



Dynamic Analysis of Circular Plates in Contact with Fluid and Resting on Two-Parameter Foundations

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ABSTRACT

The dynamic behaviour of a circular plate in contact with fluid and resting on two-parameter elastic foundations is of interest in the field of geotechnics, structure, highway, railway, oil and gas and mechanical engineering. In this work, the dynamic behaviour of circular plate in contact with fluid and resting on Winkler and Pasternak foundations is investigated. The coupled differential equation of the system is analysed using differential transformation method. Good agreements are established when the results of the analytical solutions are compared to the results of the experimental investigation as reported in literature. The analytical solutions obtained are used to investigate the effects of elastic foundation parameters on natural frequency, combine foundation parameters on natural frequency, plate in contact with fluid and that of radial and circumferential stress on mode shapes. From the results, it is observed that, increases in elastic foundation parameter increases the natural frequency in all cases. Presence of fluid reduces the natural frequency of the plate. Also, it is established that mode shapes are not altered by the presence of fluid. However, mode shape displacement occurs due to presence of radial and circumferential stresses. Since the study provides a physical insight into the vibration mode of the structure, it is expected that the study will enhance better understanding on the dynamic behaviour of a circular plate in contact with fluid and resting on two-parameters elastic foundation.

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NOMENCLATURE

		Greek Symbols	
w	Deflection		
h	Plate thickness	ν	Poisson's ratio
g	Gravity (m/s ²)	ρ	Density (kg/m ³)
λ	Aspect ratio	μ	Plane wave number
g	Slender ratio	Ω	Natural frequency

1. INTRODUCTION

Recently, research interests into the study of dynamic behaviour of circular plate in contact with fluid have gained more attentions. This is because of the wide acceptable usage in various fields of engineering. Therefore, the studies of the natural frequency and modal behaviour of circular plate are justified. Motaghian et al. [1], used exact method in obtaining the solution for free vibration analysis of plate on elastic foundation. In later

study, Rezaiefar and Galal [2] worked on free vibration of plate with nonlinear load using finite element method. Winkler's idealization represents the soil medium as a system of identical but mutually independent, closely spaced, discrete, linearly elastic springs deformation of foundation due to confined applied load to loaded regions only. The pressure-deflection relation at any point is obtained with linear relation formula. Meanwhile, for Pasternak foundation, the existence of shear interaction among the spring elements is assumed which is

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accomplished by connecting the ends of the springs to the plate that only undergoes transverse shear deformation. The load-deflection relationship is obtained by considering the vertical equilibrium of a shear layer. Hence, the pressure-deflection relationship is given by incorporation of shear layer factor to existing Winkler formula. The Pasternak foundation accounts for the deficiency part of the Winkler foundation. The adoption of two-parameters foundation has proven to be more reliable than only Winkler foundation due to the ability to take care of the shear interaction among spring element.

However, Adibi and Adibi [3] worked on forced and free convection in closed domains with emphasis on various fluids. Meanwhile, Benferhat et al. [4] determine porosity effect on free vibration analysis and bending of functionally graded plate resting on two-parameter foundations. In another work, Özdemir [5] employed finite element method in the vibration response of Mindlin plate on Winkler foundation. In a further study, Umair et al. [6] performed numerical analysis on semi-empirical relations. Likewise, Nikbakhat and Behnamfar [7] demonstrated experiment on structures under subway induced vibrations. Recently, researchers have proposed and applied several semi-analytical methods in analysing dynamic behaviour of rectangular plate resting on nonlinear foundations. Bayat et al. [8] applied Homotopy perturbation method (HPM) for nonlinear free vibration of tapered beams. Werfalli and Karoud [9] used Galerkin method for the analysis of rectangular plate. Also, variational iteration method (VIM), is adopted in analytical investigation of rectangular plate resting on two-parameter foundations by Younesian et al. [10]. Extensive literatures had studied the characteristic of immersed and submerged plate in fluid Lamb [11] carried out an analytical approach into the investigation of fluid-plate coupled system. They determined the natural frequency of clamped circular plate in contact with water using Rayleigh's method, the results were validated later with experimental results of Gascón-Pérez and García-Fogeda [12]. The reviewed of the past studies showed that the analysis of the dynamic behaviour of circular plates resting on Winkler and Pasternak foundations with the aid of differential transformation method has not been investigated. While the analytical method is considered to be more effective when compared to others like numerical and semi-analytical but the shortcomings of handling nonlinear problem have not been overcome. Differential transformation method (DTM) proposed by Zhou [13], is a reliable method of solution capable of solving linear, nonlinear, coupled and integro-differential equations. The method provides closed form solution with few iterations. The advantages of DTM over other semi-analytical method of solutions justify the application in this study. Therefore, the present study focuses on application of DTM to determine dynamic

analysis of circular plates in contact with fluid and resting on Winkler and Pasternak foundations. The solutions are used for parametric studies.

2. PROBLEM FORMULATION AND MATHEMATICAL ANALYSIS

Circular plate resting on Winkler and Pasternak foundations is considered as shown in Figure 1. The diameter of the cylinder filled with water is $2a$, height is H . The fluid is inviscid and compressible of density ρ . The bottom cavity is at $z = H$, which is resting on Winkler and Pasternak foundations, the side wall is at $r = a$ also considered rigid. However, the fluid surface is covered with circular plate at $z = 0$ and at the bottom. The motion of the fluid is presumed to be of small magnitude and irrotational. The governing differential equations of the coupled system can be expressed in dimensionless form as stated in literature [14]:

$$\frac{d^4 f}{dr^4} + \frac{2}{r} \frac{d^3 f}{dr^3} - \frac{B}{r^2} \frac{d^2 f}{dr^2} + \frac{B}{r^3} \frac{df}{dr} + \frac{A}{r^4} f + k_w f - k_s \left(\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} \right) - \Omega^2 f = \Omega p_k(r, z=0); \quad (1)$$

$$\frac{d^2 p_k}{dr^2} + \frac{1}{r} \frac{dp_k}{dr} - \frac{m^2}{r^2} p_k + \frac{1}{g^2} \frac{d^2 p_k}{dz^2} + \frac{1}{c^2} \Omega^2 p_k = 0; \quad (2)$$

where f is the transverse deflection of the circular plate, k_w is the Winkler foundation parameter, k_s is the shear of Pasternak foundation, Ω is the non-dimensional natural frequency, P_k is the hydrostatic dynamic pressure of fluid. \mathcal{G} is the slender ratio and c is the speed ratio of sound in fluid and circular plate.

$$A = m^4 - 4m^2 \text{ and } B = 2m^2 + 1 \quad (3)$$

where m is the integer number of nodal diameter.

2. 1. Boundary Conditions

Three type of boundary conditions are considered: Clamped, free and simply supported which are presented in dimensionless form according to classical theory of vibration as follows [14]:

Clamped edge,

$$f(r)|_{r=1} \Rightarrow \frac{df}{dr} \Big|_{r=1} = 0, \quad (4)$$

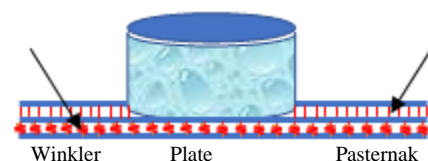


Figure 1. Circular plate with fluid resting on two-parameter foundations

Simply supported edge,

$$f(r)|_{r=1} \Rightarrow M_r|_{r=1} = -D \left[\frac{d^2 f}{dr^2} + \nu \left(\frac{1}{r} \frac{df}{dr} + \frac{m^2}{r^2} f \right) \right] = 0, \quad (5)$$

Free edge,

$$M_r|_{r=1} = -D \left[\frac{d^2 f}{dr^2} + \nu \left(\frac{1}{r} \frac{df}{dr} + \frac{m^2}{r^2} f \right) \right] = 0, \quad (6)$$

$$V_r|_{r=1} = \left[\frac{d^3 f}{dr^3} + \frac{1}{r} \frac{d^2 f}{dr^2} + \left(\frac{m^2 \nu - 2m^2 - 1}{r^2} \right) \frac{df}{dr} + \left(\frac{3m^2 - m^2 \nu}{r^3} \right) f \right] = 0, \quad (7)$$

The bending moment is represented as M_r , and the radial shear force per unit length is represented as V_r . The coupled governing Equations (1) and (2) are fourth order and second order, respectively. Invariably four conditions and two conditions are needed for solving the problem respectively. Two conditions may be obtained from the external condition of the plate while the rest is obtained from the condition at the centre of the plate. For the other equation second-order equation, one at the external as usual. The regularity conditions at the centre are given as follows:

Symmetric case,

$$\left. \frac{df}{dr} \right|_{r=0} = 0, \quad V_r|_{r=0} = \frac{d^3 f}{dr^3} = 0, \quad \frac{\partial p_k(r=0, z)}{\partial r} = 0 \quad (8)$$

where $m = 0, 2, 4 \dots$,

Axisymmetric case,

$$f(r)|_{r=0} = 0, \quad M_r|_{r=0} = \frac{d^2 f}{dr^2} \Big|_{r=0} = 0, \quad p_k(r=0, z) = 0 \quad (9)$$

and $m = 1, 3, 5 \dots$

3. METHOD OF SOLUTION

3. 1. Principle of Differential Transformation Method

Differential transformation method proposed by Zhou [13] is a very simple, powerful, reliable and very versatile method of solution. DTM involves transformation techniques which are applied to the governing equation along with the boundary conditions to form algebraic recursive expression. The resulting solution of the algebraic equations form the solution of the system in series form. This transformation makes it very simple to manipulate and convergences very fast. The accuracy of the results compared to numerical method and experimental is very high. The basic definitions and operational properties are as follows:

Considering a function $f(r)$ that is analytic in the domain R , then it will be differential continuously with rest to space r :

$$\frac{d^k f(r)}{\partial r^k} = \varphi(r, k), \quad \text{for all } r \in R \quad (10)$$

For $r = r_i$, then $\varphi(r, k) = \varphi(r_i, k)$, where k belongs to the set of non-negative integers, denoted as the k -domain. Therefore, Equation (10) is written as follows:

$$F(k) = \varphi(r_i, k) = \left[\frac{d^k f(r)}{\partial r^k} \right]_{r=r_i} \quad (11)$$

where F_k is the spectrum of $f(r)$ is at $r = r_i$ $f(r)$ expressed in Taylor's series, then $f(r)$ is presented as follows:

$$F(r) = \sum_k \left[\frac{(r-r_i)^k}{k!} \right] F(k), \quad (12)$$

Equation (12) is the inverse of $F(k)$ using the symbol "D" representing the differential transform process and combining Equation (11) and (12), we have:

$$f(r) = \sum_k \left[\frac{(r-r_i)^k}{k!} \right] F(k) = D^{-1} F(k), \quad (13)$$

3. 2. Application of DTM to the Solution of Nonlinear Equation under Investigation

Using the operational properties, the differential transformation of the governing Equations (1) and (2) are transformed along with regularity condition at the centre.

Table 1 shows operational properties of DTM. Applying the operational properties of DTM as stated in Table 1, the coupled governing differential Equations (1) and (2) are transformed as follows:

$$F_{k+4} = \frac{\Omega^2 F_k - k_w F_k + k_s \left((k+4)^2 - 6k - 16 \right) F_{k+2} + \Omega P_k}{A - B(k+4)(k+2) + (k+4)(k+2)(k+3)^2} \quad (14)$$

$$P_{k+2} = \frac{\Omega^2 P_k}{c^2 \left((k+2)^2 - m^2 \right)} \quad (15)$$

The boundary conditions stated in Equations (8) and (9) are transformed as follows:

$$F[0] = a, F[1] = 0, F[2] = b/2, P[0] = d, P[1] = e \quad (16)$$

TABLE 1. Operational properties of differential transformation method

S/N	Function	Differential Transform
1	$w(r) \pm f(r)$	$W(k) \pm F(k)$
2	$af(r)$	$aF(k)$
3	$\frac{df(r)}{dr}$	$(k+1)F(k+1)$
4	$\frac{d^2 f(r)}{dr^2}$	$(k+1)(k+2)F(k+2)$

3. 3. Application of DTM to the Conditions

Same principle is adopted in transforming other conditions. For the fluid pressure different conditions exist but, this study is investigating the effect of the fluid when in contact with the plate.

To obtain the natural frequency of the coupled equation, the condition Equation (16) is applied along with the coupled Equations (14) and (15) and non-trivial solutions obtained which is resolved to obtain the Eigen value and natural frequency.

3. 4. The Stress-Deflection Expression as [14]

To determine the radial and circumferential stress for the circular plate, the following dimensionless expression may be used:

$$\sigma_{rr} = \frac{E(r, z)z}{1-\nu^2} \left(\frac{d^2 f}{dr^2} + \frac{\nu}{r} \frac{df}{dr} \right), \tag{17}$$

$$\sigma_{\theta\theta} = \frac{E(r, z)z}{1-\nu^2} \left(\frac{1}{r} \frac{df}{dr} + \nu \frac{d^2 f}{dr^2} \right), \tag{18}$$

where E is the Young modulus of the circular plate, ν is the Poison’s ratio and z is the mid-plane of the plate. Equations (17) and (18) are the radial and circumferential stress respectively.

4. RESULTS AND DISCUSSION

The analytical solution of coupled governing equation of motion of the circular plate under various boundary conditions with differential transformation method is hereby presented. The material properties for the thin uniform thickness, homogenous circular plate used are: $E = 207GPa$, $\rho = 7850 kg/m^3$, $h = 10mm$, $\rho = 1000 kg/m^3$, Young modulus, material density, plate thickness and density of water respectively.

The analytical solution obtained is compared with experimental results reported in literature [15] and presented in Tables 2 and 3.

TABLE 2. Validation of fundamental natural frequency for symmetric condition

Edge Condition	Natural frequency	Leissa [15]	Present
Simply Supported	Ω_1	4.977	4.935
	Ω_2	29.72	29.72
Clamped Support	Ω_1	10.2158	10.2158
	Ω_2	39.7711	39.7711
Free Edge	Ω_1	9.003	9.003
	Ω_2	38.443	38.439

TABLE 3. Validation of fundamental natural frequency for axisymmetric condition

Edge Condition	Natural frequency	Leissa [15]	Present
Simply Supported	Ω_1	13.94	13.9
	Ω_2	48.51	48.48
Clamped Support	Ω_1	21.26	21.26
	Ω_2	60.82	60.83
Free Edge	Ω_1	20.475	20.555
	Ω_2	59.812	59.831

Good agreement is observed along the entire values under different boundary and regularity conditions. Generally, the natural frequency is expressed in dimensionless form Ω . Therefore, the results are valid for all thickness to radius ratio. Table 4 clearly shows that, the number of iterations needed to obtain convergence in relation to natural frequency differs. For instance, fundamental mode requires 10 iterations for DTM while the second requires 15 iterations and third iteration requires more as illustrated in Table 4. This behaviour is attributed to more complex series functions combination. Results shown in Tables 2 and 3 illustrate that, first two natural frequency gives a reasonable prediction of plate behaviour, more iterations are required for higher mode natural frequencies and also increases the fundamental frequency accuracy.

4. 1. Effect of Foundation Parameters of Natural Frequency

The analysis is performed on the three boundary conditions as discussed earlier and using regularity conditions at the centre to start the iterations. In this study, consideration is given to; Elastic Winkler type $k_w = 0, 50, 100, 150$ Elastic Pasternak type $g_s = 10, 50, 100$ Two-parameter elastic foundations ($k_p = 0, k_w = 50, g_s = 10, 50, 100$)

The values are given based on experimental investigations and as used in practice. Although, it a global behaviour of plate to be affected by presence of elastic foundation, comparing Tables 5 to Tables 2 and 3 indicates that for both plate and foundation stiffness to be

TABLE 4. Convergence study

N	Mode 1	Mode 2	Mode 3	Mode 4
15	4.9351271	29.158475		
25	4.9351358	29.719307	74.054478	
30	4.9351222	29.720113	74.194844	135.43725
35	4.9351222	29.720113	74.194693	137.30178

comparable there is a need to properly study the foundation stiffness to be chosen. As it is expected in all cases, increasing the foundation stiffness results into higher value of natural frequencies. Moreover, it is also observed that, effect of the difference in natural frequencies is more significant for higher mode of the circular plate.

4. 2. Mode Shape

According to literature [14],

for transient stress investigation, the response is normally based on modal superposition principle and the modal stress which to certain level, will expose the characteristics and content of the whole response of the plate. Based on that, study of the non-dimension radial and circumferential stress is determined using Equations (17) and (18) with results illustrated in Figures 2 and 3. The mode shape for the natural frequencies is shown in Figures 2 and 3, respectively. It is essential to note that, the mode shape obey the classical theory of vibration. For radial and circumferential stresses, location of the vibrating node and antinodes are in-away different due to the vanishing mode of the boundary condition. Figures 4 and 5 show mode shape due to the radial and circumferential stress, it is clearly shown that, the location of node and antinodes of the vibrating plate changes. Figures 4 and 5 when compared to Figures 2 and 3 different in the mode shape is clearly shown. Invariably, the extrema mode shapes location differs based on the boundary conditions. Figure 2 illustrates symmetric case vibration while Figure 3 shows axisymmetric case vibration. There is a clear difference in the shape of the mode shapes due to different modal numbers adopted. The condition at the centre play a major role in the shape modal shapes.

4. 3. Effect of Combine Winkler and Pasternak

The combine variation effect of two-parameter foundations on natural frequency as shown in Table 5 illustrate the variation of the two-parameter foundations on the natural frequencies. From the results shown, increasing the combines of the two-foundation stiffness

TABLE 5. Combine Winkler and Pasternak variation effect on natural frequency

Edge Condition	Natural frequency Mode	$(k_w = 50, m = 0)$		
		$g_s = 10$	$g_s = 50$	$g_s = 100$
Simply Supported	Ω_1	9.07	17.71	24.36
	Ω_2	33.98	48.73	63.24
Clamped Support	Ω_1	13.15	20.91	27.61
	Ω_2	42.41	56.33	69.91
Free Support	Ω_1	11.55	18.30	24.16
	Ω_2	43.23	59.74	75.90

also increases the natural frequency of the plate. The effect is more remarkable with the combined foundation parameter than when compared to only Winkler or Pasternak foundation.

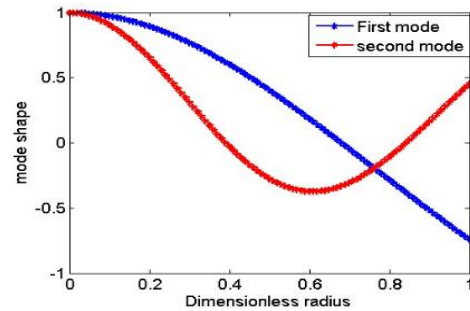


Figure 2. Modes shapes of symmetric free edge

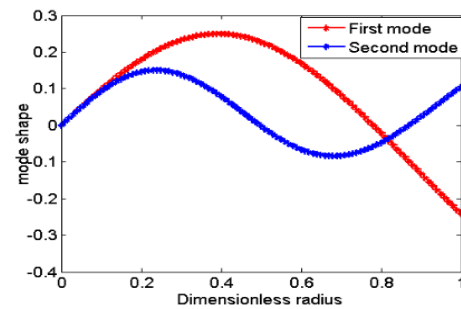


Figure 3. Modes shapes of axisymmetric free edge

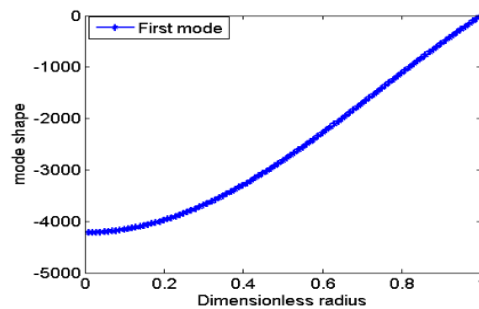


Figure 4. Radial stress for simply supported edge Symmetric case

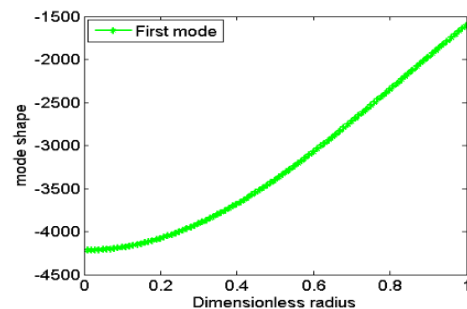


Figure 5. Circumferential stress for symmetric case first mode simply supported edge

4. 4. Effect of Fluid on the Plate Based on the analysis, it is observed that, lower natural frequency is attained while plate is in fluid as compared to when on air. The medium act as damper to the natural frequency of vibrating circular plate. This is because, both vibrating plate and fluid possess kinetic energy, when vibrating plate is in contact with fluid, the kinetic energy of both systems increases significantly. This is referred to as added virtual mass incremental factor. It is also observed from the results obtained that, there is no significant change in mode shape of the plate when immersed in fluid.

5. CONCLUSION

The study of vibration of circular plate in contact with fluid and resting on two-parameter foundations is investigated using DTM. The radial and circumferential stress determined. The analytical solutions obtained are used for the parametric study. From the results, it is concluded that, increase elastic foundation increases the natural frequencies, mode shapes are distorted due to radial and circumferential stress. Mode shapes of circular plate are not significantly affected when in contact with fluid. Natural frequencies of plate lower when in contact with fluid. The present study exposes the significant of the controlling parameters in dynamic behaviour of the plate.

6. REFERENCES

1. Motaghian, S., Mofid, M. and Akin, J. E., "On the free vibration response of rectangular plates, partially supported on elastic foundation", *Applied Mathematical Modelling*, Vol. 36, No. 9, (2012), 4473–4482.
2. Rezaiefar, A. and Galal, K., "Free vibration of thin rectangular steel plates with geometrically-nonlinear load-displacement behavior", *Thin-Walled Structures*, Vol. 129, (2018), 381–390.
3. Adibi, T., Adibi, O. and Razavi, S. E., "A Characteristic-based Solution of Forced and Free Convection in Closed Domains with Emphasis on Various Fluids", *International Journal of Engineering-Transactions B: Applications*, Vol. 32, No. 11, (2019), 1689–1695.
4. Benferhat, R., Daouadji, T.H., Mansour, M.S. and Hadji, L., "Effect of porosity on the bending and free vibration response of functionally graded plates resting on Winkler-Pasternak foundations", *Earthquakes and Structures*, Vol. 10, No. 6, (2016), 1429–1449.
5. Özdemir, Y. I., "Forced vibration analysis of Mindlin plates resting on Winkler foundation", *Challenge Journal of Structural Mechanics*, Vol. 4, No. 1, (2018), 18–26.
6. Mohd Umair, S., Gulhane, N.P., Al-Robaian, A.R.A. and Khan, S. A., "On Numerical Investigation of Semi-empirical Relations Representing Local Nusselt Number at Lower Nozzle-target Spacing's", *International Journal of Engineering - Transaction A: Basics*, Vol. 32, No. 1, (2019), 137–145.
7. Nikbakht, R. and Behnamfar, F., "Response Spectra of Structures under Subway Induced Vibrations", *International Journal of Engineering - Transaction C: Aspects*, Vol. 31, No. 12, (2018), 2009–2015.
8. Bayat, M., Pakar, I. and Bayat, M., "Analytical study on the vibration frequencies of tapered beams", *Latin American Journal of Solids and Structures*, Vol. 8, No. 2, (2011), 149–162.
9. Werfalli, N.M. and Karoud, A. A., "Free vibration analysis of rectangular plates using Galerkin-based finite element method", *International Journal of Mechanical Engineering*, Vol. 2, No. 4, (2012), 59–67.
10. Younesian, D., Saadatnia, Z., Askiri, H. and Esmailzadeh, E., "Analytical solution for oscillation of rectangular plate on nonlinear Winkler Pasternak foundation", In Proceedings of the ASME 2011, International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, USA, (2011), 1–6.
11. Lamb, H., "On the Vibrations of an Elastic Plate in Contact with Water", *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, Vol. 98, No. 690, (1920), 205–216.
12. Gascón-Pérez, M. and García-Fogeda, P., "Induced damping on vibrating circular plates submerged in still fluid", *International Journal of Applied Mechanics*, Vol. 7, No. 6, (2015), 1–18.
13. Zhou, J.K., "Differential transformation and its applications for electrical circuits", Huazhong University Press, Wuhan, China, (1986).
14. Shariyat, M. and Alipour, M. M., "Differential transform vibration and modal stress analyses of circular plates made of two-directional functionally graded materials resting on elastic foundations", *Archive of Applied Mechanics*, Vol. 81, No. 9, (2011), 1289–1306.
15. Leissa, A.W., "Vibration of plates, Office of Technology Utilization National Aeronautics and Space Administration, Scientific and Technical Information Division, Ohio, (1969).

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رفتار دینامیکی صفحه دایره‌ای در تماس با سیال ساکن بر مبنای الاستیک مورد توجه ژئوتکنیک، اتوبان، راه آهن نفت و گاز و مهندسی مکانیک قرار گرفته است. در این مقاله رفتار دینامیکی صفحه دایره‌ای در تماس با سیال ساکن بر مبنای وینکلر و پسترنک مورد تحقیق قرار گرفته است. چند معادله دیفرانسیل با استفاده از روش ترانسفرم مورد آنالیز قرار گرفته است. انطباق مطلوبی بین حل تحلیلی و نتایج تحقیق تجربی بدست آمده در پیشینه تحقیق گزارش شده است. نتایج تحلیلی بدست آمده تاثیر پارامترهای الاستیکی بر فرکانس طبیعی، پارامترهای ترکیبی بر فرکانس طبیعی، صفحه در تماس با سیال، استرس شعاعی و محیطی بر استایل شکل‌ها را بررسی نموده است. نتایج بدست آمده نشان می‌دهد در کلیه موارد افزایش پارامتر مبنای موجب افزایش فرکانس طبیعی می‌شود. حضور سیال موجب کاهش فرکانس طبیعی صفحه می‌گردد. همچنین حضور سیال موجب تغییر استایل شکل‌ها نخواهد شد. به هر حال حضور سیال موجب جابجایی استایل شکل‌ها شده که تحت تاثیر استرس‌های شعاعی و محیطی خواهند بود. مقاله حاضر نگاهی فیزیکی بر ارتعاشات سازه دارد که انتظار می‌رود موجب فهم بهتری برای رفتار دینامیکی صفحه دایره‌ای در تماس با سیال ساکن بر مبنای الاستیک دارد.

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