



Robust and Stable Flow Shop Scheduling Problem under Uncertain Processing Times and Machines' Disruption

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ABSTRACT

This paper presents a predictive robust and stable approach for a two-machine flow shop scheduling problem with machine disruption and uncertain job processing time. Indeed, a general approach is proposed that can be used for robustness and stability optimization in an m-machine flow shop or job shop scheduling problem. The robustness measure is the total expected realized completion time. The expected sum of squared aberration between each jobs' completion time in the realized and initial schedules is the stability measure. We proposed and compared two methods to deal with such an NP-hard problem; a method based on decomposing the problem into sub-problem and solving each sub-problem, and a theorem-based method. The extensive computational results indicated that the second method has a better performance in terms of robustness and stability, especially in large-sized problems. In other words, the second method is preferable because of the better manufacturer responsiveness to the customer and the production staff satisfaction enhancement.

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NOMENCLATURE

D	Downtimes (a General distribution $D \sim G(t)$); the time required to back the machine to the operational mood	λ_j	The exponential distribution rate of generation initial processing time of job j on machine 1
U	Uptimes (an exponential distribution with rate θ); The time between two consecutive machine breakdowns	μ_j	The exponential distribution rate of generation initial processing time of job j on machine 2
i	Machine index, $i = 1, 2$	p_{ij}	The initial (expected) processing time of job j on the machine i
j	Job index, $j = 1, 2, \dots, n$	C_{ij}	The expected initial completion time of job j on the machine i
r	The expected value of repair times after each breakdown	C_{ij}^r	The expected real completion time of job j on the machine i

1. INTRODUCTION

The flow Shop Scheduling Problem (FSSP) covers many real case studies in practical problems [1]. Some papers described the applications of the two-machine flow shop scheduling problem (FSSP) [2–4]. Total completion time minimizes Work in Process (WIP) costs and the rapid turnaround of jobs. The two-machine FSSP with the sum of jobs' completion time as a primary objective, this paper's focus, even in deterministic scheduling

environments, is strongly Np-hard [5]. Some effective heuristics proposed to cope with the problem's complexity do not seem superior over the other [6]. Besides, the job or machine-related uncertainties that lead to an interruption in the flow of jobs and result in unwanted delays are commonly occurring in the production environment, enhancing the problem's complexity. Arriving of an unanticipated new job [7], due date uncertainty [8], breakdown occurrence [9], uncertainty in job processing times [9, 10], etc are the

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likes of uncertainties and disruptions. In 70% of uncertainty oriented flow shop scheduling studies in past decades, the job processing time is uncertain, by 25%, the disruption is machine failure, and by 10%, both of these factors consider [11]. Machine failure and uncertain processing times discuss in this paper. Robust and stable scheduling is one of the policies in confronting uncertainty. The sensitivity of a schedule performance to its objective function is called robustness, but stability refers to the insensitivity of the start (or completion) time. Stability is a measure of changes in the sequence of jobs on a machine to the original. The concept of robustness is very close to flexibility: the ease of schedule reparability and the power of converting to new, high quality scheduling in the face of uncertainties. The expected realized total completion time has been implemented as a robustness measure by itself [8]. Here we take this definition as robustness. A function of the sum of deviation between each jobs' start/completion times in the initial and realized schedules are often the stability measure [8], and the same definition is accepted here. The value of the expected performance measure obtained by applying the righting shift policy on the initial schedule is a realized schedule. Additionally, a justified schedule with a small deviation from the initial one in the face of uncertainty and without significant degradation in the main objective is robust and stable. Simultaneous consideration of robustness and stability besides maintaining the schedule feasibility improves its flexibility against uncertainties.

Dealing with uncertainty-related deviations can be done with predictive, reactive, or predictive-reactive (hybrid) strategies [12]. In the predictive strategy, future uncertainties consider in the initial plan. Reactive or hybrid approaches are common strategies for dealing with machine breakdown. In almost all robustness-focused studies, dealing with machine failure disruption performs with reactive approaches or in the reactive phase of hybrid approaches [8]. In reactive strategies (e.g., rescheduling), especially in large-size problems, it takes a long time to deal with uncertainty. Predictive strategies can overcome this by actively preparing for any future uncertainties [12], so here we adopt a predictive approach to cope with machine breakdown and uncertainty of job processing times. The two-machine Flow Shop Scheduling Problem (FSSP) under uncertainty of processing time is referenced in many papers, commonly with makespan as a primary objective function [13, 14]. C_{max} , is also a primary objective in most FSSP under machine breakdown disruption studies [15, 16]. In addition, it has been a primary objective in the case of simultaneously considering the uncertainty of processing time and machine breakdown [9, 17]. Therefore, we define the robustness measure based on another performance measure; i.e., the total completion times. With a glimpse at the previous attempts in this

realm of research, we can state contributions of this paper:

- Although various cases of the robust and stable flow shop-scheduling problem previously raised in studies, this article discusses a (particular) case of $F2$ for the first time.
- Simultaneously considering robustness and stability to meet the requirements of producers and workers.
- Besides, predictively coping with the uncertainty of job processing times, dealing with machine breakdowns is also predictive.
- Proposing a novel robust and stable heuristics to cope with aforementioned-uncertainty conditions.
- The way of considering the uncertainty, the proposed solution method, and the primary objective function is different from the previous more related works.

The remainder of this paper is as follows; the related literature review is in section 2. In section 3, we define the problem and propose our solution method. In sections 4, 5, and 6, we presented computational results, managerial insight, and paper conclusions.

2. LITERATURE REVIEW

This paper presents robust and stable scheduling approaches for a permutation two-machine flow shop scheduling problem (PFSSP) under uncertainty with expected total completion times as a primary objective. According to the classification of Graham et al. [18], the problem denotes as $F2/prmu/\sum_{j=1}^n C_{2j}$, in the deterministic version, which is strongly Np-Hard [5]. The solution methods of $F2/prmu/\sum_{j=1}^n C_{2j}$ categorizing into exact and approximate methods. The high-performance problem-solving branch and bound algorithms and Lagrangian methods had proposed for $F2//\sum_{j=1}^n C_{2j}$ able to solve up to 50 jobs [19]. Due to the high complexity of this problem, heuristic methods have been ever the researchers' focus, for example, *MINITI* heuristic [6, 20]. Rossi et al. [21] proposed a simple high-efficiency FF-RN heuristic method by modifying the NEH heuristic and compared it with the best simple heuristics of PFSSP reviewed [21]. In the face of uncertainty, applying iterative simulation-based methods or producing robust (and stable) schedules are conventional approaches to encounter system disruptions [9]. Ghezail et al. [22] proposed a graphical robust, proactive approach to deal with uncertainty in the FSSP. Kasperski et al. [23] propose a predictive regret-based robust schedule with interval processing times. katrajani et al. [24] propose a heuristic to minimize normalized makespan and instability in a dynamic flow shop under uncertainty of machine breakdown and job-ready time variability. Ying [25] applied Iterated Greedy (IG) and Simulated Annealing (SA) heuristics to produce a predictive regret-based robust schedule in a maximum

completion time two-machine FSSP, with interval processing times. Fazayeli et al. [26] applied the Genetic Algorithm (GA) and SA to produce a robust predictive schedule in an m-machine PFSP under uncertain repair time and machine breakdown. Rahmani [9] employed GA to propose a proactive-reactive robust, and stable schedule for a two-machine PFSP under uncertain job processing time and machine failure. Also, she applied scenarios to show the uncertainty of processing times, Cmax as an efficiency measure, maximum realized completion times of jobs as robustness, and the expected sum of square deviations between the completion time in actual and initial schedules stability measure. Cui et al. [16] used a simulation-based method to propose a robust predictive schedule for two-machine PFSP under machine breakdown with Cmax as an efficiency measure. Liao and Fu [7] exploit GA to propose a robust predictive schedule for an m-machine PFSP with interval processing times. Abtahi et al. [11] employed a robust optimization method to produce efficient, robust, and stable schedules in an m-machine FSSP under uncertainty. They applied scenarios to show the uncertainty of processing times, total completion times as an efficiency measure, total realized tardiness of jobs as robustness, and the expected sum of square deviations between the completion time in actual and initial schedules as a stability measure. Here we adopt total realized completion times as robustness and the expected sum of square deviations between the completion time in actual and initial schedules as a stability measure. We propose a modified Shifting bottleneck (*SB*) to produce robust partial solutions in a two-machine FSSP in the face of job processing times uncertainty and machine breakdown. Shifting bottleneck (*SB*), a decomposition-based heuristic, performs well for job shops [27, 28]. Koulamas et al. [29] presented an efficient modified *SB* for two-machine PFSSP with total tardiness of jobs as a primary objective. Mukherjee et al. [30] showed that the modified *SB* is suitable in optimally solve a two-machine PFSSP with the makespan criterion. Elyasi and Salmasi [31] applied an adjusted *SB* in stochastic flow shop under due date uncertainty to minimize the number of tardy jobs. Allahverdi and Allahverdi [32] proposed a decomposition-based heuristics for a total completion time PFSSP with bounded processing time. As can be seen, few papers focused on

- Producing a robust and stable FSSP with total completion time as a primary objective.
- Predictively producing a robust and stable FSSP while considering the uncertainty of job processing time and machine breakdown simultaneously.

In this paper, we propose a heuristic method to produce a robust and stable schedule in a stochastic two-machine FSSP specified case with total completion time as a primary objective. Then we compare it to an exact solution method. The former (Our proposed heuristic)

employs modified *SB* and a theorem of Abtahi et al. [33], and the latter uses a theorem of Pinedo [34] to hedge against job processing time uncertainty. We employ the Right-Shifting (RS) rescheduling method to obtain a realistic schedule after machine failures occurrence.

3. PROBLEM DEFINITION AND SOLUTION METHOD

In this paper, we considered a two-machine FSSP. The uncertain job processing time and random breakdowns of machines are the system disruptions. The processing time of job j on the first and second machines respectively follow the exponential distribution with rates λ_j and μ_j . The time between two consecutive failures follows an exponential distribution with the rate of θ and at most one failure is expected on a machine in each interval $(1/\theta)$. After each breakdown, minimal repairs perform to restore machines to the operating condition (which does not affect the machine age and breakdown parameter).

The following assumptions considered:

- All jobs are available at the beginning of the schedule,
- Machines have availability restriction; i.e., random machine break down may occur during the processing of job j on machine i ,
- The time between two consecutive breakdowns follows an exponential distribution. Also, constant repair times allocate after each failure,
- The rest of the disrupted job will perform after machine repairing,
- Only non-delay schedules considered,
- The objective function is a minimization of schedules' robustness and stability simultaneously.

3.1. Solution Method According to a classification by Graham et al. [18], a problem of robust and stable two-machine FSSP under uncertainty of job processing time and machine breakdowns is represented as follows:

$$F2 \left\{ \begin{array}{l} p_{1j} \sim \exp(\lambda_j), p_{2j} \sim \exp(\mu_j); \\ \text{brkdw}n : U \sim \exp(\theta), D \sim G(t) \end{array} \right\} \alpha.RM + (1-\alpha).SM$$

Pinedo [34] showed that sorting the jobs in descending order of $(\lambda_j - \mu_j)$ optimizes the expected total completion times (i.e., the intended robustness measure) in a two-machine FSSP particular case (when job processing times on the first (second) machine pursued the exponential distribution with the rate $\lambda_j(\mu_j)$). Here, we proposed two robust and stable methods, and for each one, two policies in the face of machine failure; reactive and predictive. To predictably deal with the machine failure, the buffer time insertion method, and to encounter with reactively, the right shift rescheduling (*RSH*) is implemented to the affected jobs [13] for details.

Two algorithms are proposed in this paper to handle such a problem:

- The optimal theorem based method (OBM).
- The decomposition-based method (DBM).

Based on a theorem, the OBM considering the uncertainty of processing times acquires the optimal robust solution (of Pinedo [34]) according to the decreasing order of $\lambda_j - \mu_j$.

The DBM method employs a modified shifting bottleneck heuristics and one-machine robustness and stability optimization theorem [33]; shifting bottleneck (SB) heuristics [5] decomposes a problem into sub-problems and solve each sub-problem optimally [25] using the shortest expected processing time (SEPT) first rule.

Theorem. SEPT rule solves

$$1 \left| \begin{array}{l} p_j \sim \exp(\lambda_j); \\ brkdown : U \sim \exp(\theta), D \sim G_2(t) \end{array} \right| \alpha.RM + (1 - \alpha).SM$$

optimally [33], where θ is the rate of machine breakdown and r is the expected repair time, $RM = E \sum_{j=1}^n C_j^r$ is a robustness measure, and $SM = E[\sum_{j=1}^n (C_j - C_j^r)^2]$ is a stability measure. According to the above theorem, sorting the jobs in a no descending order of processing times over each machine seems an acceptable idea to give a robust and stable sequence for the intended uncertain two-machine FSSP. The steps of the DOM are as follows:

- Decompose the intended two-machine uncertain flow shop-scheduling problem into two one-machine sub-problems, with predefined conditions of uncertainty.
- Sequence the jobs according to the SEPT on the first machine.
- The first job on the first machine (M1) continued its process on the second machine regardless of the amount of its expected processing time (on M2).
- To determine the order of the remaining jobs on M2, do as follows.
- Whenever because of incomplete remaining (previous) jobs on M2, there is a queue with more than two jobs on M1, order the jobs queue according to the shortest expected value of their processing times on M2.
- Otherwise, the jobs keep their sequence on M1.

Figure 1 shows the flow chart of DBM. In the next section, we compared the proposed methods after the implementation of reactive as well as predictive policy. The job sequence on the two machines are the same in OBM; however, during the execution of DBM, the job sequence on $M1$ may not keep.

4. COMPUTATIONAL RESULTS

4. 1. Data Generation The job processing times on the first and second machines are uncertain and respectively follow the exponential distribution with the

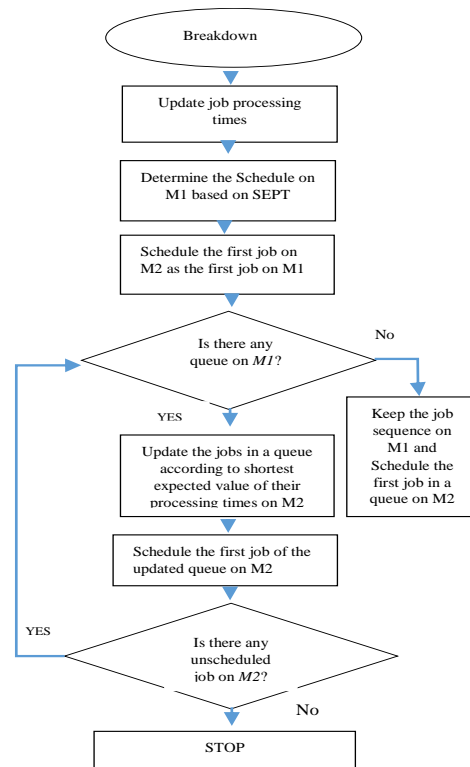


Figure 1. the flow chart of DBM

rate of λ_j and μ_j , where λ_j and μ_j are independently generated from $U[0,1,1]$. We select the number of jobs from set $j = \{3, 5, 10, 30, 50, 60, 100\}$. Then 100 instances generate for each job number. Hence, we have 700 problems. For each test problem, we chose the rate of machine breakdown from a set $\theta = \{1/50, 1/60, 1/80\}$; a higher value of θ represents a higher probability of machine breakdown disruption.

Like Nouri et al. [35], the repair times duration follows an exponential distribution based on the meantime to repair value (MTTR) at two-level. The repair times' duration calculates via $r = \exp md(MTTR)$, and the MTTR based on the machine busy time (MB); for low level, $MTTR_l \in [0.01MB, 0.05MB]$, and for high level, $MTTR_h \in [0.05MB, 0.1MB]$. Ultimately we have combination of 4200 problems. The methods compare to reach a comprehensive conclusion as follows:

1. Without considering machine breakdown,
2. Applying the reactive policy after failure,
3. Dealing with breakdown disruption predictively.

4. 2. Two Comparative Methods without Considering Machine Breakdown

Here, we examine the performance (objective function) of the two methods provide the managerial results for (the problem in question without assuming machine breakdown)

$F2/p_{1j} \sim \exp(\lambda_j), p_{2j} \sim \exp(\mu_j) / E(\sum_{j=1}^n C_{2j}^r)$. The problem coded in *MATLAB R2013b*, and the results have reported for different problems' sizes. Solving time is negligible and has not been brought. In Figures 2 to 4, *AEC*, *AECO*, and *RD* respectively represent the Average Expected Completion time of the DBM to OBM and the relative deviation of the objective function of DBM to OBM without considering machine breakdowns. Figures 2 and 3 report the *AEC*, *AECO*, and *RD* for the small size problems. It seems that DBM has a proper performance for small-size (3-30 jobs) problems (given that OBM offers the optimal solution for $F2/p_{1j} \sim \exp(\lambda_j), p_{2j} \sim \exp(\mu_j) / E(\sum_{j=1}^n C_{2j}^r)$). Although the performance of DBM as a heuristic method is still acceptable (see Figures 4 and 5), for the medium to large-size (50-100 jobs) problems, OBM thoroughly outperforms DBM.

4. 3. Two Comparative Methods Based on Applying Reactive Policy

Here, we examine two proposed methods for (the problem in question)
$$F2 \left\{ \begin{array}{l} p_{1j} \sim \exp(\lambda_j), p_{2j} \sim \exp(\mu_j); \\ brkdown : U \sim \exp(\theta), D \sim G(t) \end{array} \right\} \alpha.RM + (1 - \alpha).SM$$
 applying the reactive policy after machine breakdown. The problem coded in *MATLAB R2013b*, and outputs has been reported different problems' sizes in Table 1. *RR_DBM*, *R_OBM*, *SR_DBM*, *SR_OBM*, *Z_RDBM*,

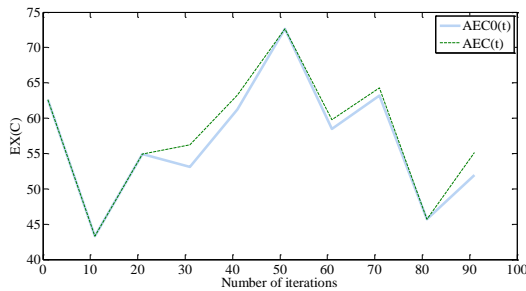


Figure 2. The comparison the expected completion time for two methods without considering machine breakdown (small-size problem)

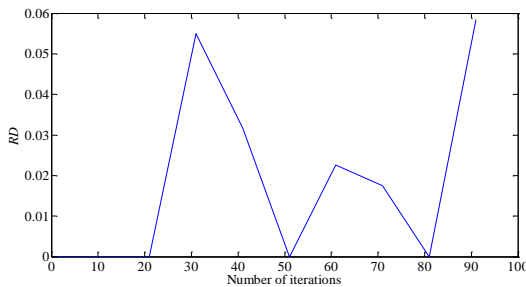


Figure 3. The related deviation between two methods without considering machine breakdown (small-size problem)

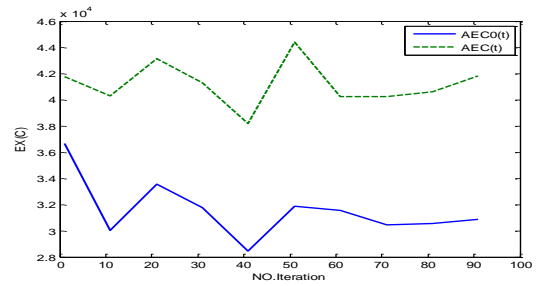


Figure 4. Comparing the two methods' expected completion time without considering machine breakdown (medium to large-size problems)

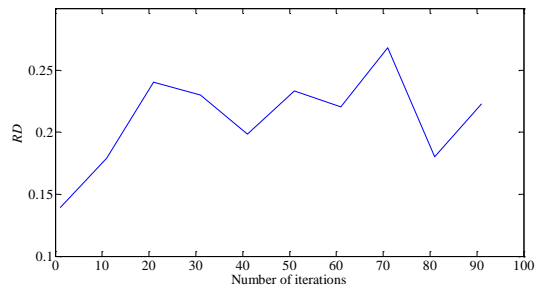


Figure 5. The related deviation between two methods without considering machine breakdown (medium to large-size problems)

Z_ROBM, *n*, and *TIME*, respectively represent the robustness of *DBM* and *OBM*, the stability of *DBM* and *OBM*, the objective function value of *DBM* and *OBM*, the number of jobs, and the problem-solving time by applying reactive policy after machine breakdown. Figures 6 to 10 illustrate the contents of Table 1. According to Figures 6 to 10, in all cases of the small-size problem ($n \leq 10$), regardless of the values of *TETA* and *MTR*; there is no significant difference between the two proposed methods' performance. Nevertheless, in medium to large-size problems ($n \geq 30$), *OBM* outperforms *DBM* by applying reactive policy after machine breakdown. These results had obtained by considering the same coefficients for robustness and stability ($\alpha = (1 - \alpha) = 0.5$).

4. 4. Two Comparative Methods' Based on Applying Predictive Policy

Here, we examine the two proposed methods for solving (the problem in question)
$$F2 \left\{ \begin{array}{l} p_{1j} \sim \exp(\lambda_j), p_{2j} \sim \exp(\mu_j); \\ brkdown : U \sim \exp(\theta), D \sim G(t) \end{array} \right\} \alpha.RM + (1 - \alpha).SM$$

by applying the predictive policy to encounter the machine breakdown. The problem coded in *MATLAB R2013b*, and the results were reported for different problems' sizes in Table 2 and Figure 11. *RP_DBM*, *RP_OBM*, *SP_DBM*, *SP_OBM*, *Z_PDBM*, *ZP_OBM*, *n*, and *TIME*, respectively represent the robustness of *DBM*

TABLE 1. Robustness, stability, and the objective function of the two methods by applying the reactive policy for different problem-parameters

No	1/TETA	MTTR	n	RR_DBM	RR_OBM	SR_DBM	SR_OBM	Z_RDBM	Z_ROBM	T
1	80	low	3	0.01	0.05	0.05	0.02	0.03	0.036	1.58
2	80	low	5	0.3	0.23	0.26	0.1	0.28	0.16	1.8
3	80	low	10	0.27	0.52	0.47	0.49	0.36	0.5	6.48
4	80	low	30	21.7	1.8	6.7	5.6	14.2	3.7	99
5	80	low	50	237	3.5	43	23.8	136	7.52	308
6	80	low	60	335	5	63	77	189	14.5	648
7	80	low	100	5858.3	15.3	226.7	79	3042.5	47	2798
8	60	low	3	0.018	0.001	0.02	0.001	0.01	0.009	27
9	60	low	5	0.05	0.33	0.18	0.12	0.11	0.22	11
10	60	low	10	0.82	0.32	0.67	0.19	0.75	0.26	11
11	60	low	30	23	4	7.7	8.3	15.5	6	202
12	60	low	50	574	4	44	15	310	10	608
13	60	low	60	1733	4	73	12	903	9	380
14	60	low	100	4279	15	223	119	1251	68	3187
15	50	low	3	0.02	0.1	0.08	0.04	0.05	0.08	1.38
16	50	low	5	0.49	0.18	0.23	0.07	0.36	0.12	1.94
17	50	low	10	0.62	0.54	0.72	0.46	0.67	0.5	5.43
18	50	low	30	18	2	7.5	3.4	12.8	2.8	51.3
19	50	low	50	475	4	45	13	260	9	213.2
20	50	low	60	1581	7.5	81	50	831	29	363
21	50	low	100	4013	13	245	93	2128	54	1567
22	80	high	3	0.07	0.17	0.11	0.17	0.09	0.17	1.4
23	80	high	5	0.43	0.4	0.4	0.41	0.42	0.41	1.8
24	80	high	10	3	1.08	1.56	1.48	2.28	1.28	6.3
25	80	high	30	147	5	23	17	85.3	11.16	72.3
26	80	high	50	42.5	46	329	8	186	27	294
27	80	high	60	2663	27	125	343	1394	185	455
28	80	high	100	12388	94.5	418	2055	6403	1075	2049
29	60	high	3	0.21	0.21	0.17	0.24	0.19	0.23	1.41
30	60	high	5	1.03	0.42	0.46	0.50	0.75	0.46	1.62
31	60	high	10	1.18	1.9	1.1	2.70	1.5	1.9	4.6
32	60	high	30	166.9	6.6	25.7	42.8	96.3	6.6	70.4
33	60	high	50	378.7	7.8	50.7	58.5	219.2	33.2	313.7
34	60	high	60	1863.8	16.2	212.5	113.4	988.6	114.3	511.6
35	60	high	100	11129.2	40.65	402	543	5765.7	291.8	2947
36	50	high	3	0.09	0.16	0.13	0.08	0.11	0.12	1.5
37	50	high	5	0.38	0.39	0.35	0.35	0.367	0.37	2.11
38	50	high	10	5.46	1.09	1.9	1.5	3.7	1.3	5.27
39	50	high	30	103.6	12.27	17.1	12.27	60.36	8.2	53.98
40	50	high	50	1384.3	19.8	97.6	170.3	740.99	95	250
41	50	high	60	2708.8	14.9	146.65	151.99	1427.7	83.47	444.9
42	50	high	100	29.6	6328	441.2	316.9	3384.7	173.2	3017

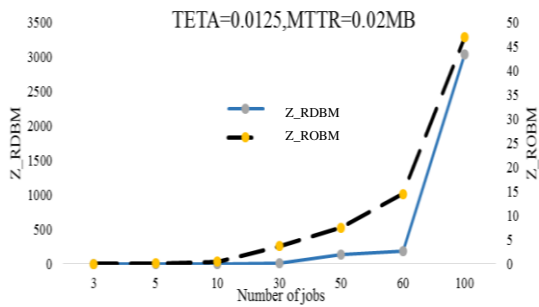


Figure 6. Comparing the objective function of the two methods by applying the reactive policy, Low expected failure, and short repair time

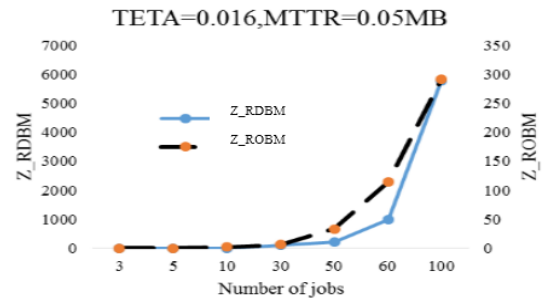


Figure 10. Comparing the objective function of the two methods by applying the reactive policy, medium expected failure, and high repair time

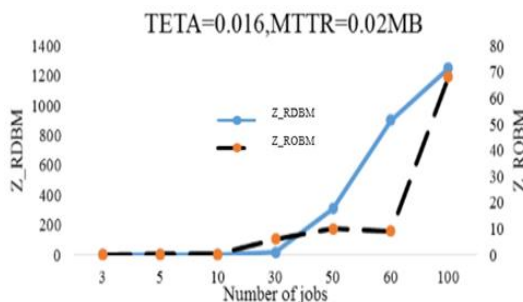


Figure 7. Comparing the objective function of the two methods by applying the reactive policy, medium expected failure, and short repair time



Figure 8. Comparing the objective function of the two methods by applying the reactive policy, low expected failure, and short repair time

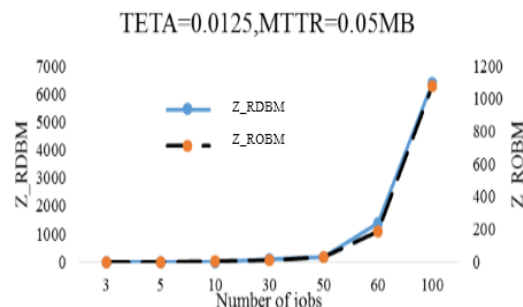


Figure 9. Comparing the objective function of the two methods by applying the reactive policy, low expected failure, and high repair time

and *OBM*. The stability of *DBM* and *OBM*, the objective function value of the *DBM* and *OBM*, the number of jobs and the problem-solving time incorporated by applying predictive policy to encounter with machine breakdown. Figure 11 illustrates the contents of Table 2. According to Figure 11 and Table 2, *DBM* is preferred to *OBM* in all cases regardless of the values of *TETA*, and *MTTR*, especially when the number of jobs increases. These results were obtained by considering the same coefficients for robustness and stability ($\alpha = (1 - \alpha) = 0.5$). In the sensitivity analysis section, we will analyze the effect of different values of α (robustness coefficient) on the performance measure of the two methods.

4. 4. Sensitivity Analysis This section provides additional tests on the methods' parameters to gauge their effects on the objective functions' values.

4. 4. 1. Testing on the Rate of Machine Breakdown and Mean Time to Repair According to Tables 3 to 8 and Figures 12 to 14, regardless of the number of jobs, the rate of a machine breakdown and the meantime to repair, *DBM* outperforms *OBM*. Also, as expected, the higher the failure rate (*TETA*) and the meantime to repair (*MTTR*), the worse the value of the robustness, stability, and two methods' objective functions.

4. 4. 2. Testing on the Stability and Robustness Coefficients In this section, different values of the robustness coefficients (α) had applied to achieve both methods' objective values. The results depicted for the low level of *MTTR* and $\theta = 0.0125$ in Figures 15 to 18. The effects of the varying coefficients of the robustness on the two methods' objective functions by applying the predictive policy showed in Table 9. According to Table 9 and Figure 17, *OBM* outperforms *DBM* when $\alpha \geq 0.7$, especially for many jobs. For values less than 0.7, *DBM* is superior to the *OBM*. In comparing two methods by applying the reactive policy, the effect of the robustness coefficient ignores. In this case, *OBM* is always almost outperformed *DBM*.

TABLE 2. Robustness, stability, and two methods' objective function by applying the predictive policy for different problem parameters

No	1/TETA	MTTR	n	RP_DBM	RP_OBM	SP_DBM	SP_OBM	ZP_DBM	ZP_OBM	T
1	80	low	3	0.01	0.069	0.067	0.015	0.039	0.042	1.58
2	80	low	5	0.27	0.2	0.27	0.07	0.27	0.13	1.8
3	80	low	10	0.26	0.95	0.73	0.32	0.5	0.48	6.48
4	80	low	30	16.6	16.6	9.5	23.7	13	20	99
5	80	low	50	221.25	85.23	44	291.7	133	188.5	308
6	80	low	60	524	164	73	1313	299	739	648
7	80	low	100	6637	667	366	9029	3501	4848	2798
8	60	low	3	0.002	0.05	0.05	0.002	0.25	0.25	27
9	60	low	5	0.04	0.26	0.25	0.08	0.17	0.15	11
10	60	low	10	0.54	0.36	0.55	0.13	0.54	0.25	11
11	60	low	30	19.5	28	13	64	16	46	202
12	60	low	50	552	134	68	936	309	535	608
13	60	low	60	1514	213	110	2086	812	1149	380
14	60	low	100	14350	1018	529	22181	7640	11600	3187
15	50	low	3	0.02	0.1	0.1	0.03	0.06	0.07	1.38
16	50	low	5	0.5	0.2	0.3	0.04	0.4	0.1	1.94
17	50	low	10	0.4	1.1	0.9	0.4	0.65	0.77	5.43
18	50	low	30	21	26	12.5	52	17	39	51.3
19	50	low	50	588	146	73.5	972	331	559	213.2
20	50	low	60	2280	265	149	1361	1214	1713	363
21	50	low	100	1241	20487	608	41905	10548	21573	1567
22	80	high	3	0.11	0.2	0.19	0.14	0.15	0.17	1.4
23	80	high	5	0.35	0.36	0.41	0.29	0.38	0.33	1.8
24	80	high	10	2.8	1.28	1.8	1.01	2.3	1.1	6.3
25	80	high	30	74.8	29	19.9	96.5	47.4	49.25	72.3
26	80	high	50	490	167	76	1399.8	283	783	294
27	80	high	60	4458	384	208.8	5710	2333.5	3047	455
28	80	high	100	30377.5	1569	705	61743	15541	31656	2049
29	60	high	3	0.16	0.21	0.2	0.18	0.18	0.2	1.4
30	60	high	5	0.91	0.38	0.51	0.29	0.71	0.34	1.62
31	60	high	10	2	1.9	1.8	1.9	1.9	1.9	4.6
32	60	high	30	159.6	54.2	28.1	244.5	93.8	149.4	70.4
33	60	high	50	973.7	202.1	96.1	2032.4	535	1117.3	313.7
34	60	high	60	2248.8	416	167.2	7344.1	1208	3880	511.6
35	60	high	100	39551	1658	815	76047	20183	38852	2948
36	50	high	3	0.06	0.15	0.14	0.05	0.1	0.1	1.5
37	50	high	5	0.26	0.38	0.37	0.22	0.32	0.30	2.11
38	50	high	10	8.6	2.16	2.46	2	5.5	2.08	5.27
39	50	high	30	201	58.87	33.97	312.85	117.5	185.86	53.98
40	50	high	50	4492.4	176.9	5718.1	95.01	2334.68	3029.5	250
41	50	high	60	6660	523.8	264.3	12592	3462.3	6558	444.9
42	50	high	100	61755.5	2149.5	1119.6	110545.6	31437.6	56347.5	3017

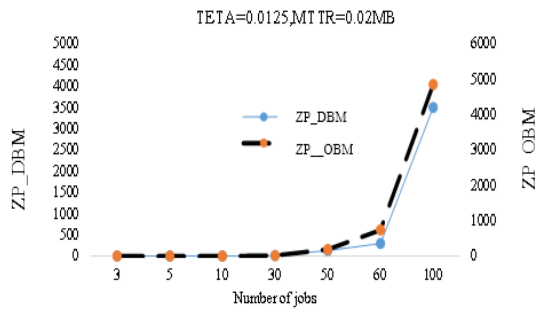


Figure 11. Comparison of two methods' objective function by applying the predictive policy, low expected failure, and short repair time

TABLE 3. Robustness, stability, and the objective function of two methods by applying the predictive policy for n=100, short MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
100	0.0125	4443	2374	8202	401	684	4293
MTTR	0.016	11600	7640	22181	529	1018	14350
0.02MB	0.02	21573	10548	41905	608	20487	1241

TABLE 4. Robustness, stability, and the objective function of two methods by applying the predictive policy for n=100, high MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
100	0.0125	31656	15541	61743	705	1569	30373
MTTR	0.016	38852	20183	76047	815	1658	39551
0.05MB	0.02	56347	31437	110545	1119	2149	61755

TABLE 5. Robustness, stability, and the objective function for the two methods by applying the predictive policy for n=50, low MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
50	0.0125	359	191	300	82	57	662
MTTR	0.016	535	309	936	68	134	552
0.02MB	0.02	559	331	972	73.5	146	588

TABLE 6. Robustness, stability, and the objective function of the two methods by applying the predictive policy for n=50, high MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
50	0.0125	783	283	1399	76	167	490
MTTR	0.016	1117	535	2032	96	202	973
0.05MB	0.02	3029	2334	95	571	177	4492

TABLE 7. Robustness, stability, and the objective function of the two methods applying the predictive policy for n=30, low MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
100	0.0125	30	21	31	12.7	21.7	29
MTTR	0.016	46	16	64	13	28	19.5
0.02MB	0.02	39	17	52	12.5	26	21

TABLE 8. Robustness, stability, and the objective function of the two methods by applying the predictive policy for n=30, high MTTR, and different values of TETA

n	TETA	ZP_OBM	ZP_DBM	SP_OBM	SP_DBM	RP_OBM	RP_DBM
100	0.0125	49.25	47.4	96.5	19.9	29	74.8
MTTR	0.016	149.4	93.8	244.5	28.1	54.2	159.6
0.05MB	0.02	185.9	117.5	312.8	33.97	58.87	201

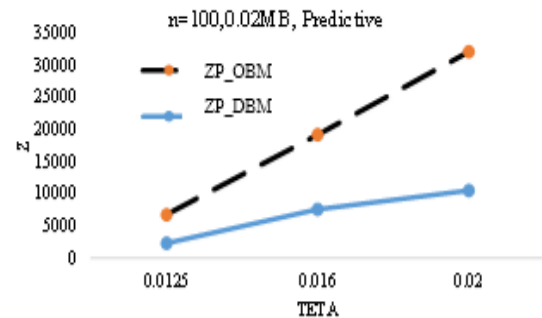


Figure 12. Comparison of two methods' objective function by applying the predictive policy for n=100, high MTTR, and different values of TETA

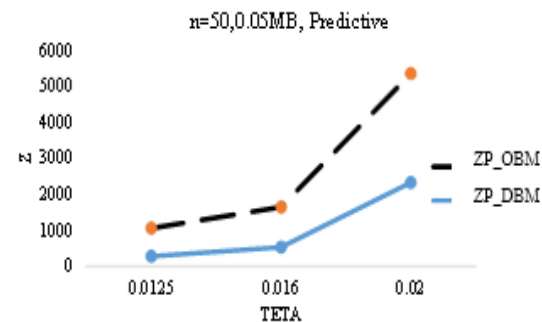


Figure 13. Comparison of two methods' objective function applying the predictive policy for n=50, high MTTR, and different values of TETA

5. MANAGERIAL INSIGHT

By increasing the importance of producer satisfaction level (robustness) to the satisfaction level of the

production environment (stability), i.e., $\alpha \geq 0.7$, OBM outperforms DBM. In other words, if the production system's interior completely aligns with the producer's goals and the producer is not worried about the reaction of the production staff, select OBM, and otherwise DBM.

If $\theta < 0.0125$, OBM outperforms DBM, i.e., if the wear of the machines is negligible, and the predictive method has a higher cost than the reactive, select OBM, and otherwise DBM.

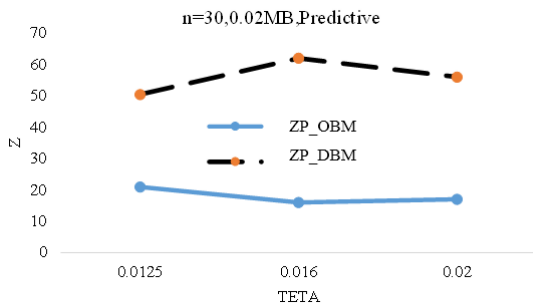


Figure 14. Comparison of two methods' objective function by applying the predictive policy for $n=30$, low MTTR, and different values of TETA

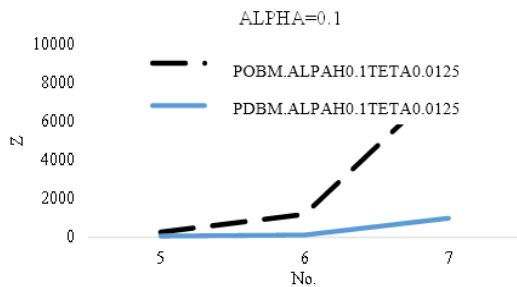


Figure 15. Comparison of two methods' objective function by applying the predictive policy for ALPHA=0.1, low level of MTTR, TETA

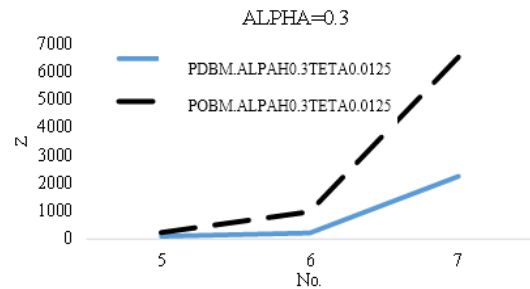


Figure 16. Comparison of two methods' objective function by applying the predictive policy, ALPHA=0.3, low level of MTTR, TETA

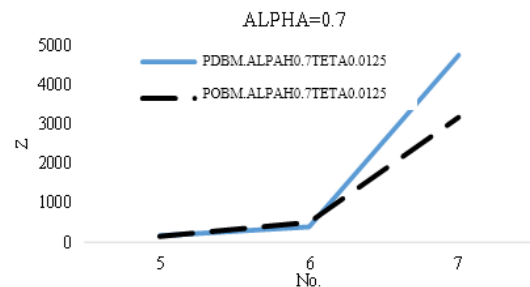


Figure 17. Comparison of two methods' objective function by applying the predictive policy, ALPHA=0.7, low level of MTTR, TETA

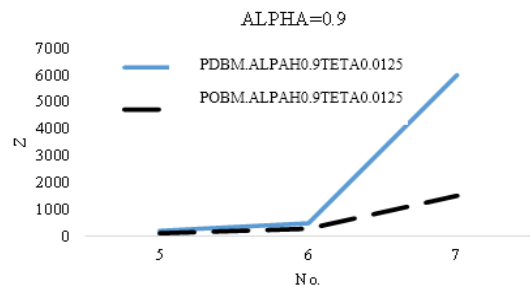


Figure 18. Comparison of two methods' objective function by applying the predictive policy, ALPHA=0.9, low level of MTTR, TETA

TABLE 9. Comparison of two methods' objective function applying the predictive policy for different values of the robustness coefficient

α	0.1	0.1	0.3	0.3	0.7	0.7	0.9	0.9
No	Z_DBM	ZP_OBM	Z_DBM	ZP_OBM	Z_DBM	ZP_OBM	Z_DBM	ZP_OBM
1	0.0613	0.0204	0.0499	0.0312	0.0271	0.0528	0.0157	0.064
2	0.27	0.083	0.27	0.109	0.27	0.161	0.27	0.187
3	0.683	0.383	0.589	0.509	0.401	0.761	0.307	0.887
4	10.21	22.99	11.63	21.57	14.47	18.73	15.89	17.31
5	61.725	271.05	97.175	229.75	168.07	147.17	203.5	105.9
6	118.1	1198.1	208.3	968.3	388.7	508.7	478.9	278.9
7	993.1	8192.8	2247.3	6520.4	4755.7	3175.6	6009.9	1503
8	0.0452	0.0068	0.0356	0.0164	0.0164	0.0356	0.0068	0.045
9	0.229	0.098	0.187	0.134	0.103	0.206	0.061	0.242

10	0.549	0.153	0.547	0.199	0.543	0.291	0.541	0.337
11	13.65	60.4	14.95	53.2	17.55	38.8	18.85	31.6
12	116.4	855.8	213.2	695.4	406.8	374.6	503.6	214
13	250.4	1898.7	531.2	1524.1	1092.8	774.9	1373.6	400
14	1911.1	20064	4675.3	15832	10204	7366.9	12968	3134
15	0.092	0.037	0.076	0.051	0.044	0.079	0.028	0.093
16	0.32	0.056	0.36	0.088	0.44	0.152	0.48	0.184
17	0.85	0.47	0.75	0.61	0.55	0.89	0.45	1.03
18	13.35	49.4	15.05	44.2	18.45	33.8	20.15	28.6
19	124.95	889.4	227.85	724.2	433.65	393.8	536.55	228.6
20	362.1	1251.4	788.3	1032.2	1640.7	593.8	2066.9	374.6
21	671.3	39763	797.9	35480	1051.1	26912	1177.7	22629
22	0.182	0.146	0.166	0.158	0.134	0.182	0.118	0.194
23	0.404	0.297	0.392	0.311	0.368	0.339	0.356	0.353
24	1.9	1.037	2.1	1.091	2.5	1.199	2.7	1.253
25	25.39	89.75	36.37	76.25	58.33	49.25	69.31	35.75
26	117.4	1276.5	200.2	1029.9	365.8	536.84	448.6	290.3
27	633.72	5177.4	1483.5	4112.2	3183.2	1981.8	4033	916.6
28	3672.2	55726	9606.7	43691	21476	19621	2741	7586
29	0.196	0.183	0.188	0.189	0.172	0.201	0.164	0.207
30	0.55	0.299	0.63	0.317	0.79	0.353	0.87	0.371
31	1.82	1.9	1.86	1.9	1.94	1.9	1.98	1.9
32	41.25	225.47	67.55	187.41	120.15	111.29	146.45	73.23
33	183.86	1849.3	359.38	1483.3	710.42	751.19	885.94	385.13
34	375.36	6651.2	791.68	5265.6	1624.3	2494.4	2040.	1108.
35	4688.6	68608	12436	53730.	27930.	23974.	35677	9097
36	0.132	0.06	0.116	0.08	0.084	0.12	0.068	0.14
37	0.359	0.236	0.337	0.268	0.293	0.332	0.271	0.364
38	3.074	2.016	4.302	2.048	6.758	2.112	7.986	2.144
39	50.673	287.45	84.079	236.65	150.8	135.06	184.3	84.3
40	5595.5	103.2	5350.4	119.6	4860.1	152.33	4615	168.7
41	903.87	11385.	2183.0	8971.5	4741.3	4144.3	6020.4	1730.
42	7183.1	99706	19310	78027	43565	34668	55692	12989

6. CONCLUSION

In this study, we simultaneously considered the uncertainty of processing time and machine breakdowns in a two-machine flow shop scheduling problem. Two methods were proposed and compared in three situations to deal with this problem; without considering machine failure disruption and considering machine breakdown applying with the reactive and predictive policy. In the first situation, decomposition-based methods have an acceptable performance compared with the optimal base one. In the second status, OBM had a higher performance than DBM except in small-size problems. In applying the

predictive policy, DBM had a higher performance than OBM, except in cases where producer satisfaction is more important than stability in the production environment. In all considering situations, the problem-solving time was acceptable and almost close to each other. Finally, OBM applying with the reactive policy, due to its lower objective function and its lower cost to DBM, seems more appropriate to solve the problem.

In this paper, a general approach proposed that can be used for robustness and stability optimization in an m-machine flow shop or job shop scheduling problem, with other measures of robustness and stability, or in the construction of predictive-reactive methods.

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Persian Abstract

چکیده

در این مقاله یک رویکرد پیش‌بینانه مقاوم و پایدار برای مسأله‌ی زمان‌بندی جریان کارگاهی دو ماشینی با فرض زمان فرآیند احتمالی کارها و اختلال خرابی ماشین ارائه شده است. در واقع، یک روش کلی ارائه می‌شود که می‌توان از آن برای بهینه‌سازی مقاوم و پایدار در مسأله‌ی زمان‌بندی محیط کارگاهی و تولید محصول استفاده کرد. مقیاس مقاومت، مقدار مورد انتظار مجموع زمان‌های اتمام کارها در زمان‌بندی واقعی است. مقیاس پایداری، مقدار مورد انتظار مربع مجموع انحرافات زمان اتمام کارها در برنامه‌ریزی اولیه و واقعی است. به منظور حل این مسأله، دو روش پیشنهاد شده و مورد مقایسه قرار گرفته است. یک روش مبتنی بر تجزیه مسأله مورد نظر به دو زیر مسأله و حل هر زیرمسأله به صورت بهینه و روش دیگر، بر پایه‌ی بر یک قضیه‌ی ریاضی بر اساس نتایج محاسباتی است. روش دوم از نظر مقاومت و پایداری، به ویژه در مورد مسائل با اندازه‌ی بزرگ، عملکرد بهتری دارد. به عبارت دیگر، روش دوم به دلیل بهبود قدرت پاسخگویی تولید کننده به مشتری و افزایش رضایت کارکنان خط تولید، برای حل این مسأله پیشنهاد می‌شود.
