



A Stochastic Model for Prioritized Outpatient Scheduling in a Radiology Center

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ABSTRACT

This paper discussed the scheduling problem of outpatients in a radiology center with an emphasis on priority. For more compatibility to real-world conditions, we assume that the elapsed times in different stages to be uncertain that follow from the specific distribution function. The objective is to minimize outpatients' total spent time in a radiology center. The problem is formulated as a flexible open shop scheduling problem and a stochastic programming model. By considering the specific distribution function for uncertain variables, deterministic mixed integer linear programming (MILP) is developed such that the proposed problem can be solved by a linear programming solver in small size. Besides an effective heuristic method is proposed for the moderate size problem. To indicate the applicability of the proposed model, it has been applied to a real radiology center. The results from the proposed optimization models indicate an increase in outpatients' satisfaction, as well as the improvement of the efficiency and productivity of the radiology center.

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NOMENCLATURE

| Indices | | Parameters | |
|-------------------------------|--|---------------------|---|
| $i, j \in \{1, 2, \dots, n\}$ | Patient indices | m, n | Number of stages, Number of patients |
| $k, l \in \{1, 2, \dots, m\}$ | Stage indices | ξ_{ik} | The length of stay of patient i in stage k |
| $r \in \{1, 2, \dots, r(k)\}$ | Index of the number of parallel resources (resources or places) in each stage | ϕ_k | The uncertainty distribution of ξ_{ik} |
| Variables | | q_i | Priority index of patient i |
| X_{ikl} | 1 if patient i passes through stage k before stage l , $k < l$ | L | A large number |
| Y_{ijk} | 1 if patient i before patient j passes through stage k , $\forall i, j, i < j$ | $(1 - \varepsilon)$ | Confidence level |
| Z_{ikr} | 1 if patient i at stage k receives healthcare from resource r | $F_{\xi_{ik}}^{-1}$ | The inverse function of the normal distribution of elapsed time of patient i in stage k |
| C_{ik} | Completion time of patient i at stage k | C^0 | The vector of objective function coefficients under uncertainty |
| C_{im} | Completion time of patient i (at the final stage) | A^0 | The matrix of objective function coefficients under uncertainty |
| | | β^0 | The vector of sources under uncertainty |
| | | u^0 | The upper bound of variables under uncertainty |

1. INTRODUCTION

Health care systems are the most costly and revenue part of each country's social security system. To provide

qualified services as well as controlled costs, health system management is vital. The related literature focuses on either the qualified patient or resource scheduling [1]. Qualified patient scheduling means

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patient satisfaction enhancement. Reducing the elapsed and waiting times of patients in healthcare systems are among the main objectives of patient scheduling [1, 2]. Unfortunately, outpatient scheduling is difficult due to natural and artificial uncertainties. However, the focus of the literature is on the uncertainty caused by patient arrival variability, variability in health service duration, and length of stay. Such uncertainties in surgical suite scheduling have been mentioned [3-5]. From the mentioned uncertainties, service duration uncertainty is considered in this paper.

In addition to the inherent uncertainties in healthcare system scheduling problems, prioritizing patients also enhances the problem difficulty [6]. The weight or priority level of patients and the necessity of using medical services are determined by the triage factor which includes five levels [6]. A higher priority level indicates a deterioration in the patient's physical condition and a more urgent need for medical care. Considering a five-level based patient's priority has approached this paper to the real cases.

Although the patient's condition is frequently examined to determine the path of each patient and this path is not settled in many cases. For instance, transfer the patient to each stage in a specialized hospital radiology center is often not predetermined (as defined in an open shop scheduling problem). Besides, each stage commonly consists of more than one device. Hence, formulating the prioritized stochastic outpatient scheduling problem in a radiology center as a stochastic flexible open shop scheduling problem is logical. Despite considerable research carried out in the field of the flexible flow shop scheduling problem [7], few studies have been addressed the flexible open shop problem [8] but no paper focuses on the stochastic flexible open shop scheduling (SFOSS) problem. In this paper, we formulate prioritized patients scheduling as an SFOSS problem under the minimization of total prioritized elapsed times (total weighted completion times). Considering specific distribution functions (as normal) for health service duration, the problem is converted to a deterministic mixed-integer programming model.

The rest of this paper is organized as follows. A concise survey of related work is presented in the next section. In section 3, a basic foundation of a flexible open shop scheduling problem, as well as a stochastic programming model, is first described. Afterward, the deterministic MILP model is developed. Sections 4 and 5 provide computational results and conclusions.

2. LITERATURE REVIEW

Elective and emergency patients have been widely considered in patient scheduling problems [1, 9]. The most interesting researches in the field of patient

scheduling concerns elective patients [2]. In this paper, we focus on prioritized elective patient scheduling. Considering patients' priority and capacity constraints, a stochastic dynamic programming model was proposed for surgical suite scheduling [6]. Considering patient priority and arrival time, a reactive scheduling method proposed to the patient's waiting time minimization [10].

From the mentioned patient scheduling problem uncertainties, service duration uncertainty is considered in this paper. To deal with healthcare service duration uncertainty, the following methods are mainly reported:

- predicting service duration using statistical methods [11, 12];
- estimation of service duration using practical percentile value [3]
- assuming deterministic service duration [4, 13-15]
- modelling uncertainty in healthcare service duration and /or patient length of stay using stochastic programming [5],
- chance constraint programming [16, 17] and
- Stochastic chance constraint programming [18].

Zhao and Li [19] presented a mixed integer programming (MINLP) model for operating room (OR) scheduling and proposed a constraint programming model to solve it. Wang et al. [20] handled the surgical suite problem subject to emergency surgery as a no-wait permutation flow shop scheduling problem and employed slack time insertion method to generate a predictive schedule. Jebali and Diabat [5] considered the OR and intensive care unit (ICU) capacity constraint as well as the uncertainty of surgery time duration and patient's stay time in the ICU and the ward, they proposed two-stage stochastic programming model and employed sample average approximation (SAA) method to solve the problem. Saadouli et al. [3] discussed the real case problem of scheduling elective surgeries by considering the uncertainty of surgery and recovery time durations. They used a realistic estimation of surgery and recovery duration to protect against overtime and under-utilization of the OR. They first employed a knapsack problem to select the operation to be scheduled, and then a MILP model was presented to assign the selected operation to ORs. Heydari and Soudi [4] formulated the surgical suite scheduling problem as a flexible flow shop scheduling problem. They considered emergency patient arrival as the source of uncertainty. They proposed a two-stage stochastic programming model to generate a robust and stable surgery schedule.

Decision making under uncertainty comes up in the lack of access to a variable probability distribution. In this case, chance constraint programming is one of the methods of uncertainty modeling in optimization. Otherwise, when a variable probability distribution it assumed to be known, applying stochastic programming is common [21-24]. Shylo et al. [16] are among the first

to apply chance constraint programming in surgery scheduling. They assume a normal distribution for surgery duration and converted the chance constraint programming model into an equivalent convex programming model. Former reports also apply a normal or lognormal distribution for OR durations [25].

The chance constraint programming model can be converted into a crisp one under the assumption of a special uncertainty distribution (e. g. normal distribution for surgery duration). In the case of general uncertainty distributions, the chance constraint programming model has generally approximated via sampling-based approaches (SAA and scenario approximation) and analytical approximation methods (e. g. robust optimization) [26]. Deng et al. [17] proposed a stochastic chance constraint programming model to allocate and schedule surgeries in parallel ORs under undetermined problem parameters. Jebali and Diabat [18] applied SAA to approximate (flexible flow shop) stochastic chance constraint programming model under uncertain OR duration and patient length of stay (LOS) in ICU.

In spite of some results reported for stochastic flexible flow shop scheduling problem [27], few studies have been stated, even in the deterministic version of stochastic flexible open shop scheduling problem [8]. From the surgical suite modeling aspect, the literature focuses on modeling the surgery scheduling problem as parallel machine scheduling problem [19], flow shop scheduling problem [20], and flexible flow shop scheduling problem [3-5]. The essence of scheduling patients in a radiology center and a flexible open shop scheduling problem has similarities. So, it seems logical to formulate patients' scheduling problems in the radiology center as a flexible open shop scheduling problem. To take advantage of effective methods of shop scheduling problem-solving, the problem is formulated as a flexible open shop scheduling problem and a stochastic programming model. Consequently, considering the specific distribution function for uncertain variables, a deterministic mixed integer programming is developed such that the proposed problem can be solved by a linear programming solver. To the best of our knowledge, no paper dealing with a stochastic flexible open shop (SFOS) as a model for the radiology scheduling problem under uncertainty.

3. PROBLEM STATEMENT& FORMULATION

In this paper, we investigate the prioritized elective radiology center scheduling problem with more than one resource in different stages such as ultrasound, magnetic resonance imaging, radiography, and computerized tomography (CT) scan. In a radiology center, patients do not need to go through all stages. That means the elapsed time of patients in some stages is zero. Also, transferring

patients to a different stage is not predetermined. Moreover, as in reality, the elapsed time of patients in stages is not deterministic. Here, the under discussion problem is formulated as a SFOSS problem. Then, considering the specific distribution function for uncertain variables, a SFOSS model is converted to a deterministic mixed-integer linear programming model. The SFOSS problem consists of n patients (jobs) that should be processed on at most m stages (machines). There is more than one resource in at least one stage. The sum of the elapsed time of all prioritized patients (sum of weighted completion time of jobs) is considered to be minimized. The notations of the models have been introduced on the first page. Problem assumptions are as follows.

- Each patient should be served on at most m stages.
- The serving route of each patient is arbitrary.
- At any time, at most one patient can be served on each resource.
- All patients are available at time 0.
- No preemption is allowed.
- All the patients' serving (processing) times at each stage is not deterministic and considered as random parameters with an independent normal distribution.

3. 1. Stochastic Programming Model When all or some of the data parameters (in a linear programming (LP) model) are characterized by random variables, we deal with (linear) stochastic programming (SP). A usual matrix form of SP model may be as follows [28]:

$$\begin{aligned} \min z &= C^0 X \\ X &\geq b^0 \\ 0 \leq X &\leq u^0 \\ \theta &= (C^0, A^0, b^0, u^0) \end{aligned}$$

It is supposed that the probability distribution of θ is known. An SP model of flexible open shop scheduling problem is as follows.

$$\min \sum_{i=1}^n q_i C_{im} \quad , \text{ s.t.} \tag{1}$$

$$C_{ik} \geq \xi_{ik} \quad \forall i, k \tag{2}$$

$$\begin{cases} C_{ik} \geq C_{il} + \xi_{ik} - L(1 - X_{ikl}) & \forall i, k, l > k \\ C_{il} \geq C_{ik} + \xi_{ik} - LX_{ikl} & \forall i, k, l > k \end{cases} \tag{3}$$

$$C_{il} \geq C_{ik} + \xi_{ik} - LX_{ikl} \quad \forall i, k, l > k \tag{4}$$

$$\begin{cases} C_{ik} \geq C_{jk} + \xi_{ik} - L(1 - Y_{ijk}) - L(2 - Y_{ikr} - Y_{jkr}) \\ \forall i, j > i, k, r \\ C_{jk} \geq C_{ik} + \xi_{ik} - LY_{ijk} - L(2 - Y_{ikr} - Y_{jkr}) \\ \forall i, j > i, k, r \end{cases} \tag{5}$$

$$C_{jk} \geq C_{ik} + \xi_{ik} - LY_{ijk} - L(2 - Y_{ikr} - Y_{jkr}) \quad \forall i, j > i, k, r \tag{6}$$

$$\sum_{r=1}^{r(k)} Z_{ikr} = 1 \quad \forall i, k \quad (7)$$

$$C_{im} \geq C_{ik} \quad \forall i, k \quad (8)$$

$$X_{ikl} \in \{0, 1\} \quad \forall i, k, l > k \quad (9)$$

$$X_{ijk} \in \{0, 1\} \quad \forall i, j > i, k \quad (10)$$

$$Z_{ikr} \in \{0, 1\} \quad \forall i, k, r \quad (11)$$

The objective function is represented in Constraint 1, which aims to minimize the prioritized total elapsed time of all patients. The relationship between completion time and elapsed time of each patient is represented in Constraint 2. Constraints 3 and 4 are sequence constraints. That is, a patient can take at most one medical care at a time. Constraints 5 and 6 are resource constraints. That is, at each resource, only one patient can be served (i.e. a patient can be served before or after another patient on a particular resource). Each patient is assigned to one resource at each stage in Constraint 7. The maximum completion time calculation of each patient is performed in Constraint 8. Constraints 9-11, define decision variables.

3. 2. Mixed Integer Linear Programming Model

We assume that patients' staying time at each stage follows from the independent normal distribution. The inverse function of the normal distribution of elapsed time of patient i in stage k ($F_{\xi_{ik}}^{-1}$) is calculated such that the confidence level is $(1-\varepsilon)$. Considering the specific distribution function for uncertain variables, a stochastic programming model is converted to a deterministic mixed-integer programming model [28]. The MILP model for SFOSS problem is presented from 12 to 22. The interpretation of the constraints is the same as the previous model. Constraints 13 and 2 are equivalent. Constraints 14, 15 illustrate sequence constraint on $(1-\varepsilon)$ level of confidence. Constraints 16, 17 ensure that each place (or service) can be assigned to the one patient at a certain time under a certain confidence level. Constraints 18-22 are the same as Constraints 7-11.

$$\min \sum_{i=1}^n q_i C_{im} \quad (12)$$

s.t.

$$C_{ik} \geq F_{\xi_{ik}}^{-1}(1-\varepsilon) \quad \forall i, k \quad (13)$$

$$\begin{cases} C_{ik} \geq C_{il} + F_{\xi_{ik}}^{-1}(1-\varepsilon) - L(1 - X_{ikl}) \\ \forall i, k, l > k \end{cases} \quad (14)$$

$$\begin{cases} C_{il} \geq C_{ik} + F_{\xi_{il}}^{-1}(1-\varepsilon) - LX_{ikl} \\ \forall i, k, l > k \end{cases} \quad (15)$$

$$\begin{cases} C_{ik} \geq C_{jk} + F_{\xi_{ik}}^{-1}(1-\varepsilon) - L(1 - Y_{ijk}) - L(2 - Y_{ikr} - Y_{jkr}) \\ \forall i, j > i, k, r \end{cases} \quad (16)$$

$$\begin{cases} C_{jk} \geq C_{ik} + F_{\xi_{jk}}^{-1}(1-\varepsilon) - LY_{ijk} - L(2 - Y_{ikr} - Y_{jkr}) \\ \forall i, j > i, k, r \end{cases} \quad (17)$$

$$\sum_{r=1}^{r(k)} Z_{ikr} = 1 \quad \forall i, k \quad (18)$$

$$C_{im} \geq C_{ik} \quad \forall i, k \quad (19)$$

$$X_{ikl} \in \{0, 1\} \quad \forall i, k, l > k \quad (20)$$

$$X_{ijk} \in \{0, 1\} \quad \forall i, j > i, k \quad (21)$$

$$Z_{ikr} \in \{0, 1\} \quad \forall i, k, r \quad (22)$$

3. 3. SWSPTB Heuristic

Open shop scheduling problems tend to be NP-hard in most cases. Particularly very little can be said concerning the total weighted completion time's objective function. The total completion time open shop scheduling problem is strongly NP-hard for more than 2 machines [29]. Therefore, the flexible open shop scheduling total weighted completion time problem will not be easier. Here, a new heuristic is proposed for total weighted completion time (TWCT) flexible open shop scheduling problem in moderate size.

Recently, the weighted shortest processing time block heuristic (WSPTB) was proposed to solve TWCT open shop scheduling problem [8]. The details of WSPTB are as follows [8].

Let $p(i, k)$ as the deterministic length of staying time of patient i at stage k . suppose that $n = lm + \theta, 0 \leq l \leq \lfloor n/2 \rfloor, 0 \leq \theta < m$,

- a. Set $P_i = \sum_{k=1}^m p(i, k), 1 \leq i \leq n, 1 \leq k \leq m, R_i = P_i/q_i$ and $r_{i,k} = p(i, k)/q_i$.
- b. Re index the patients according to the non-decreasing order of $R_i, 1 \leq i \leq n$.
- c. For the first lm patients, every m patients are assigned in one block from the beginning. Then, the last θ patients are assigned to the last block.
- d. In each patient assignment, the priority is given to the patients with the smallest $r_{i,k} = p(i, k)/q_i$.

- e. Schedule the l blocks without any delay or pre-emption.

WSPT rule has been proven to be asymptotically optimal for $P_m // \sum_{i=1}^n W_i C_i$ [8]. This result is stated as an asymptotically optimal lower bound to the total weighted completion time flexible open shop scheduling problem [8]. We modify WSPTB heuristic to solve TWCT stochastic flexible open shop scheduling problem. The details of the Stochastic Weighted Shortest Processing Time Block (SWSPTB) heuristic are as follow.

- a. Set $F_i^{-1} = \sum_{k=1}^m F_{\xi_{ik}}^{-1}, 1 \leq i \leq n, 1 \leq k \leq m, R_i = F_i^{-1}/q_i$ and $r_{i,k} = F_{\xi_{ik}}^{-1}/q_i$.
- b. Assign the first $mr(1)$ patients to the first block, the patients in $[mr(1)+1, mr(1)+mr(2)]$ to the second block, the patients in $[m \sum_{t=1}^{d-1} r(t)+1, m \sum_{t=1}^d r(t)]$ to the block d and so on.
- c. In each patient assignment, the priority is given to the patients with the smallest $r_{i,k} = F_{\xi_{ik}}^{-1}/q_i$.
- d. Schedule every block in turn without any delay or pre-emption.

3. 3. 1. Illustrative Example

Suppose that 5 outpatients pass through 3 stages in the radiology center. Each stage contains 2 identical resources. The length of stay in each stage follows from a normal distribution with a confidence level of 0.95. We formulate this, as a stochastic flexible open shop scheduling problem with $n=5, m=3$ and $r(k) = 2$.

The inverse normal cumulative distributions of service times with a confidence level of 0.95 are as follows.

| | Patient1 | Patient2 | Patient3 | Patient4 | Patient5 |
|--------|----------|----------|----------|----------|----------|
| Stage1 | 43 | 83 | 44 | 48 | 31 |
| Stage2 | 58 | 112 | 60 | 66 | 42 |
| Stage3 | 115 | 54 | 169 | 62 | 26 |
| qi | 1 | 3 | 3 | 4 | 5 |

The patients are sorted according to the value of R_i as $\{Patient_1, Patient_2, Patient_3, Patient_4, Patient_5\}$. Here there is one block (because there are $r(1) \times m = 6$ sources and all patients are included in one block). Patient₅ passes through k stages according to the increasing order of the value of $r_{5,k}$ (i.e. $r_{5,3}, r_{5,1}, r_{5,2}$).

Therefore, patient₅ will be served first in stage 3 then 1 and finally 2. In the same manner, patient₄ will be served first in stage 1 then 3 and 2. The second patient's path will be stage3-stage1-stage2. Generated schedule executing SWSPTB is given in Figure 1. As can be seen

in Figure 1, the service route for patient₃ and also patient₁ will be stage 1, stage 2 and stage 3. The objective value of the SWSPTB schedule is 3048.

Generated schedule for an illustrative example, executing the SP model in CPLEX 12.6 is given in Figure 2. The objective value of the optimal schedule is 2998.

4. COMPUTATIONAL RESULTS

The computational results will be discussed in two parts; implementation of the proposed model on a real radiology center data to evaluate the performance of the model and its applicability and the validation of the proposed model and SWSPTBT heuristic.

4. 1. Validation of SWSPTB Heuristic

To evaluate the performance of the proposed heuristic method, the relative deviation (RD) between the proposed heuristic and the exact mixed-integer linear programming (MILP) method is obtained via Equation (23) where O.F is $\sum_j W_j C_j$.

$$RD = \frac{(O.F_{SWSPTB} - O.F_{MILP})}{O.F_{SWSPTB}} \tag{23}$$

Since the run time of the stochastic model is intensively enhanced for a moderate number of jobs (even in a

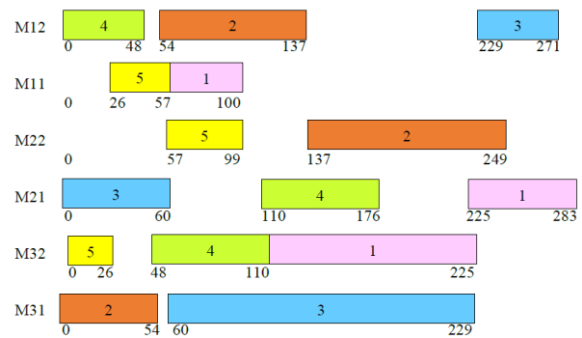


Figure 1. Generated schedule executing SWSPTB

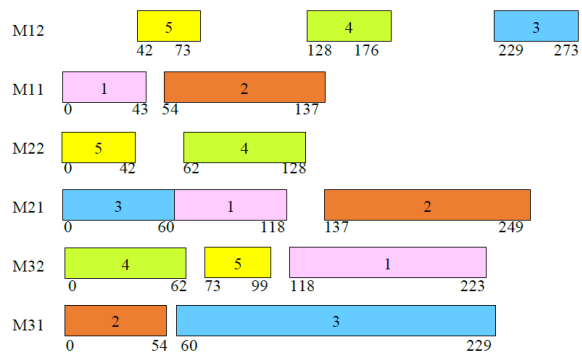


Figure 2. Optimal schedule of illustrative Example

problem with 2 stages), the objective function of the small-size exact MILP method executed by branch and bound (B&B) algorithm in CPLEX 12.6 is compared with that one of the heuristic method executed in Matlab R2013b on Intel CORE i7 2.6GHz. B&B is an exact optimization method to solve *Np-hard* combinatorial optimization problems [30] as a shop-scheduling problem [31]. We consider up to 4 stages with 2 identical devices in each stage. 100 stochastic service times are generated from a normal distribution with a confidence level of 0.95 for every case. According to experimental results from the public websites of the radiology departments in IRAN, same as Razavi Supra specialized Hospital, the average duration of each stage has been extracted. In each case, the variance was considered as one-tenth of the mean. The average duration of a general ultrasound is about 5 to 30 minutes in practice, so the average times in this stage have been extracted from a normal distribution with an average of 5 to 30 minutes. Likewise, the mean times of service in other stages are extracted as follows.

Abdominal Anomalies, Liver Ultrasound and CT-Scan are respectively considered [15, 20], [10, 15], [10, 30] minutes. Also, different types of Digestive Radiation are considered [20, 30], [30, 60], [30, 45], [120, 240], and different types of MRI as [30, 90].

The efficiency of SWSPTB is remarkable from two aspects; the elapsed time of solving problems (of different sizes) is short, and the gap between SWSPTB and the exact methods is up to 10 percent. CPLEX 12.6 is unable to solve problems with more than three stages, and in these cases, a runtime error occurs (see Table 1).

It is expected that with a lower confidence level, the total time spent by patients in the radiology department is reduced, and vice versa. The computational results confirm that considering the higher confidence level, increases the total elapsed time in the radiology center (see Table 2). The elapsed times in Tables 2 and 3 are obtained by CPLEX from MILP method. In addition, by increasing the variance of the health care service times, the total elapsed time will increase (see Table 3).

TABLE 1. The Relative Deviation between the objective function of the exact and heuristic method for the small size problems

| (m, n) | CPLEX 12.6 | Elapsed Time (S) | SWSPTB | Elapsed Time (S) | RD |
|----------|------------|------------------|--------|------------------|--------|
| (2,5) | 2826 | 0.56 | 2969 | 2.0 | 0.048 |
| (2,6) | 4533 | 0.68 | 4603 | 9.0 | 0.0152 |
| (2,7) | 4981 | 0.90 | 5213 | 3.3 | 0.0445 |
| (2,8) | 5514 | 1 | 5731 | 3.3 | 0.0378 |
| (2,9) | 6770 | 17 | 7148 | 3.3 | 0.0528 |

| | | | | | |
|--------|------|-------|------|------|--------|
| (2,10) | 6931 | 81 | 7476 | 5.6 | 0.0728 |
| (2,11) | 7936 | 2933 | 8059 | 10 | 0.0152 |
| (3,5) | 2998 | 3 | 3048 | 2.1 | 0.017 |
| (3,6) | 4338 | 10 | 4631 | 6.7 | 0.068 |
| (3,7) | 5287 | 77 | 5699 | 9.6 | 0.078 |
| (3,8) | 7425 | 602 | 7903 | 14.2 | 0.064 |
| (3,9) | - | 16200 | | 15.5 | |
| (4,5) | 4855 | 1713 | 5097 | 50 | 0.05 |
| (4,6) | - | 3624 | | | |

m: the number of stages

n: the number of patients

An increase in the variance of service times can be implicitly interpreted as an insufficient experience service provider or inappropriate physical condition of the patients, such as senility, Weakness, etc. A further reduction in total elapsed time due to a 5% reduction in the confidence level compared to a 10% reduction in service times' standard deviation (see Figure 3).

4. 1. Case Study To evaluate the performance of the proposed heuristic, it has been applied in a radiology center. In this under-studied radiology center, patients are scheduled according to a pre-reserved list.

This radiology center has two ultrasound devices, one CT-Scan device and one radiation device.

To simplify problem-solving, it's assumed that devices in each stage are similar. Also, it is assumed that patients

TABLE 2.The effect of confidence level variation on the patient's total prioritized elapsed time

| Confidence Level | 0.9 | 0.95 | 0.99 |
|------------------|------|------|------|
| (m, n) | | | |
| (2,5) | 2538 | 2826 | 4001 |
| (2,6) | 3418 | 4533 | 5650 |
| (2,7) | 3809 | 4981 | 5904 |
| (2,8) | 3852 | 5514 | 6132 |
| (2,9) | 4841 | 6770 | 7680 |
| (2,10) | 5121 | 6931 | 7959 |
| (2,11) | 6959 | 7936 | 9236 |
| (3,5) | 2841 | 2998 | 4244 |
| (3,6) | 4017 | 4338 | 5407 |
| (3,7) | 5103 | 5287 | 6509 |
| (3,8) | 6695 | 7425 | 7950 |

TABLE 3. The effect of the healthcare service times variance changing on the total prioritized elapsed time

| variance | 0.01 | 0.1 | 0.2 |
|----------|------|------|------|
| (m,n) | | | |
| (2,5) | 2678 | 2826 | 4440 |
| (2,6) | 4406 | 4533 | 4667 |
| (2,7) | 5091 | 4981 | 6353 |
| (2,8) | 5972 | 5514 | 6437 |
| (2,9) | 6300 | 6770 | 7674 |
| (2,10) | 6690 | 6931 | 8256 |
| (2,11) | 7908 | 7936 | 8786 |
| (3,5) | 2692 | 2998 | 4710 |
| (3,6) | 3627 | 4338 | 4954 |
| (3,7) | 4865 | 5287 | 6160 |
| (3,8) | 5129 | 7425 | 8555 |

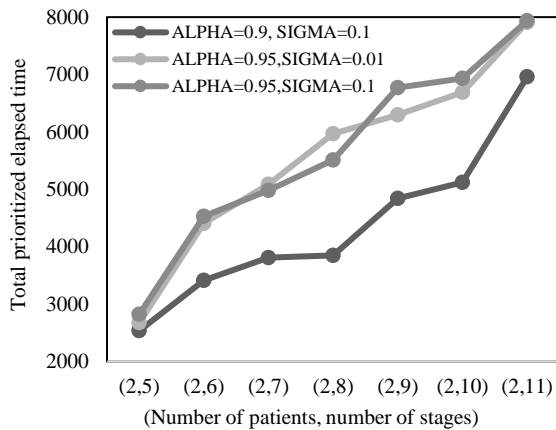


Figure 3. Comparison of the effect of reducing the service time variances with the effect of reducing the level of confidence on the total elapsed times

arrive on time as in the pre-reservation list and not-shown (a person who has made a reservation, booking, or appointment but neither keeps nor cancels it) is not considered. There are 8 working hours in the radiology center (9 -13 AM and 15-19 PM). Data are collected for 10 working days to estimate the Average Number of patients per hour (ANPH), the average number of requests for each service per hour as well as the average service times in different stages. According to the collected data, the average referrals percent per hour for different services were respectively estimated 40, 30, 20 and 10% for ultrasound, CT scan, radiography, and ultrasound together with radiography.

According to the observations, the proposed method outperforms the actual system in terms of the average total prioritized elapsed time (see Figure 4).

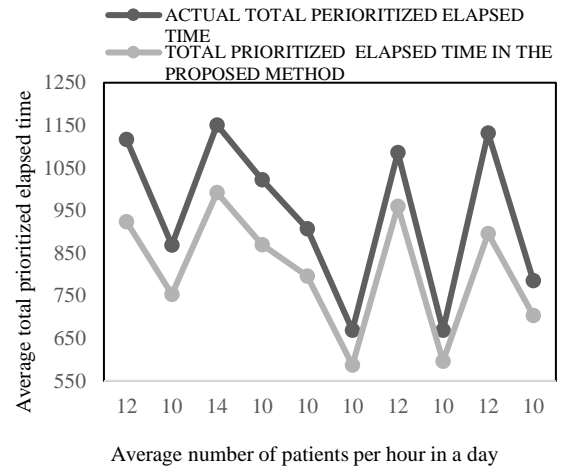


Figure 4. The comparison of the average total elapsed time of patients per hour according to the actual system and the proposed heuristic method

5. CONCLUSION

Considering service times uncertainty, scheduling of patients in a radiology department of the supra specialized hospital has been modelled in this research. Since patients did not need to go through all stages and transferring patients to different stages was not predetermined, a stochastic flexible open shop scheduling problem was proposed to model the problem. Besides, considering the priority of patients in this center has made the problem more applicable. Enhancement of patient satisfaction has been intended in this paper as a minimization of the elapsed times of all patients.

By considering normal distribution for service times, the stochastic model was converted to the deterministic one. CPLEX 12.6 has been implemented to solve the small-size proposed deterministic MILP model. Besides, a heuristic method has been proposed to solve the moderate-size flexible open shop scheduling problem.

To indicate the applicability of the proposed model, it has been applied to a real radiology center. The computational experiment has proven the efficiency and simplicity of deployment of the proposed solution method.

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Persian Abstract

چکیده

در این مقاله زمانبندی بیماران سرپائی اولویت‌بندی شده در یک مرکز رادیولوژی مورد بررسی قرار گرفته است. فرض می‌شود زمان سپری شده در هر مرحله غیر قطعی بوده و از تابع توزیع آماری مشخصی پیروی می‌کند. تابع هدف مسئله، کمینه‌سازی مجموع زمان‌های سپری شده بیماران در مرکز رادیولوژی است. این مسئله به صورت یک مسئله زمانبندی کارگاه باز و یک مدل برنامه‌ریزی تصادفی فرموله شده است. با فرض تابع توزیع آماری مشخص برای متغیرهای غیر قطعی، یک مدل برنامه‌ریزی عدد صحیح مختلط قابل حل با نرم‌افزارهای برنامه‌ریزی خطی پیشنهاد می‌شود. به علاوه، یک الگوریتم ابتکاری برای مسائل با سایز متوسط پیشنهاد می‌شود. نتایج محاسباتی حاکی از افزایش سطح رضایتمندی بیماران و همچنین بهبود کارائی و بهره‌وری مرکز رادیولوژی است.
