International Journal of Engineering

A Mathematical Model for a Blood Supply Chain Network with the Robust Fuzzy Possibilistic Programming Approach: A Case Study at Namazi Hospital

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PAPER INFO	ABSTRACT
	The main challenge in blood supply chain is the shortage and wastage of blood products. Due to the perishable characteristics of this product, saving a large number of blood units on inventory causes the spoil of these limited and infrequent resources. On the other hand, a lack of blood may lead to the cancellation of health-related critical activities, and the result is a potential increase in mortality in hospitals. In this paper, an integer programming model was proposed to minimize the total cost, shortage, and wastage of blood products in Namazi hospital by considering the different types of blood groups. The parameters in the real-world are uncertain, and this problem will be examined in the paper. The robust fuzzy possibilistic programming approach is presented, and a numerical illustration of the Namazi hospital is used to show the application of the proposed optimization model. Sensitivity analysis is conducted to validate the model for problems such as certainty level, coefficient weight, and penalty value of the objective function in the robust fuzzy possibilistic programming. The numerical results imply the model is able to control uncertainty and the robustness price is imposed on the system; therefore, the value of the objective function in the robust fuzzy possibilistic is 80% lower than probabilistic.

NOMENCLATURE

Indeces		cpu_{tf}	Maximum capacity of the hospital for blood group f in period t
i	Production life span	M	Big number
t	Time period	θ_{itf}	The fraction of hospital blood orders at age i days, period t and the blood group f $\qquad 0 \leq \theta_{(i.t.f)} \leq 1$
f	Blood groups	**Variables**	
Parameters		ss_{itf}	Confidence in the blood group f in period t with the age of i day
$\tilde{p}$	Shortage cost	u_{tf}	The amount of lesions in the blood group f in period t
$\tilde{c}$	Purchasing cost	Q_{tf}	The level of the blood group f deficiency in the period t
$\widetilde{w}$	Wastage cost	ϑ_{itf}	End-of-life inventory of the blood group f in period t with the age of i day
$\tilde{h}$	Holding cost	O_{tf}	The rate of hospital orders to the blood group f in the period t
$\widetilde{D_{tf}}$	The demand of hospital blood group f in period t	E_{itf}	The blood group f received at the beginning of period t with the age of i day
$\widetilde{cap}_{tf}$	Blood centers' capacity in period t and the blood group f	Y_{itf}	A binary variable that is 1 if the patient satisfies blood from the blood group f in the period t with the age of i days, otherwise 0.

1. INTRODUCTION

The blood supply chain is one of the few supply chains that are also in the supply sector. In addition, it is in high uncertainty in the demand segment. This complicates the management and planning of current affairs in this supply chain [1]. On the other hand, blood products are related to the health and lives of people and management of them

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has a high sensitivity; so, that the occurrence of the smallest disruption in the management of supply chain issues will result in irreparable damage [2]. In recent years, the blood supply chain has been a focus of attention due to the importance of this vital product and scarce in health systems. Providing healthy blood and enough and also its management has been from particular concern for the human race, hence the collection and management of blood distribution which is proposed in the form of supply chain management requires comprehensive and accurate management and planning. The blood supply chain has complexities so that it is distinguished from the supply chain of ordinary commodities. Blood is one of the most crucial corrosive substances in nature, which is closely related to the lives of humans. One of the most important reasons for blood and blood products is its human origin, and artificially can not be produced. In addition, blood products such as red blood cells, platelets, and plasma have a different life span and require special conditions for maintenance. On the other hand, the blood supply chain, which involves processes for collection, production, storage, and distribution of blood and blood products from donors to blood recipients, is associated with uncertainty. This uncertainty in the supply and demand process is evident because blood supply from donors is relatively unplanned and uncertain, and demand for the product is not stable. Uncertainty in supply chain issues plays a crucial role in economic performance. Therefore, adjustment of supply and demand in the blood supply chain requires the design of the appropriate supply chain network to supply blood and blood derivatives.

Robust optimization optimizes at worst so that a robust approach to optimization problems has been used since the early 1970s. It has recently been studied widely [3]. Alem and Morabito [4] cited two reasons for using robust optimization. At first robust optimization is easier than possible approaches to solve the model. Also, given that we do not need explicit knowledge of data under cognitive uncertainty, historical data, and in some cases, the experiences of decision-makers can be used to infer the uncertainty interval. The reasons for the superiority of robust possibilistic programming are compared to possibilistic programming as follows [5].

1. In robust optimization, the final answer has the stability of being optimal and stable.

2. In robust optimization, the level of confidence in satisfying the constraints is determined by the model itself and its value is optimal.

3. According to the deviations of the objective function due to the uncertainty of the parameters will be prevented heavy and irreparable costs for managers and organizations. However, it is not paid much attention to the mentioned cases in the possibilistic programming approach [6]. Therefore this paper proposes robust possibilistic programming (RPP) to solve the problem,

which has a considerable superiority when compared to a certain model.

In the next section, the literature review is addressed. The problem definition and mathematical model are presented in sections 3 and 4, respectively. The computational results and sensitivity analysis are mentioned in section 5. The conclusion and future research of the paper are finally indicated in section 6.

2. LITERATURE REVIEW

Brief literature is reviewed about the blood supply chain with solution methods and algorithms that are used to solve the problem. Ghandforoush and Sen [7] developed a review of some of the tactical and operational aspects of the collection, production, control of the inventory, the policy of shipment blood products, and delivery decisions. Haijema et al. [8] investigated a dynamic Markov and a simulation approach which two types of demand are presented in accordance with different types of patients and scarcity and waste is minimized. Ghandforoush and Sen [7] presented a system of initial decision support according to an unconfirmed integer programming in order to help regional blood transfusion centers to generate and collect platelets daily. The aim is to minimize the total daily cost including collection, production, and costs of shortages. They concluded that the supply and production should be on demand. Osorio et al. [9] have paid little attention to the relationship between the various stages of the supply chain and many single-level papers have been reviewed. Mansoori et al. [10] presented a bi-objective location-allocation model for blood supply under uncertainty. The objectives are minimizing the blood shortage in blood centers and also minimizing the operational costs including cost of transporting temporary blood facilities and the cost of blood collection and transportation in a multi-period context. They used the robust optimization approach in the model. Civelek et al. [11] presented a periodic inventory management system for platelet inspection so that demand varies from one age to another. The aim is to minimize the inventory, wastage, and shortage costs. Abdulwahab and Wahab [12] proposed a collection of methods used for vendor problems and linear programming in the blood bank inventory model and other methods of inventory. Abbasi and Hosseinifard [13] presented a model to evaluate the release of platelets and red blood cells in the blood supply service by using queuing theory. Pishvaee and Torabi [14] specified that traditional models can not apply the precision and logic of classical mathematical alone. Asadpour et al. [2] addressed a blood supply chain network with backup facilities and expiration date. They proposed a bi-

objective Mixed Integer Programming where the objectives are to minimize the total cost and detrimental environmental impacts.

The model is not real when uncertainty is ignored due to its structure. Conceptual concepts can not show some uncertainties; therefore, the theory of fuzzy sets has been introduced to express the uncertainties since 1980. Fuzzy concepts are the creation of the ability to definition ambiguous, inattentive, and unpleasant parameters that are defined based on personal beliefs [15]. Tanthatemee et al. [16] presented a single-product fuzzy inventory control system and a permanent overview. The presented model is based on the fuzzy logic control system for uncertain demand and resource availability. Selim and Ozkarahan [17] proposed an integrated multi-objective fuzzy scheduling model for designing a chain-of-distribution network. Handfield et al. [18] developed a model (Q, r) that Fuzzy concepts have been used to illustrate uncertainty of sources in a supply chain. Rajendran and Ravindran [19] developed stochastic models under uncertain demand for a single hospital. The models aimed to propose ordering policies to reduce shortage, wastage and purchase for different cost settings. Dilano et al. [20] proposed a two-stage stochastic programming model for explaining optimal periodic review policies for red blood cells inventory management. The objective is to minimize the operational costs, shortage and wastage by taking into account perishability and uncertain demand. Oserio et al. [21] developed integrated simulation-optimization model to account for uncertain supply and demand, blood group proportions, shelf life constraints, different collections and production methods in the blood supply chain.

Pishvaei et al. [22] presented a robust programming model for a supply chain network with social responsibility. They proposed a new approach and implemented the model under different assumptions and compared the performance of them. Zahiri et al. [23] proposed a model of robust possibilistic programming for location-allocation of organ transplant centers under the uncertainty. They used minimal costs to enhance the impact of their network design. Safaei et al. [24] formulated a closed-loop supply chain (CLSC) for the cardboard recycling network under the uncertainty of demand to maximize total profit. They used a robust optimization approach in the proposed mixed integer linear programming (MILP) model to combat uncertainty. Selma et al. [25] proposed a general MILP model for the multi-objective CLSC network due to the uncertainty of product demand. Haghjoo et al. [26] presented a a dynamic robust location–allocation model for designing a blood supply chain network under facility disruption risks and uncertainty in a disaster situation. Eskandari-Khanghahi et al. [27] developed a possibilistic optimization model for a multi-period and multi-objective sustainable blood supply chain with uncertain

data where the objectives are to minimize the total cost, environmental effects and maximize the sicial effects. Kazemi et al. [28] addressed blood inventory-routing problem under uncertainty and developed a mixed integer programming formulation for the problem. Zahiri and Pishvaee [29] studied blood supply chain network design under uncertainty and developed a bi-objective mathematical programming model which minimizes the total cost as well as the maximum unsatisfied demand. Ghahremani-Nahr et al. [30] proposed a MINLP location-allocation model to design a closed-loop green supply chain under uncertainty with robust fuzzy mathematical programming and solved the model with the Wall optimization algorithm.

Although some researchers have studied uncertainty in their problems, but this uncertainty is limited to some parameters like demand. In this research, all parameters such as cost, demand and capacity are assumed uncertain and trapezoidal fuzzy distribution is considered for the uncertain parameters. A robust fuzzy possibilistic programming are used to address the uncertainty. Also, according to literature review consideration of all blood groups and expiration date simultaneously is the first study in this filed.

3. PROBLEM DEFINITION

In this paper, a blood supply chain considering different types of blood groups (O+, O-, A+, A, B+, B, AB+, AB) and expiration date has been designed simultaneously. This makes the demand for blood in the hospital more efficient and can prevent the risk of blood transfusions to patients with a variety of blood groups. It also simplifies blood demand and the blood is delivered to the hospital from the blood centers without the test of compatibility so that the hospital performs the test.The supply chain of blood consists of three levels. The first level is donors who want to donate the blood to a mobile or fixed unit of the blood transfusion organization in the city. The second level in the chain is the test and production of blood products that are performed by blood test and blood test laboratories. The third level of the chain is the distribution of blood products.

In order to design the supply chain, an initial mathematical model is proposed to minimize the total cost, shortage, and wastage in blood supply chain of Namazi hospital by considering the different types of blood groups. The model has non-deterministic parameters. Therefore, the primary model is transformed into a model with some non-deterministic parameters. A chance constraint approach and robust possibilistic programming are used to address the uncertainty. After a full explanation of the approach, we introduce a new definitive and real world model that is called the second model. The model includes the objective function and the

constraints of the first model but the trapezoidal fuzzy distribution is considered for the uncertain parameters of the problem. The assumptions considered in the model of the problem are as follows:

- all blood groups are considered
- The capacity of blood centers is limited.
- The delivery time for blood supply is zero.
- The age of blood transmitted from the blood transfusion organization is known and changes over time.
- The life of red blood cells is limited and it is 35 days which is 2 days to test.
- The policy used to send the blood is the original FIFO.
- If the demand is not met, we will be in deficit.
- If the blood expires, we will have a lost cost.
- The reviewed model is a single product and a multi-period.
- The maximum hospital blood bank capacity is predetermined.

3. 1. Mathematical Model

In this section, a nonlinear integer programming model is derived from Gunpinar and Centeno [30]. In the second model, some parameters in the objective function, the technical coefficients, and the right values of some of the constraints are ambiguous therefore it is necessary to further explain. The definite model that is presented in this section is a nonlinear programming model and will be converted to a linear model. The proposed model is known NP-hard. Therefore, several algorithms are proposed to reduce the solution time for the model that will be described completely in separate sections. The mathematical model are presented and described as follows:

Objective function

$$\text{Min } z = \sum_{t=1}^{T}\sum_{f=1}^{8} \tilde{C} * O_{tf} + \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{f=1}^{8} \widetilde{H} * \vartheta_{itf} + \sum_{t=1}^{T}\sum_{f=1}^{8} \widetilde{W} * u_{tf} + \sum_{t=1}^{T}\sum_{f=1}^{8} \tilde{P} * Q_{tf} \tag{1}$$

Constraints

$$\sum_{f=1}^{8} O_{(t.f)} \leq \sum_{f=1}^{8} Cap_{(t.f)} \quad \forall t \tag{2}$$

$$\sum_{i}(V_{i,t-1.f} + SS_{i.t.f} + e_{i.t.f}) \leq cpu_{t.f} \quad \forall t,f \tag{3}$$

$$E_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \tag{4}$$

$$E_{(i.t.f)} = O_{(t.f)} * \theta_{(i.t.f)} \qquad \forall t.f.i \geq 3 \tag{5}$$

$$Y_{(i.t.f)} \geq Y_{(i-1.t.f)} \qquad \forall t.f.i \geq 3 \tag{6}$$

$$D_{(t.f)} = \sum_{i=3}^{I}\left[\left(\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)}\right) * Y_{(i.t.f)} - ss_{(i.t.f)}\right] + Q_{(t.f)} \qquad \forall t.f \tag{7}$$

$$\left(Y_{(i.t.f)} - Y_{(i-1.t.f)}\right) * \binom{\vartheta_{(i-1.t-1.f)} +}{E_{(i.t.f)}} \geq ss_{(i.t.f)} \tag{8}$$
$$\forall t.f.i \geq 3$$

$$D_{(t.f)} - \sum_{i=3}^{I}\left[\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)}\right] \leq Q_{(t.f)} \quad \forall t.f \tag{9}$$

$$Y_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \tag{10}$$

$$\vartheta_{(i.t.f)} = \left(1 - Y_{(i.t.f)}\right) * \left(\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)}\right) + \left(Y_{(i.t.f)} - Y_{(i-1.t.f)}\right) * ss_{(i.t.f)} \qquad \forall t.f.i \geq 3 \tag{11}$$

$$\vartheta_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \tag{12}$$

$$\vartheta_{(i.0.f)} = 0 \qquad \forall i.f \tag{13}$$

$$U_{(t.f)} = \vartheta_{(35.t.f)} \qquad \forall t.f \tag{14}$$

$$ss_{(i.t.f)}. U_{(t.f)}. Q_{(t.f)}. \vartheta_{(i.t.f)}. O_{(t.f)}. E_{(i.t.f)} \in Z^+ \tag{15}$$

$$Y_{(i.t.f)} \in \{0.1\} \tag{16}$$

The objective function of the problem consists of the cost of maintenance, waste, shortage, and purchasing shown in equation (1). Equation (2) indicates that the capacity of the blood center (supplier) is limited so that hospital demand cannot exceed the capacity of the blood centers. Equation (3) ensures that the amount of blood in the hospital cannot exceed the hospital's blood bank capacity. Constraints (4) and (5) ensure that the hospital never receives a unit of blood that one or two days left in its life span (since it takes 2 days to complete the test in a blood bank). Equation (6) shows the FIFO policy for blood delivery. Equation (7) responding to demand when blood supply is more than demand. In this case, the amount of blood in the system for all ages of blood in each period and each blood group is checked to see if it is used and we reduce the amount of confidence, then reduce the amount of deficiency which is continuously checked into the system to this value. Equation (8) implies that the value of the quantitative variable of confidence does not exceed the available blood units in its age group. Constraint (9) is intended to control the amount of blood deficiency and constraint (10) shows the allocation of blood units to each age group and also the amount of blood received from each blood group in each period. Equation (11) shows the level of inventory of the end of the blood cycle for each age group in each period. Constraint (12) ensures that two days of blood is not available in the stock. Constraint (13) specifies that no inventory is available at the beginning of the analysis period. Equation (14) shows the rate of hospital waste at the end of each course. The rest of the equations show the condition of the variables and parameters.

3. 2. Linearization of the Model   Equations (7), (8) and (11) are nonlinear which leads to the complexity of the model. However, using the auxiliary variables and applying some additional constraints, the extracted model can be converted to the linear model. The linearization procedure is described below.

There is a continuous variable x and binary variable y so that variable α will be defined as nonlinear variable from multiply x by y according to Equation (17).

$$\alpha = X * y \tag{17}$$

For the linearization of the Equation (17), three constraints are added instead of it as follows [30]. The three add-ons to the model ensure that if y is zero, the variable α will equal to zero. Otherwise, if y is equal one then the variable α will equal x according to Equations (18) to (20). The limitations are linearized that Gupte et al., is using [31].

$$\alpha \leq M * y \tag{18}$$

$$\alpha \leq X \tag{19}$$

$$\alpha \geq X - M(1 - y) \tag{20}$$

3. 3. Chance Constraint Programming   In order to face uncertainty, several approaches have been developed in mathematical optimization problems such as randomization, fuzzy optimization, robust optimization, and hybrid approaches. In this section, the model and concepts are derived from the paper by Pishvaee and Torabi [14] Fuzzy programming models use fuzzy confidence coefficients and membership functions to express the lack of knowledge about the parameters and are divided into possibilistic and flexible programming.

In probable planning, the lack of knowledge about the exact amounts of the model parameters with probabilistic distribution functions is modeled using available target data and decision-making knowledge. In flexible programming, the objective function and constraints are used to control the uncertainty of the flexible value, and modeling will be based on fuzzy sets or priorities. In the paper, probabilistic planning of the limits of chance is used to address the uncertainty of the various parameters in the problem.

The chance constraint method is one of the primary techniques for solving optimization problems under various uncertainties. This formulation method is an optimization problem that assures that the probability of a specific limit is higher than a certain level. In other words, it restricts space to a high level of confidence. In particular, the distribution of trapezoidal probability according to Figure 1, is used to represent non-deterministic parameters in the proposed model. For a more detailed and simplistic introduction, the compact
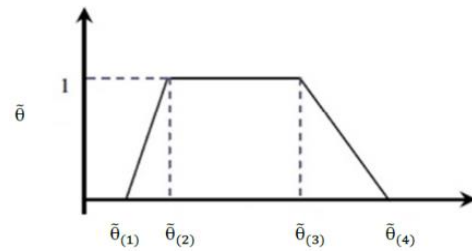
Figure 1. trapezoidal fuzzy distribution

form of the proposed model is presented as follows:

Objective function

$$Min\ Z = fy + cx \tag{21}$$

Subject to:

$$A x \geq d$$
$$Bx = d$$
$$Sx \leq Ny$$

$$Y_{i,t,f} \in \{0,1\}$$

$$x \geq 0$$

Assume that the vector f (fixed cost) is a definite parameter, and the vectors c (variable costs) and d (hospital demand) and the matrix of coefficients N (capacity of facilities) are unknown parameters of the problem. To build a basic fuzzy programming curve, chance constraint, we use the "expected value" factor to non-deterministic model parameters of the objective function and the necessity (Nec) scale for modeling losing constraints. The Nec scale can be applied directly to convert the fuzzy odds limits to equivalent equations.

Subject to:

$$Nec\{Ax \geq \tilde{d}\} \geq \alpha_m \qquad \forall\, m \in M$$

$$Nec\{Bx = \tilde{d}\} \geq \alpha_m \qquad \forall\, m \in M$$

$$Nec\{Sx \leq \tilde{N}y\} \geq \alpha_m \qquad \forall\, m \in M \tag{22}$$

$$Y \in \{0,1\}$$

$$x \geq 0$$

Since the objective function and constraints have non-deterministic parameters and are considered by fuzzy distributions, and the constraints with non-deterministic parameters must be formed with the minimum level of α_i, the model Definite can be defined as follows:

Objective function

$$Min\ E[Z] = fy + \left(\frac{c_{(1)} + c_{(2)} + c_{(3)} + c_{(4)}}{4}\right) x$$

Subject to:

$$Ax \geq (1-\alpha_m)\, d_{(3)} + \alpha_m d_{(4)}$$

$$Bx \leq \left(\tfrac{\alpha_m}{2}\right) d_{(3)} + \left(1 - \tfrac{\alpha_m}{2}\right) d_{(4)}$$

$$Bx \geq \left(\tfrac{\alpha_m}{2}\right) d_{(2)} + \left(1 - \tfrac{\alpha_m}{2}\right) d_{(1)} \qquad (23)$$

$$Sx \leq [(1-\alpha_m)\, N_{(2)} + \alpha_m N_{(1)}] Y$$

$$Y \in \{0,1\}$$

$$x \geq 0$$

According to the said articles, the definitive equivalent model of the proposed model will be as follows:

$$\text{Nec}\{\textstyle\sum_{f=1}^{8} O_{(t.f)} \leq \sum_{f=1}^{8} \widehat{CAP_{(t.f)}} \} \geq \alpha_1 \qquad (24)$$

$$\text{Nec}\{\widetilde{D_{(t.f)}} = \textstyle\sum_{i=3}^{I}\left[\left(\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)}\right) * Y_{(i.t.f)} - ss_{(i.t.f)}\right] + Q_{(t.f)}\} \geq \alpha_2 \qquad (25)$$

$$\text{Nec}\{\widetilde{D_{(t.f)}} \leq Q_{(t.f)} + \textstyle\sum_{i=3}^{I}(\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)})\} \geq \alpha_3 \qquad (26)$$

The definition of the limits of this section is the same as in the previous section, and we included uncertainty in only three constraints that included demand and capacity. The objective function, like the last section, minimizes the available costs.

Objective function

$$MinE[Z] = \textstyle\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{C1+C2+C3+C4}{4}\right) * O_{tf} +$$

$$\textstyle\sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{H1+H2+H3+H4}{4}\right) * \vartheta_{(i.t.f)}$$

$$+ \textstyle\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{W1+W2+W3+W4}{4}\right) * u_{(t.f)} +$$

$$\textstyle\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{P1+P2+P3+P4}{4}\right) * Q_{(t.f)} \qquad (27)$$

Subject to:

$$\textstyle\sum_{f=1}^{8} O_{(t.f)} \leq [(1-\alpha_1) * \sum_{f=1}^{8} CAP_2 + \alpha_1 * \sum_{f=1}^{8} CAP_1] \qquad \forall t \qquad (28)$$

$$\textstyle\sum_{i}(V_{i.t-1.f} + SS_{i.t.f} + e_{i.t.f}) \leq cpu_{t.f} \quad \forall t, f \qquad (29)$$

$$E_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \qquad (30)$$

$$E_{(i.t.f)} = O_{(t.f)} * \theta_{(i.t.f)} \qquad \forall t.f.i \geq 3 \qquad (31)$$

$$Y_{(i.t.f)} \geq Y_{(i-1.t.f)} \qquad \forall t.f.i \geq 3 \qquad (32)$$

$$\textstyle\sum_{i=3}^{I}[\alpha_{(i.t.f)} + \beta_{(i.t.f)} - ss_{(i.t.f)}] + Q_{(t.f)} \leq [\left(\tfrac{\alpha_2}{2}\right) * D_{3(t.f)} + \left(1 - \tfrac{\alpha_2}{2}\right) * D_{4(t.f)}] \quad \forall t.f.i \geq 3 \qquad (33)$$

$$\textstyle\sum_{i=3}^{I}[\alpha_{(i.t.f)} + \beta_{(i.t.f)} - ss_{(i.t.f)}] + Q_{(t.f)} \geq [\left(\tfrac{\alpha_2}{2}\right) * D_{2(t.f)} + \left(1 - \tfrac{\alpha_2}{2}\right) * D_{1(t.f)}] \quad \forall t.f.i \geq 3 \qquad (34)$$

$$\alpha_{(i.t.f)} + \beta_{(i.t.f)} - \mu_{(i.t.f)} - \varphi_{(i.t.f)} \geq ss_{(i.t.f)} \quad \forall t.f.i \geq 3 \qquad (35)$$

$$[(1 - \alpha_3) * D_{3(t.f)} + \alpha_3 * D_{4(t.f)}] \leq Q_{(t.f)} + \textstyle\sum_{i=3}^{I}[\vartheta_{(i-1.t-1.f)} + E_{(i.t.f)}] \qquad \forall t.f \qquad (36)$$

$$Y_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \qquad (37)$$

$$\vartheta_{(i.t.f)} = \vartheta_{(i-1.t-1.f)} + E_{(i.t.f)} - \alpha_{(i.t.f)} - \beta_{(i.t.f)} + \Delta_{(i.t.f)} - \gamma_{(i.t.f)} \quad \forall t.f.i \geq 3 \qquad (38)$$

$$\vartheta_{(i.t.f)} = 0 \qquad \forall t.f.i = 1.2 \qquad (39)$$

$$\vartheta_{(i.0.f)} = 0 \quad \forall t.f.i = 1.2 \qquad (40)$$

$$U_{(t.f)} = \vartheta_{(35.t.f)} \qquad \forall t.f \qquad (41)$$

$$ss_{(i.t.f)}.\, U_{(t.f)}.\, Q_{(t.f)}.\, \vartheta_{(i.t.f)}.\, O_{(t.f)}.\, E_{(i.t.f)}.$$

$$\gamma_{(i.t.f)}.\, \Delta_{(i.t.f)}.\, \varphi_{(i.t.f)}.\, \mu_{(i.t.f)}.\, \beta_{(i.t.f)}.\, \alpha_{(i.t.f)} \in Z^+ \qquad (42)$$

$$Y_{(i.t.f)} \in \{0.1\} \qquad (43)$$

3. 4. Robust Fuzzy Possibilistic   Similar to the chance constraint possibilistic model, the first part in the objective function is the expected value of z "E[z]". The second part of the objective function is the difference between the maximum and minimum possible values of z based on trapezoidal distribution. The method finds the desired value for the confidence levels, and the confidence levels are considered as a variable. In the model, the coefficient γ indicates the significance of the difference between the minimum and maximum values of the objective function and can be used in the range of [0,1]. Therefore, the presence of the section in the target function leads to a minimization of the maximum deviation of the maximum and minimum optimal values of z. It is worth noting that this optimally stable part controls the answer to the problem.

The third part, added to the objective function, indicates the level of confidence in any random constraint in which the penalty for deviating from the limit values is the uncertain parameters. It shows the difference between the worst value of the uncertain parameter and the value used in the random constraint. Therefore, using the part, the condition of the answers will be established. It should be noted that in the stable model, the expression γ (z_{max}, E[z]) minimizes the maximum deviation of the highest and lowest optimal expected value of the objective function, but in some cases, the decision-maker only select one of these. Two values of sensitivity deviation are shown as follows.

$$min E[Z] + \gamma(z_{max} - E[z]) + \delta\left(d_{(4)} - (1 - \alpha)d_{(3)} - \alpha d_{(4)}\right) + \varphi\left[\left(\beta N_{(1)} + (1 - \beta)N_{(2)} - N_{(1)}\right)\right]$$

$$Ax \geq (1 - \alpha)d_{(3)} + \alpha d_{(4)}$$

$$Bx \leq \left(\frac{\alpha_m}{2}\right)d_3 + (1 - \frac{a_m}{2})d_4$$

$$Bx \geq \left(\frac{\alpha_m}{2}\right)d_2 + (1 - \frac{a_m}{2})d_1 \qquad (44)$$

$$Sx \leq [\beta N_{(1)} + (1 - \beta)N_{(2)}]$$

$$Tx \leq 1$$

$$Z_{max} = f_4 * x + c_4 * y$$

$$x \geq 0. \quad 0 \cdot 5 \leq \alpha . \beta \leq 1$$

Therefore, the complete model is as follows so that the constraints are like 28-43.

Objective function

$$MinE[Z] = \sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{C1+C2+C3+C4}{4}\right) * O_{tf} +$$

$$\sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{H1+H2+H3+H4}{4}\right) * \vartheta_{(i.t.f)}$$

$$+ \sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{W1+W2+W3+W4}{4}\right) * u_{(t.f)} +$$

$$\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{P1+P2+P3+P4}{4}\right) * Q_{(t.f)} +$$

$$\gamma\left[\sum_{t=1}^{T}\sum_{f=1}^{8} C_4 * O_{tf} + \sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{f=1}^{8} H_4 * \vartheta_{(i.t.f)} + \sum_{t=1}^{T}\sum_{f=1}^{8} W_4 * u_{(t.f)} + \sum_{t=1}^{T}\sum_{f=1}^{8} P_4 * Q_{(t.f)}\right) - \sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{C1+C2+C3+C4}{4}\right) * O_{tf} + \qquad (44)$$

$$\sum_{i=1}^{I}\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{H1+H2+H3+H4}{4}\right) * \vartheta_{(i.t.f)} +$$

$$\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{W1+W2+W3+W4}{4}\right) * u_{(t.f)} +$$

$$\sum_{t=1}^{T}\sum_{f=1}^{8}\left(\frac{P1+P2+P3+P4}{4}\right) * Q_{(t.f)} + \delta_1 \sum_{t.f}\left(\left(1 - \frac{\alpha_2}{2}\right)d_{(4)tf} - \frac{\alpha_2}{2}d_{(3)tf} - d_{(3)tf}\right) + \delta_2 \sum_{t.f}\left(d_{(2)tf} - \left(1 - \frac{\alpha_2}{2}\right)d_{(1)tf} - \frac{\alpha_2}{2}d_{(2)tf}\right) + \delta_3 \sum_{t.f}\left(d_{(4)tf} - (1 - \alpha_3)d_{(3)tf} - \alpha_3 d_{(4)tf}\right) + \delta_4 \sum_{t.f}\left((1 - \alpha_1)cap_{(2)tf} + \alpha_1 cap_{(1)tf} - cap_{(1)tf}\right)$$

4. COMPUTATIONAL RESULTS

In this section, a numerical example from the Namazi hospital is presented to study the efficiency of the proposed model. Both programming and scheduling programs are coded with GAMS software using the CPLEX solver. It should be noted that all the necessary tests were performed on a CORE i5 computer with 1 T of RAM. The formulation of the described chain involves a large number of definite and non-deterministic parameters. Therefore, displaying all parameters is not possible due to space constraints. As a result, some essential uncertain parameters such as customer demand, shortage, holding, wastage, and purchasing costs, the capacity of the blood center and maximum hospital capacity are presented in Tables 1, 2, and 3, respectively.

To investigate the effect of the three factors of demand, the maximum capacity, and the shortage cost, various issues have been considered in which two factors are considered constant and the other one is considered variable. Figures 2 to 4 show how the objective function changes in relation to maximum capacity, demand, and shortage cost where scale objective function values according to Million. Figure 2 show the objective function changes are very high about the amount of demand, while the sensitivity of the objective function to the maximum capacity is small and negligible based on Figure 2.

TABLE 1. Fuzzy Demand for each blood group

Blood group	Demand
O$^+$	(15,22,28.35)
O$^-$	(5,7,9,11)
A$^+$	(8,12,17,21)
A$^-$	(1,2,3,4)
B$^+$	(11,15,20,24)
B$^-$	(0,1,2,3)
AB$^+$	(0,2,4,6)
AB$^-$	(1,2,3,4)

TABLE 2. Fuzzy costs

Purchasing cost	(30000,35000,40000,45000)
Holding cost	(3000,4000,5000,6000)
Wastage cost	(3000,5000,7000,9000)
Shortage cost	(400000,500000,600000,700000)

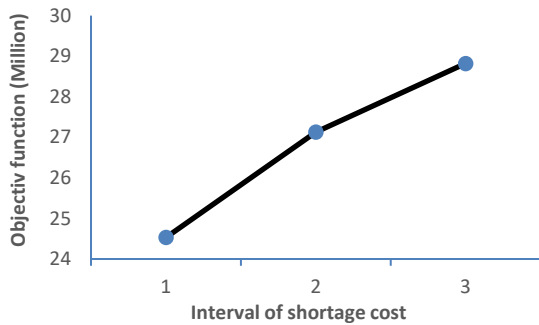**TABLE 3.** Fuzzy Capacity blood center for each blood group

Blood group	Capacity
O$^+$	(221,231,241,251)
O$^-$	(190,218,230,240)
A$^+$	(185,192,200,207)
A$^-$	(175,185,195,205)
B$^+$	(180,190,200,210)
B$^-$	(170,180,190,200)
AB$^+$	(143,150,157,202)
AB$^-$	(132,138,144,152)
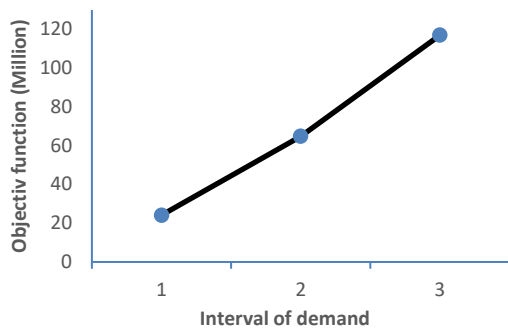
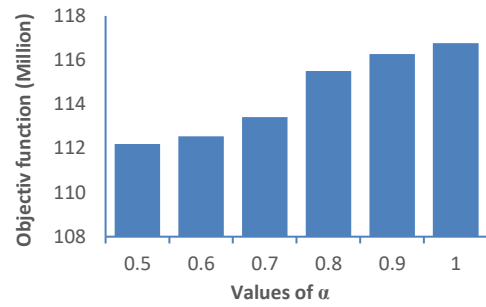**Figure 2.** Objective function changes with increasing demand
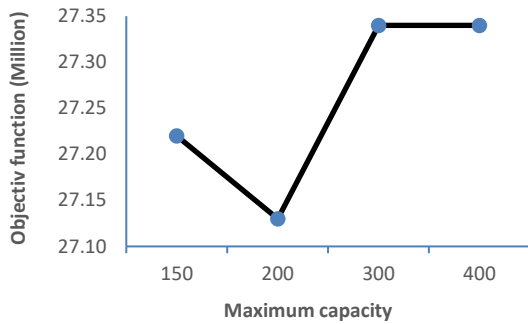
Figure 3. Objective function changes with increasing maximum capacity


Figure 4. Objective function changes with increasing shortage cost

The objective function changes with respect to increasing maximum capacity is shown in Figure 3. As shown in Figure 4 the objective function increases with increasing shortage cost.

Figure 5 also shows the comparison of the objective function obtained from the problem taking into different values of α_i. According to the Figure 5, the objective function increases with increasing α-values. Based on this diagram, it can be concluded that with increasing values of α_i, the scope of the restrictions will be smaller, and thus the value of the objective function increases.

Figure 5. Objective function changes with increasing α_i
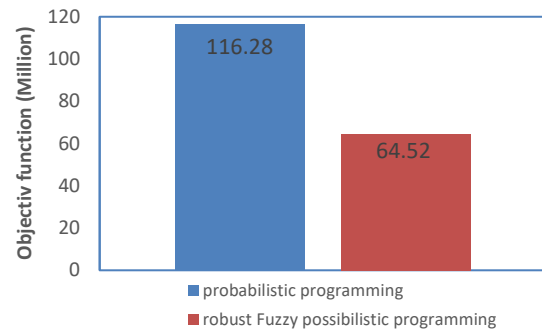
Figure 6. Comparison of the objective function of two models

Figure 6 illustrates the value of robust fuzzy possibilistic programming objective function with maximum confidence level α is less than the value of probabilistic programming objective function with the lowest level of penalty γ coefficient.

As shown in Table 4 the value of the objective function and run time of the problem is escalating with increasing the penalty coefficient. As can be seen, α_2 and α_3 have a value of 0.5 which indicates the high risk of the restriction and a low level of confidence. Therefore, the decision-maker must be risk-free against this constraint. In this case, the decision-maker must consider the maximum amount of demand so that we are less deficient in a crisis.

TABLE 4. Sensitivity analysis on weight factor parameter γ

$\delta_1 . \delta_2 . \delta_3 . \delta_4 = 10$					
$\gamma = 0.3$		$\gamma = 0.6$		$\gamma = 0.9$	
Run time (s)	60	Run time (s)	79	Run time (s)	95
E[Z]	66/77	E[Z]	71/67	E[Z]	75/99
α_1	1	α_1	1	α_1	1
α_2	0.5	α_2	0.5	α_2	0.5
α_3	0.5	α_3	0.5	α_3	0.5

5. CONCLUSION

In the paper, we will focus on a single-level inventory model for blood supply chain in the Namazi hospital and an optimization model designed to manage the resources of the hospital that ultimately reduce costs and improves patient services. The focus of this study is on the whole blood because its demand is higher and it is a non-replaceable substance. We used integer programming to the model. The first model was a nonlinear definite model which we linearized it. The second model was an uncertain model that was more closely related to the real world and solved it by a robust fuzzy possibilistic method.

The examined model was a single-product and multi-cycle model. The delivery time for blood supply was zero and the blood centers were limited and the policy was to send the product using the FIFO principle. By reducing the maximum capacity, you can not order a lot because less blood can be stored, so there will be shortage costs which will increase costs, as well as increased capacity will also increase orders. It imposes the cost of maintenance and waste on the system but its enormous increase does not affect the cost and costs constant because we can not be more than the capacity of the blood transfusion organization. Therefore, considering the appropriate limit for maximum capacity requires strong management. By increasing the shortage cost, the hospital maintains more blood units, therefore levels of deficiency reduces and inventory levels increases in leading to an increase in casualties and an increase in total hospital costs. The results showed with increasing demand for patients the total expected cost of the hospital increases, because the number of orders and the likelihood of deficiency to the hospital increases. In future research, we can use other optimization approaches. The following could be presented as suggested paths for future research.

- Extends the model and makes it a multi-product model, adding another level to the problem and reducing the cost of blood transfusion centers.
- Blood componenet or products e.g. platet and plasma is considered in the problem.
- A closed-loop supply chain is designed to accommodate a refund for blood that has not been used by the hospital and has expired.
- Patient satisfaction is also considered in the model.

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Persian Abstract

چکیده

چالش اصلی در زنجیره تامین خون، کمبود و هدر رفتن فراورده‌های خونی می‌باشد. به دلیل ویژگی‌های فاسدشدنی این محصول، نگهداری تعداد زیادی از واحدهای خونی به عنوان موجودی باعث خراب شدن این منابع محدود و نادر می‌شود. از طرف دیگر کمبود خون ممکن است باعث لغو فعالیت‌های مهم مرتبط با سلامتی شود و نتیجه آن افزایش بالقوه مرگ و میر در بیمارستان‌ها است. در این مقاله یک مدل برنامه‌ریزی عدد صحیح با هدف حداقل کردن هزینه کل، کمبود و هدر رفت فراورده‌های خونی در بیمارستان نمازی ارائه شده است. پارامترها در دنیای واقعی نامشخص هستند و این مساله در این مقاله مورد بررسی قرار می‌گیرد. رویکرد برنامه‌ریزی محدودیت احتمال ارائه شده است و یک تصویر عددی از بیمارستان نمازی برای نشان دادن کاربرد مدل بهینه پیشنهادی استفاده گردیده است. تجزیه و تحلیل حساسیت برای اعتبارسنجی مدل در مورد مسائلی مانند سطح اطمینان در مدل احتمالی، وزن ضریب و مقدار جریمه تابع هدف در مدل فازی استوار انجام شده است. نتایج عددی نشان می‌دهد که مدل قادر به کنترل عدم قطعیت می‌باشد و قیمت مقاومتی بر سیستم تحمیل می‌شود؛ بنابراین مقدار تابع هدف در فازی استوار ۸۰ درصد کمتر است.