



## A New Model of Equivalent Modulus Derived from Repeated Load CBR Test

A. Salmi<sup>\*a</sup>, L. Bousshine<sup>a</sup>, K. Lahlou<sup>b</sup>

<sup>a</sup> Structural and Material Mechanics team, National High School of Electricity and Mechanics, Hassan II University of Casablanca, Casablanca, Morocco

<sup>b</sup> Research Team in Construction Engineering, LaGCHEC Laboratory, Hassania School of Public Works, Casablanca, Morocco

### PAPER INFO

#### Paper history:

Received 23 February 2020

Received in revised form 10 Mars 2020

Accepted 12 June 2020

#### Keywords:

Modulus

CBR

Granular Material

Least-squares Method

Stiffness

Strain Gauge

Resilient Behavior

### ABSTRACT

This paper presents a new model of equivalent modulus derived from the Repeated Load CBR (RL-CBR) test without strain gauge. This model is an updated version of Araya et al. model (2011), the update consists of using the vertical strain as weighting factor instead of vertical displacement in the mean vertical and horizontal stresses calculation. The accuracy of equivalent modulus was improved by decreasing the relative error from 25% to 3%. The extra-large mold adopted by Araya et al. is used with a thickness of 8 mm instead of 14.5 mm. In experimental investigations, equivalent modulus may be calculated from experimental data and model parameters estimated by finite element (EF) simulation. There are five model parameters when the RL-CBR test is used, and three parameters when the strain gauge is not used. Model parameters are determined in two steps. First, the FE simulation of the RL-CBR test is conducted using various loading conditions (i.e., plunger penetration) and various quality ranges of unbound granular materials (UGM). In the second step, the non-linear multidimensional regression is accomplished to fit the equivalent modulus to Young's modulus. The influence of FE analysis inputs is investigated to find the optimal inputs set that make the best compromise between the model accuracy and the calculation time consumption. The calculation of model parameters is carried out based on the optimal set data. Results from the new model and those from Araya et al. model are compared and have shown the improved accuracy of the developed model.

doi: 10.5829/ije.2020.33.07a.19

### NOMENCLATURE

CBR	Californian Bearing Ratio	$abs(.)$	Absolute value function
RL-CBR	Repeated load CBR	SDS	Standard deviation sum
RLT	Repeated Load Triaxial	RD	Relative deviation ( $10^{-3}$ )
LSM	Least-Squares Method	RE	Relative error ( $10^{-2}$ )
UGM	Unbound Granular Materials	$n$	Number of data line in a set of analyses
$M_r$	Resilient modulus (MPa)	$E_i$	Young's (exact) modulus for the $i^{\text{th}}$ dat line (MPa)
$q$	Deviator stress (kPa)	$E_{eqi}$	Equivalent modulus for the the $i^{\text{th}}$ dat line (MPa)
$u$	Resilient plunger penetration (mm)	<b>Greek Symbols</b>	
$f$	Constant factor (= 2 or $\pi/2$ )	$\sigma_{r1}, \sigma_{r3}$	Resilient axial and confining pressure (kPa)
$E$	Young's modulus (MPa)	$\epsilon_{r1}, \epsilon_{r3}$	Recoverable axial and radial micro-strains
$E_{eq}$	Equivalent modulus (MPa)	$\sigma_p$	Mean stress under plunger or plate (MPa)
$d$	Plunger or plate diameter	$\nu$	Poisson's ratio (-)
$k_1$ to $k_3$	Model parameters	$\sigma_{vi}$ and $\sigma_v$	Nodal and mean vertical stress (kPa)
$u_{vi}$	Nodal vertical displacement (mm)	$\sigma_{hi}$ and $\sigma_h$	Nodal and mean horizontal stress (kPa)
$k_{q1}$ to $k_{q5}$	Model parameters	$\epsilon_{vi}$	Nodal vertical strain (-)
		$\epsilon_{hm}$	Lateral micro-strain of mold at mid-height (-)

### 1. INTRODUCTION

Stiffness modulus of soils and Unbound Granular Materials (UGM) are main input data in the Mechanistic-Empirical (M-E) design process of flexible pavements

adopted in recent decades by many countries. However, its evaluation is a challenge in road engineering. In pavement engineering, many correlations allow for the estimation of granular materials modulus based on the

\*Corresponding Author: [abdelaiz.salmi@ensem.ac.ma](mailto:abdelaiz.salmi@ensem.ac.ma) (A. Salmi)

CBR index employed worldwide. Nevertheless, using elastoplastic simulation, a recent study [1] shows that the CBR index depends on other parameters, such as yield stresses in compression and compressibility index. At times, this modulus independent of Young's modulus. The Repeated Load Triaxial (RLT) test is the most accepted and widely used test in research laboratories to study the resilient and permanent behavior of these materials [2]. However, the configuration and equipment required for this test are technically complex and very expensive. Therefore, it is not part of every laboratory's facilities, especially, those in developing countries [3–5]. To overcome this challenge, the French standard of M-E road pavement design [6] adopts a modulus based on empirical classification of UGM. In the RLT test, the specimen is loaded by a confining pressure,  $\sigma_3$ , and an axial deviator stress,  $q$ , defined by Equation (1):

$$q = \sigma_{r1} - \sigma_{r3} \quad (1)$$

To simulate traffic load repetition,  $q$  is usually a periodic stress, but  $\sigma_3$  may be periodic for Variable Confining Pressure test variant (VCP) or not periodic for Constant Confining Pressure test variant (CCP). For the RLT test, the specimen has a diameter of 160 mm or 300 mm and a height of 320 mm or 600 mm, respectively. The RL-CBR test was validated based on resilient modulus derived from a large CCP triaxial test and equivalent modulus derived from the RL-CBR test with strain gauge [2, 7]. A steel mold with a 250 mm internal diameter, 200 mm height, and wall thickness of 14.5 mm was used in the experimental validation program. In this study, we consider the same mold, except that the wall thickness is taken equal to 8 mm. This choice was made in order to reduce the mold mass for practical use in experimental tests.

Experimental characterization of granular materials is a large research field; many empirical and theoretical models were proposed to describe their resilient behavior, as reported in existing reviews of findings in the field [8-10]. Their mechanical behavior depends on multiple parameters [11, 12]. Many tests can be employed to characterize the resilient behavior of unbound granular materials [13, 14]. The use of the uniaxial compressive test is also possible in the case of cohesive or bound granular materials [15,16]. When the RLT test is used, the resilient modulus is evaluated by Equations (2) and (3) for VCP variant and CCP variant, respectively, according to the European standards [17].

$$M_r = \frac{\sigma_{r1}^2 + \sigma_{r1}\sigma_{r3} - 2\sigma_{r3}^2}{\sigma_{r1}\epsilon_{r1} + \sigma_{r1}\epsilon_{r3} - 2\sigma_{r3}\epsilon_{r3}} \quad (2)$$

$$M_r = \frac{\sigma_{r1}}{\epsilon_{r1}} \quad (3)$$

In the framework of the RL-CBR test, the stiffness of UGM is evaluated by a resilient modulus designed by Araya [2] as an "equivalent modulus", while Molenaar

[4] calls it "effective modulus". Both nomenclatures were justified by the fact that the stress state throughout the specimen is not uniform. Therefore, the resilient modulus may vary throughout it due to stiffness-stress dependency for soils and granular materials. The equivalent or effective modulus is just a bulk measurement of the specimen stiffness, rather than an intrinsic material's characteristic. This approach is also adopted by Albayati et al. [18] to calculate equivalent modulus of asphalt concrete layers. The expression used is inspired by Boussinesq's Equation (4), valid in the case of the elastic isotropic semi-infinite solid loaded by a circular plunger. A detailed review on this equation is given by Timoshenko and Goodier [19]. For the RL-CBR test, a mold with finite dimensions is employed. Araya [2] suggested modifying the Equation (4) into Equation (5). Three model parameters are introduced. They are determined by the LSM (cf. 2. 2) applied on the RL-CBR test numerical data analysis. The LSM is a statistical method widely used to find the best fit for a set of inputs and outputs data points. Recently it's served for Shen and Zhou [20] improved the constitutive modelling of clay in drained and undrained conditions. When the equivalent modulus is calculated by Equation (5), a value of Poisson's ratio,  $\nu$ , should be specified. It is generally taken equal to 0.35 for soils and UGM in pavement design [6]. This test variant has demonstrated its ability to study the effect of moisture content, dry density and stress level in the experimental investigations of Haghghi et al. [21] using the staged RL-CBR test.

$$E = \frac{f(1-\nu^2)\sigma_{p2}^d}{u} \quad (4)$$

$$E_{eq} = \frac{k_1(1-\nu^{k_2})\sigma_{p2}^d}{u^{k_3}} \quad (5)$$

In case of the RL-CBR test with strain gauge, Araya et al. [7] used the nodal vertical displacements as weighting factor to estimate the mean vertical and horizontal stresses using Equations (6) and (7). Four transfer functions, Equations (8), (9), (10) and (11) are used to establish a regression fit between the mean vertical and horizontal stresses, with Poisson's ratio and equivalent modulus on one side and the RL-CBR test outputs on the other side. The use of this approach makes possible the comparison between resilient and equivalent moduli derived from the RLT and the RL-CBR tests, respectively [2,7]. In this model, the average weighted vertical and horizontal stresses are derived from nodal vertical and horizontal stresses using Equations (6) and (7). The nodal displacements through symmetry axis are used as a weighting factor. In this paper, the nodal strains are used as a weighting factor as in Equations (12) and (13) and were shown to be the most appropriate for accurate estimation of equivalent modulus. This change improves the accuracy of the initial Araya et al. model (cf. 3. 2).

$$\sigma_v = \frac{\sum \sigma_{vi} u_{vi}}{\sum u_{vi}}, \quad \sigma_h = \frac{\sum \sigma_{hi} u_{vi}}{\sum u_{vi}} \quad (1) \& (2)$$

$$\sigma_v = k_{g1} \sigma_p \quad (3)$$

$$\nu = k_{g2} \left( \frac{\epsilon_{hm}}{\sigma_p} \right) \quad (4)$$

$$\sigma_h = k_{g3} \epsilon_{hm} \exp(k_{g4} / \nu) \quad (5)$$

$$E_{eq} = \frac{k_{g5} (\sigma_v - 2\nu \sigma_h)}{u} \quad (6)$$

$$\sigma_v = \frac{\sum \sigma_{vi} \epsilon_{vi}}{\sum \epsilon_{vi}}, \quad \sigma_h = \frac{\sum \sigma_{hi} \epsilon_{vi}}{\sum \epsilon_{vi}} \quad (7) \& (8)$$

Characteristics of used materials and research methodology are presented in section 2. After that, the optimal set of parameters is determined based on model estimation accuracy. This set will be used to validate the modified Araya et al. model at the end of this paper.

## 2. MATERIALS AND METHODS

**2.1 Materials** In the present study, a linear elastic behavior is considered for the granular materials. Large quality ranges of UGM were studied by varying the Young's modulus value from 25 to 1000 MPa and Poisson's ratio from 0.15 to 0.45. The plunger penetration, used in the present study, varies from 0.1 mm to 3 mm. It should be noted that this penetration is smaller than 1 mm in previous experimental investigations [2,22].

In the first set, we consider Young's modulus from 25 to 1000 MPa (40 values), Poisson's ratio from 0.15 to 0.45 (4 values), and the plunger penetrations from 0.1 mm to 3 mm (30 values). In total, there were 4,800 simulations of the RL-CBR test, which would take a long time to calculate. As a consequence, we had to reduce the number of simulations by choosing the ones that offer an optimal and accurate estimation of the model parameters. Optimized lengths of Young's moduli and the plunger penetration lists were determined by reducing both lists' lengths. Thus, the model's accuracy is kept at an acceptable level. Tables 1 and 2 summarize parameters sets employed at this stage of the study. The flowchart research methodology is summarized in Figure 1.

## 2.2 Methods

**2.2.1. Finite Element Model of the RL-CBR Test** Finite element simulation of the RL-CBR test is performed with CAD software. As geometry, loading, and boundary conditions are in an axisymmetric disposition, a plane axisymmetric approach is used in the modelling process of the RL-CBR set-up. A linear elastic

material behavior is assumed for the steel mold with an elastic modulus of 210 GPa and a Poisson's ratio of 0.3 (rather than the 0.2 used by Araya et al. [7]), in addition to the granular material using the various elastic characteristics presented in Tables 1 and 2. As for the standard CBR test, the RL-CBR is a strain-controlled test with a uniform downward displacement of the plunger through material specimen [1,7,23]. The contact between the plunger and the specimen is assumed to be frictionless. The hard pressure-overclosure is adopted for the normal contact property between the mold and the specimen. For the tangential interaction, frictionless contact was chosen. These considerations mean that neither penetration nor friction between both parts will take place when local contact between them is established. The use of frictionless contact assumes that the internal mold surface is very smooth; a demolding oil is used in test preparation to replicate the smoothness. The same normal and tangential interaction models are considered for the plunger-specimen contact. Figure 2 Shows the axisymmetric model used in finite element analysis of the RL-CBR test. CAX8R, 8-node biquadratic axisymmetric quadrilateral reduced integration elements type is used for specimen mesh, because it offers an accurate FE analysis of a 3D problem using plane axisymmetric model [2].

**TABLE 1.** Sets of parameters tested to reduce the Young's modulus list length

Set	E step (MPa)	$\nu$ step (-)	u step (mm)	Number of simulations
Set 1	25	0.1	0.1	4800
Set 2	50	0.1	0.1	2400
Set 3	100	0.1	0.1	1200
Set 4	125	0.1	0.1	960
Set 5	200	0.1	0.1	600
Set 6	250	0.1	0.1	480
Set 7	500	0.1	0.1	240

**TABLE 2.** Sets of parameters tested to reduce the list of plunger penetrations u

Set	E step (MPa)	$\nu$ step (-)	u step (mm)	Number of simulations
Set 5	200	0.1	0.1	600
Set 8	200	0.1	0.2	300
Set 9	200	0.1	0.3	200
Set 10	200	0.1	0.5	120
Set 11	200	0.1	0.6	100
Set 12	200	0.1	1	60

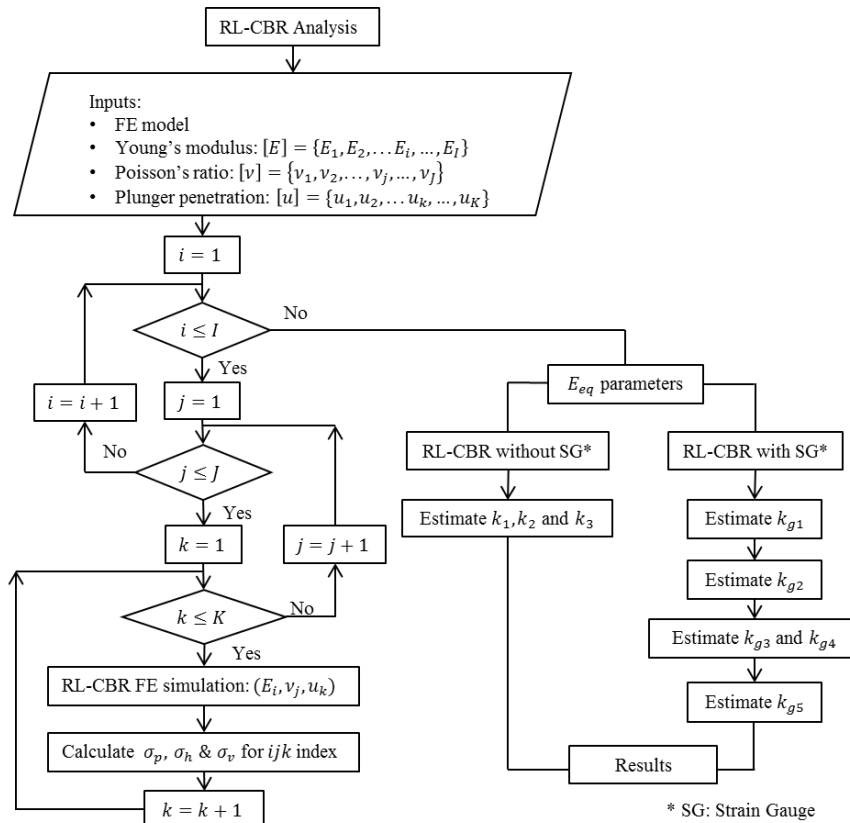


Figure 1. Flowchart of research methodology

A sensitive study of mesh size is undertaken to validate the model. A uniform mesh with a size from 0.25 mm to 8 mm for 8-node biquadratic axisymmetric quadrilateral reduced integration elements type is adopted. The granular material has a Young's modulus of 1000 MPa and a Poisson's ratio of 0.25. The plunger penetration was chosen equal to  $u=0.2$  mm. Figure 3 shows the variation of the obtained mean stress under the plunger vs. elements number in the model (in logarithmic scale). By reducing the mesh size from 8 mm to 0.25 mm, the mean stress was decreased from 3.65 MPa to 3.59 MPa with excessive start-slope that decreases to be very soft for fine mesh. The adopted mesh for this study illustrated in Figure 2 gives a mean stress under the plunger of 3.66 MPa. This value is very close to the fine mesh value, 3.95 MPa. The mesh is refined under and near the plunger where there is the stress concentration phenomenon and is made increasingly coarse away from this area. The model counts 780 elements instead of 442000 elements for the fine mesh model.

**2.2.1. Least-Squares Method** After conducting a set of RL-CBR numerical analyses (cf.2. 1), the data was organized as illustrated in Table 3. In the FE analysis of the RL-CBR test, Young's modulus  $E$ , Poisson's ratio  $\nu$

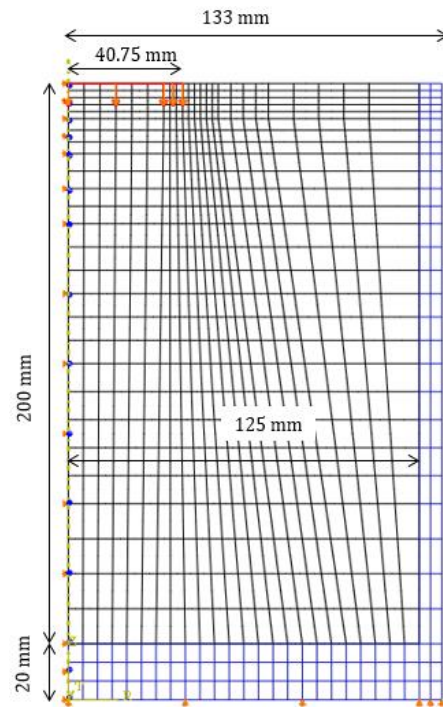
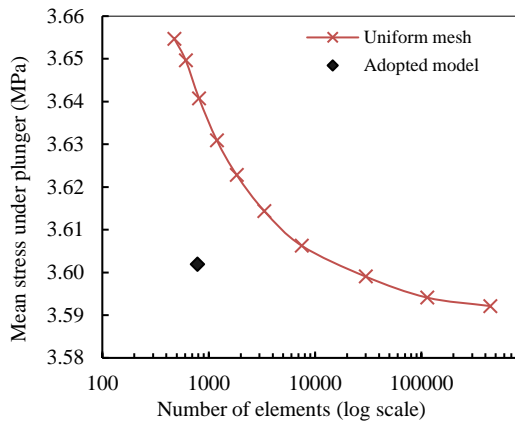


Figure 2. Axisymmetric finite element model of RL-CBR test with 8 mm thick mold



**Figure 3.** Mean stress under the plunger vs. number of elements ( $E = 1000$  MPa,  $\nu = 0.25$ ,  $u = 0.2$  mm)

**TABLE 3.** Table of variables for a set of analysis

Data line	Response $E$ (MPa)	Explanatory variables		
		$\nu(-)$	$u$ (mm)	$\sigma_p$ (MPa)
1	$E_1$	$\nu_1$	$u_1$	$\sigma_{p1}$
2	$E_2$	$\nu_2$	$u_2$	$\sigma_{p2}$
...	...	...	...	...
i	$E_i$	$\nu_i$	$u_i$	$\sigma_{pi}$
...	...	...	...	...
n	$E_n$	$\nu_n$	$u_n$	$\sigma_{pn}$

and plunger penetration  $u$  are the input parameters, and the mean stress under the plunger  $\sigma_p$  is the output parameter. In the regression analysis, with respect to Equation (5), elastic modulus is considered as the response variable and other parameters ( $\nu, u, \sigma_p$ ) as explanatory variables. However, the purpose is to derive an equivalent modulus expression to be used in the experimental characterization of granular materials using the RL-CBR test where the stiffness is researched. Resilient plunger penetration and mean resilient stress are measured during the test.

For a set of analyses, the main goal is to find the three model's parameters:  $k_1, k_2$ , and  $k_3$  that allow the estimation of the response variable given explanatory variables. These parameters should give an equivalent modulus as close as possible to the initial elastic modulus for each variable's combination. The non-linear multivariate regression problem is solved by the LSM. The LSM consists of researching parameters that minimize the Squared Deviations Sum (SDS) defined by Equation (14). The General Reduced Gradient (GRG) algorithm is applied to solve this non-linear optimization problem, which is summarized in Equation (15).

$$SDS(k_1; k_2; k_3) = \sum_{i=1}^{i=n} (E_i - E_{eqi})^2 \tag{14}$$

$$\text{Minimize } SDS(k_1; k_2; k_3) \tag{15}$$

Without technological constraints

### 3. RESULTS AND DISCUSSION

#### 3. 1. Optimal Parameters Set

Parameters of Equation (5) were determined for each data set presented in Tables 1 and 2. The values of these parameters resulting from simulations used to reduce the length of the Young's modulus list are summarized in Table 4 with the determination coefficient  $R^2$  for each set. Table 4 shows that the first parameter was decreased by increasing the step between consecutive values of Young's modulus from 1.653 for 25 MPa step set to 1.641 for 500 MPa step set. The second parameter was increased from 0.978 to 0.998, while the third parameter remained invariant for all seven sets. With respect to the determination coefficient, all correlations seem to be good except that of the 7th set, where the determination coefficient decreased to 0.990. However,  $R^2$  values presented in Table 4 cannot be used to compare the accuracy of the derived solutions because of the significant differences between the sample size of each set. To do this comparison, SDS's values are calculated for each solution with respect to the finite elements simulations' results of the largest sample (i.e., set 1). Figure 4 presents the variation of the SDS's RD when parameters derived from  $i^{th}$  set are used ( $SDS_i$ ) with respect to the SDS obtained when parameters derived from the 1<sup>st</sup> set are used ( $SDS_1$ ). RD for  $i^{th}$  set parameters is defined by Equation (16). Figure 4 shows that RD increases by increasing the Young's modulus step. For parameters obtained from set 7, RD is about 13 %, which is four times the RD obtained when parameters derived from set 6 are used and seven times the RD resulting from the use of set 5 parameters. By comparing the relative deviations related to the use of each set with respect to set 1, results show that set 5 offers an accurate estimation of the model parameters. This means that the length of Young's modulus list is divided by 8, keeping the accuracy of the estimation at the same level. This reduction may optimize the analysis time and facilitates the estimation of model parameters for other test configurations.

$$RD(set_i; set_1) = 1000 \frac{(SDS_i - SDS_1)}{SDS_1} \tag{16}$$

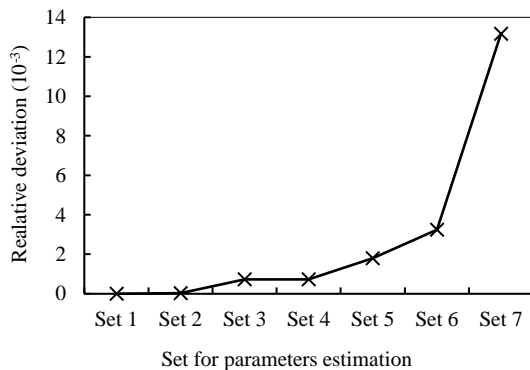
The same approach is used to reduce the length of plunger penetrations list. The starting set is set 5 (chosen above). For subsequent sets this penetration step is increased from 0.1 mm to 1 mm. Table 2 presents

adopted plunger penetration steps for each set. Table 5 summarizes obtained parameters in this part of the study. It is noted that the values of the parameters do not change much, and for the last four sets, they do not change at all.

These results indicate that reducing the length of the plunger penetration list does not significantly influence the values of the parameters of the model. This can be explained by the contact conditions between mold and specimen that make the problem linear. Then, the ratio of mean stress  $\sigma_{pi}$  by plunger penetration  $u_i$  was seen to be constant for a given specimen for each Poisson's ratio. Figure 5 shows the constancy of RD evaluated when obtained parameters from sets 5 and 8 to 12 are used. Only the parameters resulting from set 8 gave less accurate estimations. For the other cases, the same level of accuracy is maintained. To choose the appropriate set that makes the best compromise between the model accuracy and the calculation time, the non-linear character of the optimization problem had to be taken into account. Accordingly, the lengths of lists for all parameters must be at least 3, which is the case for set 12. To gain accuracy for other cases of simulation considering other analysis conditions, set 11 is chosen. In this set, plunger penetration takes 5 values, from 0.6 mm to 3 mm.

**TABLE 4.** Model parameters for sets used to reduce the list length of Young's modulus

Set	$k_1(-)$	$k_2(-)$	$k_3(-)$	$R^2$
Set 1	1.653	0.978	1.001	0.996
Set 2	1.652	0.979	1.001	0.996
Set 3	1.651	0.982	1.001	0.995
Set 4	1.650	0.983	1.001	0.995
Set 5	1.648	0.986	1.001	0.995
Set 6	1.647	0.988	1.001	0.995
Set 7	1.641	0.998	1.001	0.990



**Figure 4.** Comparison of obtained model parameters' accuracy to reduce Young's modulus list length

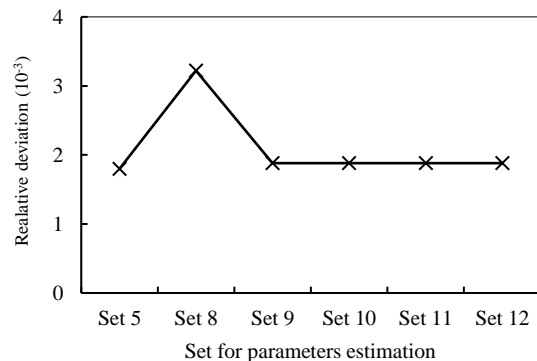
**TABLE 5.** Model parameters for sets used to reduce the list length of plunger penetrations

Set	$k_1(-)$	$k_2(-)$	$k_3(-)$	$R^2$
Set 5	1.648	0.986	1.001	0.995
Set 8	1.648	0.987	1.001	0.995
Set 9	1.647	0.987	1.000	0.995
Set 10	1.647	0.987	1.000	0.995
Set 11	1.647	0.987	1.000	0.995
Set 12	1.647	0.987	1.000	0.995

Moreover, Poisson's ratio takes four values from 0.15 to 0.45 in 0.1 steps. To reduce the number of values considered here, we removed the first value, and we saw the accuracy of the solution of the problem (15). The obtained solution is:  $k_1 = 2.062$ ,  $k_2 = 0.698$  and  $k_3 = 1.000$  with  $R^2 = 0.998$ . When these values are used and compared to set 1 data, the SDS is 3 times higher than the minimal SDS of set 1. After removing the second value (0.25), the SDS is 27 times the minimal SDS. These tests show that the reduction of Poisson's ratio list length affects the accuracy of the model, so the initial Poisson's ratio list is maintained as in the set 11. In practical use of Equation (5), the accuracy of the model must be considered in estimating equivalent modulus using Relative Error (RE) defined by Equation (17) and given in Table 6 for various combinations of Poisson's ratio and equivalent modulus.

$$RE(E; E_{eq}) = \frac{abs(E - E_{eq})}{E} * 100 \tag{17}$$

To compare the accuracy of this solution with previous studies, Table 7 summarizes the values of parameters of present and previous studies and the ratio of SDS per the reference SDS obtained for set 1 data when parameters for the set 11 are used. After this comparison, the use of Araya's solution [2] induced a model half as accurate as the solution of the current study. It is noted that for this study, a particular model for the specimen-mold contact



**Figure 5.** Comparison of accuracy of model parameters used to reduce the list length of plunger penetrations

**TABLE 6.** Relative error of equivalent modulus derived from RL-CBR test without strain gauge

(MPa)	$\nu(-)E_{eq}$			
	0.15	0.25	0.35	0.45
200	2.1	3.3	5.2	0.8
400	3.1	4.5	1.0	1.6
600	3.8	3.0	1.8	4.0
800	5.3	1.9	3.8	6.4
1000	2.0	3.6	5.8	2.3

is considered: hard for normal contact and frictionless for the tangential one.

For equivalent modulus derived from the RL-CBR with strain gauge using Equations (8) to (11), model parameters are estimated based on the set 11 data (results shown in Table 8). For comparison purposes, the values of the parameters found by Araya et al. [7] are also summarized in the same table. Regression fit of the four transfer functions to the simulation results shows a good correlation with  $R^2 = 0.999$  for average vertical and horizontal stress, 0.975 for Poisson's ratio and 0.952 for

**TABLE 7.** Comparison with previous studies for the RL-CB without strain gauge case

Study	Tangential contact	Normal contact	$k_1$	$k_2$	$k_3$	$SDS/SDS11$
Present study	Frictionless	hard	1.647	0.987	1.000	1
Salmi et al. [24]	Frictionless	exponential	1.377	1.552	1.056	6
Araya et al. [2]	intermediate friction	exponential	1.513	1.104	1.012	2

**TABLE 8.** Estimated parameters for present and previous studies

Parameter	Present study	Araya et al. [7]
$k_{g1}$ (-)	0.478	0.368
$k_{g2}$ (kPa <sup>-1</sup> )	62.027	120.927
$k_{g3}$ (kPa)	31.552	43.898
$k_{g4}$ (-)	0.095	0.072
$k_{g5}$ (mm)	0.139	0.144

the equivalent modulus. The difference between the values of the parameters in this study and that of Araya et al. [7] can be attributed to the mold thickness (8 mm instead of 14.5 mm), tangential-normal contact model (i.e. frictionless-hard instead of intermediate friction-exponential), and the accuracy of the finite element model. Particularly, the thickness effect is notable for the estimation of the Poisson's ratio: the ratio of parameters of the model for Araya et al. [7]  $k_{g2}(Ara)$  per that of the present study  $k_{g2}(ps)$  is approximately equal to the ratio of thicknesses in both studies:  $t(Ara)$  and  $t(ps)$ , respectively, as expressed by Equations (18) and (19).

$$\frac{k_{g2}(Ara)}{k_{g2}(ps)} = 1.942, \frac{t(Ara)}{t(ps)} = 1.813 \quad (18)\&(19)$$

**3.2 A New Model for Equivalent Modulus** Even if the parameters of Araya et al. model adopted for the RL-CBR test with strain gauge are derived with good determination coefficients ( $R^2$ ), it is found that this model sometimes makes inaccurate estimations of equivalent

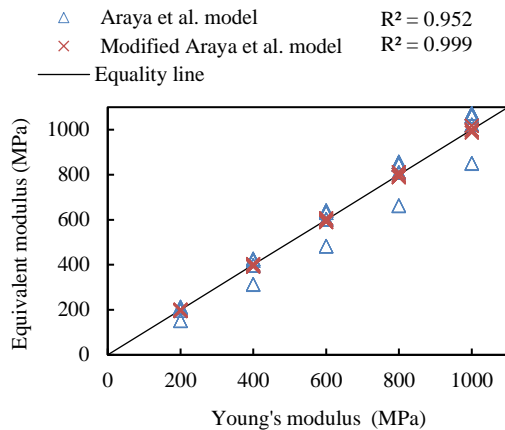
modulus, especially for  $\nu = 0.45$ , where the relative error was between 20% and 25% for many simulations.

The suggested model consists of keeping the same transfer functions, Equations (8)-(11), but nodal displacement is replaced by nodal strain in the calculation of vertical and horizontal weighted average stresses. Equations (6) and (7) are replaced by Equations (12) and (13) in the new model, named Modified Araya et al. model for the RL-CBR test with strain gauge. Table 9 summarizes estimated parameters for the new model with determination coefficient  $R^2$  for each parameter. It's noted that the second parameter,  $k_{g2}$ , is the same for both models, since the estimation of Poisson's ratio does not depend on the mean vertical and horizontal stresses expressions as in Equation (9).

The plot of equivalent moduli estimated by Araya et al. and modified Araya et al. models versus exact elastic modulus used in finite element analyses of the RL-CBR test is shown in Figure 6. For all analyses, the predictions of the developed model are more accurate than those estimated using Araya et al. model. Maximum relative error of estimated equivalent modulus for various

**TABLE 9.** Modified Araya et al. model parameters

Parameter	Value	$R^2$
$k_{g1}$ (-)	0.431	1.000
$k_{g2}$ (kPa-1)	62.027	0.975
$k_{g3}$ (kPa)	18.848	0.996
$k_{g4}$ (-)	0.140	
$k_{g5}$ (mm)	0.140	0.999



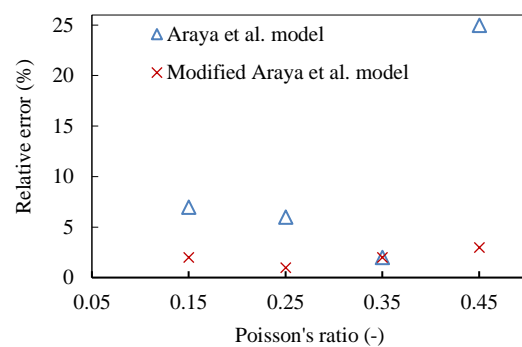
**Figure 6.** Predicted equivalent modulus vs. Young's modulus used in FE analyses

Poisson's ratios are shown in Figure 7 for both models. The obtained relative error for Araya et al. model is usually higher than 5%, except for  $\nu = 0.35$ , where it is lower than 2%. Meanwhile, it does not reach 3% for modified Araya et al.'s, even for  $\nu = 0.45$ , where the mean relative error for Araya et al. model was equal to 20% and the maximum one was equal to 25%. It's noted that for  $\nu = 0.35$ , both models' estimations are in the same accuracy level with a RE less than 2%. Meanwhile, Poisson's ratio is estimated using the same expression (4) for both models. The plot of mean estimated Poisson's ratio vs. real Poisson's ratio for various Young's moduli is shown in Figure 8.

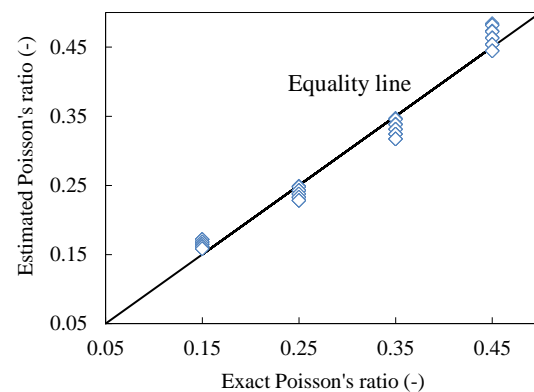
The Poisson's ratio is overestimated for high and low values (i.e., 0.15 and 0.45), while it is underestimated for intermediate values (i.e., 0.25 and 0.35) as shown in Figure 8. Additionally, the relative error of Poisson's ratio higher than 0.25 is below 10%. Accordingly, the predicted Poisson's ratio for soils and unbound granular materials around  $\nu = 0.35$  can be used in road pavement and geotechnical engineering. The modified model has demonstrated a good accuracy compared to its initial version in the equivalent modulus estimation for all Poisson's ratio. For experimental investigations, the intrinsic relative error of the model should be added to that of the equipment used. Then, the final results can be analyzed carefully. Table 10 shows relative error, for various combinations of Poisson's ratio and equivalent modulus to be considered in experimental investigations. For cases with values different than the ones given, linear interpolation can be utilized to estimate the corresponding RE.

The Poisson's ratio is overestimated for high and low values (i.e., 0.15 and 0.45), while it is underestimated for intermediate values (i.e., 0.25 and 0.35) as shown in Figure 8. Additionally, the relative error of Poisson's ratio higher than 0.25 is lower than 10%. Accordingly, the predicted Poisson's ratio for soils and unbound granular materials around  $\nu = 0.35$  can be used in road

pavement and geotechnical engineering. The modified model has demonstrated a good accuracy compared to its initial version in the equivalent modulus estimation for all Poisson's ratio. For experimental investigations, the model's intrinsic relative error should be added to that of the equipment used. Then, the final results can be analyzed carefully. Table 10 shows relative error, for various combinations of Poisson's ratio and equivalent modulus to be considered in experimental investigations. For cases with values different than the ones given, linear interpolation can be utilized to estimate the corresponding RE.



**Figure 7.** Equivalent modulus maximum relative error for Araya et al. and modified Araya et al. models



**Figure 8.** Estimated Poisson's ratio vs. exact Poisson's ratio

**TABLE 10.** Relative error (%) of Modified Araya et al. model

$\nu (-) E_{eq}$ (MPa)	0.15	0.25	0.35	0.45
200	2.1	0.7	1.1	1.2
400	1.1	0.8	1.9	1.3
600	0.4	2.5	1.4	0.1
800	3.2	1.6	0.2	1.7
1000	1.8	0.4	1.4	0.5



#### 4. CONCLUSIONS AND OUTLOOKS

The paper presented a finite element simulation of the RL-CBR test with an 8 mm thick extra-large mold. The contact between the granular specimen and steel mold was assumed to be frictionless. In case of the RL-CBR test without strain gauge, the model parameters used to calculate equivalent modulus are estimated. For the repeated load CBR test with strain gauge parameters of Araya et al. and modified Araya et al. model are estimated. The comparison between the estimations of both models showed that the accuracy was remarkably increased through this modification, especially for materials with high and low Poisson's ratio. For material with Poisson's ratio of 0.35, the accuracy of estimation is kept in the same level.

This research will continue, in one side, by investigating the effect of the contact type between the specimen and mold on parameters of the model and, in another side, by an experimental validation of the derived equivalent modulus. This validation will be based on the resilient modulus derived from the RLT laboratory test and reaction modulus derived from plate and Westergaard in-situ tests.

Equivalent modulus may be used as a comparative tool for UGM quality ranging. But its use in the M-E design of pavements requires other laboratory and full-scale investigations.

#### 5. ACKNOWLEDGMENT

- Abdelaziz Salmi is grateful to Hassan II academy of sciences and technology for its financial support of this research in the framework of the scholarships program for winners of CGST competition.

- The authors thank Y. Waddad for his support on the use of the CAD software and M. Farah, M. Esskely, H. El Fatihi, R. Ibouk El Idrissi, E. El Bahjaoui and F. Kassou for the review of the manuscript.

#### 6. REFERENCES

- Mendoza C., Caicedo B. "Elastoplastic framework of relationships between CBR and Young's modulus for granular material". *Road Materials and Pavement Design*. Vol. 19, No. 8, (2017). DOI:10.1080/14680629.2017.1347517.
- Araya A.A. "Characterization of unbound granular materials for pavements", PhD thesis, Delft University of Technology, (2011).
- Araya A.A., Huurman M., Molenaar A.A.A. "Integrating traditional characterization techniques in mechanistic pavement design approaches". In 1st Congress of Transportation and Development Institute, Chicago, Illinois, United States, March 13-16 2011, American Society of Civil Engineers, (2011), 596-606, DOI: 10.1061/41167(398)57
- Molenaar A.A.A. "Repeated load cbr testing, a simple but effective tool for the characterization of fine soils and unbound materials". In Transportation Research Board 87th Annual Meeting, January 13-17 2008, Washington, United States, (2008).
- Molenaar A.A.A. "Characterization of Some Tropical Soils for Road Pavements". *Transportation Research Record: Journal of the Transportation Research Board*, 2007, Vol. 1989-2, No. 1, 186-193. DOI: 10.3141/1989-63
- AFNOR. NF P 98-086- "Dimensionnement structurel des chaussées routières- application aux chaussées neuves". 2011.
- Araya A.A., Huurman M., Molenaar A.A.A., Houben L.J.M. "Investigation of the resilient behavior of granular base materials with simple test apparatus", *Materials and Structures*, (2012), Vol. 45, No. 5, 695-705. DOI:10.1617/s11527-011-9790-1
- Han Z., Vanapalli S.K. "State of the art: prediction of resilient modulus of unsaturated subgrade soils". *International Journal of Geomechanics*, (2016), Vol. 16, No.4, 1-15. DOI:10.1061/(ASCE)GM.1943-5622.0000631.
- Fredrick L., Ulf I., Andrew D. "State of the art. I: resilient response of unbound aggregates", *Journal of Transportation Engineering*, (2000), Vol. 126, No. 1, 66-75. DOI: 10.1061/(ASCE)0733-947X(2000)126:1(66)
- Saada A., Townsend F., "State of the art: laboratory strength testing of soils", in Laboratory Shear Strength of Soil, ed. R. Yong and F. Townsend (West Conshohocken, PA: ASTM International), (1981), 7-77. DOI:10.1520/STP28744S
- Minassian G.H. "Behavior of granular materials under cyclic and repeated loading", Ph.D thesis, University of Alaska Fairbanks, United States, (2003).
- Gidel G. "Comportement et valorisation des graves non traitées calcaires utilisées pour les assises des chaussées souples", Thèse de doctorat, Université Bordeaux I, France, (2001).
- Semmelink C.J. "The use of DRTT K-mould to determine the elastic and shear properties of pavement materials", Report No. 89/149, Department of Transport, South Africa, (1991).
- Edwards P., Thom N., Fleming P.R., Williams J. "Testing of unbound materials in the nottingham asphalt tester springbox", *Transportation Research Record: Journal of the Transportation Research Board*, (2005), Vol. 1913, No. 1, 32-40. DOI: 10.1177/0361198105191300104
- Jalili M, Ghasemi M.R., Pifloush A.R., "Stiffness and strength of granular soils improved by biological treatment bacteria microbial cements". *Emerging Science Journal*, (2018), Vol. 2, No. 4, 219-227, DOI: 10.28991/esj-2018-01146
- Ogundipe O.M., Adekanmi J.S., Akinkulore O.O., Ale P.O., "Effect of compactive efforts on strength of laterites stabilized with sawdust ash". *Civil Engineering Journal*, (2019), Vol. 5, No. 11, 2502-25014, DOI: 10.28991/cej-2019-03091428
- CEN. "EN 13286-7- Unbound and hydraulically bound mixtures - Part 7 : Cyclic load triaxial test for unbound mixtures", European committee for standardization, (2004).
- Albayati A.H, Al-Mosawe H, Fadhil A.T., Allawi A.A. "Equivalent Modulus of Asphalt Concrete Layers", Vol. 4, No. 10, *Civil Engineering Journal*, (2018), 2264-2274. DOI:10.28991/cej-03091156
- Timoshenko S, Goodier J.N. "Theory of elasticity", ed. McGraw-Hill, (1951), 532.
- J. Shen et X. Zhou, "Least Squares Support Vector Machine for Constitutive Modeling of Clay", *International Journal of Engineering Transactions B: Applications*, Vol. 28, No. 11, (2015), 1571-1578.
- Haghighi H., Arulrajah A., Mohammadinia A., Horpibulsuk S. "A new approach for determining resilient moduli of marginal pavement base materials using the staged repeated load CBR test

- method". *Road Materials and Pavement Design*, Vol. 19, No. 8 (2017), 1848-1867. DOI: 10.1080/14680629.2017.1352532
22. Abid A.N., Salih A.O., Nawaf E.A. "The influence of fines content on the mechanical properties of aggregate subbase course material for highway construction using repeated load CBR test". *Al-Nahrain Journal for Engineering Sciences*, Vol. 20, No. 3, (2017), 615-24. URL: nahje.com/index.php/main/article/view/252.
23. Gu J. "Computational modeling of geogrid reinforced soil foundation and geogrid reinforced base in flexible pavement", PhD thesis, Louisiana State University, (2011).
24. Salmi A., Bousshine L., Lahlou K., "Two unbound granular materials stiffness analysis with staged repeated load CBR test," in 14<sup>th</sup> Congress of Mechanics, Rabat, Morocco, 16-19 Apr. 2019, DOI: 10.1051/mateconf/201928606003.

---

### Persian Abstract

---

#### چکیده

این پژوهش یک مدل جدید از مدول معادل حاصل از آزمایش بار تکراری CBR (RL-CBR) بدون استفاده از کرنش سنج را ارائه می‌دهد. این مدل یک نسخه‌ی به‌روز شده از مدل آریا و همکاران (۲۰۱۱) است. مدل، به‌روزرسانی شده شامل استفاده از کرنش عمودی به عنوان ضریب وزنه به جای جابه‌جایی عمودی در محاسبه‌ی میانگین تنش‌های عمودی و افقی است. دقت مدول معادل با کاهش خطای نسبی از ۲۵٪ به ۳٪ بهبود یافت. قالب فوق‌العاده بزرگی که توسط آریا و همکاران پذیرفته شده است، به جای ضخامت ۱۴.۵ میلی‌متر از ضخامت ۸ میلی‌متر استفاده می‌شود. در تحقیقات تجربی، مدول معادل ممکن است از داده‌های تجربی و پارامترهای مدل تخمین زده شده توسط شبیه‌سازی المان محدود (EF) محاسبه شود. هنگام استفاده از آزمون RL-CBR پنج، و در صورت عدم استفاده از کرنش سنج، سه پارامتر مدل وجود دارد. پارامترهای مدل در دو مرحله مشخص می‌شوند. ابتدا، شبیه‌سازی FE از آزمایش RL-CBR با استفاده از شرایط بارگذاری مختلف (به عنوان مثال، نفوذ سمیه) و دامنه‌های مختلف کیفیت مواد گرانول نجسبیده (UGM) انجام می‌شود. در مرحله‌ی دوم، رگرسیون چندبعدی غیرخطی انجام می‌شود که برابر با مدول یانگ می‌باشد. تأثیر ورودی‌های تحلیل FE برای یافتن مجموعه‌های ورودی بهینه که بهترین سازش بین دقت مدل و صرف زمان محاسبه را دارند، بررسی شده است. محاسبه‌ی پارامترهای مدل بر اساس داده‌های مجموعه بهینه انجام می‌شود. نتایج مدل جدید و نتایج مدل آریا و همکاران با هم مقایسه شده و دقت بهبودیافته‌ی مدل توسعه‌یافته را نشان داده است.

---