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A New Method for Computation the Success Probability of Coverage for Switch Unit in the Switching Systems

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P A P E R   I N F O

A B S T R A C T

Redundancy technique is used to improve performance and achieving to increase the lifetime of a system. Nowadays, the redundancy method is applied in many industries. One of the common methods of redundancy is its utilization in the switching systems. In switching systems, one or more components are considered active mode and the others in the standby state to be used by switch if necessary. In order to be fully utilized all the components in the redundant device, the switch unit must perform its function, such as switching, perfectly. Successful coverage by switch unit is expressed with a probability. In this paper, a new approach to the likelihood of switch success is proposed, and showing that as increases switching in the system, the efficiency and performance of the switch gradually decreases. The analysis of this method was based on the stress-strength method. Finally, a few numerical examples for the validation of results were applied.

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1. INTRODUCTION

In recent years, the word "redundancy" has been considered one of the idioms that researchers widely used frequently in the reliability field. Redundancy in a system means that there is an alternative path to successful system performance and to enhance machine reliability, it used. The redundancy method is widely used in the switch component in the construction of the complex system and other mechanical systems. The switch unit in the switching systems are usually exposed to various loadings in different situations. Therefore, in order to prevent unknown failures, it is necessary to investigate and analyze the failure probability of switch before their implementation.

Generally, redundancy divided into active (static) redundancy and standby (dynamic) redundancy categories. In active redundancy, all components operate simultaneously at time zero. In other words, all components of the system are exposed to tension and failure. For the active redundancy, *k*-out-of-*n* systems are an example of these types of systems. Whereas in standby redundancy, redundant components are sequentially put into operation when the active one fails, *k*-out-of-*n* standby systems are well-known examples of these types of systems [1, 2].

Standby redundancy is classified into three types: cold, warm, and hot standby. In the cold standby state, the redundant components are completely dormant and do not fail in this mode, in fact, the failure rate in cold standby is zero. The other state is the hot standby mode. The pressure that the components tolerate in the hot state, is exactly the same as that in the fully active state. Finally, the standby mode which is far more complex in terms of mathematical modeling than the two previous ones is warm standby. In this situation, the components are in the semi-active state, hence less pressure than in the fully-active state. So, their failure rate in warm mode is lower than their failure rate in active mode [1-5].

The redundancy allocation problem (RAP) is one of the most important problems in applying the redundancy technique. RAP history goes back to the introduction of a system by Fyffe [6]. He considered a system with 14 subsystems and in each subsystem, there were three or

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four different components connected in parallel. Numerous researchers have analyzed Fyffe system reliability using various methods and assumptions. For example, it could be referred to as the works of Ardakan et al. [3], Guilani et al. [7], Sajjadi et al. [8], and Aziz Mohammadi et al. [9].

The main emphasis of this article has been to develop a method based on the stress-strength approach that can be used to find the failure probability of detector/switch unit in complex systems in the second scenario (i.e., detection and switching only at time of failure). This proposed method can be used in the optimization redundancy allocation problem to the selection of components with appropriate levels of redundancy or reliability to maximize the system reliability under some predefined constraints.

2. SWITCH UNIT PERFORMANCE

In switching systems, the switch unit is of particular importance; because it has a duty too, if necessary, inter the redundant components into the circuit for operation. In research related to switching systems, there are two mechanisms for coverage. The first mechanism is the perfect switch. That is, the switch unit, it is 100% reliable and failure-free during its mission. But in reality, the switch device will be damaged due to the stresses that are applied to it during the mission, i.e., the switch is incomplete, this means that the switch, will fail like other components of the system.

Yaghoubi et al. [1], Wu and Wu [10], Huang et al. [11], have used in their models, assuming of the perfect switch, whereas, Coit [5], Pan [12], Kececioglu and Jiang [13], Jia et al. [4], Amari [14] and Sadeghi and Roghanian [15] have used imperfect switching in their work.

2. 1. Problem Statement   Generally, there are two scenarios called the first scenario (i.e., continual monitoring and detection) and the second scenario (i.e., detection and switching only at time of failure) for switching component mechanisms in switching systems [5]. Usually, the first scenario is given more concern by the authors, whilst in the second scenario, probably nothing has been done because of its simple mathematical model, and for it, a constant probability is considered. So, in this study, we are concerned with the second scenario.

In recent decades, extensive researches have been conducted on investigating the first scenario and its application in complex systems [16-18]. But based on the second scenario, there are not a method to calculate the failure probability of switch device in switching systems. Hence, probability analysis of the switching system requires further investigations in the field of the second scenario.

The first scenario in terms of computation, is more complex than the second scenario, especially when the number of system components increases and as well as, the time-to-failure of component follows the non-exponential distributions. But in the second scenario, typically, a constant probability of success for the changeover element (denoted P_s) is considered. The P_s, is obtained ratio the number of successful switches to the total number of trials possible [19].

In practice, the switch unit is subjected to various stresses due to its dynamics during its mission. So, the probability of failure of the switch unit increase over time. Therefore, it is expected that the success probability of the switch unit will decrease at each switching, and it will not be constant during the mission. Hence, in our study, we are most concerned about the second scenario in which, the probability of the switch unit has been considered constant according to previously conducted studies, while in this study the probability of switch at each step is calculated according to the proposed method. For this purpose, in this study, the stress-strength probabilistic method is used to analyze the performance of the switch unit. In the stress-strength approach, stress or load refers to the set of environment activities which tends to increase of failure a component, whereas, strength is the ability of that component versus to the environmental loads [20]. For the applications of redundancy strategies, unlike other approaches in the second scenario, this method because of the use of non-constant probability for switch unit, to make it more realistic.

The switch performance is decreased for a variety of physical reasons, such as consecutive coverages. In other words, if the number of switching will be increased, the performance of the switch unit decreased. If the components of the system (active and standby components) represented stresses applied to the switch, and the switch component stands for strength, then, in order for the switch would not fail, strength must be dominated on stress, i.e., it must be Pr (Strength> Stress). In fact, the lifetime of the switch should be more than the sum of the lifetime of the system all components.

Let us consider a system with an active component and (n - 1) standby components. By assuming that all spare components do not fail in the standby state. In this case, the number of successful changeovers for the switch unit is (n - 1). So, if T_s and T_j, be lifetimes of the switch and j[th] active component, respectively that follows any possible distribution, then the probability that the switch can control all components of the switching system should have a longer lifetime than the total lifetime of the system components.

If ρ_i, be the probability of successful switching of the ith order, then calculating this probability is as follows:

$$\rho_i = Pr\left(T_s \geq \sum_{j=0}^{i} T_j\right) = \int_0^\infty \int_s^\infty f_{S_j}(s) f_{T_s}(t)\, dt\, ds$$

$$= \int_0^\infty \int_0^t f_{T_s}(t) f_{S_j}(s)\, ds\, dt; \quad i = 1,\ldots,n-1 \tag{1}$$

where j denotes the time of failure of system components. The probability density function (PDF), of the summation of these variables, i.e., $S_n = T_1 + T_2 + \ldots + T_n$ is obtained as follows [21]:

$$f_{S_n}(s) = \left[\prod_{i=1}^{n} \lambda_i\right] \sum_{j=1}^{n} \frac{e^{-\lambda_j s}}{\prod_{\substack{k=1 \\ k \neq j}}^{n}\left(\lambda_k - \lambda_j\right)}; \; s > 0 \tag{2}$$

In special case, where all exponential parameters are identical with a constant failure rate $\lambda_i = \lambda$, for each of i's, Equation (2) simplifies to the following form:

$$f_{S_n}(s) = \frac{\lambda^n}{\Gamma(n)} s^{n-1} e^{-\lambda s}; \; s > 0 \tag{3}$$

where $\Gamma(.)$, is the gamma function. Equation (3) shows the PDF of the gamma distribution with shape parameter n and scale parameter λ.

Now, let's consider a system with n component. By assuming that all spare components do not fail in the standby state. As mentioned, to use all the standby components in the system, (n-1) successful switching is required for the switch unit.

If T_j follows the exponential distribution with the parameter λ_j $\left(f_{T_j}(t) = \lambda_j e^{-\lambda_j t}, t > 0\right)$, then ρ_i can be expressed as:

$$\rho_i = \left[\prod_{j=0}^{i} \lambda_j\right] \sum_{j=0}^{i} \frac{1}{\left(\lambda_j + \lambda_s\right)\left(\prod_{\substack{j=0 \\ k \neq j}}^{i}\left(\lambda_k - \lambda_j\right)\right)} \tag{4}$$

$$i = 1,\ldots,n-1$$

If all system components are identical, $\lambda_j = \lambda$, then ρ_i is computation from the following equation:

$$\rho_i = \left(\frac{\lambda}{\lambda + \lambda_s}\right)^i; i = 1,\ldots,n-1 \tag{5}$$

3. NUMERICAL EXAMPLE

Regarding the results obtained in sections 2, some numerical example is solved.

Example 1. Consider a redundant system with 6 dissimilar components that are operating with one component at the first, and 5 components are available in the standby state to replace the failed component. Failure rate of components are given in Table 1.

For different values of the failure rate of the switch unit, the switch reliability in each of switching has been evaluated by using Equation (4). These results are presented in Table 2. According to Table 2, as expected, the performance of the switch unit decreases due to the increase in switching. When $\lambda_s = 0$, it means $\rho_i = 1$, for each i. In other words, the switch is the completely reliable, or so-called perfect switch. If all the components of the system are identical and their failure rate in the active mode is equal to 0.01, and the failure rate of the switch unit is equal to 0.0001 failure per hour, then result of the success probability for the switch unit in each of changeover (or ρ_i 's) according to Equation (5), are apparent in the Table 3.

Example 2. One of the continuous statistical distributions in statistics and probability theory is Gamma distribution. The probability density function (pdf) of the Gamma distribution with shape parameter k and scale parameter λ is given by $f(t) = \frac{\lambda^k}{\Gamma(k)} t^{k-1} e^{-\lambda t}$ [22]. This distribution is widely used in various fields. If $k = 1$, the exponential distribution is obtained. When k is a positive integer, then the distribution represents an Erlang distribution and for large k the gamma distribution converges to a normal distribution.

TABLE 1. Failure rate for components system

λ_0	λ_1	λ_2	λ_3	λ_4	λ_5
0.01	0.02	0.03	0.04	0.05	0.06

TABLE 2. Switch reliability with identical units

λ_s	0	10^{-6}	10^{-4}	10^{-2}
ρ_1	1.000	0.9999	0.9852	0.3333
ρ_2	1.000	0.9998	0.9819	0.2500
ρ_3	1.000	0.9998	0.9795	0.2000
ρ_4	1.000	0.9998	0.9775	0.1667
ρ_5	1.000	0.9998	0.9759	0.1429

TABLE 3. Reliability of switch with identical units

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
0.9091	0.8264	0.7513	0.6830	0.6209

TABLE 4. Reliability of switch for Example 2

k_s	ρ_1	ρ_2	ρ_3
0.5	0.6152	0.5705	0.5410
1	0.8658	0.8379	0.8174
2.5	0.9955	0.9938	0.9923
3	0.9986	0.9980	0.9975

Consider a system of 1-out-of-4 with non-identical components. In this case, three switchings are required to use all the components in the system. If the failure time of the switch unit follows the Gamma distribution with different parameters of k_s that listed in Table 4 and the parameter scale $\lambda_s = 0.01$, and time to failure of all system components in active state follows the exponential distribution with the failure rate $\lambda_i = 0.i; i = 1, 2, 3, 4$, then, according to Equation (1), the success probability of the switch at each stage of the switching is calculated. These results are presented in Table 4. It can be observed that the increase in value of k_s leads to higher probability. Moreover, calculating the probability values of ρ_i in the above table, when $k_s = 1$, can be also obtained from Equation (4). The reliability function of mentioned system is equal to:

$$R_S(t) = r_1(t) + \rho_1 I_1 + (\rho_1 \rho_2) I_2 + (\rho_1 \rho_2 \rho_3) I_3 \qquad (6)$$

Where, I_1, I_2, and I_3 are the convolution integrals, and determined as follows:

$$I_1 = \int_0^t f_0(t_0) r_1(t - t_0) dt_0$$

$$I_2 = \int_0^t \int_0^{t-t_0} f_0(t_0) f_1(t_1) r_2(t - t_0 - t_1) dt_1 dt_0 \qquad (7)$$

$$I_3 = \int_0^t \int_0^{t-t_0} \int_0^{t-t_0-t_1} \prod_{j=0}^2 f_j(t_j) r_3\left(t - \sum_{i=0}^2 t_i\right) dt_2 dt_1 dt_0$$

Now, as for Example 2 and Table 4, we want to evaluate the reliability function of the system up to the 100-hour mission. Figure 1, shows the computing of system reliability, i.e., Equation (6), up to time $t=100$, hours.

In the above figure, the reliability function of the system is plotted with different values of shape parameters in Table 4. It is clear that as the value of k_s increases, the reliability functions are equal.
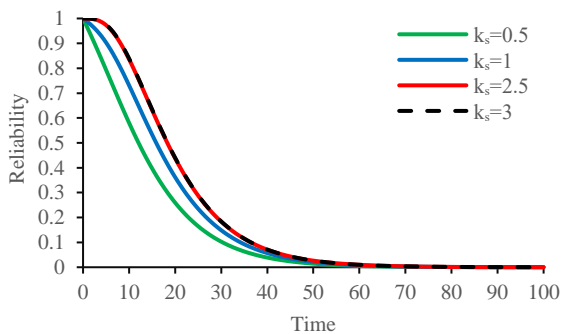

Figure 1. Reliability functions diagram

It is obvious that these values are obtained for validation of the implemented model with the system only five components subjected to a specific situation and are expected to be different for other types of systems or under different situations. As a result, it is suggested that the very complex switching systems with lots of sub-systems and the choice of multiple components be identified by the approach developed in the present study. For this purpose, due to the limited available experimental data, Taguchi method [23], which could be employed as a virtual laboratory to extract required data for a theoretical model, is proposed for future studies to determine probability analysis of switch in complex switching system with various stochastic uncertainty, in addition, to optimization redundancy allocation problem.

4. CONCLUSION

This study aimed to propose a new method for calculating the probability of the switch unit in the changeover condition (second scenario for the switch mechanism in reference [5]). It should be noted that in this method, it has been assumed that unlike the previous studies in the second scenario, the probability of the switch is not constant and the probability of switch unit at each step is calculated according to the proposed method. So, having the time-to-failure of switch unit and components, this probability assessment can be performed on the system. The evaluation of this probability was based on the stress-strength method. In this method, the switch unit represents strength, while other system components demonstrate stress, then in order for the switch to be able to overcome all system components, it must be able to properly connect all subsystem components into the system if required. The developed method was then applied to some numerical example and observed that the performance of the switch decreases as expected by increasing the switching in the system, or in other words by increasing the spare parts in the system. Thus, using developed approach, probability of any switching system under other failure mechanisms and optimization redundancy allocation problem can be investigated which is an ongoing research by the authors.

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Persion Abstract

چکیده

به منظور کارکرد بهتر و افزایش طول عمر سیستم‌ها از تکنیک افزونگی استفاده می‌شود. امروزه از روش افزونگی در بسیاری از صنایع استفاده می‌شود. بهترین استفاده از این روش، استفاده از آن در سیستم‌های سویچینگ است. در سیستم‌های سویچینگ، یک یا چند جز در حالت فعال و بقیه در حالت ذخیره هستند تا در صورت لزوم توسط سویچ از آن‌ها استفاده شود. برای آن‌که دستگاه افزونه بتواند به طور کامل از تمامی اجزای موجود خود استفاده کند، جز سویچ باید وظیفه خود مثل کلیدزنی را بدون نقص انجام دهد. کلیدزنی موفق در جز سویچ با یک احتمال صورت می‌گیرد. در این مقاله، یک روش جدید برای احتمال موفقیت سویچ پیشنهاد شده است و نشان می‌دهد که با افزایش کلیدزنی‌ها در سیستم، کارایی و عملکرد سویچ به تدریج کاهش می‌یابد. تحلیل این روش بر اساس رویکرد احتمالی قدرت – فشار بوده است. در نهایت، به منظور صحت و اعتبار نتایج به‌دست آمده، چند مثال عددی زده شده است.