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A Continuum Damage Mechanics-based Piecewise Fatigue Damage Model for Fatigue Life Prediction of Fiber-reinforced Laminated Composites

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ABSTRACT

The purpose of this study is to define a piecewise fatigue damage model (PFDM) for the prediction of damage in composite laminates under cyclic loading based on the continuum damage mechanics (CDM) model. Assuming that damage in fiber-reinforced plastic structures accumulates nonlinearly, a piecewise degradation growth function is defined and coupled with CDM and micromechanics approaches. The model divides the damage behavior of fiber, matrix, and fiber/matrix debonding at the ply scale, into three different stages. For generality, a fully multi-stage damage formulation on a single-ply level is employed. The unknown parameters of the PFDM are estimated according to obtained experimental data of damage mechanisms associated with the composites laminate under cyclic loading. To predict multidirectional composite laminates' fatigue life, the proposed model was implemented in Abaqus software by the subroutine. In a validation against experimental data on carbon fiber reinforced material, the model proves to provide a good numerical approximation of the damage during the fatigue loading. The results reveal that by considering the multi-stage process in stiffness reduction, the proposed model can estimate the fatigue life of composite laminate under multiaxial cyclic loading conditions more accurately than the similar model in the literature.

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NOMENCLATURES

Material parameter	a	**Greek Symbols**	Γ
Material constant	A	Gibbs free energy	ε
Material parameter	b	Normal strain	γ
Material constant	B	Shear strain	ν
Material constant	C	Poisson ratio	ρ
Damage variable	D	Mass density	σ
Young's modulus of elasticity	E	Normall stress	τ
Variance ratio	F	Shear stress	ν
Shear modulus	G	Poisson ratio	ω
Number of cycles	N	Piecewise degradation growth function	**Subscripts**
Probability value	p	Fiber	f
Volume fraction	V	Matrix	m
Conjugate forces of damage variable	Y	Critical	cr

1. INTRODUCTION

Composites are one of the most well-known materials that reduce the structure's weight without reducing the strength for various functions [1-4]. Polymer composites exploit a wide range of applications (such as electronic, aerospace, automobile, etc.) due to their all-around excellent performance in mechanical, thermal, and electrical properties [5-9]. One of the main concerns in using these materials is their relatively unknown and

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complicated damage process. The mechanisms of local damage usually initiate in the early stages of loading and propagate over time according to the loading, environmental, and boundary conditions. The composites' damage mechanisms can be classified into fiber breakage, matrix cracking, fiber/matrix debonding, and delamination. In the past, many researchers have made efforts to assess the damage behavior of composite structures under different loading conditions experimentally [10-12] and theoretically [13]. While considerable progress was achieved in this area, no single method is currently available that precisely estimates degradation at all levels of damage mechanism, for all loading conditions, and all types of laminated composites. On the other hand, in the last decades, due to the stochastic nature of composite damage behavior, data analysis methods [14] and Bayesian framework [15], etc. have been coupled with fatigue models to obtain the real scatter of the predicted results, similar to the scatter of experimental data.

The composites used in the structures can experience fatigue and degradation due to cyclic loads. Compared to static loading, less attention has been paid to fatigue conditions. Tools and methods that simulate the response of fiber-reinforced plastic structures under cyclic loading are required to design structures that can tolerate such failure. The response of structures made of composites under cyclic loads has been studied by several authors, which has led to the expansion of some fatigue damage models. These models can be classified into various categories; fatigue life concepts [16, 17], phenomenological models [18, 19], strength degradation models [20, 21], micromechanics models [22, 23], and continuum damage mechanics (CDM) models [24].

The fatigue life models are based on the usage of stress versus life, the so-called *S-N* procedures [25]. These approaches cannot simulate the various failure mechanisms occurring in composites [16]. These models' limitation is that they require extensive empirical data to evaluate each new laminate [26].

The phenomenological models describe material damage by modeling the reduction of specific mechanical properties (strength/stiffness) of composites during fatigue loading. Based on these approaches, some mathematical definition of strength/stiffness reduction versus the number of fatigue loading is introduced by curve-fitting on empirical data. Some methods have been developed in this area for stiffness reduction [19] and strength reduction [21] under fatigue loading. These models' main problems are the need for comprehensive empirical data and the deficiency of particular fracture criteria.

The strength degradation models are generally a composition of fracture criteria, fatigue life approaches, and degradation laws for mechanical properties [25]. Stress analysis is carried out in each load cycle and failure is investigated based on the fracture criteria. If the failure occurs, the mechanical properties of the relevant failed area are abruptly dropped; otherwise, the entire material's mechanical properties are gradually reduced based on the number of loading cycles. This procedure continues until all layers experience catastrophic fracture mode [20, 27].

To apply the micromechanical method, a suitable unit cell is selected from the composite material, and the stress and strain fields are obtained based on micromechanics [28]. At the micro-level, the stress components in the constituents need to be calculated [29]. Micromechanical models may be used in a multiscale analysis to predict the onset and propagation of cracks and stiffness reduction. These models include stress-transfer mechanisms [30], shear-lag approach [31], and variational methods [32]. The limitation of these procedures is their dependence on the micromechanical models' ability to analyze the diverse damage mechanisms associated with the composites and their limited performance with different boundary conditions, loads, and various lay-up configurations [33].

The CDM models are based on thermodynamic potential (Gibbs free energy function) to express material constitutive equations. In the composite area, the CDM approach was first introduced by Ladeve'ze [34] and Ladeve'ze et al. [35] for static loading. In recent decades, this model is developed to calculate the behavior of damage in fiber-reinforced plastic structures subjected to cyclic loading. Payan and Hochard [36] proposed a new CDM approach to predict static and fatigue damage. This approach was based on the hypothesis that fibers experience brittle failure, where the matrix experiences elastic-plastic failure. Xiong and Shenoi [37] studied the fatigue damage process in a compressive cyclic loading using a CDM base model. They presented that the reduction in stiffness occurs in two different phases. The first stage is the onset of failure, which has little efficacy on mechanical properties. The second stage is the propagation of failure, which directly affects the destruction of mechanical properties. Kawai and Honda [38] suggested a fatigue damage model for composite laminates relying on the CDM approach coupled with in situ strength and the plastic deformation of the layers. Based on the CDM approach and the definition of two variables of damage in the fiber and matrix, Shi et al. [39] presented a fatigue damage approach for the reinforced polymer lamina, which is according to the characterization process of the model proposed by Zhang and Zhao [40]. Using a single scalar damage variable in the framework of irreversible thermodynamics, Movaghghar and Lvov [41], by considering plane stress, presented an energy-based model to predict fatigue life and assessed progressive damage. Based on the CDM model and using three variable damage variables, Salimi-Majd and colleagues [42] modified Movaghghar's model to estimate intralaminar fatigue damage of fibrous composites. To evaluate the life of fiber-reinforced laminated composites under fatigue loading, Mohammadi et al. [43] by defining three damage variables for the fiber, matrix, and fiber/matrix debonding, proposed a CDM approach relying on effective average local stresses so that it can estimate the fatigue life of multidirectional composites with acceptable accuracy. The characterization of their model is based on Zhang's law, according to *S-N* and material stiffness reduction diagrams in different directions. Based on the CDM method, Mahmoudi et al. [44], using micromechanics, suggested a fatigue damage approach to estimate the life of carbon/epoxy laminate, which considers both damage variables because of static and fatigue load conditions. To validate the proposed model, they utilized the experimental method and showed that considering static damage under the cyclic loading process is necessary. Hohe and his colleagues [45] assumed that the damage of material results from microplastic work and defined a brittle damage model based on the CDM approach that describes the degradation of the laminate composite under harmonic cyclic loading.

In recent years, models based on entropy and thermodynamic laws have been developed that use the maximum entropy to predict composite laminate failure and life under fatigue load conditions. Naderi and Khonsari [46], using an empirical method based on a thermodynamic approach, showed that fatigue is an irreversible process of increasing entropy that accumulates until it achieves a significant value, called fracture fatigue entropy at the beginning of fracture. In another study, the fatigue behavior of glass/epoxy composite laminate using the dissipated energy via different mechanisms was investigated by Naderi and Khonsari [47]. Mohammadi and Mahmoudi [48] proposed a novel method relying on the thermodynamic entropy model to estimate cross-ply laminates' fatigue life and examined variables such as different stacking sequences, loading frequency, and ambient temperature. Considering that dissipated entropy during fatigue conditions can be used to measure the composite damage, they proposed [49] a theoretical approach based on the first and second thermodynamics laws, which consider work of irreversible deformations.

Because of the occurring and mixing of several damage accumulation mechanisms under fatigue loading, the damage behavior of composites must be considered as a combination of micro-damage mechanics (MIDM) and macro-damage mechanics (MADM). In the laminated composites, damage usually starts at the matrix material with the cracks and is accompanied by the fiber/matrix debonding to eventually fiber breakage occurred. On the other hand, according to experimental observations, damage evolution in fatigue conditions occurs in different stages. In the first stage, due to the rapid growth of micro-defects, stiffness reduction occurs nonlinearly. In the next step, stiffness reduction reduces linearly with partial degradation by the loading cycle. Finally, in the third stage, and before the final failure, a rapidly increasing stiffness degradation occurs. Based on a review of the literature, it can be seen that most fatigue models do not explicitly consider the multi-stage process for damage caused by cyclic loading in the formulation of the damage accumulation law. The present study is concerned with a modified model, applicable to multi-stage damage due to fatigue loading in the fatigue process, which leads to more accurate estimations about fatigue life. For this purpose, a generalization of a CDM-based constitutive model coupled with micromechanics constitutive equations and degradation growth function is proposed.

In this paper, three in-plane damage mechanisms based on the CDM approach is discussed, while the out-of-plane damage mechanism, i.e., delamination, is ignored. Based on the literature's experimental data, a degradation growth function is used to modify Mohammadi's model [43] that satisfies the laminate composite's multi-stage damage process under fatigue loading. In this study, this revised model is called the piecewise fatigue damage model (PFDM). In recent decades, damage models relying on the finite element method (FEM) have found the most attention [50, 51]. These models directly predict composite materials' damage and fracture for any stacking sequence and material properties [52]. The CDM approach has been developed increasingly in the FEM. Therefore, to show the ability of the PFDM to estimate the life of composite laminates under tension fatigue loading, a numerical approach was used and implemented in Abaqus software by the subroutine. Besides, the results of the modified model were compared to numerical and experimental data available in the literature, and it is shown that the results are consistent with the empirical data.

2. CONTINUUM DAMAGE MODEL

In the CDM approach, an irreversible thermodynamic theory is used as a logical base to formulate constitutive equations with damage. According to this concept, the free energy and the dissipation function must first be expressed. The free energy indicates the relationship between the variables of the internal variable and their conjugate forces, while the dissipation function describes the development of internal variables.

The CDM approach considered in this work is at a ply scale. In this scale, the damage is supposed to be in the form of fiber breakage, matrix cracking, or fiber/matrix interface debonding. This model describes damage by reducing the stiffness of the material. In this paper, plastic strain due to the brittle response of fiber-reinforced plastic composites is ignored. A degradation growth function is used to predict stiffness reduction in fiber-reinforced plastic composites under cyclic loading. The suggested formulas in the system are based on the materials' principal coordinates so that 1 and 2 show the fiber direction and perpendicular direction of the fibers, respectively.

2. 1. Damage Mechanism The Young's modulus of the undamaged ply in the fiber direction and perpendicular direction is expressed by E_{11}^0 and E_{22}^0, while in the -plane shear modulus is represented by G_{12}^0. In addition, the Poisson's ratio in fibers direction and in its perpendicular directions are expressed by ν_{12} and ν_{21}, respectively. To model various modes of damage, i.e., the fiber breakage, matrix cracking, and the fiber/matrix interface debonding, the values of E_{11}^0, E_{22}^0 and G_{12}^0 may be reduced. Therefore, if the damage variables of E_{11}^0, E_{22}^0 and G_{12}^0 is shown by D_1, D_2 and D_{12}, (indicating fracture in fiber, microcracking in matrix and debonding of fiber/matrix, respectively), then the Gibbs free energy for the ply in damaged state is defined as follows [53]:

$$\rho\Gamma = \frac{1}{2}\left[\frac{\sigma_{11}^2}{E_{11}^0(1-D_1)} - \frac{2\nu_{12}\sigma_{11}\sigma_{22}}{E_{11}^0(1-D_1)} + \frac{\langle\sigma_{22}\rangle_+^2}{E_{22}^0(1-D_2)} + \frac{\langle\sigma_{22}\rangle_-^2}{E_{22}^0} + \frac{(\tau_{12})^2}{G_{12}^0(1-D_{12})}\right] \quad (1)$$

where

$$\begin{aligned}&\langle a\rangle_+ = a \quad \text{if } a \ge 0; \quad \text{otherwise} \langle a\rangle_+ = 0\\ &\langle a\rangle_- = a \quad \text{if } a \le 0; \quad \text{otherwise} \langle a\rangle_- = 0\end{aligned} \quad (2)$$

The elastic constitutive equation is:

$$\varepsilon_{11}^e = \rho\frac{\partial\Gamma}{\partial\sigma_{11}} = \frac{\sigma_{11}}{E_{11}^0(1-D_1)} - \frac{\nu_{12}}{E_{11}^0}\sigma_{22} \quad (3\text{-a})$$

$$\varepsilon_{22}^e = \rho\frac{\partial\Gamma}{\partial\sigma_{22}} = \frac{\langle\sigma_{22}\rangle_+}{E_{22}^0(1-D_2)} + \frac{\langle\sigma_{22}\rangle_-}{E_{22}^0} - \frac{\nu_{12}}{E_{11}^0}\sigma_{11} \quad (3\text{-b})$$

$$\gamma_{12}^e = \rho\frac{\partial\Gamma}{\partial\tau_{12}} = \frac{\tau_{12}}{G_{12}^0(1-D_{12})} \quad (3\text{-c})$$

Damage development is controlled by three conjugate forces Y_1, Y_2, and Y_{12} calculated from the partial derivatives of the Gibbs free energy, relative to D_1, D_2, and D_{12}, respectively:

$$Y_1 = \rho\frac{\partial\Gamma}{\partial D_1} = \frac{\sigma_{11}^2}{2E_{11}^0(1-D_1)^2} \quad (4\text{-a})$$

$$Y_2 = \rho\frac{\partial\Gamma}{\partial D_2} = \frac{\sigma_{22}^2}{2E_{22}^0(1-D_2)^2} \quad (4\text{-b})$$

$$Y_{12} = \rho\frac{\partial\Gamma}{\partial D_{12}} = \frac{\tau_{12}^2}{2G_{12}^0(1-D_{12})^2} \quad (4\text{-c})$$

2. 2. Damage Variables Damage variable in terms of stiffness reduction in fiber and matrix is:

$$D_k = \frac{E_k^0 - E_k}{E_k^0} \quad (5)$$

where $k = f, m$ indicating the fiber and matrix, while E_k^0 and E_k represent the undamaged and damaged stiffness state, respectively. Damage variable in shear is:

$$D_{12} = \frac{G_{12}^0 - G_{12}}{G_{12}^0} \quad (6)$$

where G_{12}^0 and G_{12} represent the undamaged and damaged shear modulus, respectively.

To simulate the damage evolution, the micromechanical theory is used to compute the average stress distributions in each direction. The stress-strain relationship is defined [54]:

$$\left[\sigma_i^m\right] = \left[A_{ij}\right]\left[\sigma_j^f\right] \quad (7)$$

and $\left[A_{ij}\right]$ is:

$$\left[A_{ij}\right] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 & 0\\ 0 & a_{22} & 0 & 0 & 0 & 0\\ 0 & 0 & a_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & a_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & a_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & a_{66}\end{bmatrix} \quad (8)$$

where

$$a_{11} = \frac{E_m}{E_f} \quad (9\text{-a})$$

$$a_{22} = a_{33} = a_{44} = \frac{1}{2} + \frac{E_m}{2E_f} \quad (9\text{-b})$$

$$a_{55} = a_{66} = \frac{1}{2} + \frac{G_m}{2G_f} \quad (9\text{-c})$$

$$a_{12} = a_{13} = (a_{11} - a_{22})\left[\left(\frac{V_m}{E_m} - \frac{V_f}{E_f}\right)\Big/\left(\frac{1}{E_f} - \frac{1}{E_m}\right)\right] \quad (9\text{-d})$$

E_f and E_m are Young's modulus of fiber and matrix, while V_f and V_m represent the volume fractions of fiber and matrix, respectively. The stress matrix in fiber and matrix is [54]:

$$\left[\sigma_i^f\right]=\left[B_{ij}\right]\left[\sigma_j\right] \tag{10-a}$$

$$\left[\sigma_i^m\right]=\left[A_{ij}\right]\left[B_{ij}\right]\left[\sigma_j\right] \tag{10-b}$$

where $\left[\sigma_j\right]$ represents the stress of lamina, and $\left[B_{ij}\right]=\left(V_f\left[I\right]+V_m\left[A_{ij}\right]\right)^{-1}$, and $\left[I\right]$ is a unit matrix.

The undamaged elastic modulus E_{11}^0 in the longitudinal direction and E_{22}^0 in the perpendicular direction are:

$$E_{11}^0=V_fE_f^0+V_mE_m^0 \tag{11-a}$$

$$E_{22}^0=\frac{\left(V_f+V_ma_{11}\right)\left(V_f+V_ma_{22}\right)}{\left(V_f+V_ma_{11}\right)\left(\frac{V_f}{E_f^0}+\frac{a_{22}V_m}{E_m^0}\right)+V_fV_m\left(\frac{V_f}{E_f^0}-\frac{V_m}{E_m^0}\right)a_{12}} \tag{11-b}$$

The elastic modulus in the damaged state is:

$$E_{11}=V_f\left(1-D_f\right)E_f^0+V_f\left(1-D_m\right)E_m^0 \tag{12-a}$$

$$E_{22}=\frac{\left(V_f+V_ma_{11}\right)\left(V_f+V_ma_{22}\right)}{\left(V_f+V_ma_{11}\right)\left(\frac{V_f}{\left(1-D_f\right)E_f^0}+a_{22}\frac{V_m}{\left(1-D_m\right)E_m^0}\right)+V_fV_m\left(\frac{V_f}{\left(1-D_f\right)E_f^0}-\frac{V_m}{\left(1-D_m\right)E_m^0}\right)a_{12}} \tag{12-b}$$

$$G_{12}=G_{12}^0\left(1-D_{12}\right) \tag{12-c}$$

So, the damage variables in longitudinal and perpendicular directions are:

$$D_1=\frac{E_{11}^0-E_{11}}{E_{11}^0} \tag{13-a}$$

$$D_2=\frac{E_{22}^0-E_{22}}{E_{22}^0} \tag{13-b}$$

2. 3. Proposed Method In recent years, there has been much effort to develop a damage evolution model under fatigue loading conditions based on the CDM approach. In this study, to estimate the damage evolution of composite laminate material under cyclic loading, the following model is used [39]:

$$\frac{dD_k}{dN}=\frac{A_kY_k^{B_k}}{\left(1-D_k\right)^{C_k}} \tag{14}$$

where k represents the fiber, perpendicular, and fiber/matrix debonding direction, and A_k, B_k and C_k are the material constants. The conjugate forces of the damage variable are:

$$Y_k=\frac{\sigma_k^2}{2E_k\left(1-D_k\right)^2} \tag{15}$$

According to experimental observations, stiffness reduction starts with the application of fatigue loading. In the initial loading cycles, due to the rapid growth of micro-defects, stiffness reduction occurs nonlinearly. In the second stage, the initial stiffness reduction is followed by degradation as the loading cycles increase. Finally, before the final degradation, in the third stage, a rapidly increasing stiffness reduction occurs. The stiffness reduction process under fatigue loading is shown in Figure 1. Based on the mentioned degradation process in laminate composite, in the present paper, to estimate multi-stage stiffness reduction under cyclic loading, the damage evolution law in Equation (14) is expanded to:

$$\frac{dD_k}{dN}=\frac{A_k\omega\left(D\right)Y_k^{B_k}}{\left(1-D_k\right)^{C_k}} \tag{16}$$

where $\omega\left(D\right)$ is the piecewise degradation growth function:

$$\omega\left(D\right)=\begin{cases} b+a_I\left(D-D_I\right)^2 & D\le D_1 \\ b & D_I\le D\le D_{II} \\ b+a_{II}\left(D-D_{II}\right)^2 & D\ge D_{II}\end{cases} \tag{17}$$

D_I, D_{II}, a_I, a_{II} and b are the material parameters, which should be obtained based on empirical observations, considering:

$$\int_0^{D_{cr,k}}\omega\left(D\right)dD=D_{cr,k} \tag{18}$$

According to experimental results available in the literature, various models are proposed for the

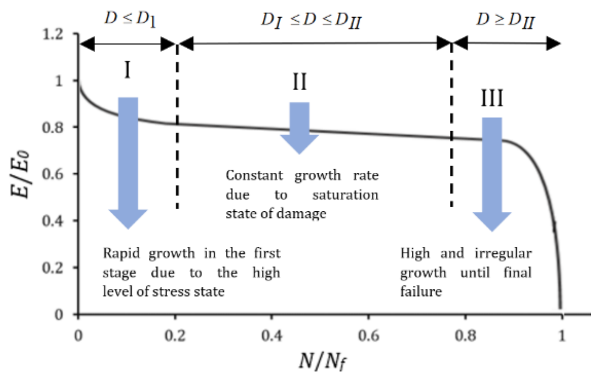


Figure 1. Overall stiffness reduction in fatigue loading

degradation growth function [45]. In this study, the introduced model is called the piecewise fatigue damage model (PFDM), which can fit each of the degradation phases.

3. PARAMETER IDENTIFICATION

Fatigue models need to characterize the material parameters, and many procedures have been developed by authors. The PFDM approach contains two type constants; (1) damage model parameters (i.e., A, B, C for fiber, matrix, and shear damage), and (2) degradation growth function parameters (i.e., a_I, b_{II}, b, D_I, D_{II}).

For the fatigue damage model presented in Equation (14), Shi and his colleagues [39] have proposed characterization parameter. Mohammadi et al. [43] suggested a modified approach for the characterization of material constants. In this study, the modified material characterization procedure suggested by Mohammadi et al. [43] is used. Based on this model, a simple form of the proposed fatigue model is considered in which the degradation growth function can be discarded by assuming $\omega(D)=1$. The damage evolution law is:

$$\frac{dD_k}{dN}=\frac{A_k}{\left(2E_k^0\right)^{B_k}}\frac{\sigma_{\max k}^{2B_k}}{\left(1-D_k\right)^{2B_k+c_k}} \tag{19}$$

where $\sigma_{\max k}$ represents the maximum applied stress in each direction. The integration of Equation (19) from $D=0$ to $D=D_{cr}$ is:

$$\sigma_{\max k}^{2B_k}\cdot N=\frac{\left(2E_k\right)^{B_k}}{A_k\left(2B_k+C_k+1\right)}\left(1-\left(1-D_{cr,k}\right)^{2B_k+C_k+1}\right) \tag{20}$$

where $D_{cr,k}$ is the critical damage parameter in each direction and is calculated experimentally. The logarithm of Equation (20) is:

$$\log N_k=\log\left[\frac{\left(2E_k\right)^{B_k}}{A_k\left(2B_k+C_k+1\right)}.\left(1-\left(1-D_{c,cr}\right)^{2B_k+C_k+1}\right)\right]-2B_k\log\sigma_{\max,k} \tag{21}$$

By plotting $\log N_k$ versus $\log\sigma_{\max,k}$, the slope B_k is obtained. On the other hand, by plotting $\log dD_k/dN$ versus $\log\left(1-D_k\right)^{-1}$, the parameters A_k and C_k can be specified.

$$\log\frac{dD_k}{dN}=\log\left(\frac{A_k}{\left(2E_k\right)^{B_k}}\sigma_{\max k}^{2B_k}\right)+\left(2B_k+c_k\right)\log\left(1-D_k\right)^{-1} \tag{22}$$

This method for characterization of material constants is applied for fiber, matrix, and fiber/matrix debonding damages, and model constants are obtained. Then, based on the experimental data, the degradation growth function is obtained for each direction to be combined with the fatigue model.

For characterization of the damage model parameters, tensile, transverse, and shear loadings are required. For this purpose, the results of experiments for measuring the residual stiffness of a unidirectional 0°, 90° plies, and cross-ply laminate under tension-tension fatigue on AS4/3501-6, obtained by Shokrieh and Lessard [27] are used. The characterization of material constants for AS4/3501-6 composites under longitudinal, transverse, and shear loading are adapted from reference [43] and listed in Table 1.

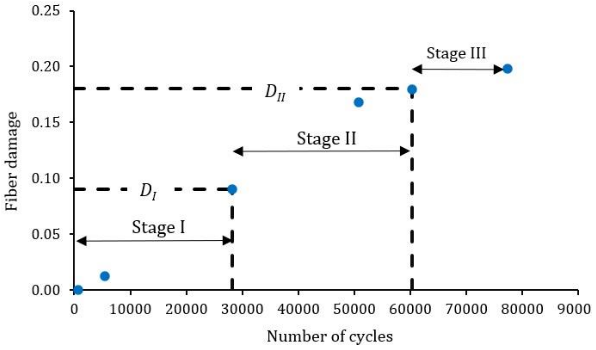
The parameters in the degradation growth function are chosen by means of diagrams of damage versus the number of cycles (*D-N*) and Young's modulus versus the number of cycles (*E-N*), so that Equation (18) is satisfied. For characterization of fiber, based on the experimental data of Shokrieh and Lessard [27] for measuring the stiffness reduction of a unidirectional $[0]_{16}$ laminate subjected to tension-tension cyclic loading, the

D_f-*N*, and *E-N* diagrams are plotted in Figure 2**.** As can be seen, the D_f-*N* diagram is divided into three stages. The values of D_I and D_{II} are obtained as 0.09 and 0.18, respectively. At the intersection of the two segments (cycles N_I and N_{II}), according to Equation (18) the constants a_I, a_{II} and b are 50.93, 91.50, and 0.38. For a description of material constants of the degradation growth function for the matrix and fiber/matrix debonding damages, according to experimental data of $[90]_{16}$ and $[0/90]_s$ laminates under tension-tension fatigue, the D_f-*N* and *E-N* diagrams are drawn in Figures 3 and 4. Based on the same previous procedure, the unknown parameters of the degradation growth function are obtained and listed in Table 1. It is worth mentioning

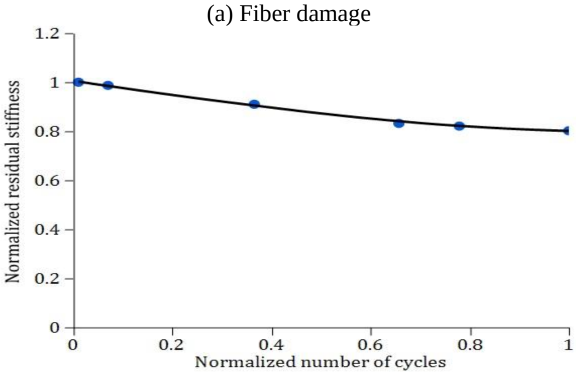
TABLE 1. Unknown parameters of PFDM approach

Function	Parameter	Fiber	Matrix	Shear
Damage model	A	3.2304×10^{-15}	3291	$1.1269\text{X}10^{-4}$
	B	7.84	7.754	6.28
	C	-19.75	-18.21	-21.72
	D_{cr}	0.2	0.56	0.28
Degradation growth	a_I	50.93	12.00	142.72
	a_{II}	91.50	43.92	110.51
	b	0.38	0.47	0.42
	D_I	0.09	0.42	0.15
	D_{II}	0.18	0.55	0.23

that these diagrams are depicted in a stress ratio of 0.1 and different maximum stresses.

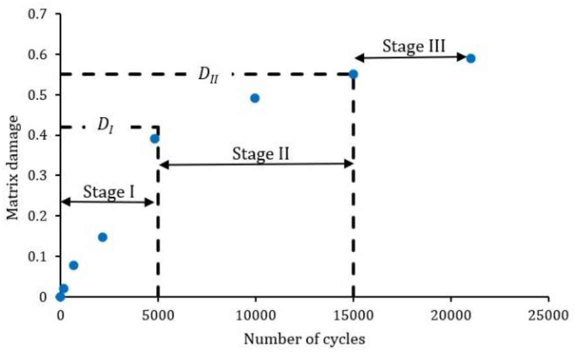


(a) Fiber damage

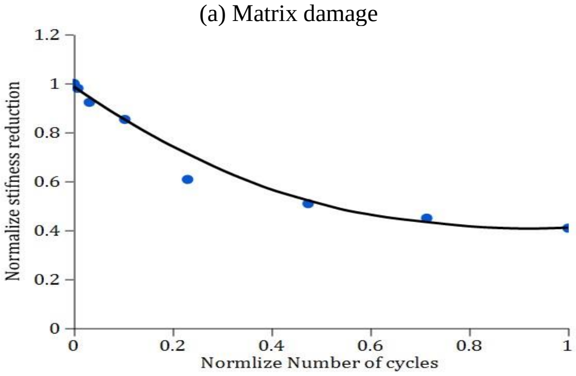


(b) Normalized residual stiffness

Figure 2. Longitudinal direction damage

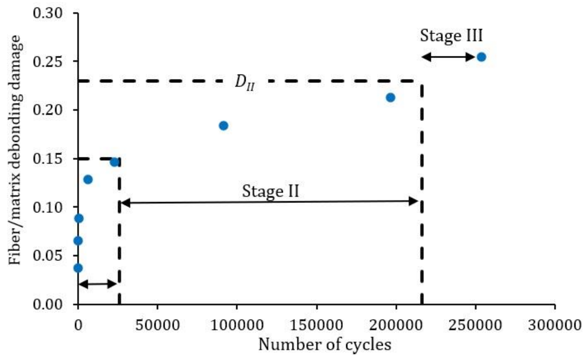


(a) Matrix damage

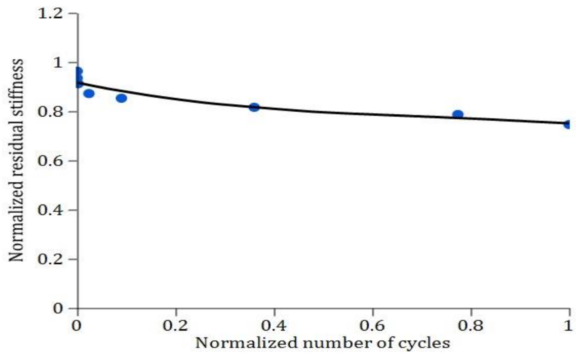


(b) Normalized residual stiffness

Figure 3. Perpendicular direction damage



(a) Shear damage



(b) Normalized residual stiffness

Figure 4. Fiber/matrix debonding damage

4. RESULTS AND DISCUSSION

To show the ability of the PFDM approach, the fatigue damage model proposed in section 2.3, the experimental results of AS4/3501-6 composite laminate [55, 56] are investigated. Based on the PFDM approach and parameter identification, for the prediction of laminated composite fatigue life, a numeral procedure is extended and implemented in Abaqus software by USERMAT subroutine. Figure 5 shows the overall flowchart of the process.

In this section, firstly a comparison between the results obtained of the PFDM approach and numerical analysis by other researchers is performed. For this purpose, the fatigue failure test results on $[30]_{16}$ AS4/3501-6 composite laminate [55] are used under uniaxial tension-tension at different stress levels, load ratio of 0.1, and frequency of 10 Hz. Figure 6 illustrates the estimated fatigue life of $[30]_{16}$ AS4/3501-6 composite laminate by different models. As shown in this figure, there is a good agreement between experimental results and estimated fatigue life by the PFDM approach. In addition, as can be seen from Figure 6, the fatigue life results of the PFDM approach same as the 3D finite element model [57] lay in the range of experimental data at different stress levels. Thus, the comparison of these two numerical models shows an acceptable agreement

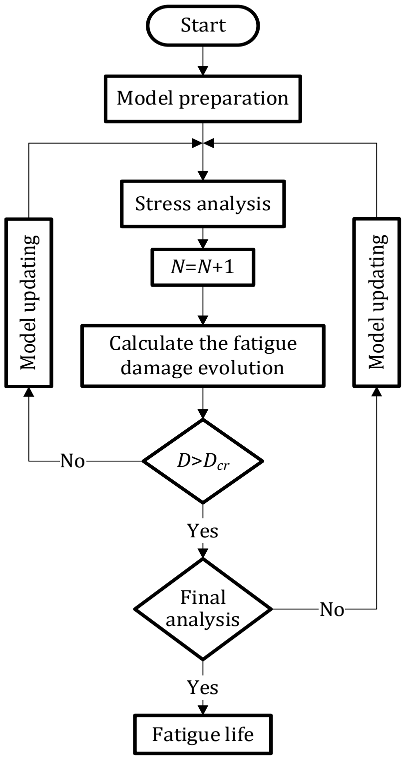


Figure 5. Flowchart of the simulation process using the PFDM approach

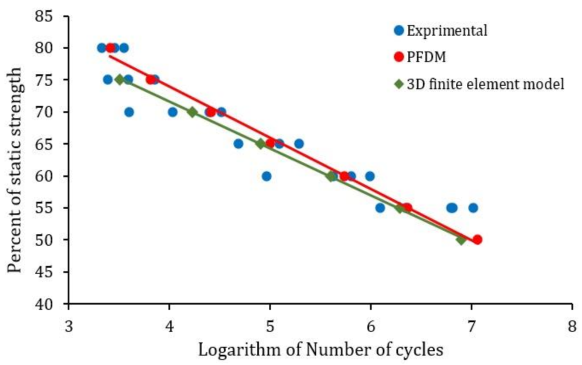


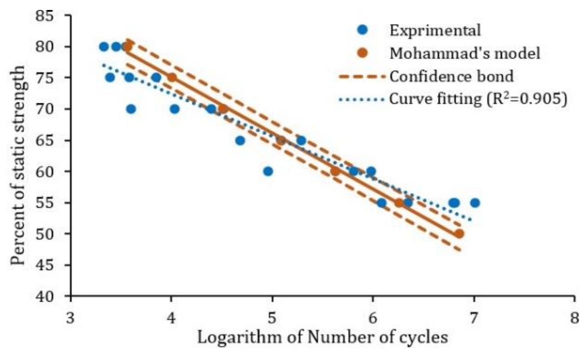
Figure 6. Comparison between fatigue life prediction of $[30]_{16}$ laminate from different numerical models

between them with less difference, and both models have made reliable predictions.

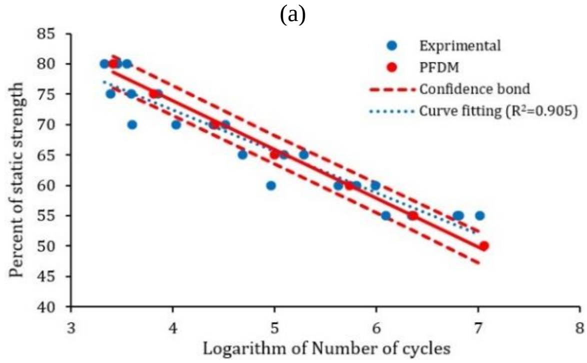
For further verification and assessments of the transferability of the PFDM approach and the parameter set exhibited in Table 1, the model with the previously specified parameter set is compared with the fatigue damage model proposed by Mohammadi et al. [43]. The fatigue life of the $[30]_{16}$ [55] and $[90/45/\text{-}45/0]_s$ [56] laminates made of AS4/3501-6 under fatigue loading conditions with stress ratio equal to 0.1 and different maximum stresses are studied. Figures 7 and 8 show the results. Furthermore, a regression model is used to fit experimental data and fatigue behavior is predicted. It can be seen from these figures that the predicted life of the PFDM approach is more accurate than the fatigue model proposed by Mohammadi et al. [43]. The differences between the two models are because of considering multi-stage stiffness reduction with the number of cycles.

In order to visualize the difference between the results obtained from the numerical approach and the experimental approach, the Analysis of Variance (ANOVA) is helpful. ANOVA is a statistically-based decision tool for detecting differences in the average performance of experimental and simulated data groups. Based on reported data in litertaure [58], some basic information about the ANOVA can be found. For an ANOVA, the null hypothesis is considered that between the groups there is no significant difference, and the alternative hypothesis considers that there is a significant difference between the groups. So, in the ANOVA after cleaning the data, assumptions of ANOVA should be tested. According to this, the *F*-ratio and the associated probability value (*p*- value) must be calculated. In general, observed probabilities of 0.05 or less are often assumed evidence that there are differences in the group means.

The fatigue life prediction of PFDM, Mohammadi's model, and experimental approach are then post-processed by using ANOVA, and the results of the implementation of ANOVA are shown in Table 2. With ANOVA for $[30]_{16}$ and $[90/45/\text{-}45/0]_s$ composite



(a)



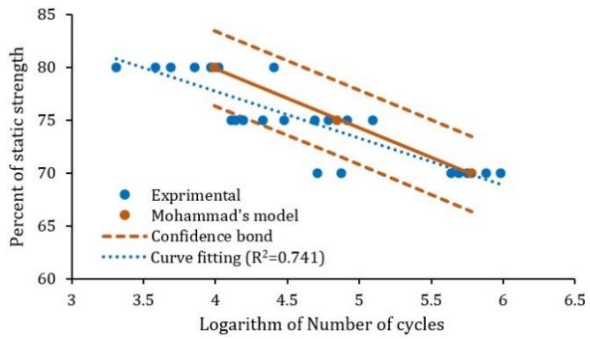
(b)

Figure 7. Fatigue life simulation and experimental results of $[30]_{16}$ laminate

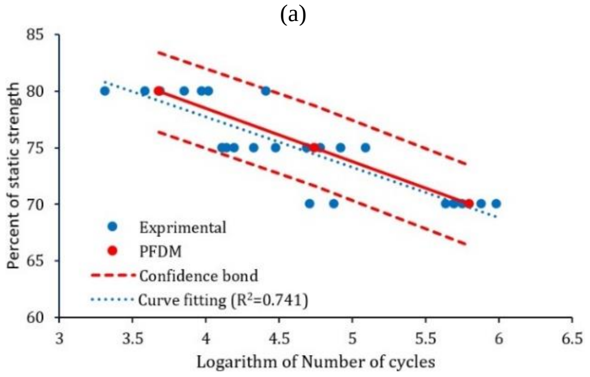
laminates, only 22 experiments are used. As shown in Table 2, the *p*-values of the numerical model are more than 0.05, showing no statistically significant differences between the PFDM approach and experimental data.

It can be concluded that considering the multi-stage stiffness reduction in longitudinal, transverse, and fiber/matrix debonding direction, more accurate results can be achieved. It is evident that if the experimental data for material characterization is more, a more exact estimate of the unknown parameters in the PFDM approach can be obtained. The prediction of the multidirectional composite laminate's fatigue life will be more consistent with the experimental data. On the other hand, by considering variations of the properties and re-running the model several times under different states of stress, the PFDM approach can be developed to achieve the real scatter of the simulation results, similar to the scatter of experimental data. Besides, the investigation should be given to the delamination analysis in the present work and can be improved by taking into account the proper cohesive zone model between the layers in the current model to simulate the separation of two layers.

The presented results are only for AS4/3501-6 Carbon-Epoxy composites under a specific load. These results will be different for other materials, loading, and geometry conditions. Investigating the fatigue life of composite laminate with various properties, considering experimental data scattering, and plastic strain and hardening process using the PFDM approach and probabilistic methods may be recommended for further studies.



(a)



(b)

Figure 8. Fatigue life simulation and experimental results of $[90/45/-45/0]_s$ laminate

TABLE 2. Results of ANOVA for the Variation of fatigue life of $[30]_{16}$ and $[90/45/-45/0]_s$ laminates

Model 1	Model 2	$[30]_{16}$		$[90/45/-45/0]_s$	
		***F* ratio**	***p*-value**	***F* rario**	***p*-value**
PFDM	Experimental	1.225	0.274	0.348	0.558
Mohammadi	Experimental	1.780	0.189	0.370	0.545
PFDM	Mohammadi	0.235	0.630	5.7×10^{-6}	0.998

4. CONCLUSION

The objective of the present work is to study the fatigue behavior of fiber-reinforced laminated composites by a continuum damage mechanics (CDM) model relying on the finite element method (FEM). The simulation results agree with the experiment well. The following concluding remarks are pertinent:

- The model is enhanced by the introduction of the degradation growth function.
- The current model is capable of taking to account multi-stage stiffness reduction in laminated composites during a fatigue loading.
- The model is capable to provide a reasonable qualitative approximation of the experimental data for a wide range of fatigue life of laminated composites.
- The prediction capability depends on the maturity of damage evolution law, micromechanics model, degradation growth function, identification of unknown parameters, etc. integrated into the proposed frame.
- In the current model, delamination has been ignored, which can be improved by applying a proper cohesive model current model to capture the delamination of different modes of fracture.

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Persian Abstract

چکیده

هدف از این مطالعه، تعریف مدل آسیب خستگی چندضابطه‌ای برای پیش‌بینی آسیب چندلایه‌های مواد مرکب تحت بارگذاری خستگی بر اساس مدل مکانیک آسیب پیوسته است. با فرض این‌که آسیب در سازه‌های تقویت_شده با الیاف پلیمری، به صورت غیرخطی انباشته می‌شود، یک تابع رشد تخریب چندضابطه‌ای تعریف شده است و با رویکرد مکانیک آسیب پیوسته و میکرومکانیک ترکیب می‌شود. این مدل رفتار آسیب الیاف، ماتریس و جدایش فصل مشترک الیاف/ماتریس در مقیاس یک لایه را به سه مرحله مختلف تقسیم می‌کند. به طور کلی، یک فرمول آسیب کاملاً چندمرحله‌ای در سطح یک لایه استفاده می‌شود. ابتدا با توجه به داده‌های تجربی به دست آمده از مکانیسم‌های آسیب چندلایه مواد مرکب تحت بارگذاری خستگی، پارامترهای ناشناخته مدل آسیب خستگی چندضابطه‌ای به‌دست می‌آید و سپس برای پیش_بینی عمر خستگی چندلایه چندجهته، این مدل با استفاده از سابروتین در نرم‌افزار آباکوس اجرا می‌شود. در یک اعتبارسنجی در برابر داده‌های تجربی مواد تقویت‌شده با الیاف کربن، مدل اثبات می‌کند که یک تقریب عددی خوب از رفتار آسیب تحت شرایط بارگذاری خستگی ارائه می‌دهد. نتایج نشان می‌دهد که با در نظر گرفتن کاهش سفتی چندمرحله‌ای، مدل پیشنهادی می‌تواند عمر خستگی چندلایه مواد مرکب تحت شرایط بارگذاری خستگی چندمحوره را با دقت بیشتری نسبت به مدل مشابه در سایر مقالات تخمین بزند.