



A General Framework for Estimating Channel of Orthogonal Frequency Division Multiplexing Systems by Utilizing Sparse Representation

S. H. Hashemi Rafsanjani ^a, S. Ghazi-Maghrebi*^b

^a Department of Digital Communication, ICT Research Center, Tehran, Iran

^b Communication Department, college of Electrical Engineering, Yadegar-e-Imam Khomeini, Shahr-e-Rey branch, Islamic Azad University, Tehran, Iran

PAPER INFO

Paper history:

Received 09 July 2020

Received in revised form 06 April 2021

Accepted 01 May 2021

Keywords:

Sparse Representation

Orthogonal Frequency Division Multiplexing

Sparse Channel Estimation

Weighted Sparse

ABSTRACT

Channel estimation is a crucial task for orthogonal frequency division multiplexing (OFDM) modulation-based systems since this estimation is used for compensating impacts of a wireless channel. Recently, sparse representation (SR) is proposed for this task as wireless channels are considered as a sparse signal. However, SR considers sparse as the main feature and omit other features of the channel while estimating the channel. In this paper, we propose a general framework for utilizing other features of the channel in sparse channel estimation for OFDM systems, while these features are omitted in conventional sparse methods. In this regard, by utilizing maximum a posterior (MAP) estimation and defining new parameters, these features are conveyed into sparse channel estimation process to improve channel estimation. The simulation results indicate that our proposed framework not only improves the estimated parameter, but also reduces the number of resources such as the number of estimation pilots or transmitted power.

doi: 10.5829/ije.2021.34.06c.11

NOMENCLATURE

P	Pilot index set	$corr$	Correlation factor
$\tilde{\mathbf{h}}_p^f$	Observed Vecotr	A	an index set
M	Mask Matrix	$C = [a_i^{(-corr)}]$	Diagonal cost matrix
A	Dictionary Matrix	B	Weighted-dictionary
z_p	Observed Noise	\mathbf{h}'	Weighted-CIR
X_{pilot}	Power of estimation pilots	T_D	Delay spread
\mathbf{h}	Channel Impulse response	a_i	Free parameter
$N(\cdot)$	Normal Distribution	$q = exp(-corr)$	Probability of locating at Δ
α	GGD parameter	$\Delta = (T_D / k) \ln(1/1-q)$	neighbor region
β	Scale parameter		

1. INTRODUCTION

OFDM modulation is a kind of multicarrier modulation (MCM) that utilizes orthogonal carriers to modulate data and overcome the multipath effects of the channels [1, 2]. This modulation divides a long stream of data into smaller data streams and modulates them on orthogonal carriers. This kind of modulation provides a narrowband channel for each carrier and reduces severe channel

conditions [1]. In order to prepare an OFDM symbol, as indicated in Figure 1, a stream of data is passed through serial to parallel block. After passing through an inverse fast Fourier transform (IFFT) block, some specific pilots, known as estimation pilots, are inserted among them. These pilots are known at the receiver side, and their position in the stream is gathered at a set $\mathbf{P} = \{p_1, p_2, \dots, p_L\}$, which L is the number of estimation pilots. After performing some other operations such as cyclic prefix

*Corresponding Author Institutional Email: s_ghazi2002@yahoo.com (S. Ghazi-Maghrebi)

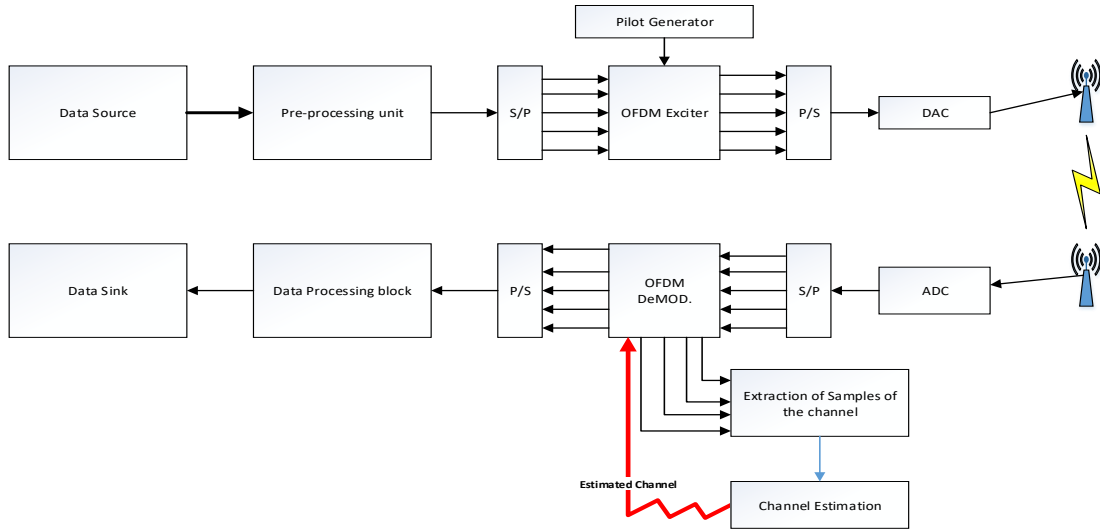


Figure 1. OFDM Modulation and Demodulation System

(CP) insertion, parallel to serial (P/S), and digital to analog conversion (DAC), an OFDM symbol is transmitted through a wireless channel [1, 3, 4]. At the receiver side, after passing through some processing blocks, estimation pilots are extracted, and the receiver estimates the wireless channel from noisy observed samples $\tilde{\mathbf{h}}_p^f = \mathbf{r}_p / X_{pilot} = \mathbf{h}_p^f + \mathbf{z}_p$, [3], [5], where \mathbf{r}_p contains received symbols at the positions of estimation-pilot \mathbf{P} , X_{pilot} is the estimation pilot, \mathbf{h}_p^f contains samples of the frequency response of the channel, and \mathbf{z}_p is the channel noise at the related positions. We utilize \mathbf{w}^f to show the Fourier transform of \mathbf{w} in this paper. To compensate effects of the channel many algorithms discussed in literature [6, 2, 7], which utilize the noisy observed samples to estimate the channel, are proposed.

As explained by Elad [8], the relationship between observation impulse response of the channel samples, \mathbf{h}_p^f , and the wireless channel impulse response (CIR), \mathbf{h} , is $\tilde{\mathbf{h}}_p^f = \mathbf{A}\mathbf{h} + \mathbf{z}_p$, $\mathbf{A} = \mathbf{M}\mathbf{F}$, where \mathbf{M} is a mask matrix, \mathbf{F} is the discrete Fourier transform (DFT) matrix, and \mathbf{A} is the DFT sub-matrix. This relation, which from now is called the observation relation, provides an underdetermined system of linear equation relation between \mathbf{h}_p^f and CIR \mathbf{h} . An underdetermined system of the linear equation has more unknowns than equations. For this kind of equation, there is either no answer or an infinite number of answers [9]. Similar to the atomic decomposition viewpoint [10], the matrix \mathbf{A} is called a dictionary matrix, because the observation vector, \mathbf{h}_p , is a linear combination of columns of the dictionary matrix, \mathbf{A} . Columns of the dictionary are called atoms, and \mathbf{h}_p^f is called the observation vector.

Nowadays sparse representation is widely used on those problems that there is an underdetermined relation

between observed parameters and the desired answer has a sparse features [11, 12]. In OFDM channel estimation problem, as wireless channels are considered as a sparse signal because their taps are related to scattering objects, and these objects are sparsely located [13], and there is an underdetermined system of linear equations between the CIR of the channel and the observation vector, sparse representation is applicable to estimate the OFDM channel. In order to estimate the sparse channel, regularization method is used which define a cost function based on desired features of the answer and try to find those answers which best matches these features. Considering sparse feature as the main feature, zero-norm function is selected as the cost function and it is tried to find the solution by solving $\min_{\mathbf{h}} \|\mathbf{h}\|_0$ s.t. $\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2$. This equation is called P_0 -problem and by substituting $\|\mathbf{h}\|_0$ with $\|\mathbf{h}\|_1$ the P_0 -problem changes to P_1 . Although zero-norm is a proper function for finding the sparse solution, it could not manipulate additional information. To clarify it, let \mathbf{C} be a diagonal cost matrix whose diagonal elements are selected based on side information. To apply side information into the estimation process, the cost matrix \mathbf{C} is multiplied to \mathbf{h} to assign costs to each element of \mathbf{h} . Therefore, the regularized term changes as, $\min_{\mathbf{h}} \|\mathbf{C}\mathbf{h}\|_0$ s.t. $\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2$. Since $\|\mathbf{C}\mathbf{h}\|_0 = \sum_i |c_i h_i|^0 = \sum_i |h_i|^0 = \|\mathbf{h}\|_0$, the zero-norm function eliminates the impact of the cost matrix. Therefore, to take the impact of the side information into account, the problem formulation should be modified which in this paper we proposed a framework to present it. Many algorithms such as Basis Pursuit (BP) [9], Matching Pursuit [14], Orthogonal

Matching Pursuit (OMP) [15], and smoothed L0, SL0 [16], are proposed to find the sparse solution. These methods consider the sparsity feature of the desired parameter as the only metric and omit other features. In addition to conventional sparse algorithms, some methods discussed by Stanković [17] are proposed to employ additional information to find the sparse solution. The above algorithm proposed [17] is mainly based on OMP [15], provides a set of candidates based on additional information, and similar to the OMP algorithm, finds each element of the sparse solution step by step. At each step, the algorithm searches for finding the most correlated candidate and compares it with other elements that are not a member of the candidate set. If the correlation of the candidate element is more than other, that element is selected as nonzero elements of the sparse solution. Otherwise, the high correlated element is selected. This algorithm is an updated version of the OMP algorithm; however, its procedure of contributing side information is not applicable to other sparse representation algorithms. Zhang et al. [18] proposed to integrate the side information via maximizing the correlation between the prior information and the desired solution and tried to find the sparse solution form, $\min_{\mathbf{h}} \|\mathbf{h}\|_1 + \lambda \langle \mathbf{h}, \boldsymbol{\varphi} \rangle$ s. t. $\|\tilde{\mathbf{h}}_p - \mathbf{A}\mathbf{h}\|_2$, where λ is a free parameter. The second term calculates the similarity between prior information and the desired solution. Similarly, Mota et al. [19] concentrated on the framework of l_1 - l_1 -minimization and tried to utilize side information by replacing l_0 -norm with l_1 -norm. These methods and similar frameworks considered l_1 -norm as the cost function to handle side information. However, the method didn't address utilization of side information in l_0 -problem. To provide a framework that contributes other features of the solution in sparse representation process in addition to sparsity, we utilize MAP estimation and select generalized Gaussian distribution (GGD) as a probability density function (pdf) of the taps of the channel.

In this paper we assume that the wireless channel is an independent and identically distributed (i.i.d) signal and sparse. Therefore, its elements should follow those pdf covering sparse characteristics. Considering a sparse signal, most elements of it are zero, and few elements gain a nonzero amount. Consequently, the pdf of the elements should have a high density at zero and heavy tail. The concentration of the mass at zero increases the probability of selecting zero, which guaranties most elements of the signal gain zero amount while this tail guaranties that few elements are gaining huge amounts. In this paper, we select GGD distribution, as a pdf of taps of the channel, because this distribution covers wide range of pdf by changing its parameters. Distributions such as Laplacian, Gaussian, uniform distributions, and distributions that represent sparse random variables are

some examples of them. This generality would also help us to present our result in a general form. The pdf of a random variable x , which obeys a GGD, is defined as follows:

$$f(x; \alpha, \beta) = \frac{\alpha}{2\beta\Gamma(1/\alpha)} \exp\left(-\frac{\|x\|^\alpha}{\beta^\alpha}\right), \quad (1)$$

$$\alpha > 0, \beta > 0$$

where $\Gamma(\cdot)$ is the Gamma function [20]. GGD depends on two parameters α and β , which α controls the shape of the GGD distribution, varying from flat distribution to semi- δ function, shown in Figure 2. Meanwhile, β at Equation (1) is a scale parameter controlling the variance of the random variable. As indicated in Figure 2, the GGD covers a wide range of probability distribution by changing α . In this regard, for the case of $\alpha > 2$, distributions are called sub-Gaussian distributions and super-Gaussian signals for the case $\alpha < 2$. Sparse distributions are achieved for $\alpha < 1$ and uniform distribution, $U(-\beta, \beta)$, is obtained while $\alpha \rightarrow \infty$ [20].

In this paper, we try to provide a general framework to overcome the limitation of SR methods, considering sparse as the only feature and omitting other features, and estimate sparse channel while other features apart from sparse are also available. To this end, firstly, we indicate the relation between GGD parameters and sparse representation and explain how sparse representation is achieved by changing GGD parameters. Then, some parameters are defined to handle additional features and the procedure of contributing these features to sparse channel estimation is explained. Finally, we propose a new method which contributes other features of the answer in addition to sparsity while estimating sparse solution.

The rest of the paper is organized as follows. Section 2 is devoted to estimate the channel of the OFDM system by utilizing MAP estimation and considering GGD as a pdf of the wireless channel in general format. The wireless channel is calculated for a different amount of α in section 3. Section 4 is devoted to define some

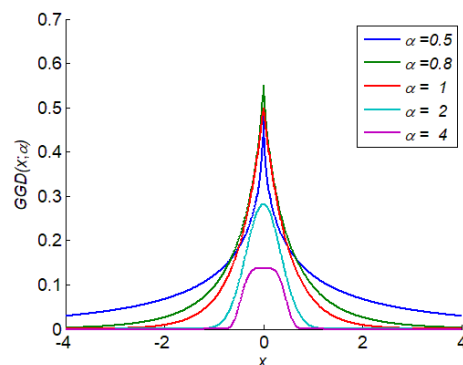


Figure 2. Generalized Gaussian Distribution

parameters to contribute other features of the answer to the channel estimation process. The simulation results are presented in section 5.

2. ESTIMATING WIRELESS CHANNEL FOR AN OFDM SYSTEM BY UTILIZING MAP

An OFDM modulation system breaks a high rate data stream into parallel low rate data streams and modulates them on orthogonal carriers [21]. In this regard, the data stream is passed through the M-QAM modulator to map digital data into digital symbols. To take samples of the channel, L deterministic symbols, which are known as estimation-pilots, are added among them. X_{pilot} indicates the transmitted power of these pilots. The position of these pilots among data samples are gathered in a set $\mathbf{P} = \{p_1, p_2, \dots, p_L\}$. The estimation-pilots and data samples then pass through serial to analog block and apply to N-point IFFT block,

$$\mathbf{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \mathbf{X}(k) \exp\left(\frac{j2\pi kn}{N}\right) \quad (2)$$

Performing some other processes such as cyclic-prefix (CP) insertion and D/A conversion, OFDM symbols are prepared to transmit through a wireless channel. At the receiver side by considering additive white Gaussian noise (AWGN), assuming semi-wide sense stationary (WSS) characteristic for the channel and omitting inter symbol interference (ISI) and inter-carrier interference (ICI) because of the CP [5], the received signal become as $\mathbf{y}[n] = \mathbf{x}[n] \otimes \mathbf{h}[n] + \mathbf{z}[n]$, where \otimes represents convolution operation. At the receiver side, the received signal is passed through an FFT block, $\mathbf{Y}(k) = \sum_{n=0}^{N-1} \mathbf{y}[n] \exp\left(-\frac{j2\pi kn}{N}\right) = \mathbf{X}(k)\mathbf{H}(k) + \mathbf{Z}(k)$, where $\mathbf{H}(k)$ is FFT of the channel impulse response. Since $\mathbf{z}[n]$ is a zero-mean white random process and FFT is a linear operator, $\mathbf{Z}(k)$ is also a zero-mean white process. By extracting the received samples of the estimation-pilots from other samples, and dividing them by X_{pilot} to remove their amount, the noisy observation of the channel is obtained by dividing $\mathbf{Y}(k)$ by X_{pilot} , $\mathbf{Y}(k)/X_{pilot} = \mathbf{H}(k) + \mathbf{Z}(k)/X_{pilot}$, $k \in \mathbf{P}$. Considering that the second term is still a noise term and the observed amounts are noisy samples of the channel frequency response. For simplicity, a new parameter, $\tilde{h}_k^f \triangleq \mathbf{Y}(k)/X_{pilot}$ is defined to represent observed samples of the channel at the position of $k \in \mathbf{P}$. Replacing $\mathbf{H}(k)$ by its Fourier transform, the observation relation becomes,

$$\tilde{h}_k^f = \sum_{n=0}^{N-1} \mathbf{h}[n] \exp\left(-\frac{j2\pi nk}{N}\right) + \frac{\mathbf{Z}(k)}{X_{pilot}}, \quad k \in \mathbf{P}. \quad (3)$$

The first term of Equation (3) is the k 'th column of the DFT transform of $\mathbf{h}[n]$. By defining $\tilde{\mathbf{h}}_p^f \triangleq [\tilde{h}_{k_1}^f, \tilde{h}_{k_2}^f, \dots,$

$\tilde{h}_{k_L}^f]^T$, $k_i \in \mathbf{P}$ as an observed vector, it is possible to rewrite the observation relation in a matrix form,

$$\begin{bmatrix} \tilde{h}_{k_1}^f \\ \tilde{h}_{k_2}^f \\ \vdots \\ \tilde{h}_{k_L}^f \end{bmatrix} = \begin{bmatrix} 1 & \exp\left(\frac{-j2\pi k_1}{N}\right) & \dots & \exp\left(\frac{-j2\pi k_1(N-1)}{N}\right) \\ 1 & \exp\left(\frac{-j2\pi k_2}{N}\right) & & \exp\left(\frac{-j2\pi k_2(N-1)}{N}\right) \\ \vdots & \vdots & & \vdots \\ 1 & \exp\left(\frac{-j2\pi k_L}{N}\right) & \dots & \exp\left(\frac{-j2\pi k_L(N-1)}{N}\right) \end{bmatrix} \mathbf{h}[n] + \mathbf{Z}, \quad k_i \in \mathbf{P}, \quad (4)$$

$$\tilde{\mathbf{h}}_p^f = \mathbf{A}\mathbf{h}[n] + \mathbf{Z},$$

where \mathbf{A} in Equation (4) is a DFT sub-matrix which its columns is the k_i 'th column of the DFT matrix. By defining a diagonal mask matrix, \mathbf{M} , which its diagonal elements are in associated with the elements of \mathbf{P} , and applying it on the DFT matrix, the matrix \mathbf{A} is achieved. Equation (4) is an underdetermined system of linear equations which provides linear relation between observed samples and the channel impulse response (CIR). At follows, we use MAP to estimate the channel.

MAP estimation tries to estimate the desired parameter by utilizing prior information. This estimation tries to find the desired answer by maximizing $f(x|y)$, where x is the desired parameter to be estimated, y is an observation parameter, and $f(\cdot|\cdot)$ is a conditional pdf [22]. To estimate an OFDM channel by MAP estimation, the desired parameter, the observation vector, and observation relation are defined as $\mathbf{x}=\mathbf{h}$, $\mathbf{y}=\tilde{\mathbf{h}}_p^f$, $\tilde{\mathbf{h}}_p^f = \mathbf{A}\mathbf{h} + \mathbf{z}_p$, respectively. Considering \mathbf{z}_p as an identical independent distributed (i.i.d) zero-mean white Gaussian noise, the conditional distribution becomes $f(\tilde{\mathbf{h}}_p^f | \mathbf{h}) = f(\mathbf{A}\mathbf{h} + \mathbf{z}_p | \mathbf{h}) = \mathbf{N}(\mathbf{A}\mathbf{h}, \sigma^2 \mathbf{I})$, where \mathbf{I} is an identical matrix, σ is the variance of observation noise, and $\mathbf{N}(\cdot)$ is the normal distribution. The pdf of $f(\mathbf{h})$, which is prior of the desired parameter, is considered as a GGD distribution. It is assumed that the desired parameter contains N i.i.d parameters $\mathbf{h}=\{h_1, h_2, \dots, h_N\}$. Consequently, $f(\mathbf{h}) = \prod_{i=1}^N f(h_i)$. By replacing normal and GGD distributions, the estimation relation changes to:

$$\hat{\mathbf{h}} = \arg \max_{\mathbf{h}} \mathbf{N}(\mathbf{A}\mathbf{h}, \sigma^2 \mathbf{I}) \prod_{i=1}^N \frac{\alpha}{2\beta\Gamma(\frac{1}{\alpha})} \exp\left(-\frac{\|\tilde{h}_i^f\|}{\beta}\right), \quad \alpha > 0, \beta > 0. \quad (5)$$

Since normal distribution and GGD are exponential functions and natural logarithm function (\ln -function) is strictly increasing, performing \ln -function on Equation (5) does not affect the maximization process. After some manipulations, the estimation relation is calculated as follows:

$$\hat{\mathbf{h}} = \arg \max_{\mathbf{h}} \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} \frac{\alpha}{2\beta\Gamma(1/\alpha)}\right) - \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} - \frac{\|\mathbf{h}\|_\alpha}{\beta^\alpha}, \quad (6)$$

where $\|\cdot\|_2$, $\|\cdot\|_\alpha$ are the norm-2 and norm- α , respectively. The first term of Equation (6) is constant and does not

affect the maximization process. Therefore, it is omitted. By omitting minus sign, maximization process changes to minimization process and the estimation relation changes as follows:

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} + \frac{\|\mathbf{h}\|_{\alpha}}{\beta^{\alpha}}. \quad (7)$$

As explained by Elad [8], Equation (7) is also achieved, while the regularization method is used to find the desired solution of an underdetermined system of linear equation, and α -norm is selected as the cost function. Therefore, it is shown that utilizing MAP and GGD distribution provides same relation for finding sparse answer similar to regularization method. This provides us a platform to consider other features of the answer in sparse estimation. In the next section, the effect of α on the channel estimation is studied.

3. ESTIMATING OFDM CHANNEL FOR DIFFERENT AMOUNT OF PARAMETERS

The GGD distribution has two main parameters α and β , controlling shape and variance of the distribution, respectively. The parameter α controls the sharpness of the GGD distribution and provides a wide range of probability density functions. For instance, by selecting $\alpha = 2$ and $\alpha = 1$ Gaussian and Laplacian distributions are obtained, respectively. In this paper, we consider GGD distribution as a pdf of the taps of the channel because the distribution of most natural signals follows GGD distribution [23]. We estimate the channel impulse response in a general form of α at Equation (7), and in this section, we find the estimation of the channel for different amount of α . By considering $\alpha = 2$ and assuming the observation noise and \mathbf{h} are i.i.d signals, the estimation relation is obtained as follow:

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} (\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h})^T (\sigma^2 \mathbf{I})^{-1} (\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}) + \mathbf{h}^T (\beta I)^{-1} \mathbf{h}, \quad (8)$$

where $(\cdot)^T$ is a transpose operator. Since Equation (8) is a convex relation, minimization relation is performed by derivative of Equation (8). After derivation and performing some straight forward calculation, the channel is estimated as $\hat{\mathbf{h}} = \mathbf{A}^T \tilde{\mathbf{h}}_p^f \beta^2 / (\sigma^2 + \beta^2)$.

By selecting $\alpha \geq 2$ non-sparse is assigned to the taps of the channel. Although this assumption is against the sparse assumption of the channel, it results in a closed-form relation for estimating the channel. Considering sparsity, α should be selected in a range $\alpha \leq 1$. By selecting $\alpha = 1$ which change GGD distribution to Laplace distribution and performing some calculation, Equation (7) changes to the following form:

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} + \frac{\|\mathbf{h}\|_1}{\beta}. \quad (9)$$

This relation is a P_1 -problem which is convex, but not strictly as discussed by Efron et al. [24]. This relation may have more than one solution, but these solutions are convex and gathered in a bounded convex set [24]. Beside it, as we are looking for the sparse answer (for example k -sparse), there exists at least one with at most k non-zero elements. The above relation can be formulated as linear programming to find the sparse solution. Algorithms such as Least Angle Regression (LARS) [24], Interior point method [25], and Least Absolute Shrinkage and Selection Operator (LASSO) [26] are used for solving it and finding the sparse solution.

Here, we consider the critical case where $\alpha \rightarrow 0$, and assume that the taps of the channel have GGD distribution with $\alpha \rightarrow 0$. In this case, the estimation relation changes as follows:

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}, \alpha \rightarrow 0} \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} + \frac{\|\mathbf{h}\|_{\alpha}}{\beta^{\alpha}}. \quad (10)$$

As indicated in Equation (10) by selecting $\alpha \rightarrow 0$, the estimation relation changes to P_0 -problem. This problem is complex as it is NP-Hard, and a straightforward approach is not applicable to solve it [23]. Find the sparse solution of the above relation; approximation methods are employed. Approximation methods are categorized into two main categories, such as greedy methods [27, 28] and relaxation methods. The greedy family methods, such as Matching Pursuit (MP) [14] and orthogonal MP (OMP) [15], try to build the nonzero elements of the answer one per time. While, the second method tries to approximate the zero-norm function by a smoother one [16, 22].

Therefore, it is shown that by considering GGD distribution and selecting $\alpha \rightarrow 0$, the obtained relation is same as the relation obtained from regularization method while zero-norm function is selected as the cost function. Then, MAP estimation and regularization function result in the same relation. In the next section, we explain how to contribute to other features of the answer in the OFDM channel estimation and discuss the impact of them on the channel estimation process.

4. CONTRIBUTING OTHER FEATURES OF CHANNEL AT SPARSE CHANNEL ESTIMATION PROCESS

Being sparse is the main feature of the channel, which makes sparse representation algorithms applicable for estimating the channel. Besides sparse, depending on the channel condition, there are other available features that could be considered for the channel estimation process. These features could be anything such as the number of channel taps, possible regions where taps are located, or side information of the channel, and so on. Conventional

sparse algorithms omit these features since these algorithms are designed based on just the sparsity assumption of the channel. In this paper, we mathematically model these features of the channel by defining some parameters and use these parameters to convey the impact of these features into the channel estimation process.

Based on the observation relation, to estimate the channel impulse response, an underdetermined system of the linear equation should be solved. As stated by Elad [8], the regularization method is suggested to find its solution. In this regard, a regularized-function is defined based on features of the desired answer, and the one which best satisfies it, is selected as the desired solution. In an OFDM channel estimation problem, the sparse feature of the channel is considered as the main feature, and regularization-function is defined such that the answer which best matches this condition is selected as the desired answer. The main function satisfying sparse condition is zero-norm function [8], and many algorithms are suggested to find the sparse solution [8]. Although these algorithms are capable of finding the sparse solution, their ability to find the solution subjected to other features of the answer is not proved. As a consequence, in this section, we propose a method in general form to cover this issue. We used MAP estimation to tackle this issue, and we also propose a new method to solve it.

MAP estimation utilizes prior information to estimate the desired parameter, and features of the channel modify these priors. Mathematical parameters such as a neighbor region Δ , an index set \mathcal{A} , and a cost matrix \mathbf{C} are defined to convey the impacts of these features into the procedure of estimating the channel. The neighbor region, Δ , indicates the length of the feasible region around nonzero taps of the channel where the next taps are more probable to be located. The index set, \mathcal{A} , contains indexes of those elements of the sparse answer located in these regions. For example, consider that $\Delta = 5$, and based on these features, it is expected that nonzero elements of the channel are located at neighborhood of $\{\lambda_8, \lambda_{55}, \lambda_{100}, \lambda_{140}\}$. Therefore, the index set, \mathcal{A} , contains indexes as $\mathbf{\Lambda} = \{\lambda_3, \dots, \lambda_{13}, \lambda_{50}, \dots, \lambda_{60}, \lambda_{95}, \dots, \lambda_{105}, \lambda_{125}, \dots, \lambda_{135}\}$, which $\{\lambda_{95}, \dots, \lambda_{105}\}$ is the feasible region around λ_{100} .

As mentioned earlier, channels are represented by their channel impulse response, $\mathbf{h}(t, \kappa) = \sum_i \mathbf{h}_i(t) \delta(t - \kappa_i)$, where $\delta(t - \kappa_i)$ indicates the taps of the channel. These taps represent paths of the channel, and the number of taps indicates the number of copies of the signal received at the receiver. Additionally, $h_i(t)$ indicates channel loss of the related path. The difference time between the first and last tap is called delay spread of the channel, T_D , which indicates maximum difference between taps of the channel. Assuming that the channel has k taps and delay spread of the channel is T_D , the mean time between two

consecutive taps is T_D/k , and the mean occurrence rate of taps of the channel is k/T_D . Since the expected time for receiving later copy of signal between two consecutive paths obeys exponential distribution with mean k/T_D [29], the probability of receiving next copy of the signal within period Δ is calculated as follow:

$$p(t < \Delta) = \int_0^{\Delta} e^{-\frac{k\tau}{T_D}} d\tau = 1 - e^{-\frac{k\Delta}{T_D}}. \quad (11)$$

Receiving the next copy of the signal within a period of Δ is similar to the event that tap is located within the region of Δ around the former tap of the channel. By considering the above explanation, the neighbor region for locating the next tap of the channel is calculated as $\Delta = (T_D / k) \ln(1 / 1 - q)$, where q is the probability of locating tap at the feasible region, T_D is the delay spread, and k is the number of taps of the channel. The probability q is related to the correlation of the channel. In case of high correlation, it is expected that taps of the channel at t_2 are located closer to taps of the channel at t_1 . However, in the case of low correlation, there is less dependence between samples of the channel. Therefore, taps of the former channel are located more independently, which results in larger Δ . To provide a relation between the correlation of the channel and the probability q , we select $q = \exp(-corr)$, where $corr$ represents the correlation of the channel and $0 \leq corr \leq 1$. In this regard, while the correlation of the channel is decreased, the wider region should be considered for the next sample. Therefore, the probability q gains a higher amount, and Δ becomes bigger.

The diagonal cost matrix assigns proper amount of cost to each atom. Costs are selected in associated with the desired features and their impacts on the desired answer. These features indicate which atoms are more probable to be selected as a nonzero element of the channel. Accordingly, atoms are not treated uniformly, and those atoms that are more probable to be selected as nonzero elements of the answer or their indexes are in the index set, \mathcal{A} , gain lower cost. On the other hand, the higher cost is assigned to other atoms that their indexes are out of the index set, \mathcal{A} . Cost selection could be performed in various ways; for example, it is possible to define different cost levels for those elements that their indexes are in the index set, such as selecting the cost in the Gaussian form, etc. Since most features of the channel are observed from its previous samples, we need a reliable factor to indicate the reliability of these features for the current sample of the channel. Correlation is a factor that indicates correlation between different samples of the channel. The low correlation indicates the observed features from previous samples are less reliable and vice versa. To assign costs base on this parameter, we define $c_i = a_i^{(corr)}$, where $0 \leq a_i \leq 1$ is a free parameter selected based on channel conditions. In this regard,

while the correlation of the channel is low $corr \rightarrow 0$, the observed features are less reliable, and the cost assigned to elements should be gained amount around 1. In this case, the estimator omits other features of the channel and considers the sparse feature as the primary and dominant feature.

Conversely, while the correlation of the channel is high, $corr \rightarrow 1$, the observed features from previous samples are reliable, and it is expected that the current samples of the channel have similar features. Therefore, those elements that their indexes are located at the index set, \mathcal{A} , gain lower cost while higher cost is assigned to others. The coefficient a_i handles the scenario of cost assignment to different elements. For example, selecting a_i based on Gaussian or stepwise formats are two forms of cost assignment. In this paper, we define two cost levels $a_i = L_1$ for $i \in \mathcal{A}$, or $a_i = L_2$ for others,

$$\mathbf{C} = \begin{pmatrix} c_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & c_N \end{pmatrix}, \quad c_i = L_1^{-corr}, i \in \mathcal{A} \\ c_j = L_2^{-corr}, j \notin \mathcal{A}. \quad (12)$$

To contribute impacts of these features into the estimation process, we convey their effects directly to atoms by assigning a cost to each of them. Consequently, Those atoms located at feasible regions are more probable to be selected, and lower cost is assigned to them. However, other elements located out of feasible regions are less probable to be selected, and higher cost is assigned to them. Therefore, by selecting the proper amount of cost and creating diagonal cost matrix $\mathbf{C} = \text{diag}(c_1, c_2, \dots, c_N)$, other features of the channel are conveyed to the procedure of estimating the sparse channel and estimation relation is changed as follows:

$$\hat{\mathbf{h}} = \arg \min_h \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} + \sum_{i=1}^N |c_i h_i|^\alpha. \quad (13)$$

By replacing $\sum_{i=1}^N |c_i h_i|^\alpha = \|\mathbf{C}\mathbf{h}\|_\alpha$, as the cost matrix \mathbf{C} is a positive diagonal matrix, Equation (13) is changed as follows,

$$\hat{\mathbf{h}} = \arg \min_h \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{h}\|_2}{\sigma^2} + \|\mathbf{C}\mathbf{h}\|_\alpha, \quad (14)$$

by defining a weight matrix which is an inverse of the cost matrix, $\mathbf{W} = \mathbf{C}^{(-1)}$ and $\mathbf{h}' = \mathbf{C}\mathbf{h}$, the estimation relation becomes

$$\hat{\mathbf{h}}' = \arg \min_h \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{A}\mathbf{W}\mathbf{h}'\|_2}{\sigma^2} + \|\mathbf{h}'\|_\alpha. \quad (15)$$

Here the feature is conveyed to the estimation process by applying the weight matrix \mathbf{W} . Since this matrix is diagonal, it is possible to combine it with the dictionary \mathbf{A} and define a weighted-dictionary $\mathbf{B} = \mathbf{A}\mathbf{W}$. Hence, additional features affect the dictionary matrix and intensify those columns of the dictionary matrix that their indexes are in the index set \mathcal{A} . Referring to the impact of

additional features of the channel on the estimation relation, their impacts are conveyed to the estimation process through substituting the dictionary matrix, \mathbf{A} , with the weighted-dictionary, \mathbf{B} , and finding the answer of the below relation for different amount of α ,

$$\hat{\mathbf{h}}' = \arg \min_h \frac{\|\tilde{\mathbf{h}}_p^f - \mathbf{B}\mathbf{h}'\|_2}{\sigma^2} + \|\mathbf{h}'\|_\alpha. \quad (16)$$

Looking closer at Equation (16), it is clearly observed that this relation is obtained while regularization is applied on $\tilde{\mathbf{h}}_p^f = \mathbf{B}\mathbf{h}' + \mathbf{z}_p$, while α -norm function is selected as the cost function. This relation which is a weighted form of the observation relation, $\tilde{\mathbf{h}}_p^f = \mathbf{A}\mathbf{h} + \mathbf{z}_p$, is also an underdetermined relation. For this relation other features are applied to the dictionary matrix, \mathbf{A} , and create the weighted dictionary, \mathbf{B} . Additionally, as \mathbf{h}' is a weighted form of \mathbf{h} , and these weights are positive non-zero, \mathbf{h}' is also a sparse signal. Therefore, the zero-norm function is a proper cost function that could be selected to find the sparse solution of $\tilde{\mathbf{h}}_p^f = \mathbf{B}\mathbf{h}' + \mathbf{z}_p$, and sparse algorithms are utilized to find the desired solution of Equation (16).

5. SIMULATION RESULT

In this section, simulation results of an OFDM system by utilizing the proposed channel estimation procedure are presented. For this purpose, we have simulated the complete system of Figure 1, with parameters given in "Table 1" by "MATLAB" software. In this system, we utilized a 4-tap Rayleigh fading channel. The input of this system is a stream of one thousand OFDM symbols, and for each symbol, the Rayleigh fading channel is created randomly. To create these channels, we randomly selected the channel taps in \mathcal{A} , the neighbor region of the previous Rayleigh channel. For example, suppose that the Rayleigh channel for the 10'th OFDM symbol has taps at time indexes of $\{1, 19, 36, 100\}$ with $\mathcal{A} = 6$. Therefore, the taps of the Rayleigh channel for the 11'th OFDM symbol, are randomly selected from $\mathcal{A} = \{1, \dots,$

TABLE 1. System Parameters

Parameters	Value
OFDM Symbol Duration T_u	200 μ
Carrier Frequency F_c	10MHz
Symbol Modulation	16-QAM
FFT Size	2048
Maximum Number of Pilots	1700
The Number of Estimation Pilots	60
Guard band	0.25

7, 13, ..., 25, 30, ..., 42, 94, ..., 106}. In this simulation, we have run this system 1000 times for each experiment, and then we computed the mean error rate as the final results. We simulated the system for two cases. For the first case, we consider sparsity as the main feature and omit other features. For the second case, although sparsity is considered as the main feature, other features are also considered.

In this regard, based on the different amounts of α , three scenarios are defined. For the first scenario, we consider $\alpha = 1$, and employ l_1 -methods to estimate the channel. We have simulated different signal to noise ratio (SNR) and the different number of estimation pilots to investigate the impact of adding other features of the channel on the estimated channel. In the second scenario, by selecting $\alpha = 0$, the estimation relation is a zero-norm problem, and SLO is selected to estimate the channel. The last scenario is devoted to employing non-sparse distribution and $\alpha = 2$. In this case, the closed-form relation for the estimated channel is provided, and we try to contribute to the impact of other features of the channel as well. To compare our proposed algorithm, we have simulated with the algorithm proposed by Stanković et al. [17] compare its result with our proposed algorithm. Since the algorithm introduced by Stanković et al. [17] is based on the OMP algorithm, we use the OMP algorithm as a sparse representation algorithm of our proposed framework and compare their results.

Figures 3 and 4 show the result of the first scenario, and present the effects of considering other features of the channel on the channel estimation, while $\alpha = 1$. Figure 3 indicates the simulation results while fixing SNR. Figure 4 presents the simulation results for the fixed number of pilots. As noted in these figures, with the same condition contributing to other features in the estimation process, channel estimation is improved. Therefore, the system experiences better performance and reduces the

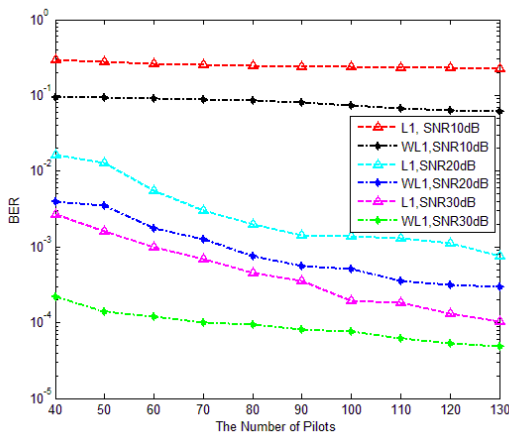


Figure 3. Comparison of performance of OFDM system, Utilizing features of the channel and fixed SNR, $\alpha=1$.

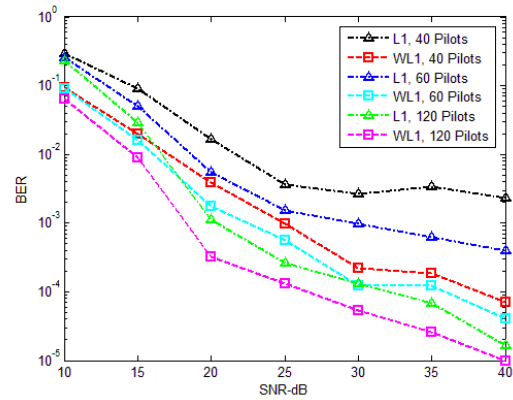


Figure 4. Comparison of Performance of OFDM system, Fixed Number of Pilots and $\alpha=1$.

bit error rate. By comparing these figures, it is observed that by considering only the sparsity feature of the channel, more resources are required to provide the same performance in comparison with the case that other features take part in the estimation process. It is also indicated that, while the system is in good condition, such as high SNR value or larger amount of estimation pilots, the need for considering other features is reduced. In other words, considering them would not increase the performance of the system the same as before. This means that the mathematical computations will be decreased and also, the processing time will be decreased, and as a result, this method will be applicable in real-time data communications.

Figures 5 and 6 indicate the effects of side information while $\alpha \rightarrow 0$. Similar to the previous scenario, results are plots for fixed SNR and a fixed number of estimation pilots in these figures, respectively. To find the answer to this case, we utilize OMP and SLO methods. Similar to the case of $\alpha = 1$, in general, form utilizing other features of the channel improves the

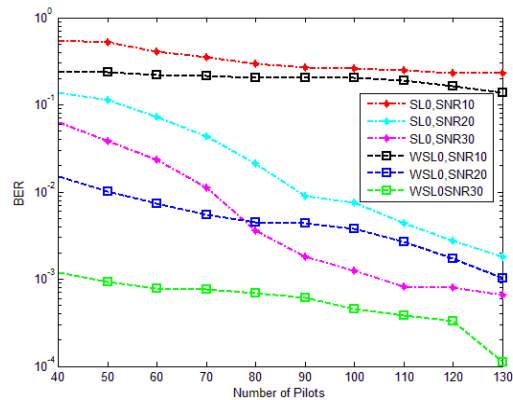


Figure 5. Comparison of Performance of OFDM system, Fixed SNR and $\alpha \rightarrow 0$.

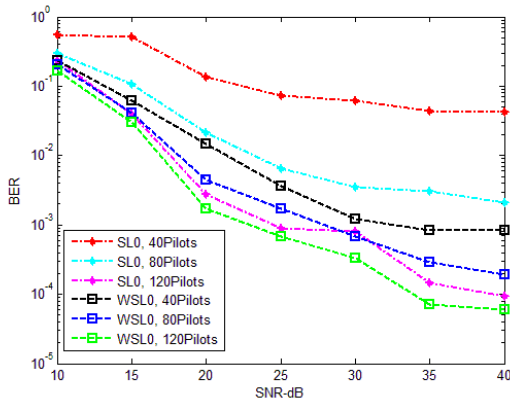


Figure 6. Comparison of Performance of OFDM system, Fixed Number of Pilots and $\alpha \rightarrow 0$

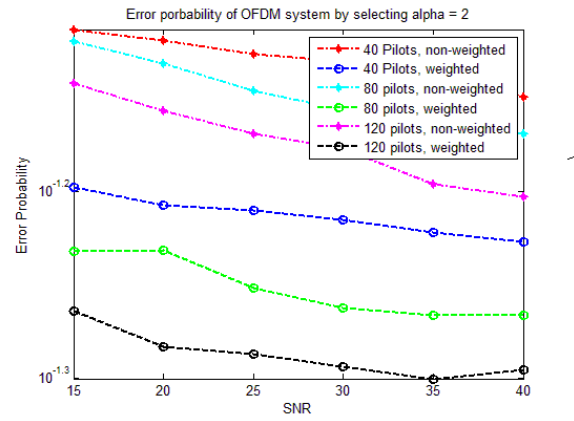


Figure 8. Comparison of Performance of OFDM system, Fixed Number of Pilots and $\alpha=2$

estimation of the channel, and fewer resources are used. There is an interpretation for this improvement, which states that, by considering these features, those atoms that are more probable to be selected are bolded, and the probability of selecting these atoms increases.

Figures 7 and 8 indicate the error probability of the OFDM system while $\alpha = 2$. By selecting $\alpha = 2$, the pdf of the channel is considered as non-sparse pdf. For this system, sparse representation algorithms are not applicable. However, the relation is convex, and the complexity of calculating the channel is very low since there is a closed-form of the estimated channel. In this case, the estimated channel is less accurate than sparse methods. Despite this less accuracy, it is observed that taking into account other features, improves channel estimation, and results in improving the performance of the system.

Compare our proposed method with Stankovi's method, which estimates the sparse signal by utilizing

side information [17]; we provided the same condition as our proposed method worked. In this regard, the index set \mathcal{A} is considered as the candidate set for Stankovi's method because \mathcal{A} indicates contains those elements that are more probable to be selected as nonzero elements of the sparse answer. Then, Stankovi's method is applied to $\tilde{\mathbf{h}}_p^f = \mathbf{A}\mathbf{h}$. Meanwhile, based on our proposed method, the dictionary matrix is updated, and the OMP algorithm is applied to estimate the sparse solution of $\tilde{\mathbf{h}}_p^f = \mathbf{B}\mathbf{h}'$. We have run simulations for a fixed number of pilots and fixed SNR and compare their results in Figures 9 and 10, respectively. As indicated at these figures, our proposed method provides better performance than Stankovi's method; however, at higher SNR, these two methods provide similar performance. Besides, it should also be considered that our proposed method is a general framework that could be used by any sparse representation algorithms.

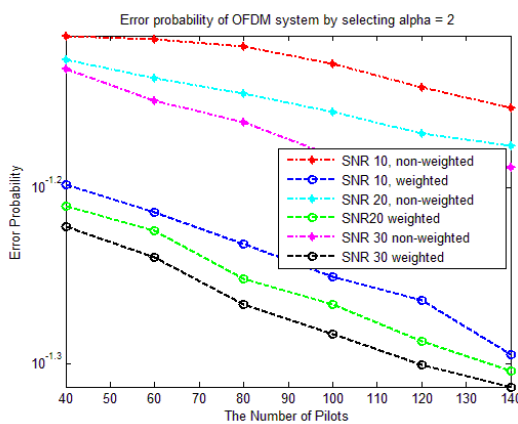


Figure 7. Comparison of Performance of OFDM system, Fixed SNR and $\alpha=2$.

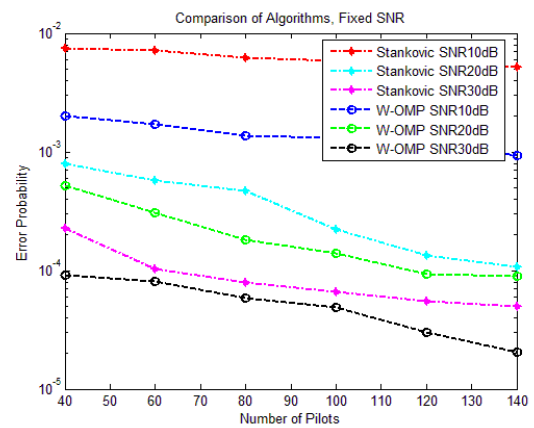


Figure 9. Comparison of the proposed algorithm and Stanikovi's algorithms, Fixed SNR

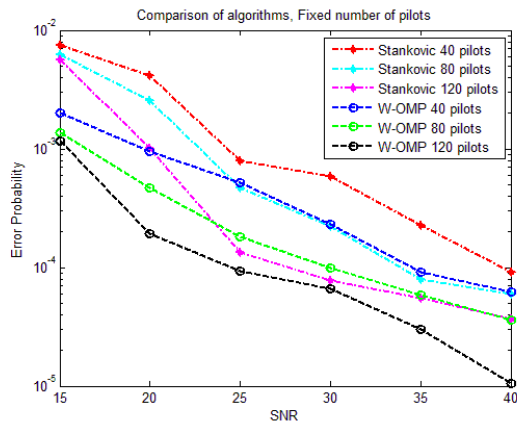


Figure 10. Comparison of the proposed algorithm and Stanikovi's algorithms, Fixed number of Pilots.

6. CONCLUSION

In this paper, we have introduced an approach to improve channel estimation for OFDM system while its complexity is not changed. In this approach, previously estimated channel samples provide information about the current status of the channel. This information is employed to provide a better estimation of the channel. To do this, we introduced a diagonal cost matrix corresponding with this information and some other related parameters such as weighted-dictionary and weighted-channel. We also utilized GGD distribution and considered different GGD's parameter amount to estimate the channel. We showed that by replacing the dictionary matrix with the weighted dictionary, not only side information is conveyed to the estimation process, but also, the computational complexity is not changed. However, in case of low correlation channel obtained information is not reliable. In this case the weight matrix is an identical diagonal matrix which doesn't have effect on the estimation process. It is also shown that utilizing this information provides the opportunity to release some parts of resources in the OFDM system, such as the number of pilots utilized to estimate the channel. For further research it is possible to utilize machine learning techniques to calculate the elements of the weight matrix. By applying the proposed method to OFDM system-based, mathematical computations and processing time will be decreased, and as a result, this method will be applicable in real-time data communications such as WiFi, WiMAX, DAB, DVB, etc.

6. REFERENCES

1. Proakis, J.G., "Digital communications, McGraw-Hill, (2001).
2. K, C.K. and J, J., "An efficient hierarchical modulation based orthogonal frequency division multiplexing transmission scheme

- for digital video broadcasting", *International Journal of Engineering, Transactions A: Basics*, Vol. 31, No. 4, (2018), 574-579. doi: 10.5829/ije.2018.31.04a.08
3. Panayirci, E. and Çirpan, H.A., "Maximum a posteriori multipath fading channel estimation for ofdm systems", *European Transactions on Telecommunications*, Vol. 13, No. 5, (2002), 487-494. doi.org/10.1002/ett.4460130509
4. Ghazi-Maghrebi, S.J.T.J.o.E.E. and Sciences, C., "An efficient ofdm-based system with an insufficient cyclic prefix via a novel constellation algorithm", *Turkish Journal of Electrical Engineering & Computer Sciences* Vol. 25, No. 3, (2017), 2278-2292. doi:10.3906/elk-1505-273
5. Pakrooh, P., Amiri, A. and Marvasti, F., "Ofdm pilot allocation for sparse channel estimation", *EURASIP Journal on Advances in Signal Processing*, Vol. 2012, No. 1, (2012), 59. doi: 10.1186/1687-6180-2012-59.
6. Fereydouni, A., Charmin, A., vahdaty, h. and nasiraghdam, h., "Channel estimation and cfo compensation in ofdm system using adaptive filters in wavelet transform domain". *International Journal of Engineering Transactions A: Basics*, Vol. 33, No. 7, (2020), doi: 10.5829/ije.2020.33.07a.09
7. Komalla, A.R., Kosaraju, S. and Ette, H., "Empirical mode decomposition based adaptive filtering for orthogonal frequency division multiplexing channel estimation", *International Journal of Engineering Transactions A: Basics*, Vol. 30, No. 10, (2017), 1517-1525. doi: 10.5829/ije.2017.30.10a.13
8. Elad, M., From exact to approximate solutions, in Sparse and redundant representations. 2010, Springer.79-109.
9. Candes, E.J., Romberg, J.K., Tao, T.J.C.o.P. and Sciences, A.M.A.J.I.b.t.C.I.o.M., "Stable signal recovery from incomplete and inaccurate measurements", *Communications on Pure and Applied Mathematics: A Journal Issued by the Courant Institute of Mathematical Sciences*, Vol. 59, No. 8, (2006), 1207-1223. doi.org/10.1002/cpa.20124
10. Chen, S.S., Donoho, D.L. and Saunders, M.A., "Atomic decomposition by basis pursuit", *SIAM Review*, Vol. 43, No. 1, (2001), 129-159. doi: 10.1137/S003614450037906X
11. Hassanpour, H. and Asadi, S., "Deblocking joint photographic experts group compressed images via self-learning sparse representation, *International Journal of Engineering, Transaction C: Aspects*", Vol. 29, No. 12, (2016), 1684-1690. doi: 10.5829/idosi.ije.2016.29.12c.07
12. Sabzaljan, B. and Abolghasemi, V., "Iterative weighted non-smooth non-negative matrix factorization for face recognition", *International Journal of Engineering, Transaction A: Basics*, Vol. 31, No. 10, (2018), 1698-1707. doi: 10.5829/ije.2018.31.10a.12
13. ETSI, T., "101 154 v2. 3.1 (2017)", Digital Video Broadcasting (DVB),
14. Mallat, S. and Zhang, Z., "Matching pursuit with time-frequency dictionaries". 1993, Courant Institute of Mathematical Sciences New York United States. doi: 10.1109/78.258082.
15. Tropp, J.A., "Greed is good: Algorithmic results for sparse approximation", *IEEE Transactions on Information Theory*, Vol. 50, No. 10, (2004), 2231-2242. doi: 10.1109/TIT.2004.834793.
16. Mohimani, H., Babaie-Zadeh, M. and Jutten, C., "A fast approach for overcomplete sparse decomposition based on smoothed ℓ^0 Norm", *IEEE Transactions on Signal Processing*, Vol. 57, No. 1, (2009), 289-301. Doi:10.1109/TSP.2008.2007606.
17. Stanković, V., Stanković, L. and Cheng, S., "Sparse signal recovery with side information", in 2009 17th European Signal Processing Conference, IEEE., (2009), 100-104.
18. Zhang, X., Cui, W. and Liu, Y., "Compressed sensing with prior information via maximizing correlation", in 2017 IEEE

- International Symposium on Information Theory (ISIT), IEEE., (2017), 221-225. doi: 10.1109/ISIT.2017.8006522.
19. Mota, J.F., Deligiannis, N. and Rodrigues, M.R.J.I.T.o.I.T., "Compressed sensing with prior information: Strategies, geometry, and bounds", *IEEE Transactions on Information Theory*, Vol. 63, No. 7, (2017), 4472-4496.
 20. Jorgensen, B., "Statistical properties of the generalized inverse gaussian distribution, Springer Science & Business Media, Vol. 9, (2012).
 21. Coleri, S., Ergen, M., Puri, A. and Bahai, A., "Channel estimation techniques based on pilot arrangement in ofdm systems", *IEEE Transactions on Broadcasting*, Vol. 48, No. 3, (2002), 223-229. doi: 10.1109/TBC.2002.804034.
 22. Berger, J.O., "Statistical decision theory and bayesian analysis, Springer Science & Business Media, (2013).
 23. Li, Y., Cichocki, A. and Amari, S.-i., "Analysis of sparse representation and blind source separation", *Neural Computation*, Vol. 16, No. 6, (2004), 1193-1234. doi: 10.1162/089976604773717586.
 24. Efron, B., Hastie, T., Johnstone, I. and Tibshirani, R., "Least angle regression", *The Annals of Statistics*, Vol. 32, No. 2, (2004), 407-499.
 25. Koh, K., Kim, S.-J. and Boyd, S., "An interior-point method for large-scale ℓ_1 -regularized logistic regression", *Journal of Machine Learning Research*, Vol. 8, No. Jul, (2007), 1519-1555.
 26. Tibshirani, R., "Regression shrinkage and selection via the lasso", *Journal of the Royal Statistical Society. Series B (Methodological)*, Vol. 58, No. 1, (1996), 267-288. doi: 10.1111/j.2517-6161.1996.tb02080.x
 27. Davis, G., Mallat, S. and Avellaneda, M., "Adaptive greedy approximations", *Constructive Approximation*, Vol. 13, No. 1, (1997), 57-98.
 28. Donoho, D.L. and Elad, M., "Optimally sparse representation in general (nonorthogonal) dictionaries via ℓ_1 minimization", *Proceedings of the National Academy of Sciences*, Vol. 100, No. 5, (2003), 2197-2202.
 29. Papoulis, A., "Probability & statistics, Prentice Hall, (1990). doi: 10.1073/pnas.0437847100.

Persian Abstract

چکیده

تخمین کانال یکی از اساسی ترین قسمت‌های سیستم‌های مودولاسیون فرکانسی متعامد (OFDM) است، که بمنظور جبران اعوجاج کانال‌های بیسیم مورد استفاده قرار می‌گیرند. اخیراً با توجه به اینکه کانال‌های بی‌سیم بعنوان سیستم‌های تنک در نظر گرفته می‌شوند، جایگزینی تنک برای این موضوع مورد استفاده و توجه قرار گرفته است. هرچند که در روش جایگزینی تنک، خصوصیت تنک بودن بعنوان تنها خصوصیت کانال در نظر گرفته می‌شود و از سایر خصوصیتها برای یافتن و تخمین کانال چشم‌پوشی می‌شود. در این مقاله راه‌کاری را پیشنهاد داده‌ایم که با استفاده از آن قادر باشیم از سایر خصوصیت‌های کانال علاوه بر تنک بودن در تخمین کانال بهره ببریم. در این رابطه با تعریف یک سری پارامتر ریاضی و بهره بردن از تخمین MAP این خصوصیتها را بمنظور بهبود تخمین در تخمین تنک کانال OFDM استفاده کرده‌ایم. نتایج شبیه‌سازی نشان می‌دهند که ساختار پیشنهادی ما نه تنها باعث بهبود تخمین شده است، بلکه قادر بوده است که منابع کمتری، از قبیل کاهش تعداد پایلوتها و توان ارسالی، استفاده نماید.
