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Robust Three Stage Central Difference Kalman Filter for Helicopter Unmanned Aerial Vehicle Actuators Fault Estimation

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ABSTRACT

This paper proposes state and fault estimations for uncertain time-varying nonlinear stochastic systems with unknown inputs. we suppose, the information about the fault and unknown inputs is not perfectly known. For this purpose, in this manuscript, we developed a robust three-stage central difference Kalman filter (RThSCDKF). We used RThSCDKF for model-based fault detection and identification (FDI) in nonlinear hover mode of helicopter unmanned aerial vehicle (HUAV) in the presence of external disturbance. In this system, actuator faults are affected by each other. The proposed method estimates and decouples actuator faults in the presence of external disturbances. This model can detect stuck and floating faults that are important to detect. At the end, this method is compared with the three-stage extended Kalman filter (ThSEKF). Simulation results show the effectiveness of the proposed robust method for detection and isolation of various actuator faults and also this shows more accuracy with respect to ThSEKF.

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NOMENCLATURE

$V^B=[u \quad v \quad w]^T$	Linear speed vector	h_{mr}	Height of main rotor hub above center of mass
$\theta=[\varphi \quad \theta \quad \psi]^T$	Euler angles	g	Gravity acceleration
a_{1f}, b_{1f}	Longitudinal and lateral stabilizer flapping angles	k_β	Main rotor blade restoring spring constant
$W^B=[p \quad q \quad r]^T$	Roll, pitch, and yaw rates in body frame	h_{tr}	Height of tail rotor axis above center of mass
T^h_{mr}, T^h_{tr},	Main and tail rotor thrust	Q^h_{mr}, Q^h_{tr}	Main and tail rotor counter-torque

1. INTRODUCTION[1]

Over recent decades, unmanned aerial vehicles (UAVs) have become an important research topic in the academic and military communities worldwide [1]. Among various UAVs, the ability of helicopter UAV (HUAV) to take off and landing vertically, hover flight, and various flight maneuvers, make them the ideal vehicles for a range of applications in a variety of environments [2]. HUAVs are categorized in different weights and sizes and used for various military and civilian purposes, such as taking photos, identifying in different areas, finding dead, or injured people by analyzing images in Hazardous environments, inspecting oil and gas pipelines, and so on [3]. In order to provide a safe flight on a helicopter, it is necessary to detect its faults and make emergency landings at the appropriate time [4]. Three main kinds of faults should be identified in aircraft or other flying vehicles that are sensor fault, actuator fault, and process fault [5]. Sensor loss makes an error or a prohibition on carrying out a mission, but in many cases, it can be compensated by control. But if the control is lost, it will lead to the crash of the HUAV [6]. Also, the possibility of an actuator's fault is more than sensors for mechanical reasons and loss of control is the most important factor in air events [7]. In this regard, dealing with the actuator's faults is a very important issue. This paper addresses the bias fault and stuck and floating actuators faults in the

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presence of external disturbance. In case of a bias fault, the control level always has a constant difference between the actual and expected deviation. In the stuck fault, the actuator is locked in a place and when a floating fault occurs, the control surface floats on its joint and may not be able to receive the control commands [8,9].In the last decade, some FDI methods have been proposed to deal with actuator faults and enhance the safety of various UAVs [6,10,11].

In general, the major problem in this paper is joint state and fault estimation in HUAV when it is under the influence of external disturbances such as wind. In this regard, some research has been done. A two-stage Kalman filter method was developed for fault and state simultaneous estimation [12]. Zhong et al. [13] presents the augmented three-stage extended Kalman filter (AThSKF) FDD scheme for a QUAV in the presence of external disturbances.

Xiao et al. [14] proposed an augmented robust three-stage extended Kalman filter (ARThSEKF) for the state estimation of Mars entry navigation under uncertain atmosphere density and unknown measurement errors. Hmida et al. [15] proposed an optimal three-stage Kalman filter (OThSKF). Based on the TKF, for the state and fault estimation of linear systems with unknown inputs, which decouple the ASKF covariance matrices. In this article, we cover various actuator faults in the presence of wind gust disturbance for a nonlinear model of HUAV in hover mode with develop ThSKF and based on central difference Kalman filter (CDKF). CDKF uses sterling polynomial interpolation to approximate the nonlinear function instead of analytical derivatives in the Taylor series. This makes the task very convenient and does not require the Jacobin and Hessian matrix, like the EKF. In this method, like the unscented Kalman filter, after considering the initial values, the Sigma points are calculated, and then time update and measurement update are considered.

In general, the main contributions of this paper include:
1) Use of nonlinear FDI for a nonlinear model of HUAV. 2) Because fault and disturbance affect the system, in the same manner, separating faults and disturbances is more difficult. So we develop actuator FDI when HUAV is under influence of external disturbance. 3) develop RThSCDKF when exact stochastic information of actuator faults and external disturbances is not available for FDI that is able to decouple the effect of actuator faults on each other because, in HUAV, roll, pitch, and yaw actuators faults are coupled and affect each other. For example, if a fault occurs in the yaw channel, it affects all channels. 4) considered stuck and floating actuator fault Which are a great danger for the HUAV and a few of the studies consider these faults.

The rest of this paper is organized as follows: Section 2 describes the model. In section 3, ThSCDKF is designed. Section 4 presents a simulation of the designed observers. Finally, the results are given in section 5.

2. HUAV MODEL DESCRIPTION

HUAVs are categorized in terms of weight and size and have four input references to perform various flight maneuvers. 1) The collective input (d_{col}) can change the reference value of the main rotor thrust. In fact, this input changes the thrust vector and HUAV flight height. 2) The longitudinal input (d_{lon}) causes the device to move forward and backward. 3) The lateral input (d_{lat}) causes the device to deflect right and left. 4) The pedal input (d_{ped}) changes value of the tail rotor thrust as a result of which, the HUAV rotates around it [16,17]. This inputs are applied to HUAV with four servo actuators. that are collective pitch servo, elevator servo, aileron servo and rudder servo.

2. 1. Mathematical Model

Equations of the helicopter are explained in literature [17,18], that cross-coupling terms are neglected as small in hover mode and summarized in a single $\dot{x} = f(x.u)$ expression as Equation (1).

$$
\begin{aligned}
\dot{u} &= -g\sin\theta + \frac{1}{m}(-T_{mr}^{h}(K_B d_{lon} + K_F a_{1f})) \\
\dot{v} &= -g\sin\varphi\cos\theta + \frac{1}{m}(T_{mr}^{h}(K_B d_{lat} + K_F b_{1f}) - T_{tr}^{h}) \\
\dot{w} &= g\cos\varphi\cos\theta + \frac{1}{m}T_{mr}^{h} \\
\dot{p} &= \frac{1}{J_{xx}}((T_{mr}^{h}h_{mr} + k_\beta)(K_B d_{lat} + K_F b_{1f}) - T_{tr}^{h}h_{tr}) \\
\dot{q} &= \frac{1}{J_{yy}}((T_{mr}^{h}h_{mr} + k_\beta)(K_B d_{lon} + K_F a_{1f}) - Q_{tr}^{h}) \\
\dot{r} &= \frac{1}{J_{zz}}(T_{tr}^{h}d_{tr} - Q_{mr}^{h}) \\
\dot{\varphi} &= p + (q\sin\varphi\tan\theta + r\cos\varphi)\tan\theta \\
\dot{\theta} &= q\cos\varphi - r\sin\varphi \\
\dot{a}_{1f} &= -\frac{a_{1f}}{\tau_f} - q + \frac{K_H}{\tau_f}d_{lon} \\
\dot{b}_{1f} &= -\frac{b_{1f}}{\tau_f} - p + \frac{K_H}{\tau_f}d_{lat}
\end{aligned}
\tag{1}
$$

The main rotor thrust (T_{mr}^{h}) and counter-torque (Q_{mr}^{h}) is in following form Equation (2).

$$
\begin{aligned}
T_{mr}^{h} &= C_{mr}^{h}(C_c d_{col} + D_c) + \frac{(D_{mr}^{T})^2}{2} - \ldots \\
& D_{mr}^{T}\sqrt{C_{mr}^{T}(C_c d_{col} + D_c) + \frac{(D_{mr}^{T})^2}{4}}
\end{aligned}
\tag{2}
$$

$$Q_{mr}^{h} = C_{mr}^{Q}(T_{mr}^{h})^{3/2} + D_{mr}^{Q} \tag{3}$$

where C_{mr}^{T}, D_{mr}^{T}, C_{mr}^{Q} and D_{mr}^{Q} are constant, and depend on the density of air and some characteristic of HUAV main rotor including the radius of disc, angular rotation rate, lift curve slope and blade chord length. The tail rotor thrust and counter-torque is in following form:

$$\begin{aligned} T_{tr}^{h} &= C_{tr}^{h}(C_t d_{ped} + D_t) + \frac{(D_{tr}^{T})^2}{2} - \ldots \\ &D_{tr}^{T}\sqrt{C_{tr}^{T}(C_t d_{ped} + D_t) + \frac{(D_{tr}^{T})^2}{4}} \end{aligned} \tag{4}$$

$$Q_{tr}^{h} = C_{tr}^{Q}(T_{tr}^{h})^{3/2} + D_{tr}^{Q} \tag{5}$$

C_{tr}^{T}, D_{tr}^{T}, C_{tr}^{Q} and D_{tr}^{Q} are constant, and depend on density of air and some characteristic of HUAV tail rotor such as the radius of disc, angular rotation rate, Lift curve slope and blade chord length.

3. THREE STAGE CENTRAL DIFFERENCE KALMAN FILTER

We assume to have a discrete-time nonlinear system with fault and unknown inputs.

$$x_{k+1} = f(x_k, u_k, b_k, d_k) + w_k^{x} \tag{6}$$

$$b_{k+1} = b_k + w_k^{b} \tag{7}$$

$$d_{k+1} = d_k + w_k^{d} \tag{8}$$

$$y_k = h(x_k, u_k, b_k, d_k) + v_k \tag{9}$$

where

$$E\left[\begin{bmatrix} w_k^x \\ w_k^b \\ w_k^d \\ v_k \end{bmatrix}\begin{bmatrix} w_j^x \\ w_j^b \\ w_j^d \\ v_j \end{bmatrix}^T\right] = \begin{bmatrix} Q_k^x & Q_k^{xb} & Q_k^{xd} & 0 \\ Q_k^{bx} & Q_k^{b} & Q_k^{bd} & 0 \\ Q_k^{dx} & Q_k^{db} & Q_k^{d} & 0 \\ 0 & 0 & 0 & R_k \end{bmatrix}\delta_{kj}, \tag{10}$$

where

$$\delta_{kj}\begin{cases} 1; k = j \\ 0; k \neq j \end{cases}, Q_k^x \succ 0, Q_k^b \succ 0, Q_k^d \succ 0, R_k \succ 0$$

Initial state, fault, disturbance estimation and estimation of covariance are in following form:

$$\begin{aligned} &\hat{x}_0 = E(x_0), \hat{b}_0 = E(b_0), \hat{d}_0 = E(d_0). \\ &\hat{P}_0^x = E\left[(x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T\right], \hat{P}_0^b = E\left[(b_0 - \hat{b}_0)(b_0 - \hat{b}_0)^T\right], \\ &\hat{P}_0^d = E\left[(d_0 - \hat{d}_0)(d_0 - \hat{d}_0)^T\right], \hat{P}_0^{xd} = E\left[(x_0 - \hat{x}_0)(d_0 - \hat{d}_0)^T\right], \\ &\hat{P}_0^{xb} = E\left[(x_0 - \hat{x}_0)(b_0 - \hat{b}_0)^T\right], \hat{P}_0^{bd} = E\left[(b_0 - \hat{b}_0)(d_0 - \hat{d}_0)^T\right]. \end{aligned} \tag{11}$$

We try that linearize the system respect to fault and unknown input.

$$B_k^x = \frac{\partial f(x_k, u_k, b_k, d_k)}{\partial b_k}\Big|_{x_k = \hat{x}_{k/k}, b_k = \hat{b}_{k/k}, d_k = \bar{d}_{k/k}} \tag{12}$$

$$E_k^x = \frac{\partial f(x_k, u_k, b_k, d_k)}{\partial d_k}\Big|_{x_k = \hat{x}_{k/k}, b_k = \hat{b}_{k/k}, d_k = \bar{d}_{k/k}} \tag{13}$$

$$B_k^y = \frac{\partial h(x_k, u_k, b_k, d_k)}{\partial b_k}\Big|_{x_k = \hat{x}_{k/k-1}, b_k = \hat{b}_{k/k-1}, d_k = \bar{d}_{k/k-1}} \tag{14}$$

$$E_k^y = \frac{\partial h(x_k, u_k, b_k, d_k)}{\partial b_k}\Big|_{x_k = \hat{x}_{k/k-1}, b_k = \hat{b}_{k/k-1}, d_k = \bar{d}_{k/k-1}} \tag{15}$$

Then, the nonlinear discrete-time varying system that was described in Equations (6) and (10) can approximate by:

$$x_{k+1} \approx f^*(x_k, u_k, \hat{b}_{k/k}, \bar{d}_{k/k}) + B_k^x b_k + E_k^x d_k + w_k^x \tag{16}$$

$$y_k \approx h^*(x_k, u_k, \hat{b}_{k/k-1}, \bar{d}_{k/k-1}) + B_k^y b_k + E_k^y d_k + v_k \tag{17}$$

where:

$$\begin{aligned} f^*(x_k, u_k, \hat{b}_{k/k}, \bar{d}_{k/k}) &= f(x_k, u_k, \hat{b}_{k/k}, \bar{d}_{k/k}) \\ &- B_k^x \hat{b}_{k/k} - E_k^x \bar{d}_{k/k} \end{aligned} \tag{18}$$

$$\begin{aligned} h^*(x_k, u_k, \hat{b}_{k/k-1}, \bar{d}_{k/k-1}) &= h(x_k, u_k, \hat{b}_{k/k-1}, \bar{d}_{k/k-1}) \\ &- B_k^y \hat{b}_{k/k-1} - E_k^y \bar{d}_{k/k-1} \end{aligned} \tag{19}$$

3. 1. Augmented State CDKF (ASCDKF)

By adding faults and unknown input, we could obtain a vector with new dimension. The state equations in the developed state are as follows:

$$x_k^a = \begin{bmatrix} x_k^T & b_k^T & d_k^T \end{bmatrix}, n^a = n + p + q \tag{20}$$

$$f^a(x_{k-1}^a) = \begin{bmatrix} f^*(x_{k-1}, u_{k-1}, \hat{b}_{k-1/k-1}) \\ + B_{k-1}^x b_{k-1} + E_{k-1}^x d_{k-1} \\ b_{k-1} \\ d_{k-1} \end{bmatrix} \tag{21}$$

$$\begin{aligned} h^a(x_k^a) &= h^*(x_k, u_k, \hat{b}_{k/k-1}, \bar{d}_{k/k-1}) \\ &+ B_k^y b_k + E_k^y d_k + v_k \end{aligned} \tag{22}$$

So we can write

$$x_{k+1}^a = f^a(x_k^a) + w_k^a \tag{23}$$

$$y_k = h^a(x_k^a) + v_k \tag{24}$$

where:

$$w_{k-1}^a = \begin{bmatrix} w_{k-1}^x \\ w_{k-1}^b \\ w_{k-1}^d \end{bmatrix}, Q_{k-1}^a = E\left[w_{k-1}^a (w_{k-1}^a)^T \right] \tag{25}$$

ASCDKF can be written in four steps.

TABLE 1. ASCDKF Algorithm

Step 1: Initialization
$x_{0/0}^a = \left[\hat{x}_0^T \quad \hat{b}_0^T \quad \hat{d}_0^T \right], \hat{P}_{0/0}^a = \begin{bmatrix} \hat{P}_0^x & \hat{P}_0^{xb} & \hat{P}_0^{xd} \\ (\hat{P}_0^{xb})^T & \hat{P}_0^b & \hat{P}_0^{bd} \\ (\hat{P}_0^{xd})^T & (\hat{P}_0^{bd})^T & \hat{P}_0^d \end{bmatrix}$
Step 2: Sigma point calculation
$\chi_{k-1,i}^a = \begin{bmatrix} x_{k-1/k-1}^a \\ x_{k-1/k-1}^a - h(\sqrt{\hat{P}_{k-1/k-1}^a})_i \\ x_{k-1/k-1}^a + h(\sqrt{\hat{P}_{k-1/k-1}^a})_{i-n^a} \end{bmatrix}$
Step 3: Time update
$\chi_{k-1,i}^{*a} = f_{k-1}^a(\chi_{k-1,i}^a), i = 0...2L$
$x_{k/k-1}^a = \sum_{i=0}^{2n^a} w_i^{(m)} \chi_{k/k-1,i}^{*a}$
$\hat{P}_{k/k-1}^a = \sum_{i=0}^{2n^a} \left\{ \begin{matrix} w_i^{(c_1)} (\chi_{k/k-1,i}^a - x_{k/k-1}^a)^2 \\ +w_i^{(c_2)} (\chi_{k/k-1,i}^a + x_{k/k-1}^a - 2x_0)^2 \end{matrix} \right\} + Q_{k-1}^a$
$\chi_{k/k-1,i}^a =$
$\left[x_{k/k-1}^a \quad x_{k/k-1}^a - h(\sqrt{\hat{P}_{k/k-1}^a})_i \quad x_{k/k-1}^a + h(\sqrt{\hat{P}_{k/k-1}^a})_{i-n^a} \right]$
$y_{k/k-1,i}^a = h_k^a(\chi_{k/k-1,i}^a)$
$y_{k/k-1}^a = \sum_{i=0}^{2n^a} w_i^{(m)} y_{k/k-1,i}^a$
Step 4: Measurment update
$\hat{P}_{xy}^a = \sum_{i=0}^{2n^a} \left\{ w_i^{(c)} [\chi_{k/k-1,i}^a - x_{k/k-1}^a][y_{k/k-1,i}^a - y_{k/k-1}^a]^T \right\}$
$\hat{P}_{yy}^a = \sum_{i=0}^{2n^a} \left\{ \begin{matrix} w_i^{(c)} [y_{k/k-1,i}^a - y_{k/k-1}^a] \\ [y_{k/k-1,i}^a - y_{k/k-1}^a]^T \end{matrix} \right\} + R_k$
$K_k^a = \hat{P}_{xy}^a (\hat{P}_{yy}^a)^{-1}$
$x_{k/k}^a = x_{k/k-1}^a + K_k^a (y_k^a - y_{k/k-1}^a)$
$P_{k/k}^a = P_{k/k-1}^a - K_k^a P_{yy}^a (K_k^a)^T$

3. 2. Robust Three Stage CDKF (RThCDKF) Design

The three stage U-V transformation is given by:

$$\hat{P}_{k/k-1}^a = U_k \bar{P}_{k/k-1}^a (U_k)^T \tag{26}$$

$$\hat{P}_{k/k}^a = V_k \bar{P}_{k/k}^a (V_k)^T \tag{27}$$

$$x_{k/k-1}^a = U_k \bar{x}_{k/k-1}^a \tag{28}$$

$$x_{k/k}^a = V_k \bar{x}_{k/k}^a \tag{29}$$

$$K_k^a = V_k \bar{K}_k^a \tag{30}$$

U and V are in the following form and determin later:

$$U_k = \begin{bmatrix} I & U_k^{12} & U_k^{13} \\ 0 & I & U_k^{23} \\ 0 & 0 & I \end{bmatrix}, V_k = \begin{bmatrix} I & V_k^{12} & V_k^{13} \\ 0 & I & V_k^{23} \\ 0 & 0 & I \end{bmatrix} \tag{31}$$

According to the inverse transformation of Equation (31), we have:

$$\bar{P}_{k/k-1}^a = (U_k)^{-1} \hat{P}_{k/k-1}^a [(U_k)^{-1}]^T \tag{32}$$

$$\bar{x}_{k/k-1}^a = (U_k)^{-1} x_{k/k-1}^a \tag{33}$$

$$\bar{x}_{k/k}^a = (V_k)^{-1} x_{k/k}^a \tag{34}$$

$$\bar{K}_k^a = (V_k)^{-1} K_k^a \tag{35}$$

$$\bar{P}_{k/k}^a = (V_k)^{-1} \hat{P}_{k/k}^a [(V_k)^{-1}]^T \tag{36}$$

According to above equation, ASCDKF equation can be transformed into:

$$\bar{\chi}_{k-1,i}^a = \begin{bmatrix} \bar{x}_{k-1/k-1}^a \\ \bar{x}_{k-1/k-1}^a - h(\sqrt{\bar{P}_{k-1/k-1}^a})_i \\ \bar{x}_{k-1/k-1}^a + h(\sqrt{\bar{P}_{k-1/k-1}^a})_{i-n^a} \end{bmatrix} \tag{37}$$

$$\chi_{K/k-1,i}^{*a} = (U_k)^{-1} f_k^a (V_{K-1} \bar{\chi}_{k-1,i}^a) \tag{38}$$

$$\bar{x}_{k/k-1}^a = \sum_{i=0}^{2n^a} w_i^{(m)} \bar{\chi}_{k/k-1,i}^{*a} \tag{39}$$

$$\bar{P}^a_{k/k-1}=\sum_{i=0}^{2n^a}\left\{w_i^{(c)}(\bar{\chi}^a_{k/k-1,i}-\bar{x}^a_{k/k-1})(\bar{\chi}^{*a}_{k/k-1,i}-\bar{x}^a_{k/k-1})^T\right\} + (U_k)^{-1}Q^a_{k-1}[(U_k)^{-1}]^T \quad (40)$$

$$\bar{\chi}^a_{k/k-1,i}=\begin{bmatrix}\bar{x}^a_{k/k-1}\\ \bar{x}^a_{k/k-1}-h(\sqrt{\bar{P}^a_{k-1/k-1}})_i\\ \bar{x}^a_{k/k-1}+h(\sqrt{\bar{P}^a_{k-1/k-1}})_{i-n^a}\end{bmatrix} \quad (41)$$

$$y^a_{k/k-1,i}=h^a_k(U_k\bar{\chi}^a_{k/k-1,i}) \quad (42)$$

$$y^a_{k/k-1}=\sum_{i=0}^{2n^a}w_i^{(m)}y^a_{k/k-1,i} \quad (43)$$

$$\hat{P}^a_{xy}=\sum_{i=0}^{2n^a}\left\{w_i^{(c)}[\bar{\chi}^a_{k/k-1,i}-\bar{x}^a_{k/k-1}][y^a_{k/k-1,i}-y^a_{k/k-1}]^T\right\} \quad (44)$$

$$\hat{P}^a_{yy}=\sum_{i=0}^{2n^a}\left\{w_i^{(c)}[y^a_{k/k-1,i}-y^a_{k/k-1}][y^a_{k/k-1,i}-y^a_{k/k-1}]^T\right\} + R_k \quad (45)$$

$$\bar{K}^a_k=(V_k)^{-1}\hat{P}^a_{xy}(\hat{P}^a_{yy})^{-1} \quad (46)$$

$$\bar{x}^a_{k/k}=(V_k)^{-1}U_k\bar{x}^a_{k/k-1}+\bar{K}^a_k(y^a_k-y^a_{k/k-1}) \quad (47)$$

$$\bar{P}^a_{k/k}=(V_k)^{-1}U_k\bar{P}^a_{k/k-1}(U_k)^T[(V_k)^{-1}]^T-\bar{K}^a_kP^a_{yy}(\bar{K}^a_k)^T \quad (48)$$

Substituting Equation (37) into Equation (38) and then Equations (38) and (39) in Equation (37), we have augmented covariance matrix. according to covariance matrix is diagonal, and with equal two matrix, we can obtain $\bar{P}^d_{k/k-1}$, $\bar{P}^b_{k/k-1}, \bar{P}^x_{k/k-1}$ and also U matrix. Subsequently, we replace Equation (41) in Equation (42) and use the first-order approximation of the Taylor series. We apply the result to Equations (44) and (45). With replacing the result in Equation (47) and by considered to relation of x doesn't have fault and unknown input and relation of fault doesn't have state and unknown input terms and relation of unknown input does not have state and fault, we can obtain equation of state, fault and unknown input and V matrix. The three-stage central difference Kalman filter is optimal when the statistical properties of models are perfectly known and when they are unknown we can use robust ThSCDKF. Accourding to equation for eliminate initial condition have to eliminate the terms of contain of fault and unknown input from equations and we can write RThSCDKF in six step in the following form:

TABLE 2. RThSCDKF Algorithm

Step 1: Initialization

$$k=0,\hat{x}_{0/0}=\hat{x}_0,\hat{P}^x_{0/0}=P^x_0\ ,V^{23}_0$$

Step 2: State sub-filter

$$\chi_{k-1,i}=\begin{bmatrix}\hat{x}_{k-1/k-1}\\ \hat{x}_{k-1/k-1}-h\left(\sqrt{P_{k-1}}\right)_i\\ \hat{x}_{k-1/k-1}+h\left(\sqrt{P_{k-1}}\right)_{i-L}\end{bmatrix}$$

$$\bar{\chi}^*_{k/k-1,i}=f^*_{k-1}\left(\chi_{k-1,i},u_{k-1},\hat{b}_{k-1/k-1},\bar{d}_{k-1/k-1}\right),$$

$$\bar{x}_{k/k-1}=\sum_{i=0}^{2n}w_i^{(m)}\bar{\chi}^*_{k/k-1,i}$$

$$\bar{P}^x_{k/k-1}=\sum_{i=0}^{2n}\begin{bmatrix}w_i^{(c_1)}\left(x_{i,k|k-1}-x_{L+i,k|k-1}\right)^2\\ ...+w_i^{(c_2)}\left(x_{i,k|k-1}+x_{L+i,k|k-1}-2x_{0,k|k-1}\right)^2\end{bmatrix}+Q^x_k$$

$$\bar{\chi}_{k/k-1,i}=\begin{bmatrix}\bar{x}_{k/k-1}\\ \bar{x}_{k/k-1}-h\left(\sqrt{P_{k-1}}\right)_i\\ \bar{x}_{k/k-1}+h\left(\sqrt{P_{k-1}}\right)_{i-L}\end{bmatrix}$$

$$\bar{y}_{k/k-1,i}=h^*_k\left(\bar{\chi}_{k/k-1,i},u_{k-1},\hat{b}_{k-1/k-1},\bar{d}_{k-1/k-1}\right)$$

$$\bar{y}_{k/k-1}=\sum_{i=0}^{2n}w_i^{(m)}\bar{y}_{k/k-1,i}$$

$$\bar{P}_{xy}=\sqrt{w_1^{(c_1)}P^-_k}\left(y_{1:L,k|k-1}-y_{L+1:2L,k|k-1}\right)$$

$$\bar{P}_{yy}=\sum_{i=1}^{2n}\begin{bmatrix}w_i^{(c_1)}\left(y_{i,k|k-1}-y_{L+i,k|k-1}\right)^2\\ ...+w_i^{(c_2)}\left(y_{i,k|k-1}+y_{L+i,k|k-1}-2y_{0,k|k-1}\right)^2\end{bmatrix} ...+Q_v$$

$$\bar{K}^x_k=\bar{P}_{xy}\left(\bar{P}_{yy}\right)^{-1},\bar{x}_{k/k}=\bar{x}_{k/k-1}+\bar{K}^x_k\left(y_k-\bar{y}_{k/k-1}\right)$$

$$\bar{P}^x_{k/k}=\bar{P}^x_{k/k-1}-\bar{K}^x_k\bar{P}_{yy}\left(\bar{K}^x_k\right)^T$$

Step 3: Fault sub-filter

$$U^{12}_k=B^x_{k-1},\ S^2_k=H_kU^{12}_k+B^y_k$$

$$\bar{P}^b_{k/k}=\left(S^{2T}_k\bar{P}^{-1}_{yy}S^2_k\right)^+$$

$$\bar{K}^b_k=\bar{P}^b_{k/k}S^{2T}_k\bar{P}^{-1}_{yy},\ \bar{b}_{k/k}=\bar{K}^b_k\left(y_k-\bar{y}_{k/k-1}\right)$$

Step 4: Unknown input sub-filter

$$U^{23}_k=V^{23}_{k-1},\ U^{13}_k=E^x_{k-1}+U^{12}_kU^{23}_k$$

$$S^3_k=H_kU^{13}_k+B^y_kU^{23}_k+E^y_k$$

$$\bar{P}^d_{k/k}=\left(S^{3T}_k\bar{P}^{-1}_{yy}S^3_k\right)^+$$

$$\bar{K}^d_k=\bar{P}^d_{k/k}S^{3T}_k\bar{P}^{-1}_{yy}\alpha_k(I-S^2_k\left(S^2_k\right))^+$$

$$\bar{d}_{k/k}=\bar{K}^d_k\left(y_k-\bar{y}_{k/k-1}\right)$$

Step 5: the correction of the state and the fault estimations

$$V_k^{12} = U_k^{12} - \bar{K}_k^x S_k^2 \,,\; V_k^{23} = U_k^{23} - \bar{K}_k^b S_k^3$$

$$V_k^{13} = U_k^{13} - V_k^{12} \bar{K}_k^b S_k^3 - \bar{K}_k^x S_k^3$$

$$\hat{x}_{k/k} = \bar{x}_{k/k} + V_k^{12} \bar{b}_{k/k} + V_k^{13} \bar{d}_{k/k}$$

$$\hat{b}_{k/k} = \bar{b}_{k/k} + V_k^{23} \bar{d}_{k/k}$$

$$P_{k/k}^x = \bar{P}_{k/k}^x + V_k^{12} \bar{P}_{k/k}^b (V_k^{12})^T + V_k^{13} \bar{P}_{k/k}^d (V_k^{13})^T$$

$$P_{k/k}^b = \bar{P}_{k/k}^b + V_k^{23} \bar{P}_{k/k}^d (V_k^{23})^T$$

Step 6: k=k+1 and return to step 1.

4. SIMULATION RESULT

In order to validate the RThSCDKF approach, four scenarios are simulated on an unmanned helicopter. In this paper, model parameters are adopted from literature [18]. Process noise and measurement noise are selected based on typical specification of low-cost sensor considering real simulation results of the system. The discrete wind gust model block implements a wind gust of standard "1-cosine" shape. fault and disturbance covariance matrix are not required. In this section, performance of RThSCDKF in the presence of some actuator bias faults is examined and are compared with RThSEKF.

Scenario 4.1: Bias Fault

Small bias faults are simulated for three actuator inputs in the presence of disturbances. So, a sequence of consecutive faults is generated. From t=4-8s, lateral servo has a bias fault between two positions of value -0.01 and 0.01 in a square-wave fashion. For t=8-10s, longitudinal servo has a bias fault near to equilibrium position in no fault mode. For t=10-14s, ruder servo has a bias fault which its value equals -0.01 that is near to control input in no fault mode. Figure 1 shows True faults, RThSEKF estimation and RThSCDKF estimation for lateral, longitudinal and yaw channels.

4. 2. Scenario 2: Floating and Stuck Fault

Floating and stuck faults are simulated for two actuator inputs in the presence of disturbances. So, a sequence of consecutive faults is generated. From t=4-8s, lateral servo has a floating fault between two positions of value 0.01 and -0.01 in a square-wave fashion. For t=14-18s, longitudinal servo has a stuck fault near to equilibrium position in no fault mode. Figure 2 shows True faults and their estimation in the presence of disturbance for lateral, longitudinal and yaw channels. As shown, ThSCDKF can accurately diagnose the faults and decouple them from each other respect to RThSEKF.

4. 3. Scenario 3: Simultaneous Faults

In this section, simultaneous faults of the model are checked.

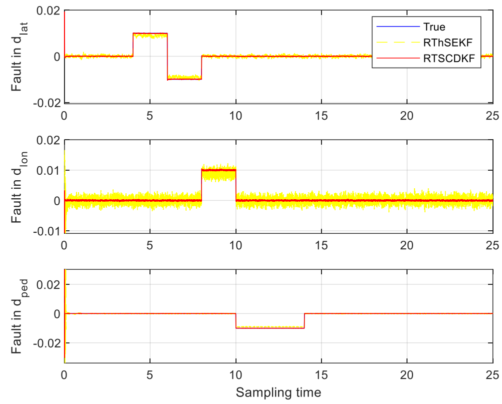

Figure. 1. Bias estimation in scenario 1

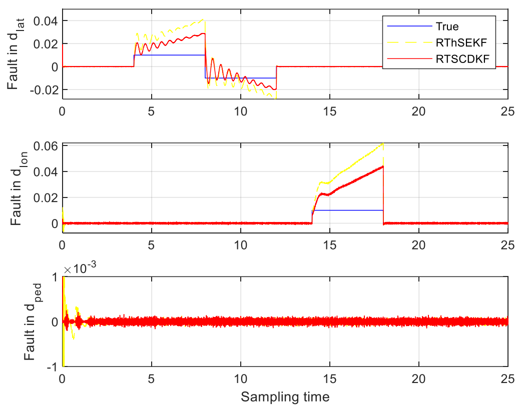

Figure 2. floating and stuck Fault Estimation of ThSCDKF in Scenario 2

Simulation results are given in Figure 3. Fault occurs in lateral and ruder servo actuators at the same time. As shown in this figure, d_{lat} has a floatting fault, and d_{ped} has a bias fault at 8-14s and RThSCDKF is able to estimate the faults better than RThSEKF.

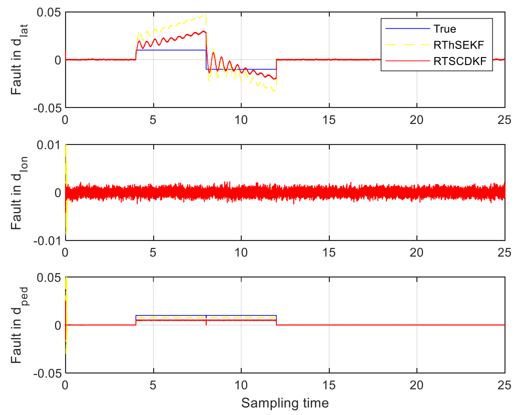

Figure 3. Simultaneous Fault Estimation in Scenario 3

5.CONCLUSION

describes FDI for HUAV actuators for detecting and isolating additive faults such as bias, floating, stuck in the presence of external disturbances in hover mode. It is important that, actuator fault detection can be decoupled and separated from disturbance. For this purpose, ThSCDKF is proposed. ThSCDKF scheme is simulated and compared with ASCDKF under different fault scenarios. Simulations deal with bias faults (scenario 1), floating and stuck faults (scenario 2), simultaneous faults (scenario 3), comparison between ASCDKF and ThSCDKF (scenario 4). Results show effectiveness of the proposed method for various additive faults in HUAV actuators and simultaneous faults in the presence of external disturbance respect to RTSEKF. The proposed method can be used for other plants with proposed faults.

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Persian Abstract

چکیده

این مقاله به تخمین حالت و عیب برای سیستمهای تصادفی غیر خطی متغیر با زمان وقتی اطلاعات مربوط به عیب و ورودیهای ناشناخته کاملاً مشخص نیست و در حضور ورودیهای ناشناخته می پردازد. برای این منظور ، فیلتر سه مرحله ای تفاضل مرکزی کالمان فیلتر (RThSCDKF) برای تشخیص و شناسایی خطای مبتنی بر مدل (FDI) در یک هلکوپتر بدون سرنشین در حضور نامعینی توسعه داده شده است. در این سیستم خطاهای عملگر تحت تأثیر یکدیگر قرار می گیرند. روش پیشنهادی در صورت وجود اغتشاشات خارجی ، عیبهای عملگر را تخمین می زند. این مدل می تواند عیب های قفل عملگر و شناور را که بسیار مهم هستند تشخیص می دهد.. در پایان این روش با فیلتر کالمن توسعه یافته سه مرحله ای (ThSEKF) مقایسه می شود. نتایج شبیه سازی نشان می دهد روش پیشنهادی نتایج خوبی داشته و دارای دقت بیشتری نسبت به ThSEKF است.