



## Ratcheting Analysis of Steel Plate under Cycling Loading using Dynamic Relaxation Method Experimentally Validated

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### PAPER INFO

#### Paper history:

Received 26 November 2020  
Received in revised form 02 April 2021  
Accepted 24 April 2021

#### Keywords:

Ratcheting  
Dynamic Relaxation Method  
Bending Loading  
Elastic-plastic Deformation

### ABSTRACT

The present study aimed to introduce a numerical method to study ratcheting strains of rectangular plates. A new numerical analysis was conducted by development of dynamic relaxation method combined with MATLAB software to evaluate the ratcheting behavior of the thin steel plate under mentioned loading condition. In order to verify the results, experimental tests were performed under stress-controlled conditions by a zwick/roell amsler HB100 machine and bending ratcheting of CK45 steel plate at room temperature was studied. Under stress-controlled conditions with non-zero mean stress, ratcheting behavior occurred on thin plate. Moreover, a finite element analysis was carried out by Abaqus using nonlinear isotropic/kinematic (combined) hardening model. The results showed that the rate of ratcheting strain decreased with an increase in cycle number. It was found that the hysteresis loops were wider in experimental method than those of other methods because of more energy dissipation. The numerical results are in a good agreement with the simulation and experimental data. Comparison of errors between these methods obviously demonstrate high accuracy of the new introduced method.

doi: 10.5829/ije.2021.34.06c.17

## 1. INTRODUCTION

Cyclic plastic deformation as an important source of the fatigue life-limiting factor can lead to catastrophic failure in critical engineering installations [1, 2]. On account of applied load and symmetric or asymmetric strain/stress cycling, materials will fail by high or low cycle fatigue, mean stress relaxation and ratcheting [3]. Ratcheting corresponds on progressive plastic deformation under stress-controlled cyclic loading with asymmetric stress (non-zero mean stress), if applied load is kept high enough for inelastic deformation [1, 4]. Ratcheting or cyclic creep is related to cyclic accumulation of strain, in the direction of mean stress [5]. Accumulation of ratcheting strain, formation and shifting of stress-strain hysteresis loop can lead to fatigue damage [2]. Therefore, simulations of hysteresis loop curves are highly important in numerical modeling of cyclic plastic deformation [6]. Investigation of

ratcheting phenomena is important to estimate remaining life, assess safety of components and design the structures [2].

As ratcheting has the highest detrimental influence on fatigue life in comparison between other forms of cyclic plastic deformation, numerous investigations have been conducted based on determination of effective controlling factors of ratcheting. Recent studies in the field of ratcheting are devoted to three categories including: (1) experimental studies on the influence of loading amplitude, temperature, loading rate, mean stress, etc. [7-10], (2) experimental studies on ratcheting behavior under uniaxial or multiaxial loading [11-13], and (3) numerical studies on development of ratcheting models [14, 15].

Considering necessity of research cost reduction and on account of ratcheting importance, it is noteworthy to numerically investigate the nonlinear ratcheting event. Dynamic Relaxation (DR) method as an approximate iterative technique, which is suitable for nonlinear analysis, explicitly solves the simultaneous system of equations and can be used for ratcheting investigation. So far, various studies have been conducted on using

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DR method to solve different problems. Collins and Cosgrove [16] used DR modeling for braced bending active grid shells with rectangular sections. Their model was validated by reference to closed form benchmark solutions and finite element models. The results exhibited major stiffness variations in various bracing and joint models. DR method was utilized by Golmakani and Kadkhodayan to solve non-linear thermo-mechanical bending behavior of thin and moderately thick functionally graded sector plates [17]. They compared the bending analysis results of sector plate based on first order shear deformation theory and classical plate theory. Zhang et al. [18] investigated the post-buckling analysis of compressed rods in cylinders by introducing a DR method. They proved that the results of numerical simulation were in good agreement with the analytical ones. DR method was used to compute the structural buckling limit load by Rezaiee Pajand and Estiri [19]. They stated that their method can successfully estimate the buckling limit load of structures. Esmailzadeh and Kadkhodayan [20] analyzed the stiffened porous bi-directional functionally graded plates by DR method.

Besides of using DR method in solving problems, its modifications and combination of this method with other numerical techniques were taken into consideration in recent literature. Kadkhodayan and Alamatian [21] developed DR method by introducing a modified fictitious time increment determined by minimizing the residual force after each iteration. They demonstrated the advantages, potentiality and high efficiency of this new method. They also showed that the modified fictitious time optimized the convergence rate of the DR method. Namadchi and Alamatian [22] combined DR method with the explicit-implicit time integrations. They found that applying the proposed strategy led to excellent accuracy in comparison with the well-known implicit Newmark and Wilson schemes. They concluded that their method reduced the computational efforts and it had a potential to combine with any explicit-implicit time integration method. Rezaiee-Pajand and Estiri [23] explored the capability of mixing DR method with load factor and displacement increments to trace the complex structural equilibrium path. They proved this suggested approach by applying it on several 3D trusses and 2D frames, with geometrical nonlinear behavior. The results demonstrated the successful trace of the complex structural equilibrium path. Rezaiee Pajand and Estiri [24] found the buckling points for nonlinear structures by implementing DR scheme. Setting the external work zero, they introduced a new scheme to calculate the load factor. The results of their study indicated that the quick proposed approach could trace the complex structural static paths, even in the snap-back and snap-through parts.

In order to generate accurate cutting patterns for tensile fabric structures, Gale and Lewis [25] proposed a new patterning method by using a discrete element model, advanced flattening methods, DR and re-meshing. They stated that the proposed model as a simple alternative was suitable for cutting pattern generation. Based on a DR algorithm, Wang et al. [26] presented a novel numerical form-finding method for cable nets in mesh reflectors. They demonstrated that the presented method could predetermine the uniform distribution of forces and also, it had a much higher calculating efficiency and a much less total calculating time for mesh reflectors than the existing algorithms.

In view of importance of ratcheting and considering precision of DR method, it is worth to use DR in the studies of ratcheting phenomena, which has not been reported yet in the literature. In this paper, ratcheting behavior of CK45 steel plate is experimentally investigated and also, ratcheting of mentioned plate is simulated by Abaqus. Then, as the main objective of this paper, a proposed numerical method by development of DR technique is introduced and the obtained results of Abaqus model, experimental investigation and proposed method were compared.

## 2. MATERIALS AND METHODS

### 2. 1. Geometrical and Mechanical Properties

The rectangular plate used in this investigation is made of CK45 steel. The chemical composition of the material is Mn 0.65%, C 0.46%, Si 0.25%, P 0.05% and S <0.05%. The geometry of the rectangular plate is taken as 180 mm length, 50 mm width and 3 mm thickness. The standard tensile test of material was performed by Shariati and Mehrabi [27]. On the other hand, the specimen of the present study is prepared from the other part of mentioned material. Therefore, the mechanical properties of specimen are extracted from stress-strain curve obtained from previous study [27] as summarized in Table 1.

### 2. 2. Experimental Setup and Procedure

The device used in this research is a zwick/roell amsler HB100 machine shown in Figure 1. The apparatus is connected to a computer for the test control as well as

**TABLE 1.** Mechanical properties of CK45 [27]

Property	Values
Young's modulus	$E = 204 \text{ GPa}$
Yield stress	$\sigma_y = 384 \text{ MPa}$
Ultimate stress	$S_u = 690 \text{ MPa}$
Poisson's ratio	$\nu = 0.3$

data acquisition. Experiments were carried out at room temperature and under stress-controlled condition with mean load of 350N and load amplitude of 350N. To apply the cyclic bending load a proper fixture is designed. It is used to exert the sinusoidal load to the center of rectangular plate. This type of loading causes bending of plate cyclically. Figure 2 shows the schematic of rectangular plate placed on the bending fixture.

**2. 3. Development of DR Method for Ratcheting Analysis**

To analyze the structures using DR method, the static system of equations is converted to fictitious dynamic space. As can be seen in Equation (1), the dynamic system achieved by adding fictitious mass and damping factor to the static system of equations,

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [S]\{X\} = \{P\} \tag{1}$$

where  $[M]$ ,  $[C]$ ,  $[S]$  and  $\{P\}$  denote mass, damping and stiffness matrices and vector of the forces, respectively. The factors of  $\{x\}$ ,  $\{\dot{x}\}$ ,  $\{\ddot{x}\}$  refer to displacement, velocity and the system acceleration. For simplification purpose, the matrix of fictitious mass is considered to be diagonal. Additionally, the factors of acceleration and velocity are obtained approximately by using the techniques of finite differences as follows:

$$\dot{X}^{n+\frac{1}{2}} = \frac{1}{\tau^{n+\frac{1}{2}}} (\{X^{n+1}\} - \{X^n\}) \tag{2}$$

$$\dot{X}^n = \frac{1}{2} (\{\dot{X}^{n-\frac{1}{2}}\} + \{\dot{X}^{n+\frac{1}{2}}\}) \tag{3}$$

$$\ddot{X}^n = \frac{1}{\tau^n} (\{\dot{X}^{n+\frac{1}{2}}\} - \{\dot{X}^{n-\frac{1}{2}}\}) \tag{4}$$

Furthermore, damping matrix is defined as a coefficient of mass matrix. This relation can be written as  $[c^n] = c^n[M]$  where the factor of  $c^n$  is the damping coefficient of  $n^{th}$  step. Substituting the values of velocity and acceleration and defining damping matrix in equilibrium equation of dynamic system of step  $n$ , the velocity is obtained in time  $t^{n+\frac{1}{2}}$  and then the result of the next step is computed as follows:

$$\{X^{n+1}\} = \{X^n\} + \tau^{n+\frac{1}{2}} \{\dot{X}^{n+\frac{1}{2}}\} \tag{5}$$

In most strategies of DR, time step is assumed to be constant (equal to 1). Mass matrix is diagonal in DR method; Hence, the results of  $[M^n]^{-1}\{R^n\}$  can be calculated in the form of  $R_i^n/m_{ii}^n$  where the  $R_i^n$  and  $m_{ii}^n$  are respectively the residual force of  $i^{th}$  degree of freedom and  $i^{th}$  diagonal entry from mass matrix in  $n^{th}$  step. The residual is computed by:

$$\{R^n\} = \{P^n\} - \{f^n\} = [M]\{\ddot{X}^n\} + [C]\{\dot{X}^n\} \tag{6}$$

In this equation, mass matrix, damping and time step are unspecified. General relation for mass values are obtained by [28]:

$$m_{ij} = \frac{(\tau^n)^2}{4} \text{Max} \left[ \sum_{i=1}^{ndof} |S_{ij}|, 2S_{ij} \right] \tag{7}$$

where the factor of  $ndof$  is the indicator of the number of degree of freedom.

In view of dynamic systems, the critical damping of the system leads to maximize the convergence rate. Therefore, the critical damping is estimated by Rezaee Pajand and Taghavian Hakkak [29]:

$$c^n = 2 \sqrt{\frac{\{X^n\}^T \{f(X^n)\}}{\{X^n\}^T [M] \{X^n\}}} \tag{8}$$

With the determination of these parameters, it is possible to utilize DR method in ratcheting analysis. In the current study, DR algorithm is developed to be able



Figure 1. Zwick/roell amsler HB100 machine

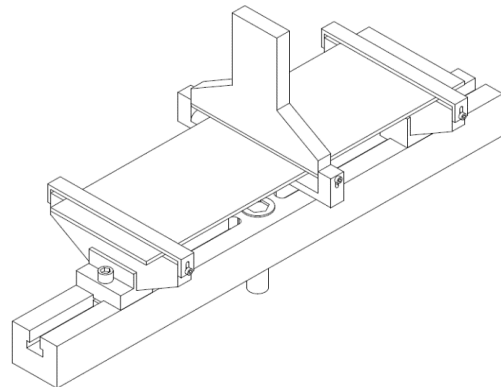


Figure 2. Schematic of rectangular plate placed on the bending fixture

to solve cyclic problems. So, when loading reaches to a certain defined maximum value, reverse loading (unloading) is applied to the specimen sinusoidally. Maximum and minimum loads are kept constant in all cycles to impose stress-controlled conditions to the ratcheting analysis. Based on kinetic or combined hardening model, yield stress is modified after each loading or unloading. Finally, some additional actions are applied to numerical code to run analysis properly. The steps of ratcheting analysis using DR process are:

1. Let initial velocity  $\{\dot{X}^0\} = 0$  and  $n = 0$ ; specify Number of Cycles (NoC), initial displacement  $\{x^0\}$ ,  $N_{max}$ ,  $e_R$  and  $e_k$  where  $e_R$  and  $e_k$  are the convergence indexes of residual force and energy, respectively.
2. Apply boundary conditions.
3. Form stiffness matrix  $[S]$  and mass matrix  $[M]$ .
4. Calculate residual force  $\{R^n\}$ .
5. If  $|R_i^{Ln}| \leq e_R$ , go to step 10; otherwise continue.
6. Calculate  $c^n$  and  $\dot{X}^{n+\frac{1}{2}}$ .
7. If the kinetic energy  $(\sum_{L=u}^w \sum_{i=1}^n (\dot{x}_i^{Ln+\frac{1}{2}})^2) \leq e_R$ , go to step 10; otherwise continue.
8. Calculate  $X^{n+1}$ .
9. Exert boundary conditions.
10. Print the displacements for this increment.
11.  $n = n + 1$ ; if  $n > N_{max}$  start new loading/unloading analysis (step12); otherwise return to step 4.
12. Apply reverse loading (unloading).
13. Modify yield stress based on desired hardening model.
14. If number of cycles  $< NoC$ , go to step 4; otherwise stop.

## 2. 4. Finite Element Simulation

The finite element simulation is carried out using Abaqus software to analyze the ratcheting behavior of rectangular plate. The plate is modeled using 3D deformable solid elements. To mesh the model, eight-noded C3D8R elements with reduced integration and hourglass control is utilized. This type of meshing improves the convergence rate [30, 31]. A mesh convergence study is conducted for model to ensure accurate simulation results and appropriate mesh density. To eliminate undesired stress concentration, distributed loading is applied to small central area of the plate. A nonlinear isotropic/kinematic (combined) hardening model is applied to obtain ratcheting data. All required data are extracted from stress-strain curve of a recent study [27].

## 3. RESULTS AND DISCUSSION

The results of ratcheting analysis of CK45 steel plate subjected to cyclically bending load are discussed in this section. Data are obtained from numerical, experimental

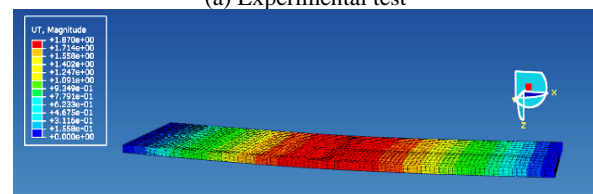
and Abaqus analyses. Moreover, numerical investigation based on developed DR method, is validated by the results of experimental and Abaqus studies. Figure 3 illustrates the experimental response of bending load on the plate during 50 cycles in comparison with the results of Abaqus simulation and numerical analysis. It is seen that similar plastic deformation occurred in the plate for all three cases. Figure 4 presents the applied sinusoidal load versus cycles. Considering stress-controlled conditions, maximum and minimum load are kept in the range of 0-700 N with mean load of 350 N and load amplitude of 350 N. The relationship between applied load and displacement in the experimental bending ratcheting study is plotted in Figure 5(a). It reveals that the hysteresis loops are converging with increase in the number of cycles. Hysteresis loops are not closed exactly at the maximum load in initial cycles, i.e. intersections of curves are not located at the maximum load as marked by circles in enlarged view of hysteresis loops, Figure 5(b). Therefore, increasing the number of cycles leads to a development in ratcheting displacement and strain accumulation as shown in Figure 5(b).

The ratcheting strain is defined as the average of maximum and minimum strain ( $\epsilon_{max}$  and  $\epsilon_{min}$ ) in each cycle of loading as stated in Equation (9):

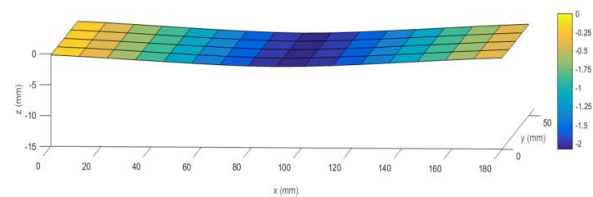
$$\epsilon_r = \frac{\epsilon_{max} + \epsilon_{min}}{2} \quad (9)$$



(a) Experimental test



(b) Abaqus simulation



(c) Numerical code

**Figure 3.** Plate response after 50 cycles, a) Experimental test b) Abaqus simulation, c) Numerical code

The variation of strain accumulation and ratcheting strain with cycles for various methods are demonstrated in Figures 6(a) and 6(b), respectively. It is seen that the ratcheting rate declines with growing number of cycles in the beginning. Then, it remains constant at a steady value and ratcheting strain increases with this constant rate. According to Figures 6(a), 6(b) and comparison between the results of numerical method, experimental study and Abaqus simulation it is obvious that the ratcheting strain values of numerical method are in a good agreement with the those of Abaqus simulation and experimental study. Besides, the results of numerical method gradually become closer to the experimental data. These results clearly validate accuracy of the proposed numerical method. At cycle 50, the error of numerical method is approximately 5.5% whereas the Abaqus simulation has an error of 9.5%.

Experimental hysteresis loops at cycles 1, 5 and 50 are shown in Figure 7. In the range of tested cyclic number, loading and unloading curves does not overlap. It means that the ratcheting strain occurs during loading process and also, strain does not reach the shakedown state. Moreover, it is observed that a rise in the number of cycles decreases the strain rate rapidly. At the higher cycles, the loading and unloading curves are closer than the initial cycles because it is more difficult to produce plastic deformation at higher cycles.

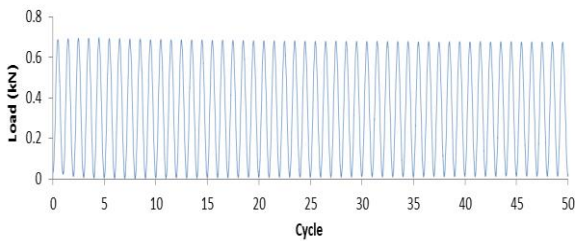
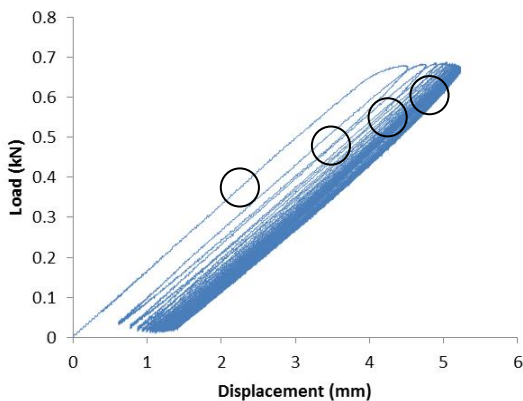
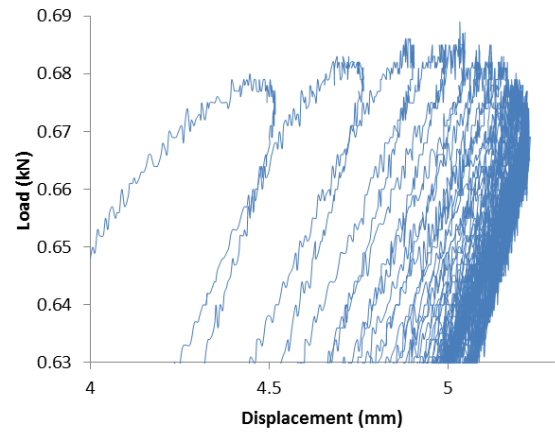


Figure 4. Applied load against cycle



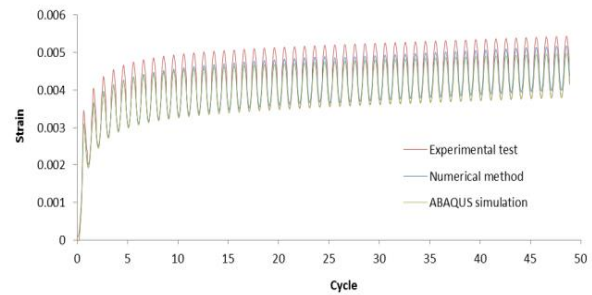
(a) Applied load versus displacement



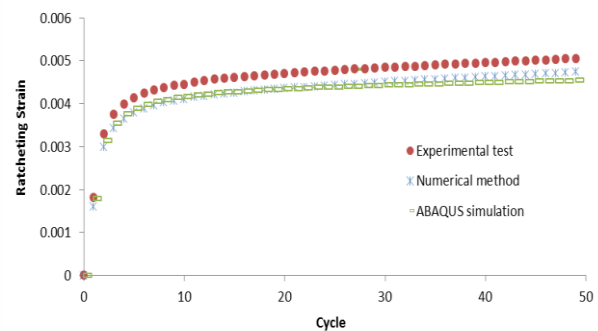
(b) Intersections of curves and development of ratcheting displacement in enlarged view of hysteresis loops

Figure 5. (a) Applied load versus displacement, (b) Enlarged view of hysteresis loops

Figure 8 presents the hysteresis loops for numerical method, experimental study and Abaqus simulation at cycles 1 and 50. The results indicate that the experimental study and Abaqus simulation precisely verify the results of the proposed numerical method, for both cycles. The area of hysteresis loops of experimental study is larger than those of other methods, which is due to existence of more energy dissipation in the experimental study.



(a) Variation of strain accumulation with number of cycles



(b) Ratcheting strain against number of cycles

Figure 6. a) Variation of strain accumulation with number of cycles, b) Ratcheting strain against cycles

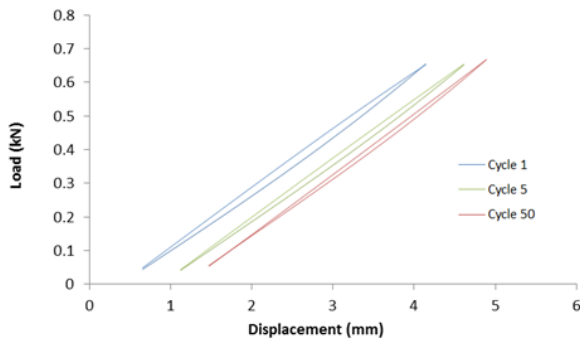


Figure 7. Experimental hysteresis loops at cycles 1, 5 and 50

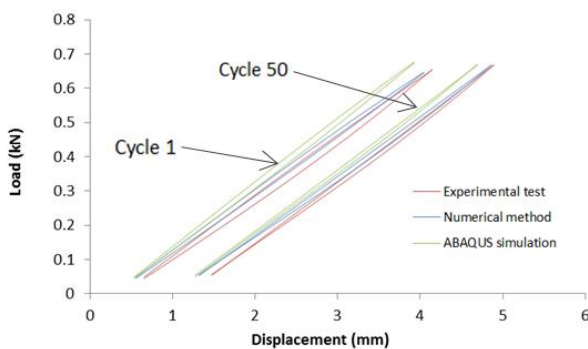


Figure 8. Comparison of hysteresis loops

#### 4. CONCLUSIONS

In this study numerical, experimental and Abaqus simulation of bending ratcheting analysis of CK45 steel plate were conducted. Numerical method was carried out by development of DR method and Abaqus simulation was performed using nonlinear isotropic/kinematic (combined) hardening model. All the mentioned methods were achieved under stress-controlled conditions. The major findings of this study are as follows:

1. The proposed numerical method for the analysis of ratcheting phenomena is able to predict the results of experimental method more accurate than the Abaqus simulation.
2. The results of numerical method gradually become closer to the experimental ones.
3. The hysteresis loops area of experimental study is larger than those of other methods due to more energy dissipation in experimental study.
4. Dissipated energy per cycle decreases when the number of cycles increases.
5. At the higher cycles, loading and unloading curves are closer and narrower than the initial cycles.
6. Ratcheting rate decreases with increase in the number of cycles.

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### Persian Abstract

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#### چکیده

در این پژوهش روش عددی جدیدی برای بررسی رچتینگ در صفحات مستطیلی معرفی می‌شود. به منظور بررسی صحت این روش، آزمایشاتی در شرایط تنش کنترل توسط دستگاه zwick/roell amsler HB100 در دمای محیط انجام و پدیده رچتینگ در صفحه فولادی CK45 تحت بارگذاری خمشی سیکلی مورد بررسی قرار گرفت. در شرایط تنش کنترل و تنش میانگین غیرصفر، رچتینگ در صفحه فولادی نازک اتفاق افتاد. سپس تحلیل المان محدود توسط نرم افزار آباکوس با مدل سخت شونده ترکیبی انجام گرفت. در نهایت یک روش عددی جدید که با توسعه روش رهایی پویا به دست آمده است، جهت تحلیل رفتار رچتینگ در صفحه نازک فولادی تحت بارگذاری اشاره شده، مورد استفاده قرار گرفت. نتایج نشان می‌دهند، نرخ رچتینگ در ابتدا زیاد و سپس با افزایش سیکل‌ها به تدریج کاهش می‌یابد. همچنین سطح حلقه‌های هیستریزس در روش آزمایشگاهی (به دلیل اتلاف انرژی)، بزرگ‌تر از دو روش المان محدود و عددی است. نتایج روش عددی هماهنگی مناسبی با نتایج روش‌های آزمایشگاهی و شبیه‌سازی دارد. مقایسه نتایج بین این سه روش دقت بالای روش عددی معرفی شده را نشان می‌دهد.

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