



## Reliability Analysis of Notched Plates under Anisotropic Damage Based on Uniaxial Loading using Continuum Damage Mechanics Approach

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### A B S T R A C T

Extensive recent researches have been underway to model the fracture mechanics degradation based on continuum damage mechanics (CDM) technique. CDM theory is a powerful tool for solving problems such as large plastic deformations that the fracture mechanics is unable to solve. This model is derived by means of the thermodynamics internal variable theory and based on the experimental results on material properties. In this paper, the reliability of rectangular plates containing a central circular hole under static tensile load using the CDM approach for ductile fracture has been studied. To investigate the initiation and evolution of damages, anisotropic damage expressed by second order damage tensor is used to derive constitutive equations. Then, these relationships together with material constants are implemented with subroutine in ABAQUS software. The reliability assessment has been investigated using first order reliability method (FORM) and second order reliability method (SORM). Based on the FORM and SORM, the limit state functions and random variables have been obtained according to the energy density release rate. The probability of failure of each plate with different hole sizes is estimated based on the anisotropic damage theory, and the results are compared with the isotropic damage model. Finally, the sensitivity analysis of the coefficient of variation is performed.

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### NOMENCLATURE

$b$	Isotropic hardening exponents	$X$	Random variable
$D$	Scalar damage variable	$Y$	Energy density release rate
$D_c$	Critical damage	<b>Greek Symbols</b>	
$D_H$	Hydrostatic damage	$\alpha$	Damage exponent
$E$	Young modulus of elasticity	$\beta$	Reliability index
$f$	Yield function of plastic criterion	$\Gamma$	Gibbs free energy
$F$	Dissipative potential function	$\varepsilon$	Uniaxial total strains
$F_D$	Damage potential function	$\varepsilon_p$	Plastic strain
$F_P$	Plastic potential function	$\varepsilon_{pD}$	Damage threshold plastic strain
$g$	Performance function	$\varepsilon_{pR}$	Rupture plastic strain
$k$	Curvature of the performance function	$\eta$	hydrostatic sensitivity parameter
$p$	Accumulated plastic strain	$\Phi$	Cumulative distribution function
$P$	Probability of failure	$\lambda$	Plastic multiplier
$R$	Isotropic hardening	$\nu$	Poisson ratio
$\mathbf{R}$	Reliability	$\rho$	Mass density
$R_\infty$	Saturated isotropic hardening parameter	$\sigma$	Uniaxial and tensorial stresses
$s$	Unified damage law exponent	$\sigma_{eq}$	Von Mises equivalent stress
$S$	Energetic damage law parameter	$\sigma_Y$	Yield stress

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## 1. INTRODUCTION

Ductile failure and its criteria are very important in fracture mechanics due to the importance of its engineering, and extensive researches have been done in this field [1]. Low weight metals under ductile fractures are assumed as a criterion for an anisotropic ductile fracture. This criterion considers the interactions between surfaces with shear connections and shear stress [2]. In this regard, comprehensive mathematical model has been extracted from the point of theoretical and experimental views to evaluate ductile fracture of anisotropic materials [3]. In structural systems, there are uncertainties in the system parameters that must be considered to describe the behavior of the system. In these systems, load variation and distribution, boundary conditions, material properties, and constants, environmental conditions and etc. are uncertainties that cause the random behavior of structures. Therefore, it is very important to study the reliability and consider the source of the uncertainties, and these propagation effects on the system reliability.

It is very common to use classical fracture mechanics models to demonstrate material degradation, and great efforts have been made to develop new models of fracture mechanics [4-7]. Most research and studies have been done to demonstrate the fracture behaviors under various stress shearing loads. In these studies, anisotropic states were investigated in the various directions of the tension test angles, and the results are examined from different angles [8]. Despite some successful applications of fracture mechanics, the results show that it is difficult to apply the theory of classical fracture mechanics to practical applications. As a result, damage models have been developed as an alternative method for modeling material degradation based on the irreversible thermodynamic process. In this context, accurate and correct failure modeling is one of the main topics in the formation of the metal. For this purpose, various categories have been extracted for failure states and modes during metal formation. In this regard, ductile fracture was no exception to this rule and has been studied in numerical, experimental and analytical forms [9]. Mechanical behavior based on the uniaxial tensile tests of the anisotropic in the ductile fracture and plastic deformation has been investigated as well [10].

Kachanov [11] proposed the concept of damage models for the first time and expressed the isotropic damage model with a scalar variable, which defined the surface density of microvoids per unit volume [12, 13]. In this model, it is assumed that the start of microvoids in the rupture process from the very beginning of loading consists of two stages. The first stage is the regular growth of microvoids and the second stage is the acceleration of fracture. Therefore, the probability of growth of microvoids is more than the surface of the body if there is an aggressive environment. As a result, the

scalar damage variable was specified as the ratio of the surface area of the damage to the whole surface. This theory is determined by considering the equivalence between the state of a body with not damaged fancied and the real damaged state. Later, the effective stress concept was proposed by Rabotnov [14] in continuum damage mechanics (CDM).

Many researchers have used continuum mechanics and scalar damage variables to properly solve many mechanical problems [15, 16]. Van Do [17] investigated the evolution of damage and the onset of failure in notched specimens using finite element model (FEM) analysis and CDM approach. The simulations were compared with the numerical and experimental results of earlier works and good agreements were found between them. Using the concepts of CDM approach, Majzoobi et al. [18] investigated the equivalent plastic strain to the failure of notched aluminum specimens and introduced a relation to express the effect of triaxial stress coefficient in the medium range of stress triaxiality. Razanika et al. [19] proposed an enhanced CDM formulation based on novel damage driving energy, which includes that involves both stored energy and dissipative contributions. The applicability of proposed model was validated by FEM analysis. Bonora et al. [20] have shown that the strain required to initiate damage in ductile materials decreases exponentially with increasing stress triaxiality. As a result, they combined this feature with the concepts of CDM approach, and provided a phenomenological relationship for the dependence of the damage threshold strain on stress triaxiality. Ganjani [21] also proposed a generalized ductile fracture model for ductile materials coupling with stress triaxiality and Lode angle parameter using CDM approach.

Nevertheless, in practical problems, it has been shown that the damage behavior of all materials are in fact anisotropic. On the other hand, by careful attention to isotropic damage models, it can be understood that these models have less ability to describe material damage than anisotropic damage models. In addition, in multiaxial loading, the results of the isotropic damage models in comparison to anisotropic damage models are very significant. Therefore, to describe the behavior of materials, it has been suggested that damage tensors be used as damage state variables, but the use of damage tensors is complex [22]. In this context, some recent studies have been done and the most practical of them is the hybrid numerical and experimental assessment in order to determine the behavioral characteristics of the plasticity of anisotropic and ductile fracture upon the high-strength materials [23]. To study the effects of anisotropic plasticity on ductile fracture, Keshavarz and Ghajar [24] proposed modified CDM approach based on the isotropic and anisotropic damage. Developed CDM formulations were implemented in Abaqus software by subroutine and the results have been verified with

experimental results. For accurate investigation, various tensile tests on different loading directions have been performed. In these studies, the mechanical model of the anisotropic damage with respect to the states of the stress and anisotropy are predicted and simulated the material fracture by some researchers. [25, 26]. Surmir et al. [27] coupled nonlinear kinematic hardening model with anisotropic damage model to predict ratcheting strain of different loading paths.

This paper represents a method for reliability analysis of a plate containing a central circular hole using CDM-based anisotropic damage model coupled with the FEM for the first time. A performance function using the energy density release rate based on the anisotropic damage is formulated within the basic thermodynamic framework. This proposed reliability relationship can be used to predict the fracture probability of structure systems. The second law of thermodynamics has been used to determine the internal energy of materials with Gibbs free energy. Then, by using Gibbs free energy, the variables that can be used to show the growth of material damage are introduced. In the framework of irreversible thermodynamic processes, thermodynamically associated variables are obtained using the dissipation potential function. Therefore, the finite element simulation of material behavior based on the evolution of damage and constitutive equation by the CDM approach during failures is implemented by a subroutine in ABAQUS software. The results of the FEM are compared with experimental results reported by other researchers and the FEM models of the notched tension tests are validated. Finally, using the FORM and SORM, the limit state functions and random variables will be obtained according to the CDM approach.

## 2. RELIABILITY ASSESSMENT

Reliability is defined as the capability of an item or equipment to perform the required activities successfully within a specified and predefined time period and operating conditions [28]. In fact, reliability refers to the probability of the proper function of a system or item without failures in specific and predetermined conditions for a given length of time. There are several ways to analyze the structural reliability of a system, the most important of which are FORM and SORM methods [29]. In reliability assessment, the failure probability measurements of a system or structure have been evaluated based on the respective failure rate/function. In this study, FORM and SORM has been used to obtain the probability of ductile fracture of the rectangular plate containing various hole size which is subjected to uniaxial tensile loading.

**2.1. First Order Reliability Method (FORM)** In the FORM model, the function of the limit state is based

on the first order Taylor expansion, which is expressed by the following relation [30]:

$$g(X) = Z(X) - S(X) \quad (1)$$

Here,  $X$  denotes the random variables of the limit state function, while  $Z$  and  $S$  are resistance and load, respectively, which are assumed to be functions of random variables.

In the FORM, at first, random variables are transferred using Rosenblatt's transformation of the main random space ( $X$ ) into the normal standard space ( $U$ ) with zero mean and standard deviation 1. The main goal in the FORM is to obtain the most probable point (MPP), i.e.,  $U^*$  as the minimal distance of the limit state surface to the origin in the normal standard space. This shortest distance is called the index of reliability or ( $\beta$ ). So, in this method, while  $g(X)$  is less than zero and failure occurs, then the  $P_f$  (failure likelihood) and subsequently reliability  $R$ , by using the reliability index is estimated as follows:

$$R = 1 - P_f = 1 - \Phi(-\beta) = \Phi(\beta) \quad (2)$$

where  $\Phi$  is the cumulative distribution function of standard normal distribution.

## 2.2. Second Order Reliability Method (SORM)

In the case of nonlinear limit state function, the probability of failure should be less than that of the linear one. In FORM approach, because of the MPP using first order approximation, the curvature of the nonlinear state function is ignored. Therefore, SORM approach was studied to consider curvature information. So, SORM uses second order Taylor expansion to calculate the failure probability as following [30]:

$$P_f = P\{g(x) < 0\} = \Phi(-\beta) \prod_{i=1}^{n-1} (1 + \beta k_i)^{1/2} \quad (3)$$

In this relation,  $k_i$  denotes the performance function of the  $i$ th main curvature at the MPP.

For linear limit state functions, FORM solution is exact. For non-linear failure functions, the exact calculation of the failure probability or the reliability generally involves mathematical and computational difficulties. Based on the number of random variables and the linearity (or not) of the failure function, FORM can be seen to have limitations for non-linear failure functions having a large number of random variables. Nevertheless, accuracy of SORM due to its approximation of the performance function is generally more than that of FORM. However, since SORM requires the second order derivative, it is not as efficient as FORM when the derivatives are evaluated numerically and its use is complex and expensive. On the other hand, several algorithms have been proposed for approximation of the most probable failure point and the  $\beta$  index,

therefore the decision as to which is the most effective algorithm depends on the limit state function of interest.

### 3. MECHANICS OF THE CONTINUUM DAMAGE

Classical fracture mechanics models that were originally developed for demonstrating material degradations required prior knowledge of location and geometry of the microcracks, which is difficult to assess before they are formed. The CDM approaches were proposed to create an alternative towards modeling material degradation based on the thermodynamics framework. The procedure of the CDM approach is to illustrate first the damage state of a material in terms of properly specified damage variables ( $D$ ) and then to explain mechanical behavior of the damaged material and further development of the damage by the use of these damage variables. Therefore, CDM approach provides a tool that can simulate damage from the beginning of loading to final fracture.

State of damage is one of the main factors that effects on mechanical properties of materials, which is determined by the density, distribution, type and direction of microvoids. According to the size and orientation of microvoids, some of them will start to develop under specified loading and environmental conditions. The CDM approach provides a new glance for the initiation and evaluation of damage. The CDM approach uses the concepts of continuum physics by defining internal field or damage variables to describe the process of material defeat and fracture.

The change of internal field of material generally depends on the direction of stress and/or strain, and therefore it can be said that the internal field is an anisotropic phenomenon. Therefore, a proper description of material behavior necessitates vectorial or tensorial damage variables. In the CDM approach, using the strain equivalence hypothesis and equivalence between the fictitious undamaged state of a body and the real damaged state, the 2<sup>nd</sup> order damage variable  $D$  by means of the effective area reduction is defined as  $(\delta_{ij} - D_{ij})n_j \delta S = \tilde{n}_i \delta \tilde{S}$ , where  $\delta S$  is total surface area with normal  $n$ , and  $\delta \tilde{S}$  is surface of effective area with normal  $\tilde{n}$ , while  $\delta_{ij}$  represent the Kronecker delta [31]. It is worth mentioning that, effective area is an area where internal force is applied. So, based on the stress  $\sigma$  applied to the total surface area, the effective stress  $\tilde{\sigma}$  on this effective area according to the tensor of the 2<sup>nd</sup> order damage is expressed as  $\tilde{\sigma}_{ij} = \sigma_{ik} (1-D)_{kj}^{-1}$ .

The CDM approach is related to thermodynamics, and the irreversible thermodynamic theory is used as a logical framework for explaining the damaged elastic-plastic material behavior. As a result, Gibbs free energy must first be defined for anisotropic damage. Then, the dissipation function is expressed to characterize the

estimation of state variables and determine the load level that denotes the elastic region.

**3. 1. Gibbs Free Energy** The free energy stored in the damaged material is determined by various factors such as strain, damage and dislocation structure state, etc. Therefore, the Gibbs free energy function according to the principle of strain equivalence between the undamaged and damaged configuration, with the definition of the anisotropic damage, may be considered as follow [32]:

$$\rho \Gamma = \frac{1+\nu}{2E} H_{ij} \sigma_{jk}^D H_{kl} \sigma_{li}^D + \frac{3(1-2\nu)}{2E} \frac{\sigma_H^2}{1-\eta D_H} \quad (4)$$

In the above equation,  $\nu$ ,  $E$ , and  $\eta$  represent Poisson ratio, Young's modulus, and hydrostatic sensitivity parameter, respectively, while  $D_H = D_{kk}/3$  is hydrostatic damage,

$\sigma_H = \sigma_{kk}/3$  hydrostatic stress,  $\sigma_{ij}^D$  deviatoric stress, and  $H_{ij} = (1-D)_{ij}^{-1/2}$  effective damage tensor.

Based on the thermodynamic formulation, the equation of constitutive elasticity and effective stress of the material under damage has the following form:

$$\varepsilon_{ij}^e = \rho \frac{\partial \Gamma_e}{\partial \sigma_{ij}} = \frac{1+\nu}{E} \tilde{\sigma}_{ij} - \frac{\nu}{E} \tilde{\sigma}_{kk} \delta_{ij} \quad (5)$$

$$\tilde{\sigma}_{ij} = \left( H_{ik} \sigma_{kl}^D H_{lj} \right)^D + \frac{\sigma_H}{1-\eta D_H} \delta_{ij} \quad (6)$$

The rate of released density energy ( $Y$ ) is depended with the damage variable  $D$ , that can be extracted from the function of Gibbs free energy as follows:

$$Y_{ij} = \rho \frac{\partial \Gamma}{\partial D_{ij}} = \frac{1+\nu}{E} \sigma_{kp}^D H_{pq} \sigma_{ql}^D A_{klmn}^{-1} H_{mi}^2 H_{jn}^2 + \frac{\eta(1-2\nu)}{2E} \frac{\sigma_H^2}{(1-\eta D_H)^2} \delta_{ij} \quad (7)$$

with  $A_{klmn} = (1/2)(H_{km} \delta_{ln} + H_{ln} \delta_{km} + H_{kn} \delta_{lm} + H_{lm} \delta_{kn})$ .

It is worth noting that the role of the rate of released density energy in the CDM approach that is a similar role as the rate of released strain energy  $G$  in the mechanics of the fracture.

**3. 2. Dissipative Potential Function** Based on the standard thermodynamics principles, the total dissipation potential function  $F$  may be presented by the plastic deformation function  $F^P$  in sum with damage function  $F^D$ . By taking account of isotropic hardening  $R$  and yield stress  $\sigma_Y$ , the plastic dissipation potential function based on the von Mises criterion is determined by [32]:

$$F^P = \tilde{\sigma}_{eq} - R - \sigma_Y \quad (8)$$

where  $\tilde{\sigma}_{eq} = (H\sigma^D H)_{eq}$  shows the effective von Mises equivalent stress. Isotropic Hardening indicates the density of the dislocations, and exponential isotropic hardening is written as  $R = R(r) = R_\infty[1 - \exp(-br)]$ , where  $r$  is isotropic hardening state variable, while  $R_\infty$  and  $b$  represent saturated isotropic hardening parameters and isotropic hardening exponents, respectively. Based on the thermodynamic rules, the evolution equation of the isotropic hardening variable is obtained in the form:

$$\dot{r} = -\dot{\lambda} \frac{\partial F}{\partial R} = -\dot{\lambda} \frac{\partial F^p}{\partial R} = \dot{\lambda} \quad (9)$$

where  $\dot{\lambda}$  is plastic multiplier. So, by using Equation (9), the constitutive equation of the strain rate of the plastic that is stated as:

$$\dot{\varepsilon}_{ij}^p = \dot{\lambda} \frac{\partial F}{\partial \sigma_{ij}} = \dot{\lambda} \frac{\partial F^p}{\partial \sigma_{ij}} = [H_{ik} e_{kl}^p H_{ij}]^D \quad (10)$$

with  $e_{kl}^p = (3\tilde{\sigma}_{ij}^D / 2\tilde{\sigma}_{eq}) (\dot{r})$ .

## 4. DAMAGE MECHNISM

**4.1. Damage Threshold** Experimental results from measuring damage in plastic region show that mechanical damage occurs when the plastic strain in the material reaches the irreversible or accumulated plastic strain  $\varepsilon_{pD}$  [33]. It can be said that this damage threshold accumulated plastic strain is almost the properties of materials. On the other hand, the strain rate of accumulated plastic  $\dot{p}$  is defined according to the yield criterion. So, exposed on the anisotropic damage and based on the von Mises criterion with Equation (10), it can be expressed as follows [32]:

$$\dot{p} = \frac{[H\tilde{\sigma}H]_{eq}}{\tilde{\sigma}_{eq}} \dot{r} \quad (11)$$

**4.2. Damage Evolution** According to thermodynamic relations in the framework of CDM approach, the evolution equations of the damage variable are governed by damage dissipation function. As result, it may be written as follows [32]:

$$\dot{D}_{ij} = \dot{\lambda} \frac{\partial F_D}{\partial Y_{ij}}; \quad r > \varepsilon_{pD} \quad (12)$$

Based on the experimental results damage dissipation function can be postulated as follows [34]:

$$F_D = \left( \frac{\bar{Y}(\varepsilon^e)}{S} \right)^s Y_{ij} \left| \frac{d\varepsilon^p}{dr} \right|_{ij} \quad (13)$$

In above,  $||$  represents the principal components in absolute form,  $s$  denotes the law unified damage, and  $S$  is defined as the parameter of the law of energy damage. On the other hand,  $\bar{Y}$  is the elastic density of the effective energy that can be expressed as follows:

$$\bar{Y} = \int \tilde{\sigma}_{ij} d\varepsilon_{ij}^e = \frac{1}{2} E_{ijkl} \varepsilon_{ij}^e \varepsilon_{kl}^e = \frac{\tilde{\sigma}_{eq}^2 \tilde{R}_v}{2E} \quad (14)$$

where  $\tilde{R}_v$  is known as the effective stress triaxiality function and can be written as  $\tilde{R}_v = \frac{2}{3}(1 + \nu) + 3(1 - 2\nu) \left( \frac{\tilde{\sigma}_H}{\tilde{\sigma}_{eq}} \right)^2$  with  $\tilde{\sigma}_H = \sigma_H / (1 - \eta D_H)$ . Then, by using Equations (9), (11) and (12), the damage evolution law is given by:

$$\dot{D}_{ij} = \left( \frac{\bar{Y}}{S} \right)^s \left| \dot{\varepsilon}_{ij}^p \right|; \quad \varepsilon_p > \varepsilon_{pD} \quad (15)$$

If damage occurred in one of the plane and causes critical conditions, it is defined as the condition of fracture in material. As a result, the onset of the fracture occurs while the damage vector in norm state ( $D_{ij} n_j$ ) or the damage biggest principal value ( $D_I$ ) reaches  $D_c$  (i.e.  $\max D_I = D_c; I = 1, 2, 3$ ). In other words, from the point view of the rate of released density energy, the main criterion of fracture in a material with damage can also be defined as follows:

$$Y_I = Y_{lc}; \quad I = 1, 2, 3 \quad (16)$$

## 5. PROPOSED METHODOLOGY

The finite element simulation of material behavior based on the evolution of damage and constitutive equation by CDM approach during failures is carried out by subroutine in ABAQUS software. In the proposed subroutine, the rate of released density energy is calculated, and the amount of damage growth in each of the principal directions is obtained and checks if damage has reached the specified critical damage or not. In the event of damage in the principal direction, the equations and behavior of the material are affected based on the updated damage mode.

A rectangular plate containing a circular hole located in the center of the plate with the principal coordinate is shown in Figure 1. If the damage is anisotropic for the uniaxial tension at direction-1, then  $D_2 = D_3 = D_1/2$  and  $D_H = 2D_1/3$ . Therefore, effective equivalent stress, effective stress triaxiality function, and evolution law lead to:

$$\tilde{\sigma}_{eq} = \frac{2}{3} \frac{\sigma_1}{1 - D_1} + \frac{1}{3} \frac{\sigma_1}{1 - D_2} \quad (17)$$

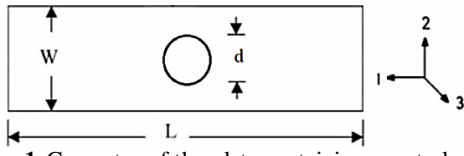


Figure 1. Geometry of the plate containing a central circular hole

$$\tilde{R}_v = \frac{2}{3}(1+\nu) + 3(1-2\nu) \times \left[ \left( 1 - \frac{2\eta}{3} D_1 \right) \left( \frac{2}{1-D_1} + \frac{1}{1-D_2} \right) \right]^{-2} \tag{18}$$

$$\dot{D}_1 = \left( \frac{\bar{Y}}{S} \right)^s \left| \dot{\varepsilon}^p \right| \tag{19}$$

Using initial condition  $\varepsilon = \varepsilon_{pD}$ ,  $D = D_0$  and fracture condition  $\varepsilon = \varepsilon_{pR}$ ,  $D = D_c$ , the integration of the damage evolution equation in direction-1 gives:

$$D_c = \left( \frac{\bar{Y}}{S} \right)^s (\varepsilon_{pR} - \varepsilon_{pD}) \tag{20}$$

Due to the symmetric conditions of the model, a quarter of the specimen was modelled in the FEM, to save computing resources. Consequently, symmetric boundary conditions are applied to the model and a unit displacement is applied to its free edge. The specimens have been meshed with four-node S4R elements, and the meshes near the notches were refined, as shown in Figure 2. The meshes were refined until the maximum equivalent stress converged. In order to analyze the reliability, a proper function for the limit state is required in accordance with the system structure. For this purpose, the fracture criterion introduced in the previous section is used to forecast damage in ductile materials based on the CDM approach. Therefore, when the rate of released density energy exceeds its threshold in each principal direction, the fracture will occur. So, the function for the limit state of this condition can be expressed as:

$$g(X) = Y_{lc} - Y; \quad I = 1, 2, 3 \tag{21}$$

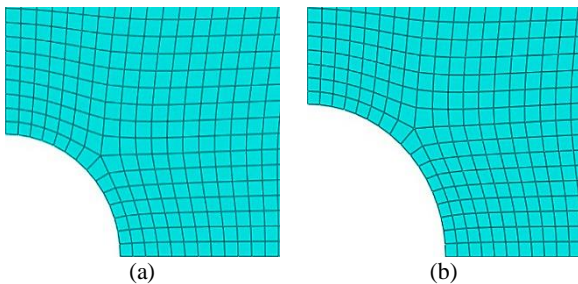


Figure 2. Mesh design of notched specimens (a) 5 mm, (b) 10 mm

TABLE 1. Statistical properties and material characteristics of aluminum alloy 2024-T3 [32, 35]

Random variables	Mean value	Coefficient of Variation	Distribution type
Critical damage, $D_c$	0.209	$8 \times 10^{-2}$	Normal
Young's modulus, $E$ (GPa)	75	$5 \times 10^{-2}$	"
Energetic damage law parameter, $S$	1.7	$10^{-2}$	"
hydrostatic sensitivity parameter, $\eta$	3	$10^{-2}$	"
Poisson's ratio, $\nu$	0.3	$5 \times 10^{-2}$	"
Rupture plastic strain, $\varepsilon_{pR}$	0.33	$5 \times 10^{-2}$	"
Damage threshold plastic strain $\varepsilon_{pD}$	0.031	$5 \times 10^{-2}$	"

According to Equations (7) and (20), in the function of the limit state, the uncertainty sources and the random variables vector is equal to:

$$X = (\nu, E, \eta, D_c, \varepsilon_{pD}, \varepsilon_{pR}) .$$

For the aluminum 2024 plate under tensile loading, mechanical properties and their statistics data (i.e. the mean value and coefficient of variation), are presented in Table 1. Sensitivity index is another parameter that is used to evaluate the failure probability of the random variables, which is obtained using Equation (22), in which  $SI_X$  is the sensitivity index of random variable  $X_i$  [36, 37].

$$SI_{X_i} = \frac{\left( \frac{\partial g(X)}{\partial X_i} \right)}{\left( \sqrt{\sum \left( \frac{\partial g(X)}{\partial X_i} \right)^2} \right)} \tag{22}$$

### 6. STATISTICAL STUDY OF RESULTS

This paper studies the fracture probability and reliability evaluation of a plate containing a central circular hole with various sizes of diameters under uniaxial tensile loading. To calibrate the FEM, the simulation results are compared with the experimental data in [38] and [39]. Figure 3 demonstrates this comparison for force-displacement responses between experiment and simulation at direction-1 for the notched specimen with a diameter of 10 mm which shows good agreement. Therefore, the FEM models of the notched tension tests are validated. The evolution of the damage variable obtained in the simulation for the specimens containing holes with diameters of 5 and 10 mm in direction-1 is shown in the contour plots in Figure 2. It can be seen that the maximum damage is detected near the edge of the

notch. As the specimen is gradually pulled, the maximum damage zone moves slowly towards the free edge of the specimen and localizes there.

Table 2 shows comparisons of the probability of failure between the isotropic and anisotropic damage model based on the FORM and SORM for various sizes of diameters. Under the same conditions and the same critical ratio ( $D/D_c$ ) for both cases of damage models, the failure probability in the damage model of anisotropic is higher than that in isotropic one. In addition, as can be seen, the SORM indicates a higher probability of failure than FORM, and by increasing diameter size, the probability of failure for both methods increases.

The effect of change in the critical ratio ( $D/D_c$ ) on the failure probability of a plate under uniaxial loading for various sizes of diameter in the principal directions -1 and -2 are depicted in Figures 5 and 6, respectively. The results show that the probability of failure in direction-1

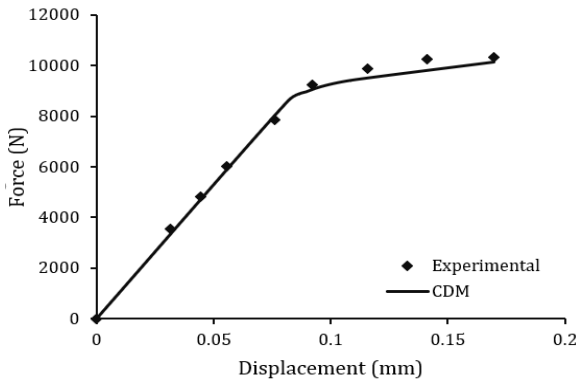


Figure 3. Comparison of force–displacement responses between experiment and simulation

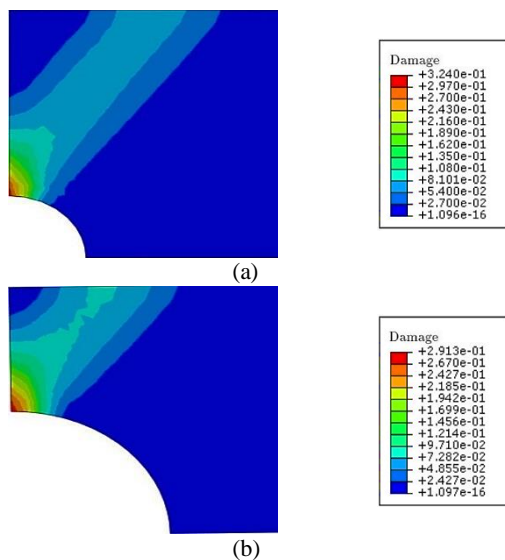


Figure 4. Damage contour plots, (a) 5 mm, (b) 10 mm

TABLE 2. Comparison of probability of failure between isotropic damage model and anisotropic damage model using FORM/SORM ( $\times 10^{-2}$ )

d (mm)	Isotropic damage		Anisotropic damage	
	FORM	SORM	FORM	SORM
2	4.8951	5.1994	5.5837	5.8025
3	5.7938	6.1994	6.3987	6.6496
5	7.9279	8.3672	8.9951	9.3477
8	8.3898	8.7268	9.4816	9.8533
10	8.6908	9.0315	9.7182	10.0992

is greater than direction-2, and as the diameter increases, the probability of failure increases. Figure 7 shows the relationship between the probability of failure and the variation coefficient for a plate under uniaxial loading. It is obvious that the critical damage  $D_c$ , rupture the strain of plastic  $\epsilon_{pR}$  and the damage threshold plastic strain  $\epsilon_{pD}$  have the lowest sensitivity; however, by increasing the coefficient of variation, other variables become more sensitive to the dispersion of the data.

Sensitivity analysis was performed according to Equation (22) and the sensitivity index for each random variable was obtained. As it is obvious in Figure 8, among the random variables determined in the problem, the critical damage  $D_c$ , rupture plastic strain  $\epsilon_{pR}$  have the highest sensitivity index and it can be said that these two variables have the most effect on the reliability of rectangular plates under uniaxial tensile loading. Other available variables have lower sensitivity index, and, in other words, the least effect on the reliability of the problem.

At the end, it should be noted that these results are obtained for a plate containing a central hole made of aluminum under specific uniaxial loading based on anisotropic damage and it may be different for other of conditions. Therefore, the proposed method provides a

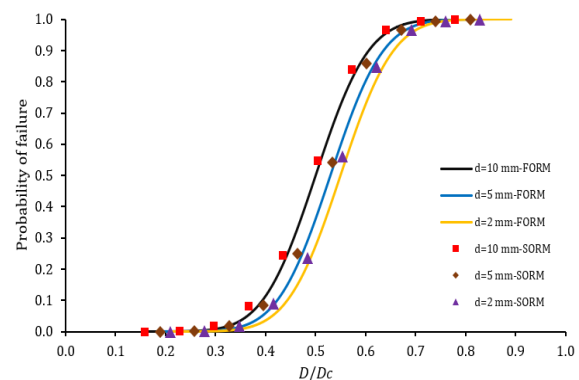


Figure 5. The effects of the changes in the critical ratio ( $D/D_c$ ) on Probability of failure in direction-1

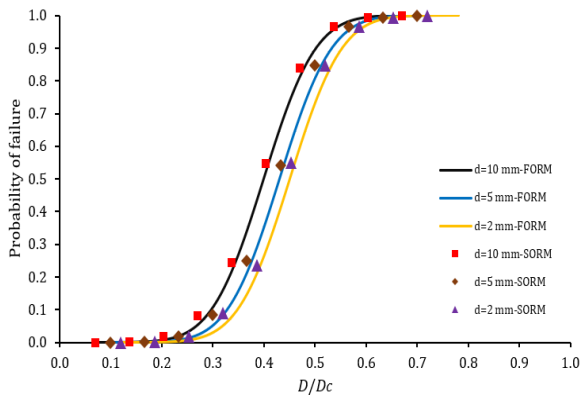


Figure 1. The effects of the changes in the critical ratio ( $D/D_c$ ) on Probability of failure in direction-2

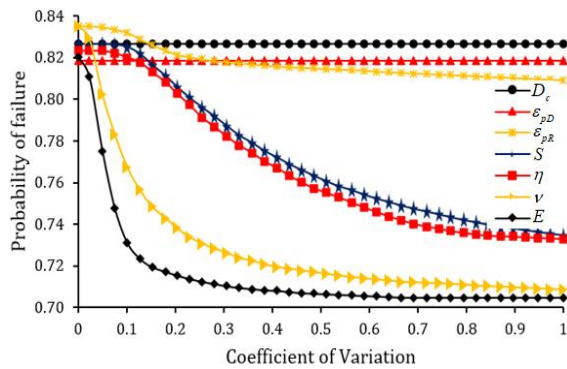


Figure 2. Relationship between the probability of failure and coefficient of variation for random variables of uniaxial loading

tool that can provide probability of damage from the beginning of the loading to final fracture. On the other hand, in this study, anisotropic damage is used which can simulate mechanical behavior in practical applications because of multiaxial loading. Therefore, use of

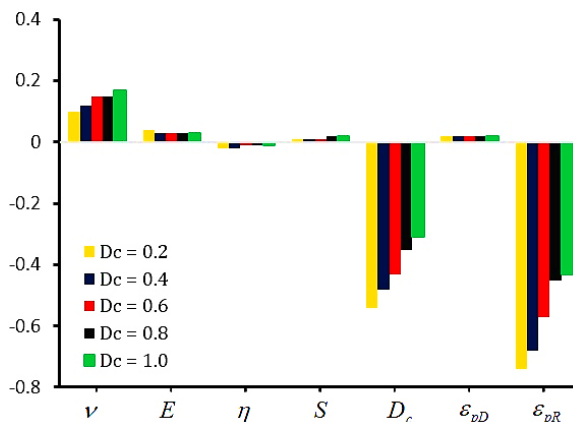


Figure 8. Sensitivity index of random variables by increasing critical damage for uniaxial loading

multiaxial loads and various material types are suggested to analyze the reliability of structural systems based on the CDM approach.

As a general conclusion, this study proposed a method for calculating the reliability of the structural system, but the result depends on the choice of performance function, selection of variables, and its approximation algorithm to obtain reliability.

### 7. CONCLUSION and FUTURE WORK

This paper introduces a framework for analyzing the reliability of a plate containing a hole on center under tensile loading using the theory of damage on anisotropic elastic-plastic based on the FORM and SORM models.

Reliability analysis was performed using the FORM and SORM. At first, based on the thermodynamic framework by the CDM approach, constructive equations, and the development of ductile elastic and plastic damage are obtained. It is then implemented using a subroutine code in the ABAQUS software to drive the stress-strain relationship and the onset of fracture. The probability of fracture of a plate containing a central circular hole with different diameters has been compared between the anisotropic and isotropic damage models. Results show that while the model of damage follows an anisotropic path, the probability of failure is higher. And, in different directions in the principal coordinates, this probability of failure is different. Next, based on the sensitivity analysis of variables, the critical damage and rupture plastic strain have the most sensitivity index. In the future work, we will extend the CDM approach more precisely over the notched composite laminates and isotropic damage under uniaxial loading to evaluate reliability analysis.

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## Persian Abstract

## چکیده

در دهه‌های اخیر، مدل‌های تخریب بر اساس رویکرد مکانیک آسیب پیوسته توسط پژوهش‌گران در زمینه‌ی مکانیک شکست مورد توجه قرار گرفته است. تئوری مکانیک آسیب پیوسته ابزاری قدرتمند برای حل مسائلی مانند تغییر شکل‌های موم‌سان بزرگی است که مکانیک شکست قادر به حل آن نیست. این مدل در چارچوب تئوری متغیر داخلی ترمودینامیک استخراج می‌شود و بر اساس نتایج تجربی که بر روی خواص ماده انجام شده است توسعه داده می‌شود. در این مقاله، قابلیت اطمینان یک صفحه‌ی مستطیل شکل حاوی سوراخ دایره‌ای مرکزی تحت بار کششی ایستا، با استفاده رویکرد مکانیک آسیب پیوسته برای شکست نرم بررسی شده است. برای بررسی شروع و پیشروی آسیب، از مدل آسیب ناهمسان‌گرد بیان شده توسط تانسور مرتبه‌ی دوم برای استخراج معادلات سازگاری استفاده می‌شود. سپس، این روابط به همراه ثابت‌های ماده با استفاده از زیربرنامه در نرم افزار آباکوس اجرا می‌شوند. ارزیابی قابلیت اطمینان با استفاده از روش قابلیت اطمینان مرتبه‌ی اول (FORM) و روش قابلیت اطمینان مرتبه‌ی دوم (SORM) انجام شده است. بر اساس روش قابلیت اطمینان مرتبه‌ی اول/دوم، تابع عملکرد و متغیرهای تصادفی با توجه به نرخ آزادسازی چگالی انرژی به دست آمده است. احتمال خرابی صفحه با اندازه‌ی سوراخ‌های مختلف بر اساس تئوری آسیب ناهمسان‌گرد محاسبه و نتایج با مدل آسیب همسان‌گرد مقایسه می‌شوند. سرانجام، تحلیل حساسیت بر روی ضریب تغییرات انجام می‌شود.