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Prediction of Mechanical Properties of Frozen Soils Using Response Surface Method: An Optimization Approach

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ABSTRACT

The present study was based on a promoting statistical method known as response surface method (RSM). RSM has been applied as an efficient method to optimize many physical applications in industry for more than two decades. In the current study, the RSM was utilized as a platform to develop models as a function of some prescribed input factors to predict mechanical properties (responses) of frozen soils (i.e. peak tensile/compressive strength, elasticity modulus). Besides, RSM makes it possible to find significant factors and probable interactions as well. A widespread literature review was conducted and three case studies were chosen to evaluate the performance of the RSM in developing precise models and finally an optimum experiment. For each case study, less than half of the available data (an average of 40.8%) was employed to develop models and the remaining part was employed to evaluate the validity of derived models. A comparison between predicted and measured data showed a good agreement with a significant level of 0.05. This indicates that upon using the model a hundred times to predict an specific property for different input factors, the maximum five predictions may diverge from the measured values with ± confidence interval. In addition, some contours were plotted to give a comprehensive presentation of any probable correlations between investigated properties and input factors. Based on the developed models with an average correlation coefficients (R^2) of 93.69, temperature was found to be the most significant factor affecting the mechanical properties of frozen fine soil, while the dry density was not as effective as the temperature.

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1. INTRODUCTION

The frost susceptibility of soils has been of great concern to civil engineers. In spite of advances gained on the knowledge of soil freezing, there is still much evidence of annual frost damage to road surfaces. Other manifestations of the frost damage are tilting of culvert walls, shallow bridge piers, and the jacking out of utility poles. Numerous contributing factors including temperature, duration of freezing period, number of freeze and thaw (F-T) cycles, texture of soil, water content, etc. can be considered in frozen soils. It seems almost infeasible, highly time-consuming and labor intensive to prepare a multidirectional research [1, 2]. Therefore, novel methods should be employed to enhance our understanding of the frost damages. On the other hand, when the problem involves data subjected to experimental errors, statistical methods are the only objective approach for analysis. The statistical approach in experimental studies can play a significant role to obtain meaningful conclusions [3]. Thus, statistical analysis may be considered as an available optimization tool for the experimental problems comprising of different interacting input factors.

In adopting a statistical approach, design of experiment (DOE) is a powerful tool for quantitative assessment in experimental efforts which is employed through Central Composite Design (CCD), response surface method (RSM), full factorial analysis, Taguchi design, and so on. The present study focuses on RSM as

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a promoting statistical approach. DOE can be successfully applied to computer simulation models of physical systems. In such applications, DOE is used to build a model -a metamodel-, and optimization is carried out on the metamodel. The assumption is that if the metamodel is an acceptable representation of the real system, then optimization of the model will result in adequate determination of the optimum conditions for the real system [4].

The models are frequently used by engineers and scientists as computer-based design tools. Typical examples are finite element analysis models for mechanical and structural design and computational models for physical phenomena such as mechanical properties of soil and concrete [72–77]. As an example, consider the case of designing an earth dam to present an optimum configuration including stable slopes under seismic loading and F-T cycles, etc. Many factors may influence the design, such as the minimum freezing temperature, water content, dry density, as well as the percentage of fine particles. Many levels for each factor are potentially important. The maximum shear stress and many other mechanical properties can be considered as engineering responses. As a practical conclusion, only a small number of these potentially important factors have significant effects on the responses. Detailed analysis or testing of the continuum is required to understand which factors are important and to quantify their effect on the design. According to above mentioned issues, performing a complete mix design aiming at effects of various F-T characteristics on engineering properties takes more than dozens of individual runs, each comprises of over 60,00 elements and takes hours of computer time. Obviously, the need to optimize simulation is great. Therefore the typical approach of factor screening followed by optimizing algorithm (i.e. RSM) might well be attractive in this scenario. It is worthy to note that DOE is commonly applied to design of concrete mixture [78–80].

The RSM was firstly developed by Box et al. [81], and then within the next 30 years it was employed vastly at manufacturing process in industry. The RSM is a collection of mathematical and statistical techniques beneficial for the modeling and analysis of the problems in which a response of interest is influenced by several variables and the objective is to optimize this response [4]. The RSM is performed through some iterative analysis to find equations (models) as a function of prescribed factors, which is capable of predicting considered properties (responses). The available and reasonable range of input factors should be firstly chosen and then equations are provided using standard multiple regression methods to be fitted to some intelligently chosen data points.

Derived equations related to response surfaces are polynomials capable of linking input factors and even their interactions to responses. It is noted that an efficient experiment is obtained by identifing important factors and their valid range, the appropriate number of levels for each factor, and the proper methods and units of measurement for each factor and response. These features are sometimes conflicting, thus judgment must often be applied in abovementioned parameters. Efficiency and simplicity of the RSM make it a novel approach in recent studies with respect to other optimization methods such as neural network

In this research, the RSM is particularly employed as a platform to achieve some advantages including:

- Proposing some practical models for the investigated mechanical properties in previous studies
- Proving the ability of RSM to effectively reduce number of tests (treatments)
- Providing a comprehensive discussion on studied factors and their interactions affecting the key engineering properties

To reach these advantages, an attempt was made to review majority of recent studies. Table 1 categorizes studies related to frozen soils. Statistical approach has

TABLE 1. Summary of studies carried out on frozen/thawed soil

Type of Analysis	Soil	Main investigated parameters			
		Mechanical properties	Hydraulic properties	Thermal/ice properties	Physical properties/durability
Experimental	Untreated soil	[2], [1], [3], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24], [25], [26]	[27]	[28], [17]	[2], [26]
	Stabilized soil	[29], [30], [31], [32], [33], [34], [35], [36]*, [37], [38], [39]	[33]	-	[29], [34], [35], [38], [39]
	Reinforced soil	[40], [41], [42], [31], [32], [43], [44]	-	-	[41], [42], [44]
Numerical/plasticity	Untreated soil	[45], [7], [11], [18], [46], [47], [48], [49],[50], [51], [52], [53], [54], [55], [56], [57], [20],[21], [24], [25]	[58], [59], [60], [61], [62], [63], [64], [65]	[66], [67], [59], [68], [47], [48], [69], [70], [53], [71], [63], [64], [57], [65],	-

* Conducted based on statistical approach

scarcely been employed within recent decades in geotechnical applications [9]. The performance of geogrid and geotextile in asphalt overlay to delay the rate of reflective crack propagation based on the RSM was investigated. However, the present study tries to examine potential capabilities of RSM in other geotechnical applications. In addition, it is worthy to note that the derived equations can be efficiently employed in some other areas in geotechnics such as calibration of numerical models, finding the governing equations on peak strength and ultimate stress. The latter can result in developing yield functions or bounding surface.

Three case studies were chosen to examine the RSM efficiency in reducing required tests for a comprehensive outlook to mechanical properties of some types of frozen soils. For each case study, less than half of available data (approximately 40%) was employed to develop prediction models and the remaining part was used to evaluate the validity of derived models. The derived statistical models enable us to quantify the level of significance of influencing factors including freezing temperature, water content, strain rate, etc. on responses such as peak strength, elastic modulus, etc. It was also proved that optimum values of some mechanical properties had been missed in original studies due to range of input factors. However, the optimum response could be considered as a secondary goal. Temperature was also found the most effective factor on mechanical properties among other factors such as induced strain rate, dry density, etc. This is in accordance with those reported by original studies. It should be mentioned that the conclusions are valid within the conditions expressed for each case study, and can be unreliable for different types of frozen soil and/or loading paths.

2. ANALYSIS METHOD

When number of input factor exceeds a specified number, traditional outlook of experimental design results in a high order test matrix (*n* factors in *m* levels requires m^n tests). While the RSM as an applicable method provides a much smaller test matrix, so that the test matrix consists of three portion; factorial portion (2^n), axial portion ($2n$), and central portion, where *n* refers to the number of input factors.

Originally, the RSM is employed to search for an optimum design that optimizes some design criterion, i.e. optimality, orthogonality and rotatability. The optimization process is basically done by means of nonlinear polynomial equations to find response surface optima. However, the process may be halted for some scientific applications as no specific optimum is required. A well discussed descriptions can be found in the literature [4, 81–84].

The RSM is performed in a staged manner to reach the highest order of precision. Thus, levels of input factors are determined at center, ends and/or other required levels of the studied range (known as *design points*) and then polynomial equations are regressed on the design points as a function of the input factors and even their interactions. The process is continued with enhancing equations by omission of non-significant factors and regeneration of the equations. Finally derived equations are verified by means of some other points (known as *verification points*) within the considered domain of factors. It is worthy to note that the process is enriched with complementary information such as correlation coefficient (R^2), confidence interval, lack of fitness, etc. The equations are developed in terms of normalized values, as shown in Equation (1). The absolute and normalized values of the parameters are presented in Table 3.

$$Coded\ values = \frac{(absolute\ value - center\ value)}{(Max.value - center\ value)} \tag{1}$$

where center value represents the center of studied range of the input factors corresponding to normalized value of zero. There are some other essential points to develop a more reliable equation, i.e. factorial points corresponding to normalized values of -1 and +1 and star points corresponding to normalized values higher than +1 and lower than -1. It should be mentioned that the abovementioned points are essential in response surface methodology if CCD is employed to derive quadratic equations while these points were not necessarily available in three chosen case studies.

Statistical models presented in this study are established by multi-regression analysis employing the least-square method as:

$$Y = \beta_0 + \sum_{i=1}^{n} \beta_i X_i + \sum_{i<j}^{n} \sum_{j}^{n} \beta_{ij} X_i X_j \tag{2}$$

where Y is the predicted response, X_i and X_j are the normalized values of the modeled variables, β_0 is a constant coefficient, β_i is linear coefficient, β_{ij} is coefficient of the interaction, and n is the number of the modeled variables.

The model described in Equation (2) is known as the Scheffé polynomials which is able to find the interaction between investigated factors. The significance of variables and their interactions are determined by the analysis of variance (ANOVA) using the least-square approach.

3. DISCUSSION

Three case studies in companion with input factors (independent parameters), their domains and response factors (dependent parameters) are shown in Table 2. As mentioned before, nearly half of the existing data of each case study was excluded and the remaining part was employed for derivation of statistical models. The remained and excluded parts are known as *design portion* and *validation portion*, respectively. The size of design portion was intentionally kept smaller than the validation portion, as reduction of required mixtures for an optimum design of experiment is one of the most popular advantages of the RSM. The ratio of the numbers of treatments engaged in the model derivation to the total number of tests is called "*efficiency factor*", which represents efficiency of the RSM to reduce the number of required tests for a comprehensive perspective about studied properties.

Design points are selected to cover the considered domain of variables. However, some design points which are partly important in a perfect design of experiments could not be easily found in existing data bank, i.e. center point (design points with normalized value of zero), star points (design points beyond the design space). Contrary to inevitable limitations, well fitted models can mostly be reached utilizing the available design points, as will be illustrated in validation section. It could be partly attributed to the second order interactions included in the derived models.

Again, it should be noted that equations are developed in an evolutionary process. Thus, insignificant factors or interactions are sequentially emitted and remaining terms are again recalculated to find more reliable models. The presented coefficients are successive estimated values for each three case studies.

3. 1. Case Study 1 Li et al. [1] performed laboratory tests on a remolded clayey soil classified CL according to the Unified Soil Classification System (USCS). The clay liquid limit and plastic limit were 28.8 and 17.7%, respectively. The case study aimed to measured uniaxial compressive strength (UCS) by a screw-driven universal material testing machine. Tests were performed on frozen specimens with length of 150 mm and diameter of 61.8 mm. The obtained results are presented in Table 3 for 96 treatments, as reported by Li et al. [1]. In the current research, the model was prepared using 39 treatments which, in turn, validated using 57 remaining treatments. It is interesting to note that the efficiency factor was found 40.6%.

Table 4 shows the coefficients obtained from the statistical analysis to predict the experimental program. Statistical models were performed by full regression analyses. All factors are expressed in terms of normalized values. The coefficients are expressed by significant factors which have a p-value less than presumed significant level ($\alpha = 0.05$). The correlation coefficients (R^2) and adjusted correlation coefficient (R^2_{adj}) of the proposed models are 98.52 and 98.18%, respectively, indicating reliable models. Therefore, quadratic equation to predict peak strength of frozen-thawed clay regressed as follows, which is valid within the accepted conditions. As mentioned before, the validation of derived equations will be demonstrated subsequently.

$$q_u = -0.551T - 0.454S \times S + 0.225D + 0.188S - 0.140T \times D - 0.138T \times S + 0.106D \times D + 0.038 \quad (3)$$

where T, S, and D denote temperature, strain rate and dry density, respectively.

The coefficients of the equations are presented in the order of magnitude. The estimated coefficients for each factor refer to its contribution to the modeled response. Thus, the higher value represents more effective contribution. A negative coefficient in Equation (3) indicates that an increase in the input factor results in the reduction in predicted response. For instance, compressive strength was primarily affected by temperature (-0.551) and lightly influenced by dry density (0.225). The compressive strength was found to increase under higher dry density and strain rate. This is in agreement with those shown by Li et al. [1]. In addition, they demonstrated that strain rate and dry density also had significant effects on the compressive strength compared with temperature. It is noted that logarithmic functions had been derived by Li et al. [1].

To achieve a better understanding of existing interactions between factors, some trade-offs were drawn based on the derived equation in uncoded (absolute) format (Figure 1). It should be mentioned that allocated value for the third input factor is shown on corresponding figures.

TABLE 2. Studied factors for chosen case studies

Case study	Design factors	Applied range	Response factors
Case study 1 [1]	Dry density (gr/cm^3)	1.28 – 1.88	Peak compressive strength
	Temp (ºC)	-15 – -2	
	Strain rate (1/sec)	1.00E-6 – 7.03E-4	
Case study 2 [13]	Water content (%)	30.3 – 50.0	Initial yield strength , peak strength, $E_{50\%}$†
	Dry density (gr/cm^3)	1.08 – 1.43	
	Temp (ºC)	-10 – -0.5	
	Strain rate (1/sec)	6.14E-3 – 8.10E-7	
Case study 3 [19]	Water content (%)	15 – 30	Compressive peak strength, deformation modulus, compressive failure strain, tensile peak strength
	Temp (ºC)	-20 – -2	

†: $E_{50\%}$ is tangent modulus on stress-strain curve, corresponding to half of peak strength

TABLE 3. Experimental design matrix for case study 1

Design points						Validation points					
Absolute values			Normalized values			Absolute values			Normalized values		
temp. (ºC)	strain rate (1/sec)	Density (gr/cm^3)	temp.	strain rate	density	temp. (ºC)	strain rate (1/sec)	Density (gr/cm^3)	temp.	strain rate	density
-2	6.67E-04	1.38	1.00	0.90	-1.00	-2	1.04E-04	1.38	1.00	-0.71	-1.00
-2	1.10E-06	1.38	1.00	-1.00	-1.00	-2	1.04E-04	1.38	1.00	-0.71	-1.00
-10	6.05E-04	1.38	-0.23	0.72	-1.00	-2	9.10E-06	1.38	1.00	-0.98	-1.00
-10	9.27E-05	1.38	-0.23	-0.74	-1.00	-2	9.10E-06	1.38	1.00	-0.98	-1.00
-10	1.00E-06	1.38	-0.23	-1.00	-1.00	-10	6.10E-04	1.38	-0.23	0.74	-1.00
-5	6.67E-04	1.38	0.54	0.90	-1.00	-10	9.37E-05	1.38	-0.23	-0.74	-1.00
-5	9.10E-06	1.38	0.54	-0.98	-1.00	-10	8.40E-06	1.38	-0.23	-0.98	-1.00
-15	5.58E-04	1.38	-1.00	0.59	-1.00	-10	8.00E-06	1.38	-0.23	-0.98	-1.00
-2	6.67E-04	1.58	1.00	0.90	-0.20	-10	1.00E-06	1.38	-0.23	-1.00	-1.00
-2	9.10E-06	1.58	1.00	-0.98	-0.20	-5	6.67E-04	1.38	0.54	0.90	-1.00
-10	6.67E-04	1.58	-0.23	0.90	-0.20	-5	1.04E-04	1.38	0.54	-0.71	-1.00
-10	1.40E-04	1.58	-0.23	-0.60	-0.20	-5	9.10E-06	1.38	0.54	-0.98	-1.00
-10	8.91E-06	1.58	-0.23	-0.98	-0.20	-5	1.10E-06	1.38	0.54	-1.00	-1.00
-5	1.04E-04	1.58	0.54	-0.71	-0.20	-5	1.10E-06	1.38	0.54	-1.00	-1.00
-15	6.73E-04	1.58	-1.00	0.91	-0.20	-15	5.58E-04	1.38	-1.00	0.59	-1.00
-15	8.61E-06	1.58	-1.00	-0.98	-0.20	-15	1.26E-04	1.38	-1.00	-0.64	-1.00
-2	7.03E-04	1.88	1.00	1.00	1.00	-15	8.30E-06	1.38	-1.00	-0.98	-1.00
-2	1.09E-06	1.88	1.00	-1.00	1.00	-15	8.33E-06	1.38	-1.00	-0.98	-1.00
-10	6.43E-04	1.88	-0.23	0.83	1.00	-15	1.01E-06	1.38	-1.00	-1.00	-1.00
-10	8.88E-06	1.88	-0.23	-0.98	1.00	-2	1.04E-04	1.58	1.00	-0.71	-0.20
-5	6.74E-04	1.88	0.54	0.92	1.00	-2	1.04E-04	1.58	1.00	-0.71	-0.20
-5	9.05E-06	1.88	0.54	-0.98	1.00	-2	9.10E-06	1.58	1.00	-0.98	-0.20
-15	6.57E-04	1.88	-1.00	0.87	1.00	-2	1.10E-06	1.58	1.00	-1.00	-0.20
-15	1.10E-06	1.88	-1.00	-1.00	1.00	-10	6.60E-04	1.58	-0.23	0.88	-0.20
-2	6.67E-04	1.38	1.00	0.90	-1.00	-10	1.38E-04	1.58	-0.23	-0.61	-0.20
-2	1.10E-06	1.38	1.00	-1.00	-1.00	-10	9.11E-06	1.58	-0.23	-0.98	-0.20
-5	1.04E-04	1.38	0.54	-0.71	-1.00	-10	1.10E-06	1.58	-0.23	-1.00	-0.20
-15	8.94E-05	1.38	-1.00	-0.75	-1.00	-10	1.10E-06	1.58	-0.23	-1.00	-0.20
-15	1.02E-06	1.38	-1.00	-1.00	-1.00	-5	6.67E-04	1.58	0.54	0.90	-0.20
-2	6.67E-04	1.58	1.00	0.90	-0.20	-5	6.67E-04	1.58	0.54	0.90	-0.20
-2	1.10E-06	1.58	1.00	-1.00	-0.20	-5	9.10E-06	1.58	0.54	-0.98	-0.20
-5	1.04E-04	1.58	0.54	-0.71	-0.20	-5	9.10E-06	1.58	0.54	-0.98	-0.20
-15	9.94E-05	1.58	-1.00	-0.72	-0.20	-5	1.10E-06	1.58	0.54	-1.00	-0.20
-15	1.10E-06	1.58	-1.00	-1.00	-0.20	-5	1.10E-06	1.58	0.54	-1.00	-0.20
-2	6.94E-04	1.88	1.00	0.97	1.00	-15	6.62E-04	1.58	-1.00	0.88	-0.20
-2	1.08E-06	1.88	1.00	-1.00	1.00	-15	1.01E-04	1.58	-1.00	-0.72	-0.20
-5	1.07E-04	1.88	0.54	-0.70	1.00	-15	8.74E-06	1.58	-1.00	-0.98	-0.20
-15	1.05E-04	1.88	-1.00	-0.70	1.00	-15	1.07E-06	1.58	-1.00	-1.00	-0.20
-15	1.13E-06	1.88	-1.00	-1.00	1.00	-2	1.04E-04	1.88	1.00	-0.71	1.00
						-2	1.09E-04	1.88	1.00	-0.69	1.00

-2	9.05E-06	1.88	1.00	-0.98	1.00
-2	9.17E-06	1.88	1.00	-0.98	1.00
-10	6.72E-04	1.88	-0.23	0.91	1.00
-10	1.03E-04	1.88	-0.23	-0.71	1.00
-10	1.02E-04	1.88	-0.23	-0.71	1.00
-10	8.94E-06	1.88	-0.23	-0.98	1.00
-10	1.10E-06	1.88	-0.23	-1.00	1.00
-10	1.10E-06	1.88	-0.23	-1.00	1.00
-5	6.76E-04	1.88	0.54	0.92	1.00
-5	1.20E-04	1.88	0.54	-0.66	1.00
-5	9.04E-06	1.88	0.54	-0.98	1.00
-5	1.09E-06	1.88	0.54	-1.00	1.00
-5	1.08E-06	1.88	0.54	-1.00	1.00
-15	6.33E-04	1.88	-1.00	0.80	1.00
-15	1.07E-04	1.88	-1.00	-0.70	1.00
-15	9.20E-06	1.88	-1.00	-0.98	1.00
-15	8.87E-06	1.88	-1.00	-0.98	1.00

TABLE 4. Parameter estimates of derived models for peak compressive strength in normalized format

Term	Coef.	p value
Constant	0.038	0.429
T	-0.551	0.000
S	0.188	0.000
D	0.225	0.000
S*S	-0.454	0.000
D*D	0.106	0.000
T*S	-0.138	0.000
T*D	-0.140	0.000

T: Temperature , S: Strain rate , D: Dry density

Figure 1(a) illustrates the effects of temperature and dry density on the compressive strength. As expected, the lower temperature resulted in the higher compressive strength for a given density. A further increase was found for dry mass density higher than 1.65 g/cm^3. Although, there is a threshold for dry mass density beyond that soil experiences more expansion as there is no space to dissipate ice pressure. The contour diagram of compressive strength in Figure 1(b) illustrates the same trade-offs between temperature and strain rate for soil with dry mass density of 1.63 g/cm^3. Peak strength tends to increase as freezing temperature decreases regardless of strain rate. Moreover, compressive strength exhibited a maximum value at temperatures lower than -15ºC and strain rate of approximately 0.0005 (1/s). It should be noted that investigated range of factors can be redefined to reach an optimum response, if required.

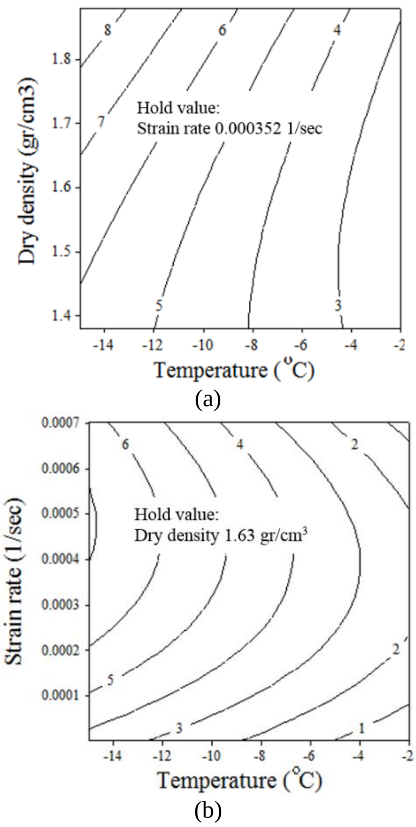

Figure 1. Absolute peak compressive strength (MPa) as a function of (a) dry density and temperature, (b) strain rate and temperature

3. 2. Case Study 2 Yuanlin and Carbee [13] have chosen a silt soil from the USA CRREL experimental permafrost tunnel at Fox, near Fairbanks, Alaska. They conducted uniaxial compressive strength tests on remolded, saturated Fairbanks frozen silt under various constant machine speeds, temperatures and dry densities. The soil was classified as ML in the USCS. Prepared specimens had 70 mm diameter and 152 mm height. Table 5 presents absolute and normalized

TABLE 5. Experimental design matrix for case study 2

Design points								Validation points							
Absolute values				Normalized values				Absolute values				Normalized values			
water content (%)	**dry density (gr/cm^3)**	**Temp (ºC)**	**strain rate (1/sec)**	**water content**	**dry density**	**temp**	**strain rate**	**water content (%)**	**dry density (gr/cm^3)**	**Temp (ºC)**	**strain rate (1/sec)**	**water content**	**dry density**	**temp**	**strain rate**
43.0	1.177	-0.5	5.85E-03	0.29	-0.46	1.00	0.91	43.6	1.184	-0.5	1.06E-03	0.35	-0.42	1.00	-0.65
42.9	1.205	-1.0	5.73E-03	0.28	-0.30	0.89	0.87	45.2	1.155	-0.5	1.00E-04	0.51	-0.59	1.00	-0.97
42.7	1.187	-1.0	1.00E-05	0.26	-0.41	0.89	-1.00	43.9	1.177	-0.5	9.23E-06	0.38	-0.46	1.00	-1.00
45.4	1.155	-2.0	1.07E-03	0.53	-0.59	0.68	-0.65	41.6	1.206	-1.0	1.10E-03	0.15	-0.29	0.89	-0.64
42.8	1.206	-2.0	1.11E-04	0.27	-0.29	0.68	-0.96	42.6	1.198	-1.0	1.10E-03	0.25	-0.34	0.89	-0.64
40.8	1.227	-2.0	1.14E-04	0.07	-0.17	0.68	-0.96	42.8	1.195	-1.0	1.12E-04	0.27	-0.36	0.89	-0.96
42.9	1.187	-3.0	5.63E-03	0.28	-0.41	0.47	0.83	42.2	1.203	-1.0	1.11E-04	0.21	-0.31	0.89	-0.96
42.8	1.195	-3.0	1.08E-06	0.27	-0.36	0.47	-1.00	41.7	1.211	-1.0	1.06E-05	0.16	-0.26	0.89	-1.00
43.1	1.184	-5.0	1.11E-03	0.30	-0.42	0.05	-0.64	41.9	1.202	-1.0	1.01E-05	0.18	-0.32	0.89	-1.00
42.5	1.195	-5.0	1.12E-03	0.24	-0.36	0.05	-0.64	42.4	1.200	-1.0	1.06E-06	0.23	-0.33	0.89	-1.00
40.6	1.229	-5.0	1.15E-04	0.05	-0.16	0.05	-0.96	41.3	1.219	-2.0	1.15E-03	0.12	-0.22	0.68	-0.63
42.0	1.203	-5.0	1.06E-05	0.19	-0.31	0.05	-1.00	42.3	1.200	-2.0	1.13E-04	0.22	-0.33	0.68	-0.96
41.4	1.221	-5.0	1.13E-05	0.13	-0.21	0.05	-1.00	43.2	1.189	-2.0	1.12E-05	0.31	-0.39	0.68	-1.00
44.6	1.165	-7.0	5.57E-03	0.45	-0.54	-0.37	0.81	41.7	1.213	-2.0	1.11E-05	0.16	-0.25	0.68	-1.00
42.4	1.202	-7.0	1.11E-03	0.23	-0.32	-0.37	-0.64	42.2	1.206	-2.0	1.11E-06	0.21	-0.29	0.68	-1.00
40.7	1.222	-7.0	1.14E-06	0.06	-0.20	-0.37	-1.00	41.8	1.208	-2.0	1.03E-06	0.17	-0.28	0.68	-1.00
41.8	1.210	-10.0	1.11E-03	0.17	-0.27	-1.00	-0.64	42.0	1.190	-3.0	1.12E-03	0.19	-0.39	0.47	-0.64
42.6	1.198	-10.0	1.23E-03	0.25	-0.34	-1.00	-0.60	41.2	1.221	-3.0	1.12E-03	0.11	-0.21	0.47	-0.64
50.0	1.086	-2.0	1.00E-04	1.00	-1.00	0.68	-0.97	41.3	1.219	-3.0	1.13E-04	0.12	-0.22	0.47	-0.96
48.3	1.104	-2.0	1.01E-05	0.83	-0.89	0.68	-1.00	41.5	1.216	-3.0	1.12E-04	0.14	-0.24	0.47	-0.96
30.3	1.426	-2.0	6.14E-03	-1.00	1.00	0.68	1.00	41.5	1.216	-3.0	1.10E-05	0.14	-0.24	0.47	-1.00
31.7	1.394	-2.0	1.24E-03	-0.86	0.81	0.68	-0.60	41.9	1.208	-3.0	1.11E-05	0.18	-0.28	0.47	-1.00
31.6	1.389	-2.0	1.22E-05	-0.87	0.78	0.68	-1.00	42.4	1.198	-3.0	1.03E-06	0.23	-0.34	0.47	-1.00
31.3	1.389	-2.0	1.19E-06	-0.90	0.78	0.68	-1.00	42.0	1.211	-5.0	1.07E-06	0.19	-0.26	0.05	-1.00
30.4	1.422	-2.0	1.18E-06	-0.99	0.98	0.68	-1.00	42.3	1.203	-7.0	1.11E-03	0.22	-0.31	-0.37	-0.64
								43.3	1.174	-7.0	1.05E-04	0.32	-0.48	-0.37	-0.97
								41.4	1.211	-7.0	1.15E-05	0.13	-0.26	-0.37	-1.00
								43.1	1.187	-7.0	1.04E-05	0.30	-0.41	-0.37	-1.00
								41.7	1.210	-10.0	1.15E-04	0.16	-0.27	-1.00	-0.96
								45.9	1.168	-10.0	1.13E-05	0.58	-0.52	-1.00	-1.00
								43.7	1.179	-10.0	1.12E-05	0.36	-0.45	-1.00	-1.00
								42.3	1.202	-10.0	1.09E-06	0.22	-0.32	-1.00	-1.00
								49.3	1.099	-2.0	8.10E-07	0.93	-0.92	0.68	-1.00
								31.5	1.395	-2.0	1.24E-04	-0.88	0.82	0.68	-0.96
								31.2	1.408	-2.0	1.23E-05	-0.91	0.89	0.68	-1.00

specifications of treatments employed in the current statistical analysis. The model was derived using 25 treatments which, in turn, validated using other 35 treatments (*efficiency factor* of 41.7%). Tables 6 summarizes coefficients and their p-values. Again, probability values less than 0.05 are considered to realize significant influences on the modeled responses. Quadratic equations are derived in a sequential manner such that non-significant factors should be eliminated to attain a more precise equation.

For instance, water content exhibited no significant effect on $E_{50\%}$ while minimum temperature, strain rate and related second-order interactions had significant effects. Table 6 shows final successive estimations for remaining terms. Some logarithmic correlations were introduced by Yuanlin and Carbee [13]. Estimated coefficients are presented in descending order in Table 7. As mentioned before, the higher value represents more effective contribution. A negative coefficient indicates that an increase in the input factor results in a reduction of the predicted response. To elaborate the descriptions, corresponding coefficients are given in parenthesis.

The initial yield strength decreases with increasing the temperature, while an increasing trend is observed with increasing the strain value. Although, strain rate seems to be a neutral factor for freezing temperatures greater than -5ºC (Figure 2(c)). As seen, the temperature (-0.876) is more effective on the initial yield strength compared with the strain rate (0.535) and other interactions including S*S (-0.598) and S*T (-0.352). It is clear from the corresponding equation that the peak strength of frozen silt significantly increases as temperature falls down (-0.8546) and strain rate increases (0.5510). However, dry mass density (0.7206) is more effective than induced strain rate (0.5510). Similarly, Yuanlin and Carbee [13] stated similar results with those obtained for peak strength. Regarding $E_{50\%}$, the contribution of strain rate (0.5090) is nearly similar to its contribution in other two responses (0.5510 and 0.5350). The R^2 values of the proposed models vary in the range of 96.49-98.85, indicating models can properly predict validation points. It should be mentioned that deformation modulus is not precisely estimated from the RSM analysis. Undoubtedly, this is partly due to lack of key data in existing database to perform a perfect statistical analysis, i.e. center of studied ranges (equivalent to normalized value of zero). However, the RSM takes into account the interactions which have been neglected in previous studies.

Based on equations given in Table 7, following trade-offs are shown to attain a better understanding of variations. It should be mentioned that responses were drawn as a function of strain rate and temperature while other significant factors were remained constant amidst corresponding range. As shown in Figures 2(a) and 2(b), the variation of initial yield strength and peak strength

TABLE 6. Parameter estimates of derived models in normalized format

Initial yield strength			Peak strength			$E_{50\%}$		
Term	**Coef.**	**p value**	**Term**	**Coef.**	**p value**	**Term**	**Coef.**	**p value**
Constant	0.6220	0.000	**Constant**	0.6058	0.000	**Constant**	0.0390	0.579
T	-0.8760	0.000	**W**	0.6947	0.047	**D**	-0.2398	0.000
S	0.5350	0.000	**D**	0.7206	0.037	**T**	-0.8078	0.000
S*S	-0.5980	0.000	**T**	-0.8546	0.000	**S**	0.5091	0.000
S*T	-0.3520	0.000	**S**	0.5510	0.000	**S*S**	-0.1991	0.041
			T*T	0.1040	0.041	**T*S**	-0.5473	0.000
			S*S	-0.5388	0.000			
			T*S	-0.3605	0.000			

T: Temperature, S: Strain rate, W: Water content, D:dry density

TABLE 7. Derived estimation model for experimental program

Dependent variable	**Derived equation (normalized units)**	R^2	R^2_{adj}
Initial yield strength	$-0.8760T + 0.6220 - 0.5980S \times S + 0.5350S - 0.3520S \times T$	97.99	97.59
Peak strength	$-0.8546T + 0.7206D + 0.6947W + 0.6058 + \ldots$ $\ldots + 0.5510S - 0.5388S \times S - 0.3605T \times S + 0.1040T \times T$	98.85	98.38
$E_{50\%}$	$-0.8078T - 0.5473T \times S + 0.5090S - 0.2398D - 0.1991S \times S + 0.0390$	96.49	95.57

is more affected by the temperature compared with the strain rate at high level of induced strain (shaded area). Figure 2(c) demonstrates that $E_{50\%}$ increased as the temperature drops and the strain rate increases, as contours are getting closer at top-left corner of the plot. It should be mentioned that those significant factors which were not employed as input factors in plotting trade-offs, were hold on amidst of their corresponding domains. As a practical conclusion, each three responses experience optimum values for input variables beyond the chosen domains in original studies. However, values of input factors beyond the investigated range may be practically unfeasible. It is worth noting that a more efficient design of experiment would not necessarily require more treatments.

3. 3. Case Study 3

The objective of case study 3 reported by Christ and Kim [19] was to evaluate the mechanical properties of frozen Siberian silt. They determined unfrozen water content, uniaxial compressive strength and direct-tensile strength of frozen silt samples at different water contents and temperatures. Absolute and normalized values of modeled parameters are presented in Table 8. It is interesting to note that the *efficiency factor* was kept 40.0% (8 from 20 treatments).

The derived coefficients and corresponding p-values are summarized in Table 9. The equations are stated as a function of factors with significant influences on modeled responses listing in descending order. The R^2 value of the proposed models ranges 84.85-98.62. As seen in Tables 9 and 10, probability values are relatively low which can be attributed to fewer available design points. However, the modeled responses are still reliable and reasonably validated (Figures 3-5).

Based on the equations presented in Table 10, trade-offs between temperature and water content on values of the modeled properties are plotted in Figure 6. Based on the equations presented in Table 10, trade-offs between temperature and water content are plotted in Figure 6. As seen, temperature has the most significant effect on the compressive strength and deformation modulus. However, tensile strength is significantly affected by water content. As seen in Table 10, the temperature and water content exhibit conflicting influences on all responses. Contours demonstrate a maximum value as temperature and water content decreases and increases, respectively. This is in accordance with those reported by Christ and Kim [19]. In addition, Christ and Kim [19] found an exponential and linear increase in tensile and compressive strength, respectively, as temperature dropped especially at high water content. Moreover, a linear correlation between deformation modulus and input factors was found which is also in accordance with those presented by Christ and Kim [19].

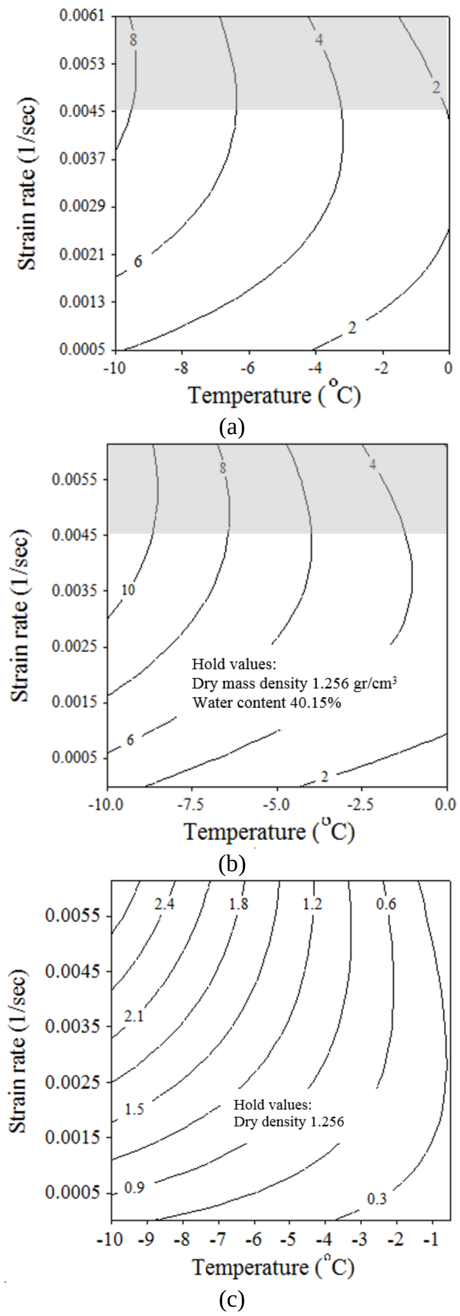

Figure 2. Trade-offs plotted for absolute values: (a) Initial yield strength (MPa), (b) Peak strength (MPa) and (c) $E_{50\%}$ (GPa)

4. REPEATABILITY AND VALIDATION OF DERIVED STATISTICAL MODELS

As mentioned above, large part of available data was intentionally excluded for validation points as listed in Tables 3, 5 and 8. The estimated relative errors corresponding to 95% confidence limit are shown in Table 11. Based on the calculated errors, Figures 3-5

TABLE 8. Details of experimental program

Design points				Validation points			
absolute values		normalized values		absolute values		normalized values	
water content (%)	temp. (ºC)	water content	temp.	water content (%)	temp. (ºC)	water content	temp.
15.0	-2	-1.00	1.00	15.0	-5	-1.00	0.67
15.0	-20	-1.00	-1.00	15.0	-10	-1.00	0.11
19.8	-5	-0.36	0.67	15.0	-15	-1.00	-0.44
19.8	-15	-0.36	-0.44	19.8	-2	-0.36	1.00
25.0	-5	0.33	0.67	19.8	-10	-0.36	0.11
25.0	-15	0.33	-0.44	19.8	-20	-0.36	-1.00
30.0	-2	1.00	1.00	25.0	-2	0.33	1.00
30.0	-20	1.00	-1.00	25.0	-10	0.33	0.11
				25.0	-20	0.33	-1.00
				30.0	-5	1.00	0.67
				30.0	-10	1.00	0.11
				30.0	-15	1.00	-0.44

TABLE 9. Parameter estimates of derived equations for modelled responses

Compressive strength			Deformation modulus			Failure strain			Tensile strength		
Term	Coef.	p value	Term	Coef.	p value	Term	Coef.	p value	Term	Coef.	p value
Constant	-0.200	0.094	**Constant**	-0.332	0.030	**Constant**	0.383	0.012	**Constant**	-0.918	0.002
W	0.450	0.018	**W**	0.465	0.025	**W**	0.893	0.000	**W**	0.495	0.003
T	-0.586	0.004	**T**	-0.580	0.008				**T**	-0.444	0.004
									T*T	0.457	0.034
									W*T	-0.477	0.004

W: Water content, T: Temperature

TABLE 10. Derived estimate model for experimental program

Dependent variable	Derived equation (normalized units)	R^2	R^2_{adj}
Compressive strength	$-0.586T + 0.450W - 0.200$	87.88	83.04
Deformation modulus	$-0.580T + 0.465W - 0.332$	84.85	78.79
Failure strain	$0.893W + 0.383$	86.33	84.05
Tensile strength	$-0.918 + 0.495W - 0.477W \times T + 0.457T \times T - 0.444T$	98.62	96.79

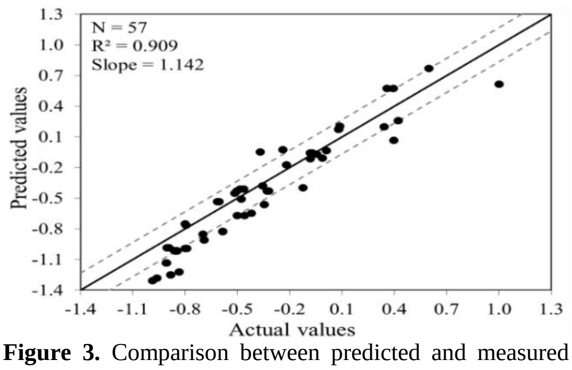

Figure 3. Comparison between predicted and measured normalized compressive strength values for case study 1

present a comparison between predicted and measured values for all three case studies to prove that the RSM can reliably be employed in experimental studies. The number of validation points is given on each plot.

In Figure 5, data points below the continuous line indicate that derived equations underestimate and those above the line overestimate the measured values. Two parallel dotted lines were drawn to present the 95% confidence interval. The majority of the predicted responses were within the 95% confidence limits which can be found in Table 11. These limits constitute experimental errors for the measurements. In case study

1, despite a small confidence limit, the predicted and measured values relatively lied within the confidence interval. The predicted-to-measured ratio and R^2 values are 1.142 and 0.909, respectively. The majority of responses for case study 2 are in close proximity with 1:1 diagonal line. This indicates good accuracy of the models for prediction of peak strength, initial yield strength and $E_{50\%}$. Although, $E_{50\%}$ was not predicted as accurate as was expected. This can be rooted in lack of key data. In the case study 3, except peak tensile strength, other modeled responses including peak compressive strength, compressive failure strain and

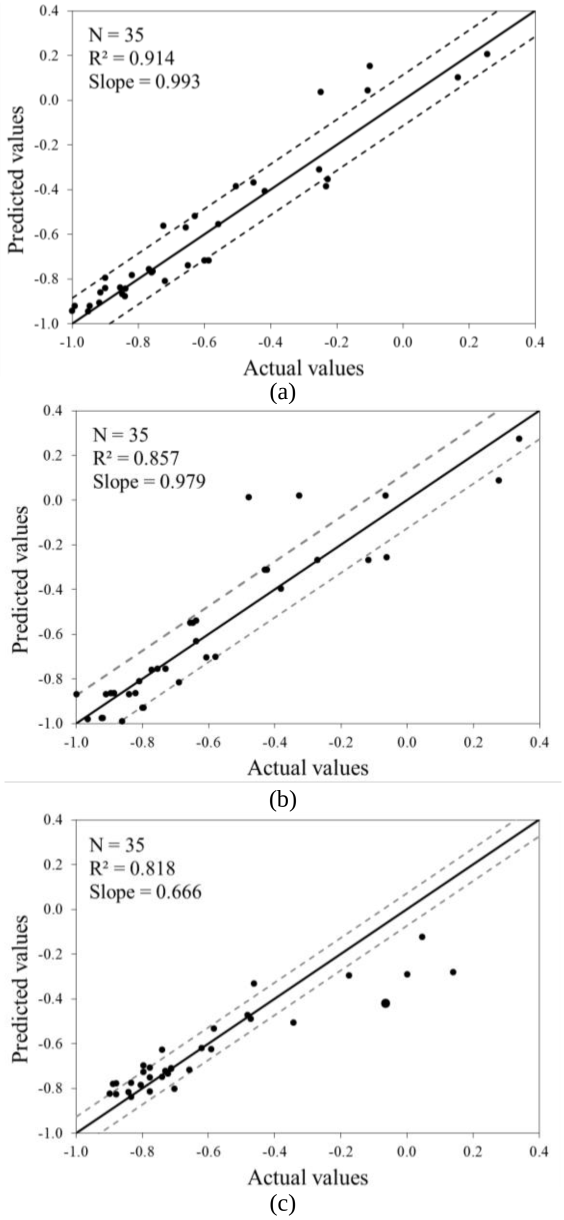

Figure 4. Comparison between predicted and measured normalized responses for in case study 2: (a) Peak strength, (b) Initial yield strength, (c) $E_{50\%}$

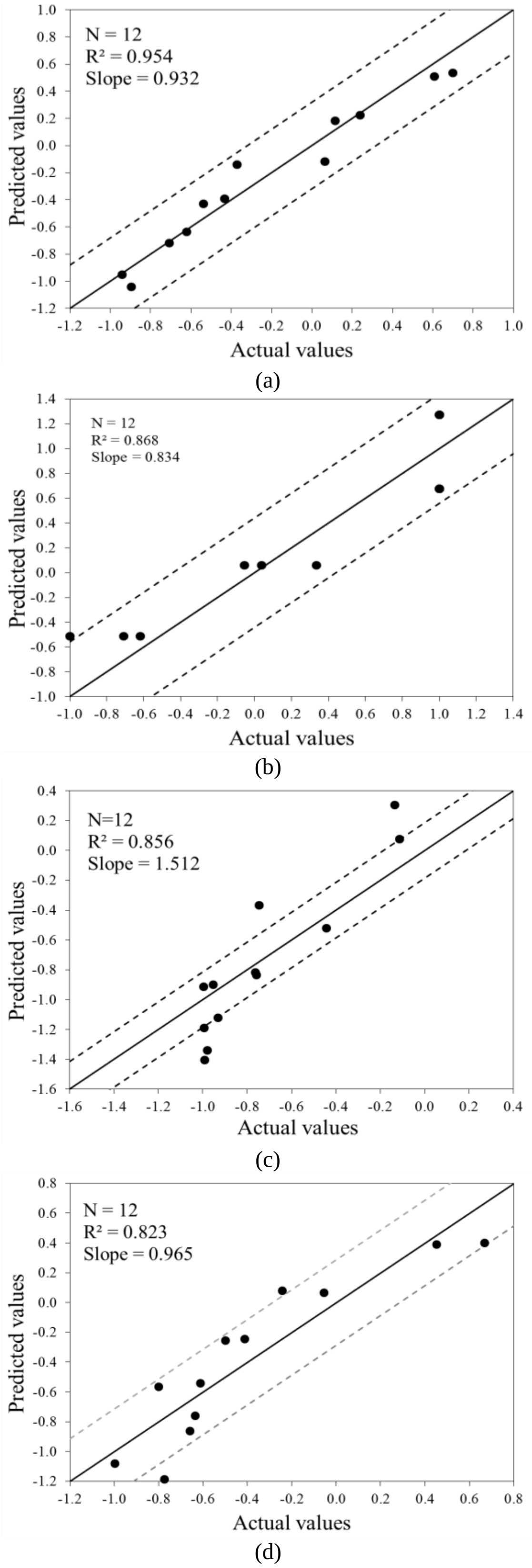

Figure 5. Comparison between predicted and measured normalized responses for case study 3: (a) Peak compressive strength, (b) Compressive failure strain, (c) Peak tensile strength (d) Deformation modulus

deformation modulus were within their corresponding confidence intervals. The ratio of predicted-to-measured values ranged between 0.834 and 0.965 for abovementioned properties. In summary, all derived equations prepared acceptable predictions within the investigated range of input factors. It is noted that the modeled responses remain valid as far as characteristics of raw material and test procedures tolerate small variations.

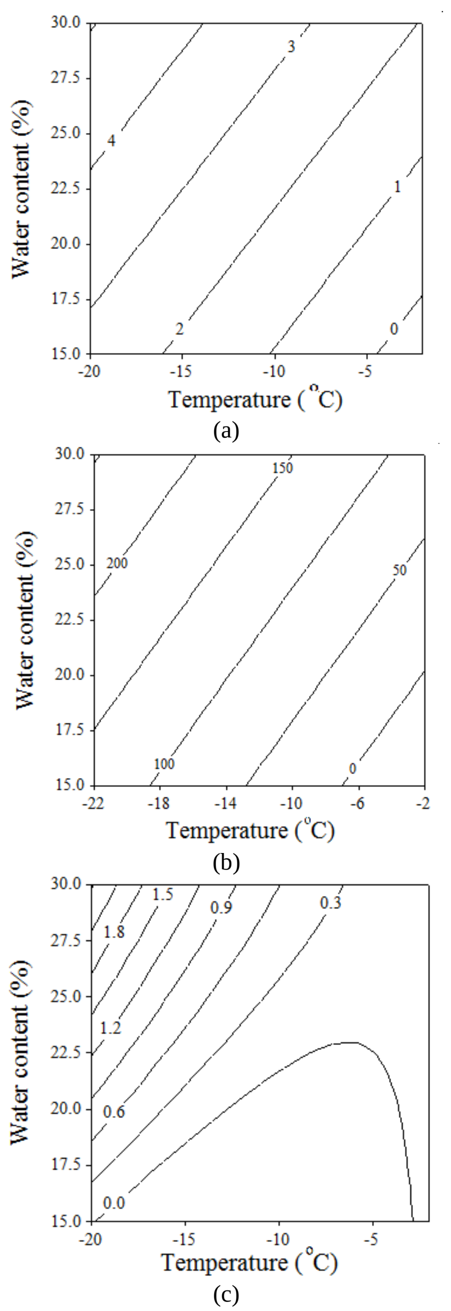

Figure 6. Trade-offs expressing the absolute values of: (a) Compressive peak strength (MPa); (b) Deformation modulus (MPa); (c) Peak tensile strength (MPa), as functions of temperature and water content

TABLE 11. Standard deviation and confidence interval for validation points based on measured normalized values (95% confidence level)

Case study	Response	Standard deviation	Confidence interval
1st	Peak compressive strength	0.566	0.167
2nd	Peak compressive strength	0.331	0.114
	Initial yield strength	0.345	0.119
	$E_{50\%}$	0.323	0.111
3nd	Peak compressive strength	0.563	0.318
	Compressive failure strain	0.779	0.441
	Peak tensile strength	0.509	0.288
	Deformation modulus	0.328	0.186

5. CONCLUSIONS

The main aim of this paper was to minimize the number of tests required to predict mechanical characteristics of frozen soil as well as to present a more comprehensive perspective about the influencing factors and their interactions on the mechanical properties of frozen soil.

RSM was employed as a promoting statistical method to evaluate three case studies. Based on the obtained results, the following conclusions can be extracted. It should be mentioned that the conclusions are valid within the conditions expressed for each case study and may be unreliable for different types of frozen soil and/or loading paths.

- Temperature showed the most effective factor on mechanical properties among other factors.
- In spite of existing limitations, less than half of available data in the investigated case studies (average *efficiency factor* of 40.8%) was enough to provide reliable models, indicating efficiency of RSM in optimization.
- The key data such as response at the centre point was not readily available and it caused more efforts to develop reliable models.
- Optimum values of mechanical properties occur beyond of investigated domain of input factors in case study. However, values of input factors beyond the investigated range may be practically unfeasible.
- As a practical conclusion, three investigated experimental programs were not efficiently cover all probable correlations. While, the RSM can provide an optimum experimental design which can cover nearly all probable interactions. It should be noted that a RSM-

designed experiment necessarily requires less treatments (tests) with respect to traditionally designed experiments.

- Under the adopted conditions for the case studies, the RSM can potentially be considered to design optimum experiments and find significant factors affecting prescribed responses.
- It should be mentioned that size of original database plays an important role in precision of the models, so that a minimum threshold should be considered to minimize any probable errors.

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Persian Abstract

چکیده

بررسی آزمایشگاهی دقیق در خصوص خواص مهندسی خاک عموما بسیار زمان بر و احتمالا پر هزینه است. این روش به عنوان یک روش کارآمد برای بهینه سازی بسیاری از مسائل فیزیکی همچون کنترل میزان تولید و ساخت در صنعت بیش از دو دهه مورد استفاده بوده است. در تحقیق حاضر روش سطح پاسخ به عنوان یک ابزار برای توسعه مدل هایی جهت پیش بینی برخی خواص مکانیکی برای خاک های منجمد (مانند مقاومت پیک کششی و یا فشاری، مدول ارتجاعی، کرنش گسیختگی و غیره) مورد استفاده قرار گرفت. همچنین روش سطح پاسخ یافتن فاکتورهای معنی دار و اندرکنش های محتمل را میسر می‌سازد. پس از انجام یک بازبینی گسترده ادبیات فنی، سه مطالعه موردی به منظور ارزیابی عملکرد روش سطح پاسخ در تولید مدل‌های دقیق آماری انتخاب گردید. برای هر مطالعه موردی کمتر از نیمی از داده‌های قابل دسترس (تقریباً ٤٠/٨ درصد) به منظور توسعه مدل‌های آماری به کار گرفته شدند و باقی مانده داده‌ها به جهت صحت‌سنجی مدل‌های استخراج شده مورد استفاده قرار گرفتند. یک مقایسه میان مقادیر پیش بینی شده و اندازه‌گیری شده تطابق مناسبی با سطح معنی داری ۰/۰۵ مشاهد شد، که این امر نشان دهنده عملکرد قابل اعتماد مدل‌ها در محدوده فرضیات در نظر گرفته شده برای هر یک از نمونه‌های موردی است. به علاوه کانتورهایی به منظور ارائه یک توصیف جامع از همبستگی‌های محتمل میان خواص و فاکتورهای بررسی شده ارائه گردیده است. بر اساس مدل‌های استخراج شده که به طور متوسط دارای ضریب همبستگی (R^2) ۹۳/٦۹ می‌باشند، دما معنی‌دارترین فاکتور اثرگذار بر خواص مکانیکی خاک‌های ریزدانه منجمد به شمار می‌رود؛ در حالی که دانسیته خشک خاک به اندازه دما اثرگذار نمی‌باشد.