



Orthotropic Coefficients and Continuity Factors of Reinforced Concrete Slabs Supported on 3-sides

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ABSTRACT

The analysis and design of reinforced concrete slabs supported on 3-sides in masonry or reinforced concrete construction involve analytical formulations. In published analytical formulations, orthotropic coefficients and continuity factors are unknown parameters. To obtain moment carrying capacity of the slabs using available formulations, these factors must be required. In this research work, these orthotropic coefficients and continuity factors are presented for transverse loaded RC rectangular slabs supported on three sides under uniform area loading at top face of the slab. These coefficients were obtained using FEA (Finite element Analysis) based Structural Analysis Program (SAP) software. It is also validated with FEA (Finite element Analysis) based SCIA Engineer software and published formulations. It has been observed that obtained results are well comparable with published literature and FEM based software. Results presented in this research paper are conducive to predict the moment-field of the reinforced concrete rectangular slabs supported on three sides having one edge is unsupported. These coefficients will be very helpful for structural designers dealing with reinforced concrete slabs supported on three sides.

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1. INTRODUCTION

The Slab is one of the integral parts of the construction practices, integrity of the same affects the analysis and design of the structures. Due to different support conditions, like discontinuous and continuous edges and various aspect ratios, moment field of the slabs may vary. While analyzing, the slabs having variation in moment-field (bending moment) in orthogonal directions are considered to be non-isotropic slabs. In general practice of reinforced concrete construction, tensile steel is placed parallel to the edges due to which these slabs were also considered as orthotropic slabs. Moment-field develops at the continuous edges of the slab arise a need to calculate continuity factors and difference in moment field in orthogonal directions arise a need to calculate orthotropic coefficients.

Nowadays, finite element method (FEM) analysis is adopted worldwide and researchers are also using FEM

in conjunction with yield line theory for analyzing slabs. Ingerslev [1] firstly came with the concept of yield line theory. Johansen [2-4] contributes a lot in this field and proceeds the Ingerslev work by introducing geometrical unknown variables to predict the failure mechanisms. After that several researchers [5-10] worked in the field of yield line analysis of reinforced concrete slabs and their work was accepted by world community.

With the increase in use of FEM in engineering several researchers start using this technique in analyzing slabs. Al-Sabah and Falter [11, 12] presented a non-iterative lower bound finite element method using rotation free elements to analyze the isotropic and non-isotropic slabs which is safer than yield line theory due to lower bound approach. Gohnert [13] presented the yield-line elements with the help of FEA to calculate the load carrying capacity of the slabs using both elastic and inelastic analysis. The theory proposed is called an over-shoot method. Firstly, an elastic theory was used to

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formulate the flexibility matrix in the form of compatibility equations using principle of virtual work. After that elastic analysis was transformed into an in-elastic analysis by adapting the elastic flexibility matrix. Obtained results were validated within a maximum percentage difference of 10 percent with johansen yield line theory and within 20 percent with experimental data.

Famiyesin et al [14] incorporate the membrane effect in the conventional yield line theory to mobilize the results obtained by the conventional yield line theory by using available test results. By using parametric study of FE based analysis, charts have been developed which are helpful to determine collapse load of the slab with different percentage of steel. Kwan [15] presented a new method for defining or predicting the yield line patterns of the slabs. In this method, yield line patterns were attained in terms of dip (rotation) and strike (orientation of axis of rotation) of slab surface using work method. Based on the above methodology, a new computer program was produced in which users no longer need to input any assumed yield line pattern however, program automatically adjusts the yield line pattern to obtain critical load factor. The limitation of this method is that it is not applicable to convex polygonal slabs.

Singh and Kumar [16] presented a simplified approach using yield line theory for analyzing the three sides supported slabs by using equations of four sides supported slabs. Gupta and Singh [17] formulated the design aids for the analyzing of the three-side supported RC rectangular slabs for different support conditions which were obtained with the help of yield line theory and FEA. Gupta and Naval [18] presented a simplified approach to analyze the slabs supported on two adjacent edges with the help of yield line theory.

Abdul-Razzaq et al. [19] studied the behavior of post-tensioned two way slab under flexure. Parametric study carried out by the author reveals that moment carrying capacity of the slabs having tendons placed in two directions is more than slabs having tendons in one direction. Wenjiao et al. [20] studied the crack response of reinforced concrete two way slab subjected to dynamic loading using finite element analysis. Authors observed that existence of initial cracks in slabs damage more than normal slabs under same loading condition. Colombo et al. [21] presented a new strip method considering tensile membrane action of the slabs when large deflections reached, where tensile membrane action plays a key role. This method was used to obtain ultimate load carrying capacity of reinforced concrete two way slabs. The analytical model was validated for laterally restrained strips and slabs supported on four laterally unrestrained edges with experimental data.

If we want to determine the moment-field of RC slabs correspond to any value of load, we have to know the orthotropic coefficients and continuity factors which we can't obtain directly from the yield line theory. Singh et

al. [16] stated that it is desirable to use orthotropic coefficients and continuity factors correspond to elastic distribution to determine bending moment for better performance under service conditions. Detailed literature reveals that many mathematical equations are available to obtain the moment-field of reinforced concrete slabs having different boundary conditions. However, in these formulations these parameters are still unknown. To encounter this problem, authors proposed the orthotropic coefficients and continuity factors of reinforced concrete slabs supported on 3-sides with one edge is unsupported carrying uniform area loading to obtain moment field of the RC slabs under service conditions. The proposed coefficients are very helpful for structural designers to analyze and design the slabs supported on three sides.

2. NUMERICAL MODELLING

Numerical modelling of the RC rectangular slabs supported on three sides was carried out using FEM based SAP software. SAP is useful for analyzing and designing the structural and non-structural elements. In this research work, SAP was used to obtain the moment field of the slabs which is helpful to procure the orthotropic coefficients and continuity factors. The whole methodology is shown by means of flow chart in Figure 1.

Let us consider a slab having span L_x in x direction and L_y in y direction giving an aspect ratio r equals to L_y/L_x . L_x is parallel to free edge and is taken as 3m which is freeze for all cases whereas L_y is variable depending on the aspect ratio. The negative moment of resistance of the slabs was observed at continuous edges in both directions i.e. m'_x in x direction and m'_y in y direction as shown in Figure 2. Similarly, the positive moment of resistance was observed in both directions i.e. m_x in x direction and m_y in y direction. From these moments of resistance, orthotropic coefficients which is defined as the ratio of m_y/m_x and continuity factors i_1 and i_2

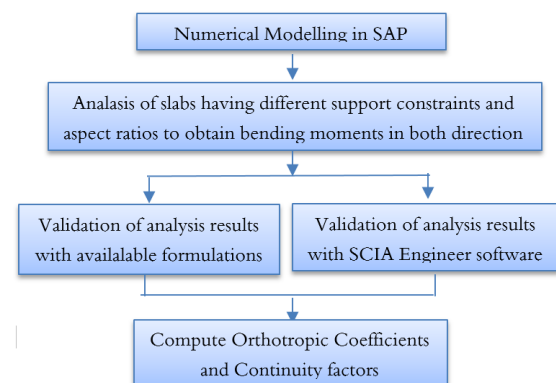


Figure 1. Flow chart of research methodology

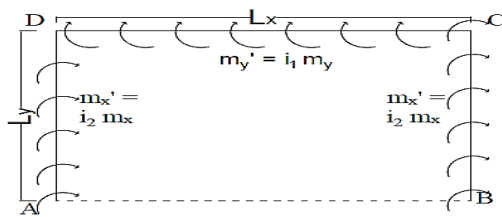


Figure 2. Moment field acting on slab

which are the ratio of m'_y/m_y and m'_x/m_x respectively were determined.

2. 1. Description of Models Slabs having different support conditions (Three sides discontinuous, three sides continuous, parallel sides discontinuous, parallel sides continuous and two adjacent sides discontinuous) and different aspect ratios (1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9 and 2.0) were modelled. For modelling, four node shell type section was used and thickness of the slab was taken as 130mm. Mesh size is very critical while analyzing so, to get the precise results, sensitivity analysis was carried out to obtain an optimum mesh size which comes out to be 0.05 meter equally in both directions. This was obtained by reducing the size of mesh till it does not affect the results after further reducing the mesh size.

2. 2. Material Properties For modelling of the slabs, concrete material was used having compressive strength 20 MPa, Modulus of elasticity was obtained from the equation given in design code BIS: 456 [22] i.e.

$$\text{Modulus of elasticity} = 5000\sqrt{f_{ck}} \quad (1)$$

where, f_{ck} is compressive strength of concrete which is taken as 20 MPa. Using equation (1) we get modulus of elasticity as 22360.67 MPa. Other properties of concrete such as Poisson ratio was taken as 0.15 and density of concrete as 25 KN/m³.

3. SAP RESULTS

Let us consider a transverse loaded slab carrying uniform area loading 10 KN/m² on the top face of the slab to obtain the moment-field of the slab. Figure 3 shows moment-field in x direction for the 3-sides supported continuous slab and Figure 4 shows moment-field in y direction for the 3-sides supported continuous slab. Results for different boundary conditions and different aspect ratios were presented in Table 1.

4. VALIDATION OF SAP RESULTS

Results obtained from SAP were validated with available formulations and FEM based SCIA Engineer software.

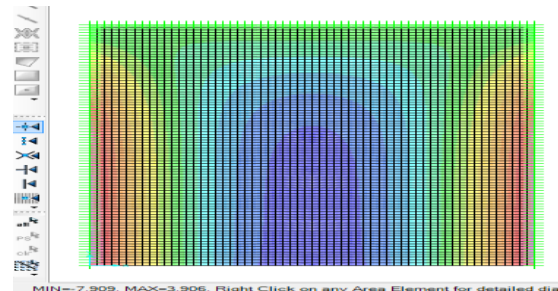


Figure 3. Moment field in x direction for the 3-sides continuous slab for aspect ratio 1.5 from SAP

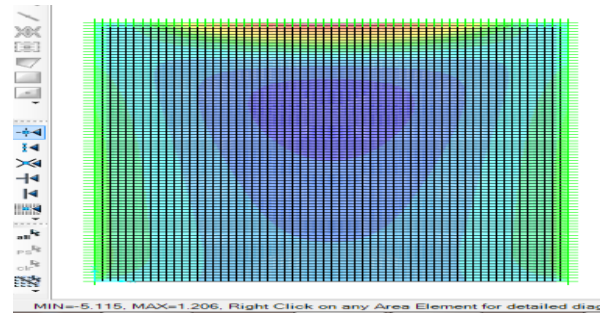


Figure 4. Moment field in y direction for the 3-sides continuous slab for aspect ratio 1.5 from SAP

4. 1. Validation with Timoshenko and Krieger

Results obtained from SAP were validated with results published by Timoshenko and Krieger [23] for the case of 3-sides continuous slab having one edge is free for aspect ratio 1.0. The moment in x and y direction can be obtained from moment coefficients given in Table 2 which were proposed in published literature.

4. 2. Validation with SCIA Engineer

In SCIA Engineer, orthotropic RC rectangular slab supported on three sides was modelled having aspect ratio 1.5 i.e. L_x was taken as 3m and L_y was taken as 4.5m with five different support conditions. For validation of results, different material properties like compressive strength, modulus of elasticity, poisson's ratio and density of concrete and mesh size were kept same as used in SAP software. For analysis, uniform area load of 10 KN/m² was applied at the top surface of the slab. Figure 5 depicts the moment field for the 3-sides supported continuous slab in x direction and Figure 6 shows the moment field for the 3-sides supported continuous slab in y direction.

Comparative analysis given in Table 3 shows that results obtained from numerical simulation are well validated with available formulations given in the literature. This shows that results obtained from SAP can be used to obtain the orthotropic coefficients and continuity factors. The case of 3-sides continuous slab is only validated because the other cases like 3-sides Discontinuous, Parallel Supported sides Continuous and

TABLE 2. Moment coefficients given in Timoshenko and Krieger [23]

b/a	$M_x = \beta_1 qa^2$ β_1	$M_y = \beta_2 qa^2$ β_2	$M_x = \beta_3 qa^2$ β_3	$M_y = \beta_4 qa^2$ β_4
1	0.0444	0.0138	-0.0853	-0.0510

TABLE 3. Validation of Results with published literature

S. No.	Support Conditions and moment field	Aspect Ratio, r	
		1.0	
1	3-sides Continuous slab	Results from SAP	Results from literature [23]
	Positive moment in x	3.85	3.99
	Negative moment in x at continuous edge	8.08	7.67
	Positive moment in y	1.17	1.24
	Negative moment in y at continuous edge	5.09	4.59

TABLE 4. Validation of Results with SCIA Engineer

S. No	Moment-Field and Boundary Condition	SAP	SCIA
1	3-sides Discontinuous		
	Positive moment in x	11.18	11.33
	Positive moment in y	2.95	3.36
2	3-sides Continuous		
	Positive moment in x	3.91	3.91
	Moment in x at continuous edge	7.91	7.76
	Positive moment in y	1.21	1.35
3	Parallel Supported sides Continuous		
	Positive moment in x	3.90	3.90
	Moment in x at continuous edge	7.88	7.74
	Positive moment in y	1.47	1.63

TABLE 5. Orthotropic coefficients and continuity factors

S. No	Support Conditions and moment field	Aspect ratio, r										
		1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1	3-sides Discontinuous											
	Value of μ	0.293	0.285	0.278	0.272	0.268	0.264	0.261	0.260	0.258	0.256	0.255
2	3-sides Continuous											
	Value of μ	0.303	0.305	0.306	0.307	0.308	0.309	0.309	0.309	0.310	0.310	0.310
	Value of i_1	4.358	4.290	4.258	4.244	4.240	4.241	4.242	4.246	4.246	4.250	4.250
	Value of i_2	2.097	2.065	2.045	2.033	2.027	2.025	2.023	2.023	2.023	2.023	2.023
3	Parallel Supported sides Continuous											

	Moment in y at continuous edge	-	-
4	Parallel Supported sides Discontinuous		
	Positive moment in x	10.80	10.94
	Moment in x at continuous edge	-	-
	Positive moment in y	2.59	2.96
5	Moment in y at continuous edge	11.15	10.95
	Two adjacent sides Discontinuous		
	Positive moment in x	6.56	6.61
	Moment in x at continuous edge	11.91	11.74
	Positive moment in y	2.01	2.27
	Moment in y at continuous edge	-	-

5. RESULTS AND DISCUSSION

Numerical simulation of the three sides supported reinforced concrete rectangular slabs for different boundary conditions and different aspect ratios carrying uniform area load was carried out with the help of SAP and results are presented by the authors in Table 1. Results presented in Table 1 were validated with available literature and with SCIA Engineer software and those results were presented in Tables 3 and 4 respectively.

Results obtained from SAP were used to discover orthotropic coefficients and continuity factors and results are given in Table 5. The moment distribution across the slab varies with boundary constraints, aspect ratio and loading condition. However, the orthotropic coefficients and continuity factors are function of bending moment itself. And, results presented in Table 5 shows that these coefficients change with boundary constraints and aspect ratio.

These factors will be helpful for designers dealing in the slabs supported on 3-sides to calculate moment-field of the slabs using available analytical formulations in which these parameters are unknown.

	Value of μ	0.372	0.373	0.374	0.376	0.376	0.377	0.377	0.377	0.378	0.378	0.378
	Value of i_1	-	-	-	-	-	-	-	-	-	-	-
	Value of i_2	2.051	2.035	2.028	2.024	2.023	2.022	2.022	2.022	2.023	2.023	2.023
4	Parallel Supported sides Discontinuous											
	Value of μ	0.265	0.259	0.254	0.249	0.244	0.240	0.237	0.234	0.232	0.231	0.229
	Value of i_1	4.752	4.564	4.446	4.373	4.326	4.298	4.282	4.273	4.270	4.269	4.268
	Value of i_2	-	-	-	-	-	-	-	-	-	-	-
5	Two adjacent sides Discontinuous											
	Value of μ	0.316	0.312	0.310	0.309	0.308	0.307	0.307	0.307	0.306	0.306	0.306
	Value of i_1	-	-	-	-	-	-	-	-	-	-	-
	Value of i_2	1.898	1.866	1.845	1.831	1.823	1.816	1.812	1.809	1.808	1.806	1.806

6. CONCLUSION

In this research work, the orthotropic coefficients and continuity factors have been presented by the authors for the transverse loaded reinforced concrete rectangular slabs supported on 3-sides having one edge is unsupported for different boundary conditions carrying uniform area loading with the help of numerical simulation of FEM based Structural Analysis Program (SAP) software. It is possible to find these factors from published literature but all cases were not mentioned and if anyone wants to use the theory of plates, the equations used in these concepts are very cumbersome so it is not feasible to use these in daily routine. Yield line analysis is also a widely accepted tool for analysis of the slabs but it is not feasible to obtain orthotropic coefficients and continuity factors from yield line theory. The factors proposed by the authors can be used by the designers dealing with slabs supported on 3-sides to obtain the moment field of the slab.

Results obtained from Structural Analysis Program (SAP) software were validated with available literature with a maximum percentage difference of 9.82 percent. Results obtained from SAP were also validated with FEM based SCIA Engineer software and results are in favour with a maximum percentage difference of 14.06 percent.

The results presented in table 5 are simple to apply in equations of yield line theory in routine flow of calculations for determining moment carrying capacity, and would save computational time for analyzing and designing of the slabs.

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Persian Abstract

چکیده

تجزیه و تحلیل و طراحی صفحات بتن آرمه که از ۳ طرف در ساخت و سازهای بتونی یا بتن مسلح پشتیبانی می شوند، شامل فرمول های تحلیلی است. در فرمولهای تحلیلی منتشر شده، ضرایب ارتوتروپی و عوامل تداوم پارامترهای ناشناخته ای هستند. برای بدست آوردن ظرفیت حمل لحظه ای اسلبها با استفاده از فرمولاسیون های موجود، این عوامل باید مورد نیاز باشد. در این کار تحقیقاتی، این ضرایب ارتوتروپیک و فاکتورهای تداوم برای اسلبهای مستطیلی RC بارگذاری عرضی که از سه طرف تحت بارگذاری منطقه یکنواخت در بالای صفحه دال پشتیبانی می شوند، ارائه شده است. این ضرایب با استفاده از نرم افزار (FEA) تحلیل اجزای محدود مبتنی بر برنامه تحلیل ساختاری (SAP) بدست آمد. همچنین با نرم افزار مهندس SCIA و فرمولهای منتشر شده (FEA) تحلیل اجزا محدود) تأیید می شود. مشاهده شده است که نتایج به دست آمده با ادبیات منتشر شده و نرم افزار مبتنی بر FEM قابل مقایسه است. نتایج ارائه شده در این مقاله تحقیقاتی برای پیش بینی میدان گشتاور دالهای مستطیل بتونی مسلح که از سه طرف دارای یک لبه پشتیبانی می شوند، پشتیبانی نمی شود. این ضرایب برای طراحان سازه ای که با دال های بتونی تقویت شده از سه طرف پشتیبانی می شوند بسیار مفید خواهد بود.
