International Journal of Engineering

Journal Homepage: www.ije.ir

Application of Random Radial Point Interpolation Method to Foundations Bearing Capacity Considering Progressive Failure

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PAPER INFO

Paper history:
Received 16 July 2022
Received in revised form 15 October 2022
Accepted 23 October 2022

Keywords:
Bearing Capacity of Foundation
Radial Point Interpolation Method
Probabilistic Analysis
Monte Carlo Simulation
Progressive Failure

ABSTRACT

In conventional analyzes of foundations failure, strengh parameters are assumed constant. However, during the failure, soil resistance exhibits maximum and residual amounts, and its strength decreases prematurely by increasing the plastic strain. In addition to change soil strengh parameters in the progressive mechanism, the non-uniform nature of the soil also causes spatial variations of these parameters. Therefore, geotechnical systems should be considered in terms of the uncertainty of soil parameters values, uncertainly using the concepts of statistics and probabilities. The purpose of this study is to investigate foundations in meshless method. In this article, radial point interpolation method (RPIM), a meshless method is proposed for simulation of soil foundation. Difficulties of methods related to mesh are solved by using this method. A code has been developed based on this method and some examples are solved for analyzing the code. In this research, a RPIM in combination with a random field was used to model the spatial variations of soil strengh properties and foundation bearing capacity analysis. For probabilistic analysis, random field is also used to determine the cohesion and the friction angle as well as the dilation angle based on their mean values and standard deviation. In order to investigate the application of the point interpolation method with randomized radial functions, a foundation with definite geometry has been analyzed deterministic and probabilistic and its safety factor has been investigated. Based on the analysis of the progressive failure modeling, it is concluded that the actual failure of the soil and the occurrence of continuous displacements occur simultaneously with the formation of a progressive mechanism of soil failure and the arrival of the slipping path to the ground. In the following, probabilistic distribution functions of the safety factor were determined by probabilistic analysis and the production of random fields, and then the statistical parameters are calculated.

***doi**: 10.5829/ije.2023.36.02b.07*

NOMENCLATURE

κ^{ps}	development of stiffness strain	c'_p	maximum cohesion
$\Delta\varepsilon_1^{ps}$ $\Delta\varepsilon_3^{ps}$	development of the plastic strain along the direction of maximum and minimum stress are the main	c'_r	residual cohesion
$\Delta\varepsilon_m^{ps}$	development of the volumetric plastic shear strain correction factor	φ'_p	maximum internal friction angle
κ_r^{ps} κ_p^{ps}	strain values are threshold	φ'_r	residual internal friction angle

1. INTRODUCTION

Analysis of soil bearing capacity under foundations is always one of the challenging problems that has been remained in geotechnical engineering and it has been the subjected by numerous researches over the past years [1]. In such issues, the occurrence of soil rupture and design failure can be caused by not paying attention to locative changes in soil properties and complexity of the deterioration mechanism. Or it can be caused due to the problems and limitations of the modeling tools under consideration, which could leads to financial and fatality loss in engineering projects.

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Please cite this article as: S. Hashemi, R. Naderi, Application of Random Radial Point Interpolation Method to Foundations Bearing Capacity Considering Progressive Failure, ***International Journal of Engineering, Transactions B: Applications***, Vol. 36, No. 02, (2023), 264-275

Numerous studies had been done in the field of numerical modeling soil interaction and behavior at failure and analysis of its deterioration mechanism [2, 3]. However, the real concept of soil failure mechanism is not fully understood and its modeling is always accompanied by ambiguities and uncertainties. In most studies, it is assumed that the failure occurs simultaneously along the slip surface in the soil mass. However, the plastic strains were not uniform due to an increase in loading or decrease in soil strength and thereby the process of failure will be progressive. In addition, in most studies it was assumed that the soil parameters remain unchanged even at large strains. Considering this assumption in issues such as analysis soil bearing capacity under foundations is incorrect, Because soil strengh parameters show maximum and residual values, also soil strength process decreases with increasing plastic strain. According to this concept, numerical analyzes of progressive failure has been used in various geotechnical issues, until now [4-6].

In addition to changes of soil strengh parameters during the progressive mechanism, the non-uniform nature of the soil causes spatial changes in these parameters. Soil has always been recognized as heterogeneous material and its spatial specification changes has important role in soil behavior. Therefore concepts of statistics and probabilities should be used in geotechnical systems which have uncertainty in soil parameters values. In purpose of checking the locative changes of soil strengh parameters effects on soil behavior, soil modeling done in form of one multidimensional process along with several random parameters in probabilistic analysis. Random field theory is the basis of providing such model. Details of random field theory and its application in geotechnical engineering are fully described by Zdravkovic et al. [7]. By use of this theory numerous researchers have examined the impact of spatial changes in soil parameters [8, 9].

It should be noted that deterministic and probabilistic analysis of the progressive failure process are only possible by using numerical techniques such as finite elements method that are able to simulate the creation and development of a shear zone with a focus on strain. Although finite elements method (FEM) widely used at analysis of foundation bearing capacity. However, this method has problems like stress discontinuity at boundary element and low accuracy at analysis of large deformations and weakness in convergence caused by entanglement elements. This category of finite elements method problems basically related to meshing.

In the context of deformation problems, the finite elements method suffers from several problems, which are mainly caused by its complexity of mesh element. In fact, main weakness of methods which is perform their analysis based on mesh is by every changes in the geometry of the problem, mesh needs to regenerated and this is a time-consuming task and in addition it increases complexity and decreases the accuracy of results. Other problem of these methods include low accuracy in stress calculation, especially in the case of complex phenomena such as crack propagation or phase change (due to severe discontinuities).

Therefore one suitable way to get ride of this difficulties is using meshless methods (MFMs) to analysis the stability issues with enough accuracy. Meshless methods developed by Lucy [10] using the smoothed particle hydrodynamics (SPH) method for the modeling physical astronomy phenomena. Nowadays, this method is known as an effective numerical tool for analyzing various engineering problems and several studies have been conducted on the application of this method in various branches such as geotechnical engineering [11-13].

Another issue in analysis of instability of soil problems is necessity of combining desire numerical method by concepts of statistics and probability. Soil in its natural is considered as a material with the most changes in behavioral characteristics among engineering materials. Therefore, uncertainty in geotechnical engineering and soil mechanics is considered a reality and considering it has made the engineering perspective more open in analysis of stability issues.

In this research, it has been used point interpolation method with radial functions in combination with random field for modeling locative variations of soil strengh parameters and analysis of soil bearing capacity under foundations. In order to consider the progressive failure of soil, the elastoplastic solution method has been used with the extended Mohr-Coulomb model in terms of strain-softening behaviour. Firstly, strengh parameters such as cohesion and internal friction angle are considered indefinitely with mean values and standard deviation to perform this analysis. Then random fields of indefinite parameters are generated by examining the correlation between domain points. These data along with other parameters values use as input to point interpolation method with radial functions in analysis of soil bearing capacity of foundations. Probabilistic analysis of this method is placed in combination with Monte Carlo simulation. In other words, stability analysis is repeated as much as the number of random fields created. The output of this process is probabilistic distributions for soil bearing capacity safety factor.

In meshless method, due to creation of large number of unknowns in equation that should solved simultaneously, the volume calculations is large. So first step in using this method is using computer programming to control this system of equations. In this research, MATLAB programming software is used as a matrix calculator for analysis in combination with elastoplastic theory and progressive failure model to analyze instability in soil problems. Key features of using MATLAB are simplicity and ease of working with it, a huge library of predefined functions, high ploting power and finally, having a comprehensive and complete guide on how to execute commands.

In this paper authors focused on analyzing the influence of some relevant aspects of random characterization of soil by means of numerical algorithm, as follows:

- the rule of anisotropy in random field approach to soil parameters, implemented by analyzing different values of correlation length along vertical and horizontal direction;
- to investigate random variability of soil properties based on progressive failure data resulting.

Hereinafter is organized as follows. The next sections briefly describes the progressive rupture. Then formulations of RPIM method are described and random field is explained. In following sections, we perform numerical model, deterministic and probabilistic analysis of foundation bearing capacity which are described. Then results of the analysis are presented. We finalized this article by the conclusions section.

2. PROGRESSIVE FAILURE IN SOIL

The failure caused by large displacements in soil problems is made by the progressive expansion of inelastic shear bands. Over time, many efforts have been made to identify the spread of failure in soil, and until now, non-uniform distribution of strain is known as one of main causes of progressive failure. Suitable conditions of progressive failure provides by reduction of shear strength in proportion to shear plastic strain, from its maximum to residual value. Analysis of such issues is possible by applying a model considering strain-softening behaviour. Some of complex problems in geotechnical engineering are analyzing the slope, bearing capacity and other soil problems in regard to strain-softening behavior. In such problems, specificity of material is changed at different stages from maximum to residual value, and failure is occured by applying strength reduction technique with increasing the strain. In general, this type of failure, failing happen in part of soil in which strains are locally formed. Soil strength decreases from a maximum value to residual value by increasing strain in this area. The application of reprocessing stress method causes expansions on shear zone and its penetration into adjacent soil. Therefore, the slip surface is following progressive expansion along with area by mean strength between maximum and residual value.

Various strain-softening behaviours have always been proposed to calculate soil strength parameters during strain changes. Among these, we can refer to extended model of Mohr-Columbus, which allows materials to behave with strain-softening. In this model, the properties are defined as linear functions of a piece of plastic shear strain κ^{ps}. The development of hardening strain is also presented as follows:

$$\Delta\kappa^{ps} = \frac{1}{\sqrt{2}}\sqrt{(\Delta\varepsilon_1^{ps} - \Delta\varepsilon_m^{ps})^2 + (\Delta\varepsilon_m^{ps})^2 + (\Delta\varepsilon_3^{ps} - \Delta\varepsilon_m^{ps})^2} \qquad (1)$$

where $\Delta\varepsilon_1^{ps}$ and $\Delta\varepsilon_3^{ps}$ represented plastic strain of maximum and minimum main stress. $\Delta\varepsilon_m^{ps}$ is development of the volumetric plastic shear strain that is defined as follows:

$$\Delta\varepsilon_m^{ps} = (\Delta\varepsilon_1^{ps} + \Delta\varepsilon_3^{ps})/3 \qquad (2)$$

According to studies in soil problems, a model with three-component partial linear function with strain-softening behaviour according to Figure 1 is often used [7].

The characteristics of this model are presented in the form of following relations:

$$c' = \begin{cases} c'_p & \kappa^{ps} \le \kappa_p^{ps} \\ c'_r + \dfrac{\kappa^{ps} - \kappa_r^{ps}}{\kappa_p^{ps} - \kappa_r^{ps}}(c'_p - c'_r) & \kappa_p^{ps} \le \kappa^{ps} \le \kappa_r^{ps} \\ c'_r & \kappa^{ps} \le \kappa_r^{ps} \end{cases} \qquad (3)$$

$$\varphi' = \begin{cases} \varphi'_p & \kappa^{ps} \le \kappa_p^{ps} \\ \varphi'_r + \dfrac{\kappa^{ps} - \kappa_r^{ps}}{\kappa_p^{ps} - \kappa_r^{ps}}(\varphi'_p - \varphi'_r) & \kappa_p^{ps} \le \kappa^{ps} \le \kappa_r^{ps} \\ \varphi'_r & \kappa^{ps} \le \kappa_r^{ps} \end{cases} \qquad (4)$$

In these relations c'_p, c'_r are the maximum and residual cohesion, as well as φ'_p and φ'_r which is the maximum and residual internal friction angle, respectively. Also κ_r^{ps}, κ_p^{ps} are threshold strain values. The values of these parameters are obtained by performing conventional tests [7, 14].

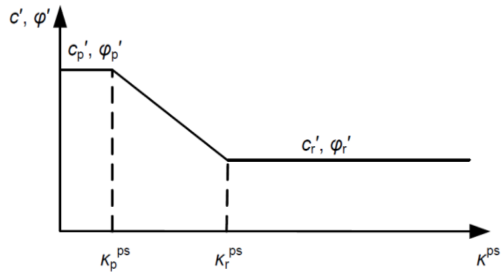


Figure 1. Strain-softening model

3. POINT INTERPOLATION METHOD WITH AMPLIFIED RADIAL FUNCTION

Point interpolation method is one of kind meshless methods that uses finite series form to represent the approximation function. For this purpose the scalar function u(x,y) is considered in two-dimensional space created by a bunch of scattered nodes. The point interpolation relationship of the function u(x,y) at the point (x,y) is given as follows:

$$u(x,y)=\sum_{i=1}^{m}B_i a_i \tag{5}$$

In this relation Bi(x,y) is base function in two-dimensional coordinates, m is the number of the base function and ai is the coefficient related to the base function. In the point interpolation method, basic functions can be selected as polynomial functions. As a result, derivation shape functions are easily performed. Simplicity and appropriate accuracy of results are key features of this method. However, point interpolation method with polynomial basic functions always suffers from solvation of individual torque matrix. For fixing this, interpolation method used radial functions. On the other hand, in order to take advantage of polynomial functions, we can strengthen the model by adding polynomial phrase as basic functions until desire order. In this case, point interpolation equation with the amplified radial basis functions for u(x,y) is written as follows:

$$u(x,y)=\sum_{i=1}^{n}R_i(x,y)a_i+\sum_{j=1}^{m}P_i(x,y)b_i=\mathbf{R}^T(x,y)\mathbf{a}+\mathbf{P}^T(x,y)\mathbf{b} \tag{6}$$

In this equation, R and P are radial basic functions and polynomial of n number points nodes at local support domain point with (x,y) coordinates and m is a number of polynomial phrase items use to basic functions. The phrases of polynomial functions in specific spatial coordinates are selected using the Pascal triangle [15]. To determine the values of ai and bj, it is necessary to form n + m equation. In this regard, n equations are created by applying node values to the function u(x,y) as follows:

$$u_k=u(x_k,y_k)=\sum_{i=1}^{n}R_i(x_k,y_k)a_i+\sum_{j=1}^{m}p_j(x_k,y_k)b_j \quad k=1,2,\dots,n \tag{7}$$

Equation (7) rewrite in Matrix form as follows:

$$\mathbf{U_s}=\mathbf{R_Q a}+\mathbf{P_m b} \tag{8}$$

In this equation, RQ and Pm are from matrices in the following form in two-dimensional space, respectively:

$$\mathbf{R_Q}=\begin{bmatrix} R_1(x_1,y_1) & R_2(x_1,y_1) & \dots & R_n(x_1,y_1)\\ R_1(x_2,y_2) & R_2(x_2,y_2) & \dots & R_n(x_2,y_2)\\ \vdots & \vdots & \ddots & \vdots\\ R_1(x_n,y_n) & R_2(x_n,y_n) & \dots & R_n(x_n,y_n)\end{bmatrix} \tag{9}$$

$$\mathbf{P_m}=\begin{bmatrix} p_1(x_1,y_1) & p_2(x_1,y_1) & \cdots & p_m(x_1,y_1)\\ p_1(x_2,y_2) & p_2(x_2,y_2) & \cdots & p_m(x_2,y_2)\\ \vdots & \vdots & \ddots & \vdots\\ p_1(x_n,y_n) & p_2(x_n,y_n) & \cdots & p_m(x_n,y_n)\end{bmatrix} \tag{10}$$

In radial functions, the only available variable is ri , which is distance between two spatial coordinates (x,y) and (xi,yi). Different radial functions provided for performing analysis. In this research, Multiquadratics radial function with following form of equation has been used:

$$R_i(x,y)=(r_i^2+c^2)^q=[(x-x_i)^2+(y-y_i)^2+c^2]^q \tag{11}$$

In this relation, c and q are shape parameters. The best value for these parameters is obtained based on type of problem and performing numerical tests. In this study, according to analysis on shape parameters in solid mechanics, the values of 1.42 and 0.98 have been used for c and q , respectively [15]. The m remaining equation will be obtained from unique actions conditions answer as follows:

$$\sum_{i=1}^{n}p_j(x_i,y_i)a_i=0 \quad j=1,2,\dots,m \tag{12}$$

Or in matrix form:

$$\mathbf{P_m^T}a=0 \tag{13}$$

Therefore, Equation (8) is rewritten in the following form:

$$\mathbf{\bar{U}_s}=\begin{Bmatrix}\mathbf{U_s}\\ \mathbf{0}\end{Bmatrix}=\begin{bmatrix}\mathbf{R_Q} & \mathbf{P_m}\\ \mathbf{P_m^T} & \mathbf{0}\end{bmatrix}\begin{Bmatrix}\mathbf{a}\\ \mathbf{b}\end{Bmatrix}=\mathbf{Ga_0} \tag{14}$$

In this regard:

$$\mathbf{a_0}=[a_1\ \ a_2\ \dots\ a_n\ \ b_1\ \ b_2\ \dots\ b_m] \tag{15}$$

$$\mathbf{\bar{U}_s}=[u_1\ \ u_2\ \dots\ u_n\ \ 0\ \ 0\ \dots\ 0] \tag{16}$$

Therefore, according to Equation (14), we can write:

$$\mathbf{a_0}=\begin{Bmatrix}\mathbf{a}\\ \mathbf{b}\end{Bmatrix}=\mathbf{G^{-1}\bar{U}_s} \tag{17}$$

Finally, by combination of Equations (17) and (6) we have:

$$u(x)=\{\mathbf{R}^T(x)\ \ \mathbf{P}^T(x)\}\mathbf{G^{-1}\bar{U}_s}=\mathbf{\bar{\Phi}^T}(x)\mathbf{\bar{U}_s} \tag{18}$$

In this regard:

$$\mathbf{\bar{\Phi}^T}=\{\varphi_1(x)\ \varphi_2(x)\cdots\varphi_n(x)\ \varphi_{n+1}(x)\ \varphi_{n+2}(x)\cdots\varphi_{n+m}(x)\} \tag{19}$$

After calculating vector of shape functions, for desire domain of support with n number of nodes in it, the vector of main shape function is considered as follows:

$$\mathbf{\Phi^T}=\{\varphi_1(x)\ \varphi_2(x)\cdots\varphi_n(x)\} \tag{20}$$

4. RANDOM FIELD

Soil is one of the materials in which its characteristics are related to the location. In other words, properties of these materials vary from one place to other place. So it is not possible to use usual methods of statistics and probability, which are based on independence observations of samples. On the other hand, during soil exploration operations, its characteristics are obtained only in sampled places. However, values of these specifications remain unknown in other parts of the area. In this regard, random field theory is known solution used to obtain random values in different parts of the domain area to deal with uncertainty [16]. Random field theory can effectively explain the spatial variation of soil properties by correlation function. In fact, this theory is a forecasting method to predict desired characteristics of other points based on limited available information. In this method, a specific soil feature is almost identical at very close points and will not be related at distant points. According to this purpose, relationship between the points in domain area are defined by the correlation function. Among the existing correlation functions. We can mention Gaussian, triangular, etc. correlation functions. In this research, exponential correlation function is used to construct a correlation matrix as follows:

$$\rho = \exp\left(-\frac{\tau_x}{\theta_x} - \frac{\tau_y}{\theta_y}\right) \tag{21}$$

In this equation, τx and τy are distances in x and y directions between two points under consideration and θx and θy are correlation lengths in the x and y directions, respectively. The length of correlation represents the threshold distance that shows effect of parameters correlation. The correlation function is applied to all points in relation to other points and the correlation matrix for n points system is performed as follows:

$$\boldsymbol{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{12} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & \rho_{nn} \end{bmatrix} \tag{22}$$

In next step, by decomposing this matrix using Cholesky method, a top and bottom triangular matrix is obtained:

$$\boldsymbol{\rho} = \mathbf{L}\mathbf{L}^{\mathbf{T}} \tag{23}$$

Then, desire values of constructing a random field are obtained by multiplying the lower triangular matrix L and normal random values of standard Z that generated by random numbers in standard norm with mean values of zero and standard deviation of one, as follows:

$$\mathbf{G} = \mathbf{L} \times \mathbf{Z} \tag{24}$$

Finally, by using G matrix values and using the following equation, random field for variable x is obtained by the following expresion:

$$\mathbf{X}_{\mathbf{R.F}} = \mu(x) + \mathbf{G}.\sigma(x) \tag{25}$$

In this equation, $\mu(x)$ and $\sigma(x)$ are mean and standard deviation of the random field of x respectively.

5. NUMERICAL MODELING AND ANALYSIS METHOD

The intended specifications for soil behavior at maximum and residual conditions are given in Table 1. The soil under the foundation including a layer of soil with the characteristics of conventional parameters are stated in Table 2. In this research, the bearing capacity of the foundation is investigated in the form of plain strain conditions. In this type of analysis, geometry, problem properties, and field variables are defined in terms of two spatial coordinates, x and y.

5. 1. Geometry and Boundary Conditions

In order to investigate the application of point interpolation method with random radial functions, a foundation with the geometry shown in Figure 2 was analyzed deterministic and the probabilistic. The soil under the foundation with a depth of 5 meters and a radius of 10 meters from the center of the foundation is examined. This geometry is determined based on formation of a failure wedge under foundation and depth of its stress impact [17]. The boundary conditions are as follows: the bottom of soil domain (BD) is fixed in both directions, while on other sides of domain (AB and CD), horizontal

TABLE 1. Values of soil resistance parameters at maximum and residual state

Parameter	Maximum amount	Residual amount
Internal friction angle (°)	20	15
Cohesion (kN / m 2)	20	5
Dilation angle (°)	10	0.0005

TABLE 2. Values of definite soil parameters

Parameter	The amount of
Modulus of elasticity (kN/m^2)	100000
Specific gravity (kN/m^3)	16
Poisson ratio	0.3

displacements are fixed only which is allowing nodes for a vertical displacement.

5. 2. Methodology The first step in analyzing point interpolation method with radial functions is to define domain of problem by distributing its node. The number and arrangement of nodes are chosen so that simulated body is as close to reality as possible. The choice of node arrangement is often dictated by the geometry of problem and number of independent points needed to define the scope of problem. In order to perform problem analysis, soil amplitude under the studied foundation is modeled by nodes according to Figure 3 by point interpolation method with radial functions.

Integration with surface or volume is required in order to estimate stiffness and force matrix. Therefore, it is necessary to use appropriate numerical integration method to calculate relationships in problem domain. In this research, stress point method due to high convergence power has been used to perform numerical integration. For this purpose, we defined points in domain problem between nodes of point interpolation method with radial functions. Then voronoi cells are created around these points, so that area allocated to each stress point by each cell represents the integral weight of that stress point. Figure 4 shows defined stress points along with their Voronoi cells.

The result of bearing capacity of foundation analysis is safety factor parameter. This quantity is equal to coefficient on which main parameters of strengh, c and φ , are divided and thus decreased shear strength under a constant weight force, resulting in failure. For this purpose, during analysis, weight load is obtained by integral of each support domain according to considered value of specific gravity of materials and applied to problem during its development. Then a strength reduction loop is considered in program, which gradually reduces the soil strength to perform failure by performing elastoplastic analysis of soil. Accordingly, multiplied resistance parameters of soil are expressed as follows:

$$\begin{aligned} \varphi_f &= \arctan(\tan\varphi/srf) \\ c_f &= c/srf \end{aligned} \tag{26}$$

The second step is to perform probabilistic analysis, choosing appropriate distribution for the input parameters, which in this research are cohesion, internal friction angle and dilation angle, which are considered as probabilities with specified mean values and standard deviation.

Correlation between different points of domain is checked in third step of this method. In fact, at this stage, relationship between different points can be created to desire parameter based on their spatial distance defined by building a correlation matrix on a suitable random field . In this research, integral points or stress points are used in meshless method to define correlation. We should mention that these points represent specified region of corresponding Voronoi cell. Based on these by using the appropriate correlation function such as the Markov function, the correlation matrix is created and random field related to each parameter is created as described.

In the next step, Monte Carlo simulation method is used after constructing a random field for selected non-deterministic parameters. In fact, constructed random fields used as input of numerical solution method. After analysis, a value is calculated for considered output parameter, which is used as safety factor in this research. Based on Monte Carlo method concepts, this process is repeated 5000 times and according to repetitions number, different values are obtained for the desired output. In other words, by using Monte Carlo method, according to repetitions number of solution process for different input random fields, the final output of problem will be different values for which a probability distribution that can be obtained after statistical analysis.

In last step, after determining probability distribution value of random variable of safety factor, performing a probability analysis in form of calculating target parameters such as reliability index, probability of failure or coefficient of variation are examined.

Figure 5 shows methodology flowchart.

6. DETERMINISTIC ANALYSIS OF FOUNDATION BEARING CAPACITY

In this section, soil under desired foundation is definitively analyzed using parameters of Table 2 and

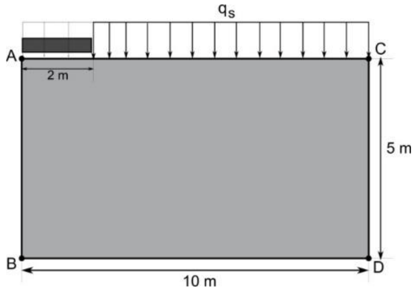


Figure 2. The geometry and boundary conditions foundation

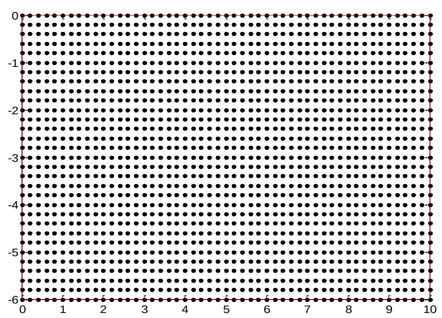


Figure 3. Domain separated by knot

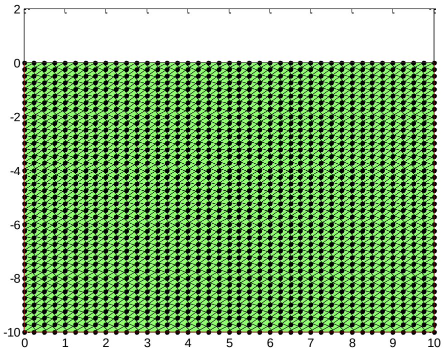


Figure 4. View stress points and related Voronoi cells

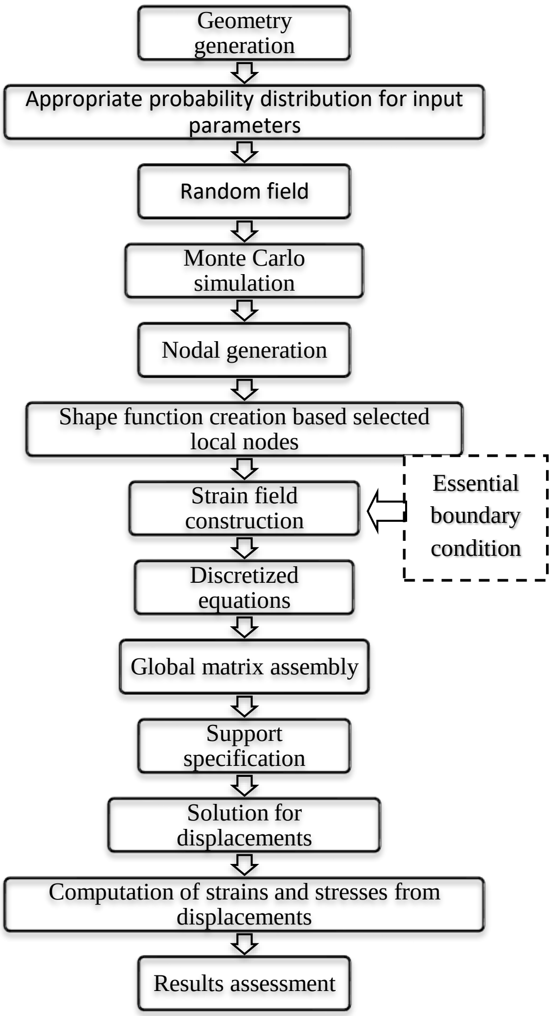
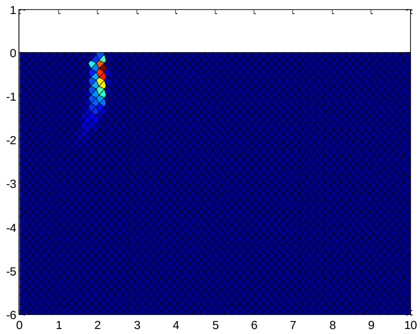


Figure 5. Methodology flowchart

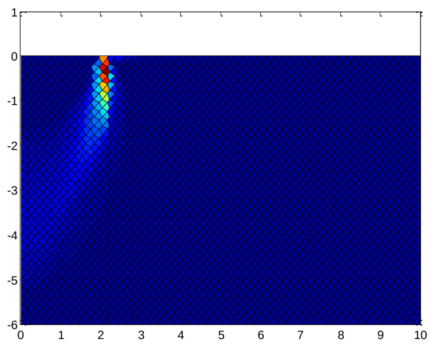
soil parameters in maximum and residual conditions presented in Table 1 separately. The safety factors related to analysis of bearing capacity of desired foundation have been done separately using point interpolation method with radial functions using soil parameters in maximum and residual state without considering progressive failure.

In continuation of deterministic problem analysis on desired foundation is examined in terms of progressive failure model. According to model with strain-softening behaviour in simulation of progressive failure, values of strengh parameters (friction angle and cohesion) are modified using Equations (3) and (4) with a value between maximum and minimum. Figures 6(a) to 6(h) show progression of a deviant plastic strain during repetition of elastoplastic solution in soil slope under foundation. According to Figure 6(a), it is observed that approximately in repetition number 10, plastic strains have formed in edge of foundation area. Also, it shows increasing repetition in soil area under foundation in proportion to failure mechanism on slip path (Figure 6(b)). The plastic strains according to Figure 6(c) in repetition number less than 30 is to a depth of 3 meters under foundation formed failure mechanism. Then, according to Figures 6(d) to 6(y), it expands with increasing repetition failure advances, so that large displacements occur in different parts of soil under foundation.

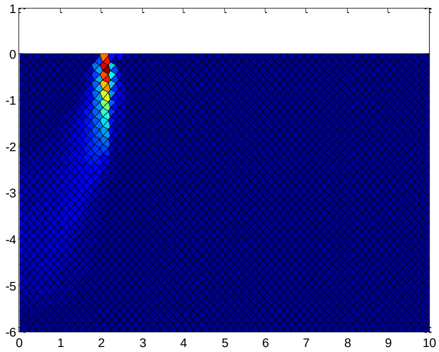
6. 1. Verification The desired foundation has been investigated by the finite elements method using Plaxis software and results are presented in Table 3 in order to



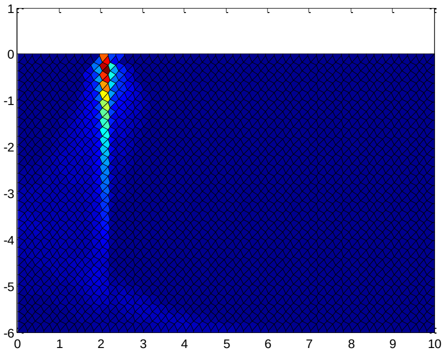
(a) 10 Repetition



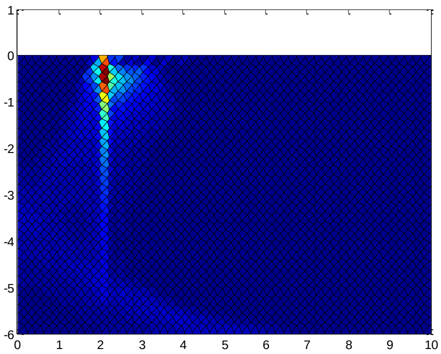
(b) 20 Repetition



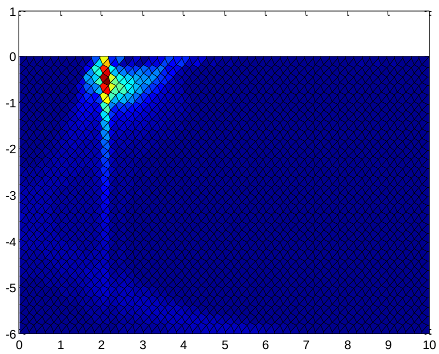
(c) 30 Repetition



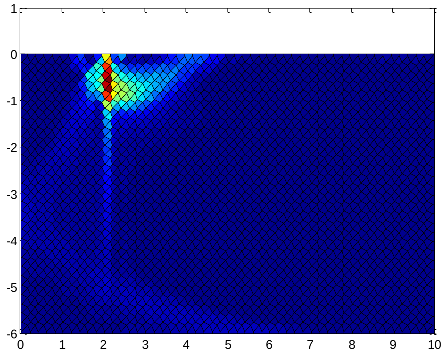
(d) 50 Repetition



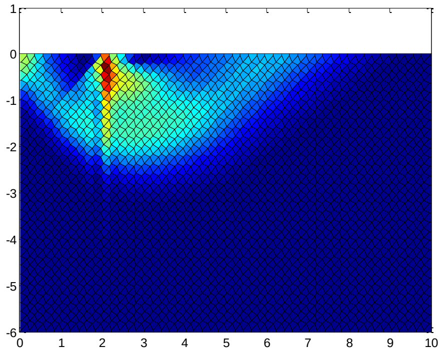
(e) 80 Repetition



F) 150 Repetition



G) 200 Repetition



H) 400 Repetition

Figure 6. progressive shear plastic strain at levels of elastoplastic solution

validate numerical program of solution by point interpolation method with radial functions. The deformed amplitude of soil under foundation is shown in Figure 7 by performing analysis on maximum soil parameters. According to safety factors presented in Table 3, it is possible to compare results of above methods. According to results presented in table, it can be seen that safety factor changes obtained from point interpolation method with radial functions are in a more limited range than finite elements method. It is also known that safety factor of foundation bearing capacity decreases by changing values of parameters from maximum to residual state. Therefore, type of failure in soil under investigated foundation is progressive.

TABLE 3. Comparison of values of safety factors obtained by point interpolation methods with radial functions and finite elements

Method of analysis	Maximum mode	Residual mode
Interpolation with radial functions	4.08	2.18
Finite elements	4.31	1.97

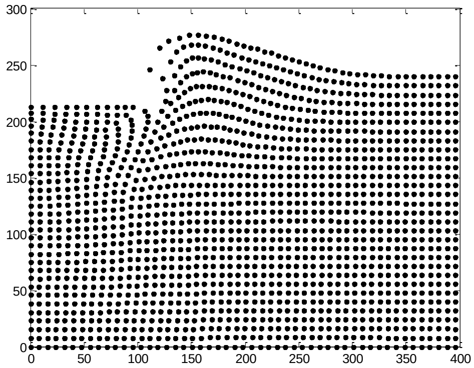


Figure 7. Domain change shape found soil under foundation

7. PROBABILISTIC ANALYSIS OF FOUNDATION BEARING CAPACITY

Influential parameters in process of modeling soil problems have inherent uncertainties. Therefore, using only one value will not represent changes in these parameters. Therefore, solution to this problem is generating random field using statistical distribution for each parameter as a input of computational algorithm in stability problem analysis. Hence, in order to perform probabilistic analysis using point interpolation method with random radial functions, appropriate distribution for input parameters is selected, which is considering by mean value and standard deviation. Soil parameters are often defined using constrained normal distributions or normal logs. Table 4 shows mean values and standard deviation of probabilistic parameters. In this research, definition of correlation between used domain points under integral points or so called stress points used in point interpolation method with radial functions. It should be noted that these points represent identified region of Voronoi cell. Next, a correlation matrix is constructed by using Markov correlation function and a random field corresponding to each parameter is generated.

TABLE 4. Mean values and standard deviation of probabilistic soil parameters

Parameter	Average	Standard deviation
Maximum internal friction angle (°)	20	2
Residual internal friction angle (°)	15	2
Maximum cohesion (kN/m ^{2})	20	2
Residual cohesion (kN/m ^{2})	5	2
Maximum dilation angle (°)	10	2
Residual dilation angle (°)	0.0005	2

Figures 8 to 13 show random field generated of Monte Carlo iteration on parameters of internal friction angle, cohesion and dilation angle of both maximum and residual modes, respectively. Random fields in this analysis are generated in terms of correlation length of 10 meters. As shown in Figures 8 and 9, the values of friction angle in random field of maximum state in range between 16 to 22 degrees and in random field of residual state in range between 13 to 19 degrees. The difference between values of generated fields between maximum and residual state of cohesion parameter was greater according to Figures 10 and 11. This is related to significant reduction of cohesion parameter during progressive failure.

8. RESULTS

Finally, appropriate distribution of calculated safety factors is obtained and main necessary parameters of

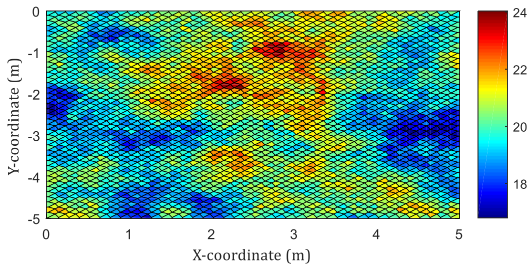


Figure 8. Square by accident angle friction internal maximum (°)

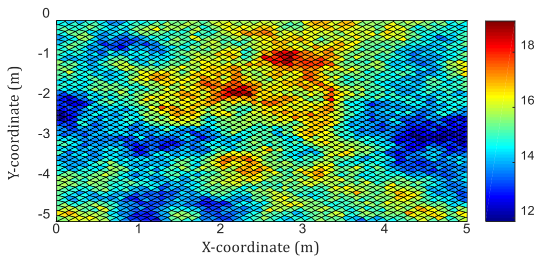


Figure 9. Square by accident angle friction internal residual (°)

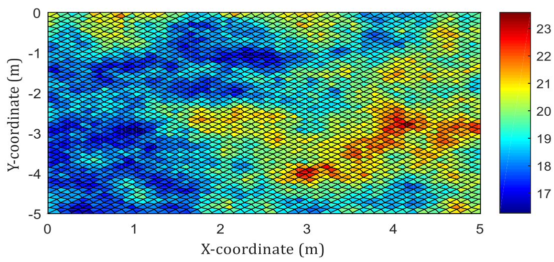


Figure 10. Square by accident cohesion maximum (kN/m^2)

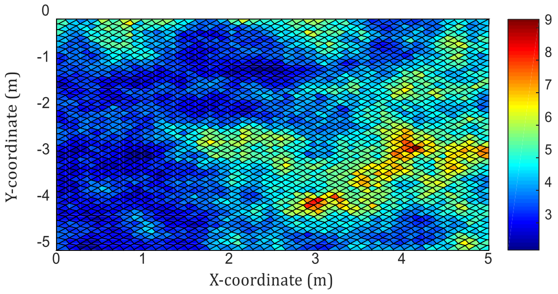


Figure 11. Square by accident cohesion residual (kN/m^2)

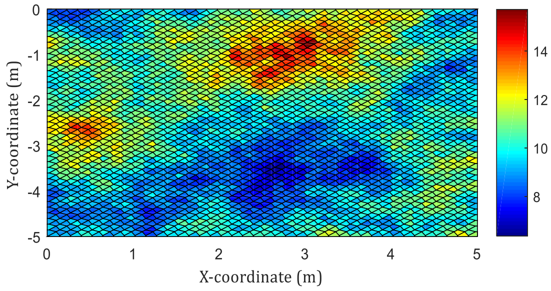


Figure 12. Square by accident dilation angle maximum (°)

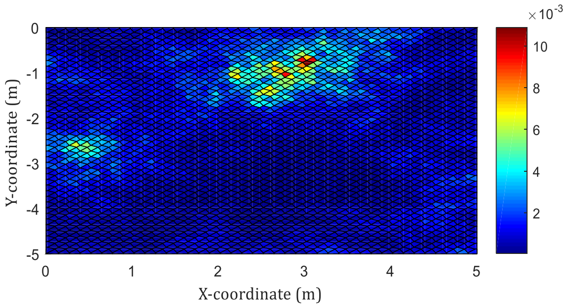


Figure 13. Square by accident dilation angle residual (°)

analysis can be obtained by applying statistical concepts. After determining probability distribution for target value of random variable (safety factor), performing probability analysis in form of calculating parameters such as reliability index, failure probability or coefficient of variation are examined.

After repeating the steps of stability analysis 5000 times, histogram diagram of the 5000 reliability numbers obtained is drawn according to Figure 14.

According to the figure, results obtained from random field are often associated with fluctuation. Therefore, continuous distribution function is obtained through curve fitting and makes probabilistic analysis possible. Since all uncertain parameters are assumed to be normal log input distribution, probability density function of safety factor follows the same distribution and fits histogram. By determining probability density function, cumulative distribution function can also be determined according to Figure 15. It can be concluded that probability of safety factor used for bearing capacity of foundation in terms of progressive failure and spatial variation of strengh parameters is less than 3 will be about 14% by using this diagram. This number actually indicates probability of foundation failure under consideration. Then, according to probabilistic distribution functions, statistical parameters of problem such as mean, standard deviation, reliability index, failure probability and coefficient of variation are determined and presented in Table 5.

It should be noted that coefficient of variation represents dispersion of probability distribution function related to random variable and reliability index (β) indicates distance of average distribution to failure threshold.

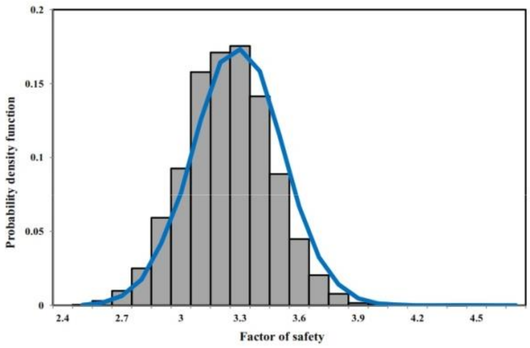


Figure 14. Density probability density function

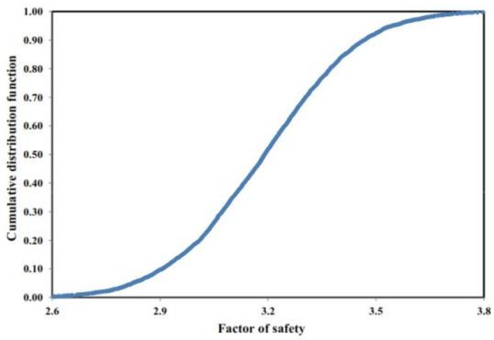


Figure 15. Cumulative reliability distribution function

TABLE 5. Probabilistic parameters of reliability distribution

Average	3.19
Standard deviation	0.22
Coefficient of change	0.07
Probability of failure	0.14
Reliability	0.19

9. CONCLUSION

In this paper, point interpolation method with random radial functions has been used to analyze failure of soil under foundation by modeling spatial variation of soil strengh properties to taking into account progressive failure. In order to evaluate application of this method, The foundation with definite geometry is analyzed deterministic and probabilistic. Then, its safety factor is calculated. According to presented results, it can be seen that safety factor changes obtained from point interpolation method with radial functions are in a more limited range than other methods. Also, values of this safety factor are reduced by changing values of the parameters from maximum to residual state. Therefore, in this study, progressive failure in soil is occured. Based on results obtained in progressive failure, when failure mechanism is formed and has advanced to ground, large deformations have been occurred in different points under foundation, which always will increase with increasing repetition. In other words, actual failure of soil and occurrence of continuous displacements arises simultaneously with formation of progressive mechanism of soil failure and arrival of slip surface to ground. In this study, it can be concluded that by increasing value of correlation length parameter, probability of failure under foundation decreases by examining effect of correlation length on probability analysis. In other words, not considering dispersion of soil properties under problem can lead to conservative results. It should be noted that in this analysis, same correlation length is considered for all uncertain parameters. Therefore, according to all presented results, point interpolation method with random radial functions can be used as a suitable numerical tool with possibility of probabilistic modeling of changes in main soil parameters in various geotechnical problems.

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Persian Abstract

چکیده

در تحلیل‌های مرسوم گسیختگی در زیر پی، پارامترهای مقاومتی بدون تغییر و به‌صورت قطعی فرض می‌شوند. این در حالی است که در حین گسیختگی، مقاومت خاک مقادیر بیشینه و پسماند از خود نشان داده و استحکام آن به‌صورت پیش‌رونده با افزایش کرنش خمیری کاهش می‌یابد. علاوه بر تغییرات پارامترهای مقاومتی خاک طی مکانیسم پیش رونده، ماهیت غیریکنواخت خاک نیز سبب ایجاد تغییرات مکانی این پارامترها می‌شود. ازاین‌رو، بایستی با لحاظ عدم قطعیت مقادیر پارامترهای خاک به‌صورت غیرقطعی و با استفاده از مفاهیم آمار و احتمالات بررسی شوند. در این تحقیق از روش درون‌یابی نقطه‌ای با توابع شعاعی در ترکیب با میدان تصادفی جهت مدل‌سازی تغییرات مکانی خصوصیات مقاومتی خاک و تحلیل ظرفیت باربری پی استفاده‌شده است. برای انجام تحلیل احتمالاتی، میدان تصادفی پارامترهای چسبندگی و زاویه اصطکاک و همچنین کرنش خمیری حد آستانه بر اساس مقادیر میانگین و انحراف معیار آن‌ها تولید می‌شوند. به‌منظور بررسی کاربرد روش درون‌یابی نقطه‌ای با توابع شعاعی تصادفی، یک پی با هندسه مشخص به‌صورت قطعی و غیرقطعی مورد تحلیل قرارگرفته و ضریب اطمینان مربوط به آن بررسی‌شده است. بر اساس تحلیل به‌واسطه مدل‌سازی گسیختگی پیش‌رونده، نتیجه می‌شود که گسیختگی واقعی خاک و وقوع جابجایی‌های ادامه‌دار به‌طور هم‌زمان با شکل‌گیری مکانیسم پیش‌رونده زوال خاک و رسیدن مسیر لغزش به سطح زمین به وقوع می‌پیوندند. در ادامه با انجام تحلیل احتمالاتی و تولید میدان‌های تصادفی، توابع توزیع احتمالاتی ضریب اطمینان تعیین‌شده و پس‌ازآن پارامترهای آماری محاسبه‌شده‌اند.