Usually, $C_1 = C_2 = 2$, which controls the amount of displacement of a particle in one repetition. r_1 and r_2 are two uniform random numbers in the interval [0,1], the parameter w represents the inertia weight, which takes an initial value in the interval [0,1].

4. 2. 1. Discrete PSO Algorithm The discrete mode of PSO algorithm was presented by Pan et al. [32]. In this article, the discrete mode of the PSO algorithm is used to solve the problem. The way to solution representation and decode each particle is the same as the genetic algorithm presented in section 4-1. The equation of the algorithm for the discrete state is according to Equation (23) in which the mutation and crossover operators are used.

$$X_i^t = c_2 \otimes F_3(c_1 \otimes F_2(\omega \otimes F_1(X_i^{t-1}), P_i^{t-1}), G^{t-1}) \tag{23}$$

In relation 24, X_i^t is the position of the particle, P_i^t is the best solution of the particle and G^t is the best global solution. $\gamma = \omega \otimes F_1(X_i^{t-1})$ is the speed of the particle and F_1 is the mutation operator with probability ω. The expression $\beta = c_1 \otimes F_2(\gamma, P_i^{t-1})$ is part of the particle and F_2 is the crossover operator with probability c_1. The expression $\alpha = c_2 \otimes F_3(\beta, G^{t-1})$ shows the general part and F_3 performs the crossover operator with probability c_2. In each iteration of the algorithm, each particle is updated based on Equation (23) and a new solution is created.

4. 2. 2. Mutation Operator In order to create diversity in the solutions of the problem, the mutation operator has been used. The mutation operator used here is swap, an example of which is shown in Figure 4 [33]. For this purpose, two genes are randomly selected from the chromosome and the jobs in them are moved. According to Figure 4, two jobs 2 and 3 are randomly selected and moved with each other.

4. 2. 3. Crossover Operator In order to create a new solution, the crossover operator has been used in the equation of the problem. There are different modes of this operator, the order crossover (OX) mode is used here [28]. In this case, two points are randomly selected on two parents. The sequence between two points is directly transferred from the first parent to the first child and from the second parent to the second child. The sequence of jobs is transferred from the second point of the second parent to the first child. An example of this mode is shown in Figure 5. Two points are randomly selected on the parents. The numbers 5, 6 and 7 are directly transferred from the first parent to the first child. In the same way, the sequence of jobs is transferred to the first child after the second point of the second parent. It is done in the same way for the second child.

4. 3. Stop Condition After performing operations on the solution and creating a new population, the

Parent	4	7	8	2	5	10	6	3	9	1
Child	4	7	8	3	5	10	6	2	9	1

Figure 4. Swap Mutation operation

Parent 1	1	2	3	4	5	6	7	8	9	10
Parent 2	2	7	8	5	4	9	6	3	10	1
Child 1	*	*	*	*	5	6	7	*	*	*
Child 2	*	*	*	*	4	9	6	*	*	*
Child 1	2	8	4	9	5	6	7	3	10	1
Child 2	*	*	*	*	4	9	6	*	*	*
Child 1	2	8	4	9	5	6	7	3	10	1
Child 2	2	3	5	7	4	9	6	8	10	1

Figure 5. OX crossover operation

condition for stopping the algorithm execution is checked. In this article, the stop condition is 600 seconds. It means that after reaching the stopping condition, the execution of the algorithm is stopped and the best solution obtained is selected as the solution of the algorithm.

5. IMPROVEMENT OF GA AND PSO ALGORITHM

The In order to improve the results and performance of the GA and PSO algorithm, the shortest processing time (SPT) method is used in this section. The problem investigated in this article consists of three stages. Therefore, each job has three processing times. In order to use the SPT method, different modes of combining processing times can be considered [34]:

1. Considering the processing times of the first stage for each job
2. Considering the processing times of the second stage for each job
3. Considering the processing times of the third stage for each job
4. Considering the total processing times of the first and second steps for each job
5. Considering the sum of the second and third processing times for each job
6. Considering the total processing times of the first, second and third steps for each first

In this method, in order to create a new population, one of the solutions is created using SPT. Each of the 6 states mentioned is calculated for an initial solution. The value of the objective function is calculated for each state and the best state is selected and considered as a solution.

For example, suppose we have 3 jobs. Their processing times are shown in Table 2. In the first case, the jobs are sorted in ascending order of the processing time of the first stage. Therefore, job 2 is processed first, then job 1 and finally job 3. In the second case, the jobs

TABLE 2. An example of local search

Job	Processing time		
	Stage 1	Stage 2	Stage 3
1	12	31	57
2	2	87	23
3	91	76	14

are sorted in ascending order of the processing time of the second stage. Therefore, job 1 is processed first, then job 3 and finally job 2. In the third mode, the jobs are arranged in ascending order of the processing time of the third stage. Therefore, job 3 is processed first, then job 2 and finally job 1. In the fourth mode, the jobs are arranged in ascending order of the total processing time of the first and second stages. Therefore, job 1 (31+12) is processed first, then job 2 (87+2) and finally job 3 (76+91) is processed. In the fifth mode, the jobs are arranged in ascending order of the total processing time of the second and third stages. Therefore, job 1 (57+31) is processed first, then job 3 (14+76) and finally job 2 (23+87). In the sixth mode, the jobs are sorted in ascending order of the total processing time of the first, second and third stages. Therefore, job 1 (57+31+12) is processed first, then job 2 (23+87+2) and finally job 3 (14+76+91) is processed. It is created from the combination of GA and PSO algorithms with the local search of the combined Hybrid Genetic Algorithm (HGA) and Hybrid Particle Swarm Optimization (HPSO) algorithms. In HGA and HPSO algorithms, the steps of the algorithm are the same as GA and PSO respectively, with the difference that one of the initial solutions is checked using the mentioned 6 states and the sequence with the best value of the objective function is considered as an initial solution.

6. CALCULATION RESULTS

In this section, the calculation results are presented in two sizes, small and large. For the small size of the presented model, it has been solved in GAMS software and by using Cplex solver, and for the large size, it has been implemented in Java software. The different values of the parameters for creating different samples in small and large sizes are shown in Table 3. Due to the time limit applied to the presented algorithms, the maximum number of jobs for the large size is 100 in order to achieve the appropriate solutions in a reasonable number of repetitions. For the flow shop problem, based on the reported data of Xiong et al. [34], processing times are set in the range of 0 to 100. Also, the number of cars of the second and third stages according to the article. Jolai et al. [35] is considered equal to 2, 3 and 4.

According to the model's ability to solve problems accurately, the sizes of 3 to 10 jobs are considered as small size problems and the problems with size 15 to 100 are considered as large size problems.

6. 1. Parameter Setting

Different parameter values of GA and PSO algorithms have been used to solve the problem using one-way analysis of variance (ANOVA). For GA algorithm parameters, three different levels are considered according to Table 4.

For sample, with the values of $n = 9$, $m_2 = 4$ and $m_3 = 4$ from the combination of different levels of parameters using the orthogonal matrix according to Table 5, number of 9 different combinations of parameters are shown.

Each combination has been implemented 5 times and the average deviation from the best solution has been calculated using Equation (24).

$$ARE = \frac{obj - bestobj}{bestobj} \tag{24}$$

In relation 24, Average Relative Error (ARE) shows the amount of relative deviation. obj specifies the value of the objective function in each execution of the algorithm and $bestobj$ determines the best value obtained by all algorithms for each instance.

TABLE 3. Parameter values to create problem examples

Parameter	Range of parameters values	
	Small size	Large size
n	{3,4,5,6,7,8,10}	{15,20,25,35,45,50,70,90,100}
m_2	{2,3,4}	{2,4,6,8}
m_3	{2,3,4}	{2,4,6,8}
tf	U(1,100)	U(1,100)
ts	U(1,100)	U(1,100)
tt	U(1,100)	U(1,100)

TABLE 4. GA algorithm parameter levels

Factor	Level	Value
PS	1	30
	2	50
	3	70
P_{Cross}	4	0.7
	5	0.8
	6	0.9
P_{mut}	1	0.1
	2	0.2
	3	0.3

TABLE 5. Relative deviation mean values for ANOVA samples for GA algorithm

Experiment number	Parameters level			Ave
	PS	P_{Cross}	P_{mut}	
1	1	1	1	0.0303
2	1	2	2	0.0357
3	1	3	3	0.0638
4	2	1	2	0.0328
5	2	2	3	0.0491
6	2	3	1	0.0203
7	3	1	3	0.0682
8	3	2	1	0.0442
9	3	3	2	0.0679

Average and scatter diagrams were drawn for each of the parameters. For PS parameter, as the population increases, the average value first decreases and then increases. Therefore, the best value of this parameter is at level 2 and is equal to PS=50.

For P_{Cross} parameter, as the rate of crossover parameter increases, the average value first decreases and then increases. Therefore, the best value of this parameter is at level 2 and is equal to $P_{Cross} = 0.8$.

For P_{mut} parameter, by the increasing of the rate of mutation parameter, the average value increases in levels 2 and 3, but the dispersion value first increases and then decreases. In level one, the average values are lower than the other two levels. Therefore, level one is selected with the value of $P_{mut} = 0.1$.

According to the results, the values of $\text{PS} = 50$, $P_{Cross} = 0.8$ and $P_{mut} = 0.1$ are considered. For the parameters of PSO algorithm, three different levels are considered according to Table 6.

For example, with the values of $n = 10$, $m_2 = 2$ and $m_3 = 2$ from the combination of different levels of parameters using the orthogonal matrix according to Table 7, 9 different combinations of parameters are shown.

TABLE 6. PSO algorithm parameter levels

Factor	Level	Value
PS	1	70
	2	80
	3	90
C_1 , C_2	4	0.7
	5	0.8
	6	0.9
ω	1	0.1
	2	0.2
	3	0.3

Each combination has been executed 5 times and the average deviation from the best solution has been calculated using Equation (22). For the PSO algorithm, average and scatter diagrams were drawn for each of the parameters. For PS parameter, as the population increases, the mean value increases. Therefore, the best value of this parameter is at level 1 and equal to $\text{PS} = 70$.

For C_1 and C_2 parameters, as the parameter rate increases, the average value increases. Therefore, the best value of this parameter is at level 1 and equal to $C_1 = C_2 = 0.7$.

For parameter ω, as the mutation rate increases, the average value decreases. Therefore, the best value of this parameter is at level 3 and is equal to $\omega = 0.3$.

Therefore, the values of $PS = 70$, $C_1 = C_2 = 0.7$ and $\omega = 0.3$ were specified. We have considered the values. Experiments have been carried out for two sizes, small and large, which are shown in the following calculation results.

6. 2. The Results of Small Size Calculations

In order to check the performance of the presented algorithms, the results of calculations in small-sized problems have been compared with the results of GAMS to solve the problem accurately. GAMS calculations have been done with a time limit of 3600 seconds and for other algorithms with a time limit of n seconds. In order to check the performance of GAMS to solve the problem, different sizes have been considered. As the number of jobs increases, the solution time increases and it becomes difficult to find the solution. Therefore, up to 20 jobs have been produced for a small sample size and 44 samples of the combination of different amounts of jobs, second-stage machines and third-stage machines have

TABLE 7. Relative deviation mean values for ANOVA samples for PSO algorithm

Experiment number	Parameters level			Ave
	PS	C_1, C_2	ω	
1	1	1	1	0.0442
2	1	2	2	0.0581
3	1	3	3	0.0514
4	2	1	2	0.0544
5	2	2	3	0.0501
6	2	3	1	0.0789
7	3	1	3	0.0673
8	3	2	1	0.0702
9	3	3	2	0.0807

been created. Each algorithm has been executed 5 times in each sample. In order to compare the results of the algorithms, the deviation from the best solution has been calculated from Equation (25).

The calculation results are shown in Table 8. In this table, the percentage deviation of the CPLEX solution from the best solution of the algorithms is calculated from Equation (25).

$$RD = \frac{CplexObj - bestobj}{bestobj} \quad (25)$$

In the above equation, $CplexObj$ is the value of the objective function obtained from solving the model and $bestobj$ is the best obtained solution from solving the model and algorithms.

The value of the objective function (obj), the time it takes the model to reach the optimal solution or stop the solution (Time), the minimum (Min), maximum (Max) and average (Ave) values of the relative deviation (ARE) have been calculated. The average values obtained for all samples are shown at the end of Table 8. The average deviation of the total samples for Cplex, GA, HGA, PSO, HPSO methods is 0.197, 0.055, 0.038, 0.068 and 0.046, respectively. The lowest amount of deviation is related to the HGA algorithm. The obtained results of HGA algorithm are better than Cplex, which indicates the effectiveness of the algorithm. Also, due to the fact that the solution has been reached in less time, the efficiency of the algorithm is also high.

The average solving time is 10 seconds for the algorithms and 3102 seconds for Cplex. In 6 sizes, The model will be able to solve the problem in large dimensions, but according to the specifications of the processor system, with the increase in the problem size, the processor may not be able to process the model. Therefore, due to the inability of the processor to solve the model, an "out of memory" error occurs. the model is not able to reach the optimal solution (NA) and has a memory error, which is shown in the table with Out of Memory (OM). Also, considering the time limit of 3600 seconds, no solution has been received by Cplex. Therefore, it is possible to use algorithms to solve large size problems.

TABLE 8. Comparison of different algorithms with exact solution results in small size

Instance	n	m_2	m_3	CPLEX			GA			HGA			PSO			HPSO			Time (s)
				RD	Obj	Time (s)	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	
1	3	2	2	0.000	704	1.7	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	3
2	4	2	2	0.000	973	22.5	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	4
3	4	3	2	0.000	973	14.8	0.000	0.002	0.011	0.000	0.000	0.000	0.000	0.004	0.018	0.000	0.000	0.000	4
4	5	2	2	0.000	1087	980	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.068	0.000	0.000	0.000	5
5	5	3	3	0.000	1087	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.013	0.022	0.000	0.026	0.064	5
6	5	4	3	0.000	1087	1865	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.018	0.044	0.000	0.009	0.022	5
7	6	2	2	0.000	1524	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6
8	6	3	3	0.000	1524	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	6
9	6	4	3	0.000	1524	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.014	0.000	0.004	0.018	6
10	6	4	4	0.000	1524	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.010	0.019	0.000	0.003	0.010	6.0
11	7	2	2	0.012	1640	3600	0.000	0.023	0.038	0.020	0.024	0.038	0.020	0.024	0.038	0.020	0.072	0.111	7.0
12	7	3	2	0.000	1635	3600	0.000	0.007	0.036	0.000	0.000	0.000	0.000	0.001	0.006	0.000	0.000	0.000	7
13	7	3	3	0.000	1621	3600	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.082	0.000	0.008	0.022	7
14	7	4	2	0.003	1640	3600	0.000	0.007	0.024	0.000	0.000	0.000	0.000	0.003	0.007	0.000	0.011	0.047	7
15	7	4	3	0.000	1621	3600	0.000	0.004	0.013	0.000	0.001	0.006	0.000	0.004	0.018	0.000	0.000	0.000	7
16	8	2	2	0.058	2202	3600	0.000	0.010	0.020	0.006	0.006	0.008	0.006	0.020	0.042	0.022	0.063	0.096	8
17	8	3	2	0.002	2052	3600	0.000	0.002	0.006	0.000	0.001	0.003	0.000	0.004	0.022	0.030	0.032	0.041	8
18	8	4	2	0.014	2052	3600	0.006	0.012	0.031	0.008	0.008	0.008	0.000	0.013	0.061	0.000	0.010	0.029	8.0
19	8	4	4	0.002	2009	3600	0.000	0.002	0.002	0.000	0.001	0.002	0.002	0.011	0.041	0.002	0.015	0.068	8
20	9	2	2	0.043	2770	3600	0.019	0.066	0.087	0.000	0.034	0.075	0.015	0.038	0.061	0.009	0.083	0.131	9

21	9	3	2	0.021	2679	3600	0.005	0.016	0.029	0.000	0.020	0.051	0.006	0.039	0.099	0.032	0.053	0.080	9
22	9	3	3	0.000	2613	3600	0.000	0.016	0.030	0.000	0.024	0.050	0.004	0.032	0.064	0.033	0.037	0.051	9
23	9	4	2	0.019	2644	3600	0.000	0.009	0.025	0.002	0.020	0.046	0.040	0.057	0.078	0.000	0.028	0.069	9
24	9	4	3	0.000	2581 (OM)	2608	0.006	0.025	0.046	0.002	0.028	0.060	0.039	0.061	0.081	0.000	0.021	0.064	9
25	9	4	4	0.047	2581	3600	0.049	0.051	0.053	0.050	0.073	0.096	0.000	0.072	0.125	0.047	0.077	0.097	9
26	10	2	2	0.002	2947	3600	0.000	0.029	0.057	0.029	0.065	0.101	0.005	0.058	0.168	0.038	0.074	0.117	10
27	10	2	3	0.000	2836	3600	0.012	0.039	0.067	0.000	0.024	0.038	0.029	0.057	0.109	0.027	0.068	0.122	10
28	10	2	4	0.000	2730	3600	0.032	0.061	0.101	0.017	0.041	0.086	0.000	0.022	0.039	0.000	0.050	0.121	10
29	10	3	2	0.086	2990 (OM)	773	0.035	0.075	0.094	0.000	0.027	0.073	0.058	0.069	0.084	0.010	0.045	0.089	10
30	10	3	3	0.032	2863 (OM)	2633	0.004	0.017	0.044	0.005	0.023	0.044	0.000	0.034	0.108	0.005	0.023	0.037	10
31	10	4	4	0.025	2764	3600	0.001	0.028	0.066	0.018	0.036	0.053	0.003	0.020	0.035	0.000	0.018	0.083	10
32	15	2	2	0.231	7650	3600	0.195	0.274	0.360	0.178	0.202	0.237	0.100	0.301	0.518	0.000	0.040	0.105	15
33	15	2	3	0.208	7640	3600	0.144	0.195	0.256	0.059	0.142	0.202	0.095	0.203	0.291	0.000	0.048	0.087	15
34	15	3	2	-0.097	5920	3600	0.058	0.150	0.235	0.000	0.041	0.087	0.114	0.252	0.345	0.128	0.154	0.180	15
35	15	3	3	0.292	7589	3600	0.065	0.103	0.185	0.077	0.120	0.167	0.082	0.139	0.216	0.000	0.029	0.075	15
36	15	4	2	0.058	6184 (OM)	1603	0.029	0.055	0.080	0.055	0.059	0.070	0.069	0.116	0.186	0.000	0.027	0.069	15
37	15	4	3	0.100	6247	3600	0.016	0.056	0.114	0.000	0.020	0.054	0.040	0.125	0.211	0.025	0.079	0.210	15
38	15	4	4	0.138	6343	3600	0.045	0.083	0.123	0.029	0.048	0.066	0.000	0.093	0.162	0.003	0.036	0.074	15
39	20	2	2	0.813	20359	3600	0.257	0.345	0.473	0.030	0.065	0.111	0.120	0.180	0.269	0.000	0.072	0.240	20
40	20	3	2	0.806	19893	3600	0.020	0.066	0.122	0.060	0.133	0.175	0.069	0.128	0.246	0.000	0.163	0.265	20
41	20	3	3	NA	NA	3600	0.099	0.198	0.426	0.000	0.111	0.210	0.007	0.165	0.331	0.090	0.190	0.315	20
42	20	3	4	0.696	17735	3600	0.072	0.191	0.370	0.000	0.071	0.096	0.174	0.265	0.341	0.030	0.196	0.318	20
43	20	4	3	2.579	34663 (OM)	3600	0.059	0.095	0.140	0.000	0.112	0.180	0.058	0.151	0.250	0.035	0.110	0.182	20
44	20	4	4	2.180	30558 (OM)	3600	0.001	0.112	0.201	0.000	0.072	0.166	0.056	0.150	0.227	0.034	0.069	0.104	20
Average				0.197	-	3,102	0.028	0.055	0.090	0.015	0.038	0.060	0.028	0.068	0.117	0.014	0.046	0.087	10

6. 3. Large Size Calculation Results After checking the performance of the presented algorithms in small sizes, the calculation results of large sizes are presented in Table 9. As the size of the problem increases, the number of iterations of the algorithm decreases. Due to the time limit considered in order to solve the problems in large size, the number of jobs is considered up to 100 and calculations have been done in 35 different samples. Each sample has been run 5 times and the deviation from the best solution has been obtained. The average deviation of all samples for GA, HGA, PSO, HPSO methods is 0.277, 0.0677, 0.038, 0.2665 and 0.1215, respectively. The lowest amount of deviation is related to the algorithm, which is equal to 0.038.

TABLE 9. Comparison of different algorithms in order to solve the problem in large size

Instance	n	m_2	m_3	GA			HGA			PSO			HPSO		
				Min	Ave	Max	Min	Ave	Max	Min	Ave	Max	Min	Ave	Max
1	25	2	2	0.0524	0.1427	0.2157	0.0000	0.0200	0.0355	0.0869	0.2189	0.3889	0.0110	0.0308	0.0550
2	25	2	4	0.0270	0.2539	0.5199	0.0026	0.0139	0.0423	0.0737	0.1411	0.3046	0.0000	0.0211	0.0287

3	25	4	2	0.0577	0.0899	0.1636	0.0112	0.0802	0.1893	0.0000	0.0598	0.1386	0.0192	0.1191	0.2138
4	25	4	4	0.0080	0.0864	0.1340	0.0000	0.1121	0.1891	0.0377	0.1124	0.2088	0.1479	0.1878	0.2407
5	35	2	2	0.3270	0.4234	0.5283	0.0000	0.0061	0.0225	0.2675	0.3449	0.4579	0.0000	0.2392	0.4796
6	35	2	4	0.3637	0.4388	0.5376	0.0552	0.0894	0.1409	0.2104	0.3699	0.5489	0.0000	0.0669	0.1409
7	35	4	2	0.0113	0.1861	0.2962	0.0024	0.0158	0.0372	0.1346	0.2703	0.3474	0.0000	0.0423	0.1063
8	35	4	4	0.0000	0.0522	0.1727	0.0241	0.0464	0.0594	0.0607	0.1244	0.2546	0.0210	0.0411	0.0683
9	45	2	2	0.4757	0.5287	0.5635	0.0133	0.0637	0.1141	0.4651	0.5218	0.6352	0.0000	0.0363	0.0952
10	45	4	2	0.2182	0.3000	0.3854	0.0000	0.0077	0.0264	0.2451	0.3012	0.3345	0.0047	0.1090	0.4196
11	45	4	4	0.4019	0.4333	0.4705	0.0000	0.0565	0.1257	0.3786	0.4204	0.4692	0.0364	0.1440	0.3757
12	45	6	2	0.1678	0.2074	0.2628	0.1164	0.1592	0.2039	0.1731	0.2040	0.2630	0.0000	0.1375	0.3407
13	45	6	4	0.1290	0.1542	0.1753	0.0000	0.0855	0.1801	0.0761	0.1464	0.1969	0.0036	0.1019	0.2176
14	65	2	2	0.4325	0.4463	0.4562	0.0000	0.0537	0.1148	0.4165	0.4385	0.4671	0.0440	0.3425	0.5060
15	65	4	2	0.2484	0.2664	0.2906	0.0000	0.0794	0.1481	0.2354	0.2707	0.3129	0.0233	0.1169	0.2810
16	65	4	4	0.5721	0.6049	0.6542	0.1055	0.2954	0.5532	0.2455	0.3966	0.6357	0.0000	0.2648	0.7134
17	65	6	2	0.1170	0.1666	0.1867	0.0089	0.0768	0.1139	0.1544	0.1915	0.2073	0.0000	0.0665	0.1861
18	65	8	6	0.1235	0.1462	0.1645	0.0253	0.0527	0.0845	0.1129	0.1621	0.1883	0.0000	0.0299	0.0532
19	75	2	2	0.3290	0.3689	0.3948	0.0000	0.1270	0.3680	0.3286	0.3700	0.4136	0.0021	0.0945	0.2987
20	75	4	2	0.2997	0.3818	0.4140	0.0026	0.0099	0.0201	0.3320	0.3707	0.4107	0.0000	0.1063	0.3513
21	75	4	4	0.3660	0.3847	0.3945	0.0000	0.0330	0.0879	0.3784	0.4187	0.4559	0.0136	0.1894	0.3098
22	75	6	2	0.2486	0.2647	0.2912	0.0539	0.1616	0.2738	0.2747	0.3073	0.3295	0.0000	0.1211	0.2883
23	75	6	6	0.3339	0.3519	0.3781	0.0824	0.1370	0.2038	0.3324	0.3621	0.3819	0.0000	0.2007	0.3475
24	85	2	4	0.3293	0.3877	0.4161	0.0000	0.0152	0.0272	0.4087	0.4357	0.4789	0.0122	0.1263	0.4459
25	85	4	2	0.2292	0.2549	0.2933	0.0000	0.0646	0.2304	0.2511	0.2739	0.2906	0.0282	0.1399	0.3079
26	85	6	4	0.1963	0.2062	0.2139	0.0000	0.0134	0.0410	0.1984	0.2150	0.2310	0.0033	0.0435	0.1953
27	85	8	4	0.1361	0.1477	0.1586	0.0000	0.0648	0.1677	0.1485	0.1743	0.1888	0.0016	0.1075	0.1867
28	95	2	4	0.3610	0.4201	0.4715	0.0000	0.0204	0.0689	0.2069	0.2224	0.2359	0.0245	0.2009	0.4296
29	95	4	2	0.2363	0.2448	0.2646	0.0272	0.0807	0.1544	0.2349	0.2665	0.2987	0.0000	0.0937	0.2724
30	95	6	4	0.1181	0.1312	0.1452	0.0325	0.0568	0.0929	0.1130	0.1411	0.1569	0.0000	0.0717	0.1525
31	95	8	4	0.0913	0.1039	0.1134	0.0700	0.0953	0.1253	0.1051	0.1145	0.1271	0.0000	0.0708	0.1284
32	100	2	4	0.4646	0.4891	0.5038	0.0000	0.0544	0.2084	0.2599	0.2861	0.2937	0.0090	0.2009	0.5093
33	100	4	2	0.2648	0.2804	0.3009	0.0000	0.0115	0.0179	0.2906	0.2999	0.3242	0.0394	0.1539	0.3056
34	100	6	4	0.2264	0.2361	0.2519	0.0413	0.0803	0.1087	0.2510	0.2616	0.2754	0.0000	0.1435	0.2736
35	100	8	4	0.0988	0.1136	0.1232	0.0000	0.0276	0.0753	0.0830	0.1137	0.1395	0.0386	0.0901	0.1345
Average				**0.2304**	**0.2770**	**0.3230**	**0.0193**	**0.0677**	**0.1329**	**0.2163**	**0.2665**	**0.3255**	**0.0138**	**0.1215**	**0.2703**

6. 4. Sensitivity Analysis In order to analyze and investigate the effect of each of the parameters of the problem, in this section, the comparative diagrams of the algorithms are presented. According to Figure 6, deviation values for 4 algorithms are shown. The average values of all samples corresponding to the minimum, average and maximum for all algorithms have been calculated.

As can be seen, the HGA algorithm has the lowest values for Ave and Max and it has the best performance compared to other algorithms.

According to Figure 7, the average values of deviations have been calculated and plotted for different standards of work.

By increasing of jobs number, the amount of deviation increases on average, and the slope of the graph

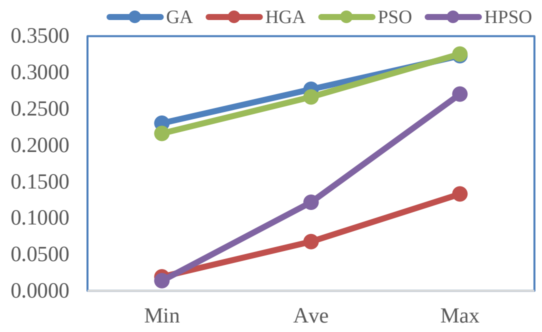

Figure 6. Comparison of mean of Min, Ave and Max deviation values from the best solution of different algorithms

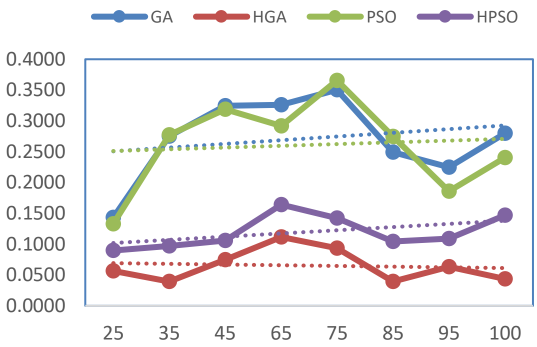

Figure 7. Comparison of the average values of the average deviation based on the number of jobs

is upward for all algorithms. For the HGA algorithm, the slope of the graph is less. Therefore, it performs better than other algorithms.

According to Figures 8 and 9, the average values of deviation for different values of the machines in the second and third stages have been calculated and plotted.

An increase in the number of machines in the second stage, the amount of deviation decreases for all

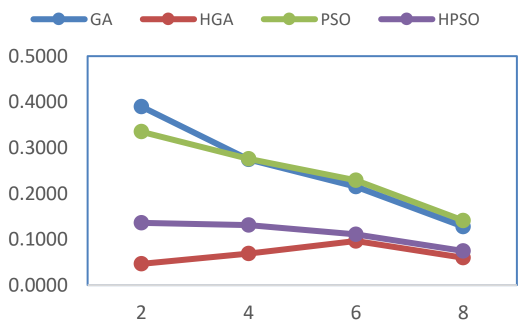

Figure 8. Comparison of the mean values of the average deviation of the machines number in the second stage

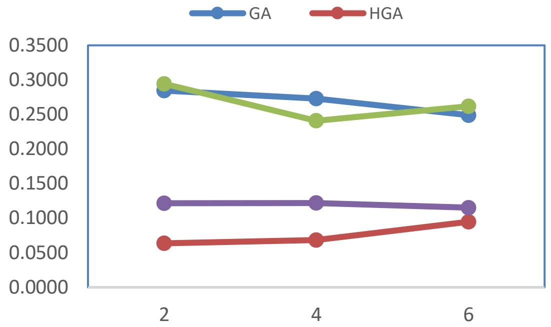

Figure 9. Comparison of the mean values of the average deviation of the machines number in the third stage.

algorithms. Therefore, the more the number of machines increases, the better, but the number of machines should be increased according to the available capital. The values related to the HGA algorithm are lower than the another algorithms.

According to learning-based algorithm [36], the Adaptive Polyploid Memetic Algorithm (APMA) [37, 38] alternative method of solutions were presented. Finally, the universal island-based metaheuristic algorithm (UIMA) [39], the nature-inspired evolutionary algorithm [40], the discrete gravitational search algorithm [41], multi-objective scheduling were discussed and compared.

7. CONCLUSION

Equipment overhaul is a fundamental issue in improving the performance of any system. In this article, we investigated the equipment overhaul by a three-stage flow shop system. The works done and the articles presented in this field were reviewed.

A number of equipments are available for repairs and overhaul. A new mathematical model was presented. The planning and scheduling of the overhaul of this equipment in a flow shop system with parallel machines in the second and third stages should be done in such a way that the total time of the equipment overhaul is minimized. In order to schedule, for small sizes with few equipment, a new mathematical model is presented. Considering that the problem is NP-hard, it is not possible to solve it in large dimensions by the model. Therefore, approximate methods are provided to solve the problem in large size. Genetic algorithms (GA) and Particle Swarm Optimization (PSO) were used for this purpose. These algorithms were improved using local search. In the small size, the results were compared with the Cplex method, and the efficiency and effectiveness of the presented algorithms were determined. Then the problem was solved in large sizes. According to the obtained results, the improved genetic algorithm has better results than other algorithms and is the best algorithm that can be used for the relevant problem.

Some cases can be mentioned as future suggestions:
1. Using other functions such as earliness and tardiness
2. Other solution methods can be used and the results can be compared.
3. Parallel machines can also be considered for the first stage.

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Persian Abstract

چکیده

در این مقاله، تعمیرات اساسی تجهیزات در یک مسئله زمان‌بندی جریان کارگاهی چند مرحله‌ای در نظر گرفته شده است. در این مسئله در مرحله اول تجهیزات دمونتاژ شده و در مرحله دوم توسط کارگاه های موازی تعمیرات اساسی روی تجهیزات انجام می شود و در مرحله سوم عملیات مونتاژ در کارگاه های موازی صورت می‌گیرد. در نظر گرفتن اورهال سه مرحله ای با ماشینهای موازی در مراحل دوم و سوم در صنعت اورهال جدید است. توالی پردازش تجهیزات در مرحله اول تعیین می شود و همچنین تخصیص و توالی تجهیزات در مرحله دوم و سوم باید به گونه ای انجام شود که کل زمان اتمام کارها به حداقل برسد. برخلاف اغلب مقالات توالی پردازش کارها در تمام مراحل یکسان نیست و با استفاده از رمزگشایی تغییر میکند. برای نوآوری بعدی: به منظور حل مسئله، یک مدل ریاضی جدید ارائه شده است. دو الگوریتم بهبودیافته جدید به منظور حل مسئله در ابعاد بزرگ ارائه شده است. با توجه به اینکه مسئله NP-hard است و حل مسئله در ابعاد بزرگ توسط مدل زمان بر است، دو الگوریتم فراابتکاری الگوریتم ژنتیک و بهینه سازی ازدحام ذرات همراه با بهبودهای لازم برای حل ابعاد بزرگ ارائه شده است. با استفاده از روش ابتکاری کوتاه‌ترین زمان پردازش (SPT)، این دو الگوریتم بهبود یافته و الگوریتم‌های ژنتیک ترکیبی و بهینه سازی ازدحام ذرات ترکیبی ارائه شده‌اند. به منظور دستیابی به نتایج بهتر با شرایط فعلی تنظیم پارامترها توسط آنالیز واریانس یک طرفه (ANOVA) انجام شده است. نهایتا بهبود عملکرد تجهیزات با بکارگیری مسئله ارائه شده میسر میگردد.