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A Joint Optimization Model for Production Scheduling and Preventive Maintenance Interval

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A B S T R A C T

Machine maintenance is performed in production to prevent machine failure in order to maintain production efficiency and reduce failure costs. Due to the importance of maintenance in production, it is necessary to consider an integrated schedule for production and maintenance. Most of the literature on machine scheduling assumes that machines are always available. However, this assumption is unrealistic in many industrial applications. Preventive maintenance (PM) is often performed in a production system to prevent premature machine failure in order to maintain production efficiency. Machine maintenance plan is often performed in a production system to prevent premature machine failure in order to maintain production efficiency In this paper, a parallel machine scheduling problem with individual maintenance operations is considered. Then, a mathematical model is formulated including scheduling and maintenance operation optimization. The objective is to assign all jobs to machines so that the completion time and the average cost are minimized, jointly. Maintenance is considered in time intervals. To solve the proposed problem, a branch and bound (B&B) algorithm is adapted and proposed. The results showed the applicability of the mathematical model in production systems and efficiency of the adapted B&B in compare with Gams optimization software.

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NOMENCLATURE

Indices	
i	index of jobs, $i = 0,1,2,\dots,n$;
k	index of machines, $k = 1,2,\dots,K$;
m_k	index of maintenance operations, $m_k = 1,2,\dots,m$;
Parameters	
p_{ik}	Processing time of the ith job on machine k
M	A large positive number
M_k	Number of maintenance activities on machine k
C_{ik}	Completion time of the ith job on machine k
L_{mk}	The latest maintenance start time for the mth maintenance activity on machine k
C_{ik}	Completion time of the ith job on machine k
C_{pm}	Cost of a PM action
C_{Cm}	Cost of a CM action

B	A positive number between 0 and 1
μ_{pmk}	Average time for executing a PM action
μ_{cmk}	Average time for executing a CM action
$g_{pm}(t_{mk})$	Probability density function associated with the duration of a PM action
$g_{cm}(t_{mk})$	Probability density function associated with the duration of a CM action
ε	Instant of failure in the machine
$F(t)$	Probability distribution function for time to failure of machines
$R(t)$	Machine reliability $R(m) = 1 - F(t)$
Decision Variables	
x_{ijk}	1 if job i precedes job j on machine k
Y_{imk}	1 if the m^{th} maintenance activity is performed prior to the i^{th} job on machine k
C_{max}	Completion time of the last job $C_{max} = \max\{C_{jk}\}$
t_{mk}	Start time of maintenance operations on the machine k

1. INTRODUCTION

Determining the schedule and sequence of operations in a production cycle is utmost important and plays a substantial and effective role as one of the key success factors in any production system. Production scheduling minimizes the accumulation of capital, reduces waste, reduces or eliminates machine downtime and tries to

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make better use of the resources leading to satisfying customer orders in a timely manner. Time-based production and manufacturing have been proposed as a rational approach to production and operations management that also established a time-based orientation in manufacturing companies. Manufacturing companies need to transfer the elements of this concept to their operations management. An organization that can provide a variety of products to its customers in a shorter period of time with a desirable level of competitive price will be a successful organization. Time-based systems with a focus on shortening process time will lead to a reduction in overall delivery time. As a result, the inventory level will decrease and it will be possible to increase the responsiveness.

In many manufacturing industries, equipment failure is one of the main reasons for reduced production efficiency [1]. Various examples can be found in the real world, such as thermoplastic industry (such as molds in hydraulic presses), the semiconductor industry [2], cutting tools in drilling (machines, cooling systems), and sensors [1]. In some cases, equipment and machinery can be repaired and maintenance is an effective strategy to maintain machine performance and improve production efficiency.

Maintenance schedule is one of the most important issues in the manufacturing industry [3]. Despite the importance of maintenance in production, maintenance activities may interfere with production timing and cause conflicts during the execution of jobs and maintenance. Irregular maintenance may cause a complete shutdown of the production unit and reduce its availability.

There are two challenges in a production system. The first one is the production planning problem, which determines the optimal production lots, the accumulated size and evaluates the required production capacity; and the second is the scheduling and sequence of production operations, which allocate the existing production capacity to the jobs, determine the sequence of production operations and their start time. Maintenance in a production system is basically of two types. In the first type, there is no control over the condition of the machine meaning that the machines are repaired only in case of failure. Therefore, in this type of maintenance, practically no planning is done for maintenance. In the second type of maintenance, there is a partial control over the condition of the machine meaning that the maintenance includes both corrective and preventive tasks. In PM, it is possible to repair the machines to prevent the occurrence of failure. Therefore, in this situation, optimal maintenance planning is important with the aim of minimizing maintenance costs or maximizing the availability or reliability of the production system [1]. However, ignoring maintenance planning may lead to a complete stoppage of machines and the whole production system. Therefore, using machine failure information to simultaneously optimize production and maintenance schedules is a challenging problem.

The number of researches dedicated to single-machine production units is more than any other production unit. There are several reasons for this focus in single-machine environments. The first reason is the simplicity of the single-machine production unit, and the second reason is that some studies focus on bottlenecks when analyzing production lines. That is, according to the machine which failure has a great impact on the level of efficiency and operational strength of the production line [4].

From the above discussion, it can be concluded that in a production system, decisions are divided into two general groups. The first group is decisions related to the production and the second one is related to the maintenance. PM models can be broadly classified into two types: time-based and condition-based models [5]. Numerous studies have addressed the issue of integrating maintenance planning and production scheduling with time based maintenance activities. For example, in a two-machine flow shop environment with the aim of minimizing production completion time, a precautionary maintenance is planned on one of the two machines in the first period [6]. The integration of corrective maintenance (CM) and PM based on time and production schedule on a single machine production system was studied by Wang and Liu [7]. Also, the issue of fixed and flexible preventive maintenance in a job shop scheduling problem with fuzzy processing time has been discussed by Li and Pan [8]. Integration of maintenance and production scheduling has been investigated in the periodic mode, in which a single machine scheduling problem was considered and maintenance activities were performed periodically over predetermined periods [9]. The previous problem is scheduling a single machine with periodic maintenance and random processing and repair time that was developed by Shen and Zhu [10]. Mosheiov and Sidney [11] reached a similar point, that is, the problem of scheduling for a single machine with maintenance activities, but assuming that maintenance activities decline and if it is done later, it will take frailer and more time. Yazdani et al. [12] addressed the issue of scheduling of tool replacement activities in a multi-factor machine production system. Similarly, the tool was replaced after a predetermined period of time. Numerous articles have been published in the field of joint optimization of parallel machine production schedule and maintenance, which are briefly reviewed in the following.

Wang and Liu [13] investigated a multi-objective parallel machine scheduling problem with two kinds of resources (machines and moulds) and with flexible PM activities on resources. The objective was to simultaneously minimize the makespan for the production, the unavailability of the machine system, and the unavailability of the mould system for the maintenance. A multi-objective integrated optimization method with NSGA-II adaption was proposed to solve this problem. Gara-Ali et al. [14] considered a general model for scheduling jobs on unrelated parallel-machines with maintenance interventions. The processing times deteriorated with their position in the production sequence and the goal of the maintenance was to help to restore good processing conditions. The maintenance duration was dependent on the time elapsed since the last maintenance intervention. Lee [15] considered a problem of scheduling on parallel machines where each machine required maintenance activity once over a given time window. The objective was to find a coordinated schedule for jobs and maintenance activities to minimize the scheduling cost represented by either one of several objective measures including makespan, (weighted) sum of completion times, maximum lateness and sum of lateness. Zhang et al. [16] studied linear deteriorating jobs and maintenance activities under the potential disrupted parallel machines. Potential disruption means that there exists an unavailable interval at a particular time under a certain probability on some machines. Shen and Zhu [17] studied a parallel-machine scheduling problem with PM. Because of the existence of indeterminacy phenomenon, the processing and maintenance times were assumed to be uncertain variables. Branda et al. [18] have examined a flow shop scheduling problem in which machines are not available during the whole planning horizon and the periods of unavailability are due to random faults. To solve their problem, proposed two novel meta-heuristic algorithms obtained modifying a standard Genetic Algorithm (GA) and Harmony Search (HS).

Kalay and Caner [19] investigated a tactical level production planning problem in process industries under costly sequence dependent family setups, which drives the need for manufacturing of product families in campaigns. They investigated the question by implementing a multidimensional Global Optimization branch and bound algorithm with the help of three frameworks with a different level of abstraction. Daneshamooz et al. [20] proposed exact methods are based on branch and bound (B&B) approach to minimize the total completion time of products. Some numerical examples are used to evaluate the performance of the proposed methods.

Rahimi et al. [21] presented a mathematical model to address the integrated cell formation and cellular rescheduling problems in a cellular manufacturing system. As a reactive model, the model is developed to handle the arrival of a new job as a disturbance to the system. They used Gams software to solve their model. Abtahi and Sahraeian [22] presented a predictive robust and stable approach for a two-machine flow shop scheduling problem with machine disruption and uncertain job processing time. A general approach is proposed that can be used for robustness and stability optimization in an m-machine flow shop or job shop scheduling problem. A method based on decomposing the problem into sub-problem and solving each sub-problem, and a theorem-based method. The extensive computational results indicated that the second method has a better performance in terms of robustness and stability, especially in large-sized problems.

In this paper, a new mathematical model is formulated for the problem of parallel machine production scheduling and preventive maintenance (PM). In addition to preventive maintenance, corrective maintenance (CM) is also included when machines experience accidental failures. To prevent these failures, reliability has been considered for each machine, which is another major contribution of this paper.

2. PROPOSED MODEL

2. 1. Proposed Optimization Model

The description of the problem under study is given as follows: There are n independent jobs in the job set $\mathrm{N} = \{\mathrm{J}_1, \dots . \mathrm{J_n}\}$ which are going to be processed on K identical parallel machines, over a scheduling period. All the n jobs are available for processing at time zero. Each job needs to be processed only on one machine and each machine is capable of processing any job but at most one job at a time. Also, it is assumed that two parallel machines are similar and independent. Maintenance operations are performed periodically. After each maintenance operation, the machine returns to its original state. Maintenance operations are not performed at time zero. If the system fails between maintenance intervals, it will undergo repair work. In this model, maintenance can be performed on both machines simultaneously.

2. 2. The Proposed Model

The objective function of the model consists of two parts. The first is to minimize the completion time and the second is to minimize the average cost of maintenance and repairs per unit of time. The model is formulated as follows:

$$\min \mathrm{Z}_1 = C_{max} \tag{1}$$

$$\min \mathrm{Z}_2 = \sum_{k=1}^{2} \frac{C_{pm} * R_k(t_{mk}) + C_{cm} * F_k(t_{mk})}{\mu_{pmk} * R_k(t_{mk}) + \mu_{cmk} * F_k(t_{mk}) + \int_{m-1}^{m} R_k(u)du} \tag{2}$$

$$\min Z : \; W_1\left(\frac{Z_1 - Z_1^*}{Z_1^*}\right) + W_2\left(\frac{Z_2 - Z_2^*}{Z_2^*}\right) \tag{3}$$

$$C_{\max} \geq \mathrm{C_{jk}} \quad \forall\, K \in \{1,2,\dots,n\}, \quad \forall \; j \in \{0,1,2,\dots,N\} \tag{4}$$

$$\sum_{k=1}^{n}\sum_{i=0}^{N} x_{ijk} = 1 \quad \forall\ i \neq j\ \ and\ j \in \{1,2,\dots,N\} \tag{5}$$

$$\sum_{k=1}^{n}\sum_{j=1}^{N} x_{ijk} \leq 1\ \ \forall\ i \neq j\ \ and\ \ i \in \{0,1,2,\dots,N\} \tag{6}$$

$$\sum_{h=0}^{N} x_{hik} \geq x_{ijk} \quad \forall\ i \neq j, i \neq h\ \ and\ \ i,j, \in \{1,2,\dots,N\} \quad K \in \{1,2,\dots,n\} \tag{7}$$

$$C_{0k} = 0 \qquad \forall \in \{1,2,\dots,n\} \tag{8}$$

$$C_{jk} + M\left(1 - \mathrm{x}_{ijk}\right) \geq\ C_{ik} + \mathrm{p}_{jk} + \quad (t_{m_K k} + g_{pm}(t_{m_K k}) * R(t_{m_K k}) + g_{cm_K}(t_{m_K k}) * F(t_{m_K k}) * Y_{im_k k} \tag{9}$$

$$i \in \{0,1,..,N\}\ \ i \neq j\ \ \forall j \in\ \{1,2,\dots,N\}\ \ ,$$

$$\forall\ K \in\ \{1,2,\dots,n\}, and\ \forall\ m_k \in\ \{1,2,\dots,m\}$$

$$\sum_{m_k=1}^{m} Y_{im_k k} \leq 1\ \ \forall\ K\ \in \{1,2,\dots,n\}, i \in \{0,1,..,N\} \tag{10}$$

$$\sum_{m_k=1}^{m} Y_{im_k k} \leq \sum_{j=1}^{N} x_{ijk}\ \ \forall\ K \in \{1,2,\dots,n\},\ \ i \in \{0,1,2,\dots,N\}, i \neq j\ and\ j \in \{1,2,\dots,N\} \tag{11}$$

$$\sum_{i=1}^{N} Y_{im_k k} = 1\ \ \ \forall\ K\ \in\ \{1,2,\dots,n\}, m_k \in \{1,..,m\} \tag{12}$$

$$t_{mk} = L_{mk}\ \forall\ K \in \{1,2,\dots,n\},\ \ m_K \in \{1,2,\dots,m\} \tag{13}$$

$$L_{m+1,k} = t_{m,k} + g_{p(m)}(t_{(m)k}) * R(t_{(m)k}) + L_k \quad \forall\ K \in \{1,2,\dots,n\}, \quad m_K \in \{1,2,\dots,M_K - 1\} \tag{14}$$

$$p(\varepsilon \geq L_k) = \beta \tag{15}$$

$$x_{ijk}\ and\ Y_{imk} \in \{0,1\}\ \ \ t_{m,k} \geq 0\ \forall\ K \in \{1,2,\dots,n\},\quad m \in \{1,2,\dots,M_K\}\ \forall\ i,j\ \in\ \{1,2,\dots,N\} \tag{16}$$

Equation (4) defines the maximum completion time. Equation (5) ensures that every job is assigned to only one machine and has exactly one predecessor. Equation (6) ensures that the maximum number of successors of every job to be one. Equation (7) limits the number of successors of each job to a maximum of one on each machine. Equation (8) indicates the completion time of the job zero. Equation (9) is used to calculate the completion times of the jobs on machines. Basically, if the job j is assigned to the machine k after the job i (i.e., $\mathrm{x}_{\mathrm{ijk}} = 1$), its completion time C_{jk}must be greater than the completion time of the job i, i.e., C_{ik}. If $\mathrm{x}_{\mathrm{ijk}} = 0$, as the constant M is a large positive number, the constraint will become redundant and can be removed. Equation (10) ensures that the m^{th} maintenance task must be performed on each machine exactly after the job i. Equation (11) shows that the m^{th} maintenance task after the job i will be done on the machine k when the job i is assigned to the machine k. Equation (12) indicates that only one maintenance task is performed on each machine at a time. Equations (13) and (14) show how to obtain the latest start time for maintenance tasks. Equation (15) shows how to obtainL_{k}, which indicates the upper limit of the maximum time that the machine can operate without performing preventive maintenance. Equation (16) indicates decision variables.

3. NUMERICAL SOLUTION ALGORITHM

The following is a numerical method for determining decision variables that minimize completion time and minimize total maintenance costs per unit time. Initially, according to the definition of reliability, the value of $L_k$which indicates the working time of the machine without performing maintenance operations, is calculated to be in the constraint (10); in other words, the latest start time of maintenance operations, then the latest start time and the earliest start time of maintenance operations are calculated. After assigning each job to each machine, the latest start time and the earliest start time for maintenance operations are calculated. If the processing time of the jobs does not intersect with the time of maintenance operations, the jobs will be assigned to the machine; otherwise, that job will not be allocated and maintenance operations will be performed. This operation continues until no jobs are left. The values of the objective functions are then calculated. The branch and bound algorithm is adapted to assign jobs to machines and the solution steps are given. Figure 1 shows the solution algorithm.

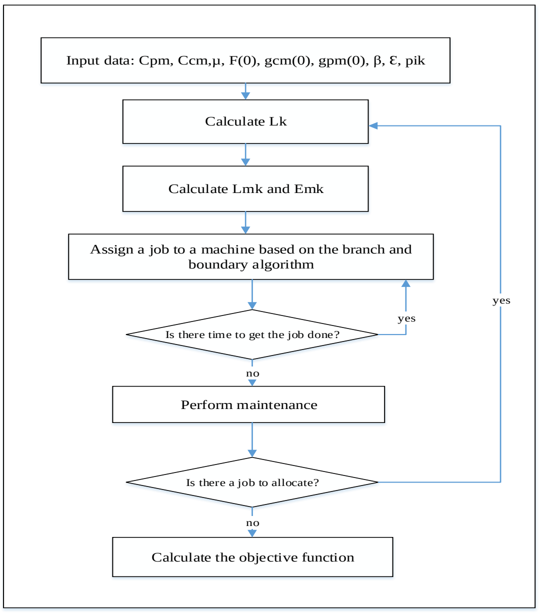


Figure 1. Algorithm for solving the proposed model

3. 1. Adapted Branch and Bound Design The branch and bound algorithm is a well-known exact algorithm for solving an optimization problem, especially combinational optimization. This algorithm was introduced by Land and Doig [23] to solve discrete optimization problems for the first time. This method finds possible answers to the problem in a state space search. Here the set of probabilistic answers is considered as a tree whose root corresponds to all the answers and its branches are subsets of probabilistic answers. Before navigating the set of answers of a sub-branch, the algorithm checks the set of answers of the branch with the lower and upper bounds of the optimization problem in general, and if the sub-branch is not able to generate a more optimal answer to the problem, it scans the whole sub-branch discard.

3. 1. 1. Features of the Method To solve optimization problems for which a polynomial time algorithm has not been found, algorithms with exponential complexity are used that have low execution time, branching and bounding method in this style of problems is a good option. The complexity of these algorithms is usually exponential.

3. 1. 2. Proposed Branching Strategy In each node, one job is assigned to one machine**.** The sequence of jobs is determined by the parent-child relationship in the search tree. The children in this search tree include members of jobs that have not been assigned to any machine up to that node (node *N*).That is, all cases of assigning jobs to machines are considered, for each allocation, a child node is created in *N*. Suppose that node *N* assigns job *i* to machine *k* (*N (i, k*)). Given the path from root to node, which represents a partial scheduling scheme, start time of job *i* is equal to the earliest time when both the machine is available and no maintenance is performed on the machine. Profile-machine shows the busy times of the machine for processing job. In other words, it shows the history of the jobs performed by the machine until the moment of assigning another job, which shows both the to-do list and the maintenance performed up to that moment. And these values are calculated according to the path from the root node to the parent when assigning the job.

$$\textbf{Profile} - \textbf{machine} = \begin{cases} 1 & \text{When the machine is processing jobs or maintenance operations} \\ 0 & \text{otherwise} \end{cases}$$

The start time of job *i* in machine *k* is then calculated as follows. And is equal to the earliest time the first machine is released.

$$\boldsymbol{min\ start\ time\ job\ i} = \left\{\left\{start\ time\ job\ i \,\middle|\, \text{Profile} - \text{machine} = 0;\ C_{i-1,1}, C_{i-1,2}\right\}\right\}$$

The symbol in Figure 2 is used to display nodes in the search tree.

3. 1. 3. Boundary Strategy To avoid duplicate nodes or the development of nodes that we know do not improve the best answer found, rules must be designed for boundaries. In the following, 3 features are presented as boundary strategies.

Feature 1: If we are in node *N* and the moment of arrival of job i on the machine is *k*, it should be checked that if during the processing of job *i*, the time for maintenance operations is reached, job *i* should not be assigned to that machine. And that branch is removed. See Figure 3 for clarity. Suppose the machine is at time 28 and it is time to assign job 7 to machine 1, which has a processing time of 14. If Job 7 is done, the end time of Job 7 is 42, while the machine must be maintained at 38.Therefore, in that node, Job 7 is not assigned to machine 1 and that branch is deleted.

Feature 2: If we are in node *N* and the moment of entry of job *i* is on machine k, and at that moment the machine has a corrective failure, job *i* may not be assigned to machine *k*, in which case the branch will be deleted, or job *i* may wait for the maintenance operation to be completed and then perform it on the same machine,

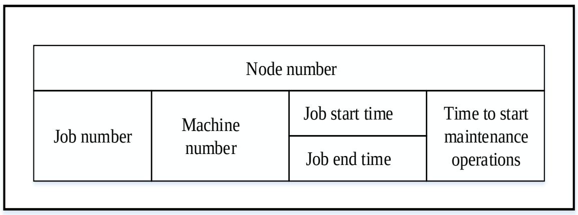


Figure 2. Nodes in the search tree

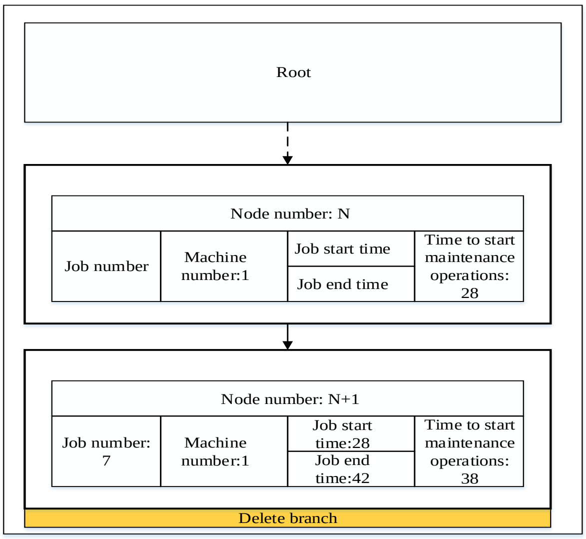


Figure 3. The branch according to feature 1

in which case the start time will be delayed as much as the maintenance operation (See Figure 4).

Feature 3: At the beginning of assigning tasks to machines, we consider the time to complete the jobs infinitely, and then we assign all the jobs to one machine. We get the maximum completion time of jobs by taking into account the processing time of works and maintenance operations which is specified as the upper bound, jobs are then assigned to both machines. If the job is completed in a branch longer than this value, the branch is removed. In addition, if a branch has been completed and a value is obtained for the time of completion, and in another branch that has not yet been completed, the completion time is longer than that branch, the branch that has not been completed will be removed (See Figure 5).

3. 1. 4. General Structure of the Method If we want to present an algorithm that minimizes the function f and the function g is the lower bound for the value of f at the vertices of a sub tree of state space. The general structure of this method will be as follows:

1) First find an arbitrary answer x and set the value of B to *f (x)*, from now on, the value B will indicate the best answer found up to this point.
2) Consider a row of vertices of the state space and add the root of the state space tree to it.
3) Repeat the next steps until the queue is empty.
- A vertex is pulled out of the queue.
- If this vertex represents a specific answer to the problem such as *x* and *f (x) <B*, this answer is the best answer ever found, so the value *f (x)* is placed inside B.
- Otherwise for all branches of this vertex, for example *Ni*.

4) If *g (Ni) <B:*
- This branch may lead to a better answer, so we add *Ni* to the list.
- Otherwise this branch has no value because the lower limit of its answers is larger than the upper limit of the problem answer. Return to command 3.

3. 2. Numerical Examples In this section, the input data for solving the model by the proposed algorithm is given. This model is solved by 10 problems with the same dimensions and different processing times. An example of an issue is given below. The results for comparing 10 problems are given in Table 1.
- Costs: Cost of PM = 2, Cost of CM = 4;
- The duration of PM task ~LogNorm (mean μ_P=10, standard deviation $\delta_P = 1.5$);
- The duration of CM task ~LogNorm (mean μ_C=20, standard deviation $\delta_C = 2$);
- The time to machine failure ~ Weibull distribution (shape parameter = 2, scale parameter = 100), i.e., the average lifetime is μ =8.86 time units;
- β = 90%
- Number of machines =2;
- The number of jobs and their processing times, as well as the start time and duration of CM tasks are given in Tables 2, 3, and 4, respectively.
- In this example, the jobs are assigned to machines and there are different sequences between the jobs of each machine.

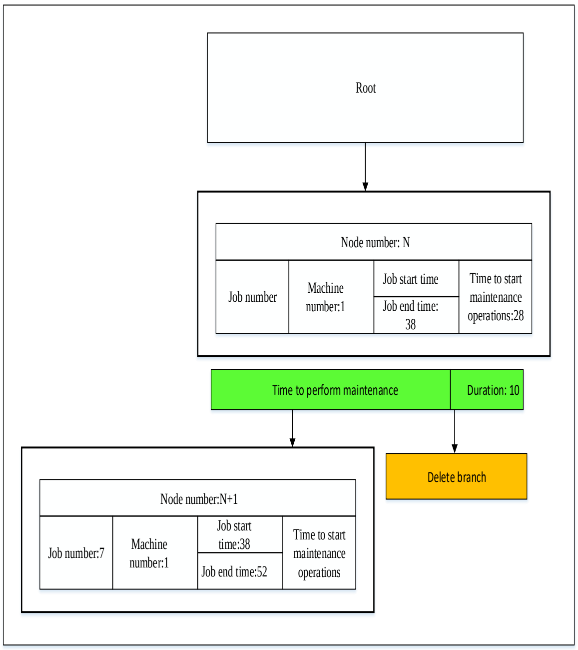


Figure 4. The branch according to feature 2

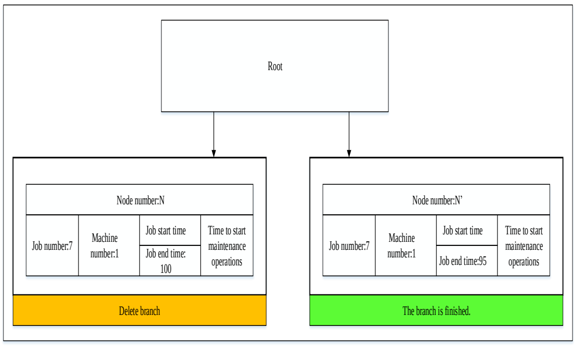


Figure 5. The branch according to feature 3

4. COMPUTATIONAL RESULTS

The proposed algorithm is implemented in MATLAB software. By studying several articles in this field, random data is generated to evaluate the algorithm. In this model assumed in the worst case, all jobs are assigned to one of the machines, and after each job,

TABLE 1. Processing time for 10 jobs in 10 problems

Job	Problem number	1	2	3	4	5	6	7	8	9	10
	1	5	8	12	9	18	14	14	-	-	-
	2	6	10	7	12	18	17	5	-	-	-
	3	6	20	7	19	8	19	20	-	-	-
	4	11	8	5	6	10	6	13	-	-	-
	5	18	20	18	9	12	14	5	8	18	16
p_{ik}	6	6	11	8	12	6	7	14	12	17	5
	7	5	10	16	20	17	15	15	14	6	14
	8	13	12	15	15	15	17	20	15	16	10
	9	19	10	9	5	12	19	15	14	19	8
	10	11	13	7	9	17	5	5	15	20	14

there is a need for preventive maintenance, so there are 10 maintenances for each machine. The numerical algorithm first determines the required time interval of machine reliability, In fact, the upper limit of this interval, L_k, indicates the maximum useful life of some parts that must be replaced after 90% operation because if they are not replaced, the machine may fail and lead to many breakdowns. After determining L_k, the earliest and latest time for maintenance operations is determined. Jobs are then assigned to machines, and this continues until the first maintenance operation is completed. It then stops and undergoes maintenance.

The important point is that if the job time to be allocated to a machine interferes with the time of preventive maintenance, the machine may be idle and not in production until the end of the maintenance operation or the machine may perform the desired job and then preventive maintenance. However, delays are not allowed for corrective maintenance, and the machine must be stopped at any time according to the schedule. After the maintenance operation, for each machine again L_k, the earliest and latest time for maintenance operations is determined and jobs are reassigned, and this continues until no jobs are left and the maximum time of the machines is set to C_{max}.

The results of solving one of the problems by the proposed algorithm and assigning jobs to two machines are given in Tables 2 and 3. In these tables, (*pcm*) indicates corrective maintenance operations; (*pm*) indicates preventive maintenance operations.

TABLE 2. Schedule on the machine 1

Job	7	Pm	2	Pm	5	pm	8	Pcm	4
Start time	0	14	14.45	23	23.61	42	42.55	57.24	58.23
End Time	14	14.45	22.45	23.61	41.61	42.55	54.55	58.23	63.55

TABLE 3. Schedule on the machine 2

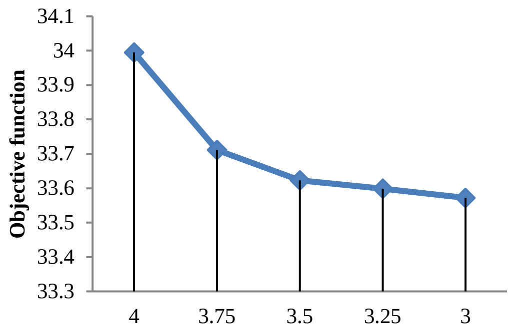
Job	1	3	Pm	6	Pm	10	Pcm	9
Start time	0	5	17	17.68	32	32.73	48.23	49.36
End Time	5	17	17.68	31.68	32.73	40.73	49.36	66.36

TABLE 4. Results of solving problems

Problem Number	Objective Function	Cmax Gams	Cmax Matlab	Solution Time Matlab	Solution Time Gams
1	20.623	41.05	41.05	121.18	524.12
2	19.713	39.23	39.23	99.89	335.92
3	26.273	52.35	52.35	101.22	529.32
4	15.723	31.25	31.25	133.37	658.18
5	33.995	-	66.53	143.269	-
6	37.043	-	72.58	187.191	-
7	47.478	-	93.45	135.594	-
8	48.373	-	95.24	258.427	
9	33.328	-	65.15	227.686	-
10	40.448	-	79.39	167.517	-

4. SENSITIVITY ANALYSIS

In order to perform the sensitivity analysis, by keeping the other parameters constant, the costs are reduced and the objective function is evaluated. According to Figure 3, as the corrective maintenance cost decrease, the value of the objective function decreases more than preventive maintenance cost. Because the amount of maintenance costs was higher than preventive maintenance and the amount of objective function was more sensitive to it. The results of Figure 4 show that by increasing the value of *w* the objective function increases from 0.197 to 66.53. It can be concluded that if the completion time of the jobs



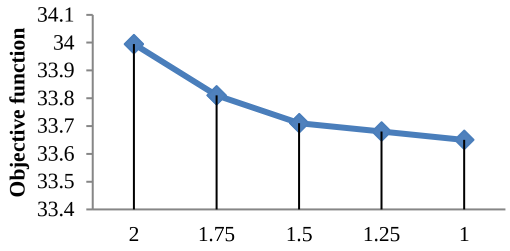


Figure 3. Diagram of changes in the objective function by reducing corrective and preventive maintenance costs

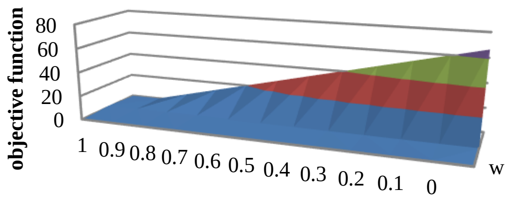


Figure 4. Variation of objective function problem 5 with changes in the value of w

is more important than the cost, the amount of the objective function will be higher.

5. CONCLUSION

In this paper, a model for scheduling a parallel machine production system and an exact solution algorithm is presented. In this case, considering maintenance operations in the schedule complicates the issue. The objective function consists of two parts, the first part is to minimize the maximum completion time of the jobs and the second part is to minimize the cost of maintenance. After presenting the model, in order to check the accuracy of its performance, several sample problems with a size of 10 jobs were solved by Gams software. Following combined branch and bound algorithm are presented. The resulting algorithm is a exact solution algorithm and the answers obtained from it on a small scale are the same as the answers obtained from Gams software. With the difference that the solution time in Gams software is much longer than the proposed algorithm. In the section of sensitivity analysis, it can be concluded that the objective function is more sensitive to the cost of corrective maintenance and decreases to a greater extent by reducing it.

Maintenance for future studies can be considered to be based on the conditions or quality control can be used to detect the maintenance operation. By changing (reducing) the parameter value of preventive maintenance costs and running the algorithm, the value of the objective function from 33.995 to 33.651 and by changing (reducing) the value of the parameter of the cost of corrective maintenance of the algorithm, the value of the objective function from 33.995 to 33.572 decreases, which indicates the correct operation of the algorithm.

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Persian Abstract

چکیده

نگهداری و تعمیرات ماشین در تولید برای جلوگیری از خرابی ماشین به منظور حفظ کارایی تولید و کاهش هزینه های خرابی انجام می‌شود. با توجه به اهمیت نگهداری و تعمیرات در تولید، لازم است یک برنامه یکپارچه برای تولید و نگهداری و تعمیرات در نظر گرفته شود. اکثر پیشینه تحقیق مربوط به زمانبندی ماشین فرض می‌کنند که ماشین‌ها همیشه در دسترس هستند. با این حال، این فرض در بسیاری از کاربردهای صنعتی غیر واقعی است. نگهداری و تعمیرات پیشگیرانه (PM) اغلب در سیستم تولید برای جلوگیری از خرابی زودرس ماشین به منظور حفظ کارایی تولید انجام می شود. در این مقاله، یک مساله زمان بندی ماشین موازی با عملیات نگهداری و تعمیرات در نظر گرفته شده است. سپس، یک مدل ریاضی شامل زمانبندی و بهینه‌سازی عملیات نگهداری و تعمیرات تدوین می‌شود. هدف این است که همه کارها به گونه‌ای به ماشین‌آلات اختصاص داده شود تا زمان اتمام و هزینه متوسط به طور مشترک به حداقل برسد. نگهداری و تعمیرات در فواصل زمانی منظم در نظرگرفته می‌شود. برای حل مساله پیشنهادی، یک الگوریتم شاخه و کران (B&B) پیشنهاد شده است.