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Mathematical Model for Bi-objective Maximal Hub Covering Problem with Periodic Variations of Parameters

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ABSTRACT

The problem of maximal hub covering as a challenging problem in operation research. Transportation programming seeks to find an optimal location of a set of hubs to reach maximum flow in a network. Since the main structure's parameters of the problem such as origin-destination flows, costs and travel time, change periodically in the real world applications, new issues arise in handling it. In this paper, to deal with the periodic variations of parameters, a bi-objective mathematical model is proposed for the single allocation multi-period maximal hub covering problem. The ε-constraint approach has been applied to achieve non-dominated solutions. Given that the single-objective problem found in the ε-constraint method is computationally intractable. Benders decomposition algorithm by adding valid inequalities is developed to accelerate the solution process. Finally, the proposed method is carried out by CAB data set, and the results confirm the efficiency of it regarding optimality and running time.

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1. INTRODUCTION

In recent years, many shipping and telecommunication companies have tended to use hub networks for transferring flows between origins and destinations. Hub facilities are located in network nodes and provide services such as switching, sorting, and consolidation of flows. Using hubs can reduce network connections and thus reduce network construction costs. Also, with the use of special transport facilities between hubs, economic savings will be made in traveling costs. Hub location issues are commonly applied in air transport industries, postal delivery services, telecommunication services, container, and maritime transportation systems.

In general, the hub location problems (HLPs) can be divided into four categories, including p-hub median location problem, hub location problem with fixed costs, p-hub center problem and hub covering problems [1]. For the first time, Campbell [2] presented the mathematical model of hub covering problem with single and multiple allocations. The difference between hub covering problems (HCPs) with other HLPs is the limitation of length or time or cost of the path of origin-destination (O/D) pairs. HCPs can be applied to the design of the distribution network of perishable goods and also the design of transportation networks with driving time limitation. HCPs are divided into two categories: Hub set-covering problem (HSCP) and Maximal-hub covering problem (MHCP). HSCP aims to determine the location of the hubs so that all O/D pairs are covered, and the total costs of establishing hubs are minimized. In MHCP, due to the limitations such as the number of fleets and available budgets, the goal is to maximize the amount of covered demand by locating a certain number of hubs.

Campbell [2] defined three types of coverage in HCPs. Each O/D pair (i,j) is covered by hubs k and l if:

- The flow transmission cost (time or distance) from origin i to destination j does not exceed a predetermined value.
- The flow transmission cost (time or distance) from origin i to hub k and from hub k to hub l and from

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hub l to destination j does not exceed a predetermined value.

- The flow transmission cost (time or distance) from origin i to hub k and from hub l to destination j does not exceed a predetermined value.

Karimi and Bashiri [3] presented models for HSCP and MHCP with type 2 coverage and provided two heuristic algorithms. Hwang and Lee [4] proposed a new mathematical formulation for the MHCP with fewer constraints and variables than the existing models. Also, they presented two heuristic algorithms to solve MHCP and confirmed them on the Civil Aeronautics Board (CAB) Dataset. The results showed that the algorithms are efficient regarding to solution quality and solving time. Jabalameli et al. [5] proposed two mathematical formulas for MHCP with a single allocation and developed a simulated annealing algorithm to solve it. Ebrahimi-zade et al. [6] presented a non-linear multi-objective model for MHCP by considering uncertainty. The model was provided for single and multiple allocation types and it was also linearized. Since there is the possibility of interruptions in each O/D path in the real world, maximizing the reliability in the weakest network path were also considered along with the common goal of maximizing the amount of covered flow. Also, the modified non-dominated sorting genetic algorithm II (NSGAII) was used to solve the problem. Pasandideh et al. [7] presented a bi-objective model for uncapacitated single allocation MHCP considering time-dependent reliabilities. They used the second type of coverage, and the objectives of the problem including maximizing the flow and the reliability of the network. They transformed the proposed model to a single-objective model by goal attainment method. Because of the NP-hardness of the problem, the genetic algorithm was developed.

Bashiri and Rezanezhad [8] presented a multi-objective model for uncapacitated single allocation HSCP and used ε-constraint and NSGAII algorithms to solve the problem. The model aims to minimize the total investment and transportation costs, minimizes the maximum traveling time between pair of nodes, maximizes the total reliability of available paths and forces to allocate near nodes to more reliable hubs. Karimi, et al. [9] presented a mathematical formulation for capacitated single allocation HSCP in multi-modal network. They presented six valid inequalities to tight the linear programming lower bound and developed a heuristic based on the tabu search algorithm to solve the problem. Ebrahimi-zade, et al. [10] presented a bi-objective model for uncapacitated single allocation MHCP with uncertainty. They also used fuzzy multi-objective linear programming to solve the problem. The model aims to maximize flow and maximize reliability in the weakest path of the network. They assumed that the transportation time is a normal random variable. Janković and Stanimirović [11] proposed a mathematical model for uncapacitated r-allocation MHCP. To solve the model, they used the general variable neighborhood search algorithm. Janković, et al. [12] presented different mathematical models for uncapacitated MHCP with single and multiple allocations. They used two different types of coverage (binary and partial) and general variable neighborhood search algorithm to solve the proposed models. Madani, et al. [13] presented a reliable bi-objective mathematical model for the single allocation MHCP. The objectives include maximizing the covered flow and minimizing congestion in the network. They used NSGAII algorithm to solve the proposed model and examined its effectiveness against multi-objective particle swarm optimization (MOPSO) and Epsilon constraint algorithms.

Several parameters influence the design of hub networks, such as O/D flows, hub capacity, the capacity of transportation facilities, costs, and traveling time. These parameters may change periodically in the future by some reasons such as seasonal variations, inflations and technology improvements. Regarding the periodic variations of parameters during the planning horizon, HLPs can be divided into static and multi-period (dynamic) issues. In static HLPs, it is assumed that the effective decision parameters are fixed and remain unchanged over the planning horizon. Therefore, the optimum locations of the hub facilities are fixed in the planned horizon. While in multi-period HLPs, it is assumed that the effective decision parameters change periodically and their values are constant in each period. In this situation, the planning horizon is divided into different periods, while the location of hub facilities can change in different periods. In this case, the obtained solution may not be optimal for each of the periods but it will be the best solution throughout the planning horizon, indeed.

Gelareh [14] thesis is one of the first studies in designing a multi-period hub network in public transport. He considered parameters such as demand, discount factor, the operational cost of the hubs, and the cost of opening and closing hubs are changed periodically. A few research has been conducted on multi-period hub location problems. In hub covering problems, only Ebrahimi-zade et al. [15] presented a mathematical formulation for multi-period hub set-covering problem, in which the coverage range is a decision variable. They considered parameters such as travel cost, opening and closing costs of hubs, hubs covering costs, and the income of closing hubs are changed periodically. Moreover, because of the NP-hardness of the proposed model, they developed a genetic algorithm to solve it.

Reviewing the literature on the MHCPs showed that most of the proposed models provide conditions in which decision-making parameters will not change in the future, and mathematical models have been developed in a static environment. One of the primary reasons for this, is the high cost of constructing and launching hubs, which has made it impossible to make changes into the hub network. However, in some cases, the establishing cost of hubs is meager in comparison with the cost of routing flows (such as telecommunications networks) or the hub network service providers are not the infrastructure owner (such as airlines). So, in these contexts, the structure of the hub network can be changed over the planning horizon [16]. Also, even if changing the location of the hubs is impossible, we can adequately determine the flow path according to the periodic variations of the parameters.

In this study, to deal with the periodic variations of parameters, a mathematical formulation for the bi-objective multi-period maximal hub covering problem (BOMMHCP) is developed. Another contribution in the model is the simultaneous consideration of the goals of maximizing the covered demand of all O/D Pairs and minimizing the cost of hub establishment. The purpose of most MHCPs is maximizing the flow due to the coverage limits and the number of hubs, while most network owners seek to reduce the cost of the network construction. Therefore, by designing a bi-objective model and presenting non-dominated solutions, managers can choose one of them according to their preferences. The ε-constraint method has been developed for obtaining non-dominated solutions. Given that the single-objective problem found in the ε-constraint method is computationally intractable, Benders decomposition algorithm is developed to accelerate the solution process by adding valid inequalities.

This article is arranged as follows. In section 2 the mathematical model of bi-objective multi-period MHCPs is proposed. The ε-constraint mixed integer linear programming (MILP) model is presented in section 3. We define a valid inequality to speed-up the solving process. In the following, the Benders decomposition (BD) algorithm is developed and improved to quicken the implementation of the BD. In section 4, we show the experimental results. Finally, The conclusion is presented in section 5.

2. MATHEMATICAL FORMULATION

To provide the proposed BOMMHCP model, we considered the following assumptions:

- The planning horizon is divided into some limited and equal periods.
- The amount of flow between O/Ds, the cost of establishing and closing hubs are changes periodically.
- Due to the use of special facilities for transferring flows between hubs, it is assumed that traveling time (cost) between two hubs is less than usual. Accordingly, the discount factor α is used ($0<\alpha\leq 1$).
- The time (cost) of the direct connection from node i to node j is equal to the time (cost) of the direct connection from node j to node i.
- Transmission between two non-hub nodes is not possible directly and the O/D route passes at least one and at most two hubs.
- Hub nodes can be selected from all network nodes.
- It is possible to open the hubs in different periods, but hubs can be closed from the second period.
- In each period, the number of hubs is predetermined and the number of hubs in different periods can be various.
- Non-hub nodes can be allocated to one hub node at most.
- There is no capacity constraint in the network. In other words, the capacity of the hubs and arcs of the network are unlimited.

Notations and Parameters:

N: Node sets

T: Period sets

i,j: indices of Origin (destination) nodes	$i,j=1,...,N$
k,l: indices of Hubs	$k,l=1,...,N$
t: indices of periods	$t=1,...,T$
P_t : Number of hubs in period t	$t=1,...,T$
D_{ij} : time of direct path from node i to node j	$i,j=1,...,N$
W_{ijt} : the amount of demand flow from origin node i to destination node j in period t	$i,j=1,...,N$ $t=1,...,T$
α: The discount factor for transferring flow between two hub nodes	
β: Coverage radius (allowable travel time/cost between O/D nodes)	
OP_{kt} : the fixed setup cost for establishing a hub at node k in period t	$k=1,...,N$ $t=1,...,T$
CL_{kt} : The fixed cost of closing a hub that located at node k in period t, $CL_{kt}\leq OP_{kt}$	$k=1,...,N$ $t=2,...,T$

Decision variables:

$y_{ijklt} = 1$ if flows of nodes i and j routed through hubs k and l respectively and otherwise equal 0. $\quad i,j,k,l = 1,...,N \quad t = 1,...,T$

$x_{ikt} = 1$ if node i is allocated to hub k in period t, and unless equal 0.
If $x_{kkt} = 1$, this indicates that at the beginning of period t, a hub established in node k. $\quad i,k = 1,...,N \quad t = 1,...,T$

$r_{kt} = 1$ if at the beginning of period t a hub established in node k and otherwise, equal 0. $\quad k = 1,...,N \quad t = 1,...,T$

$s_{kt} = 1$ if at the beginning of period t, hub node k is closed and otherwise, equal 0. $\quad k = 1,...,N \quad t = 2,...,T$

Mathematical formulation of BOMMHCP as follows:

$$Max \sum_{i=1}^{N} \sum_{j=1,\neq i}^{N} \sum_{k=1}^{N} \sum_{l=1}^{N} \sum_{t=1}^{T} W_{ijt} y_{ijklt} \tag{1}$$

$$Min \sum_{k=1}^{N} \sum_{t=1}^{T} OP_{kt} r_{kt} + \sum_{k=1}^{N} \sum_{t=2}^{T} CL_{kt} s_{kt} \tag{2}$$

s.t.

$$\sum_{k=1}^{N} x_{ikt} \leq 1 \qquad i = 1,...,N, t = 1,...,T \tag{3}$$

$$\sum_{k=1}^{N} x_{kkt} = P_t \qquad t = 1,...,T \tag{4}$$

$$x_{ikt} \leq x_{kkt} \qquad i,k = 1,...,N, t = 1,...,T \tag{5}$$

$$2y_{ijklt} \leq x_{ikt} + x_{jlt} \qquad i \neq j, k, l = 1,...,N, t = 1,...,T \tag{6}$$

$$(D_{ik} + \alpha D_{kl} + D_{lj}) \times y_{ijklt} \leq \beta \qquad i \neq j, k, l = 1,...,N, t = 1,...,T \tag{7}$$

$$r_{k1} \leq x_{kk1} \qquad k = 1,...,N \tag{8}$$

$$r_{kt} - s_{kt} \leq x_{kkt} - x_{kk(t-1)} \qquad k = 1,...,N, t = 2,...,T \tag{9}$$

$$y_{ijklt}, x_{ikt}, r_{kt}, s_{kt} \in \{0,1\} \qquad i,j,k,l = 1,...,N, t = 1,...,T \tag{10}$$

The objective function (1) maximizes the total covered flows during the planning horizon. The objective function (2) minimizes the total cost of establishing and closing hubs during the planning horizon. Constraints set (3) ensure that in each period, each non-hub node can be allocated to at most one hub. Constraints set (4) show the number of hubs which are going to be operated in each period. Constraints set (5) state that in each period, node i can be allocated to node k, if node k is served as a hub. Constraints set (6) show that in each period, the path $i \to k \to l \to j$ will be established where node i is allocated to hub k and node j is allocated to hub l. Constraints set (7) ensure that in each period, the path $i \to k \to l \to j$ is established, where traveling time is less than the coverage radius of β. Constraints set (8) and (9) indicate the possibility of creating and closing hubs over different periods. It is possible to close a hub if it had been located in previous periods and it can be opened if it had not been established in the previous periods. Constraints set (10) represent the type of decision variables. In the next section, the method of solving the proposed model is investigated.

3. SOLUTION APPROACH

Epsilon constraint method is one of the most appropriate solution methods for multi-objective programming models and is one of the recommended ways when there is no access to the decision makers (DM) [17]. In this method, after selecting one of the objectives as the primary objective function, others are moved into the problem constraints and an upper or lower limit (ε) is considered for them. As a result, the multi-objective problem is converted into a single-objective problem and Pareto solutions can be determined by assigning different values to Epsilon [18]. Consider the multi-objective programming problem (11) which consists of m different objectives and S represents the feasible region [17].

$$\begin{aligned} & Max / Min \ (f_1(x), f_2(x), ..., f_m(x)) \\ & s.t. \quad x \in S \end{aligned} \tag{11}$$

By choosing k ($k \in \{1,...,m\}$) as the primary objective, the other objectives are moved to the constraints.

$$\begin{aligned} & Max / Min \ f_k(x) \\ & s.t. \ x \in S \end{aligned} \tag{12}$$

$$f_i(x) \geq \varepsilon_i \quad i \in \{1,...,m\}, i \neq k \ \text{for max objectives} \tag{13}$$

$$f_j(x) \leq \varepsilon_j \quad j \in \{1,...,m\}, j \neq k \ \text{for min objectives} \tag{14}$$

3. 1. ε-constraint Method To implement the ε-constraint method in the proposed BOMMHCP model, two single-objective problems are considered as follows.

Problem 1:

The objective function (1)

s.t.

Constraints (3)-(10)

$$\sum_{k=1}^{N}\sum_{t=1}^{T} OP_{kt} r_{kt} + \sum_{k=1}^{N}\sum_{t=2}^{T} CL_{kt} s_{kt} \le \varepsilon \tag{1}$$

The parameter *Obj 1* is equal to value of the optimal objective function of *problem 1*.

Problem 2:

The objective function (2)

s.t.

Constraints (3)-(10)

$$\sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} W_{ijt} y_{ijklt} \ge Obj\,1 \tag{2}$$

The following steps are implemented to obtain the set of Pareto optimal solutions:

Step 1: Set the Pareto optimal solutions to the empty and the ε value to infinity.

Step 2: Solve Problem 1. If the problem has an optimal solution, then set the Obj 1 and go to step 3. Otherwise, stop.

Step 3: Solve Problem 2. Put Obj 2 equal to the value of the optimal objective function and go to step 4.

Step 4: Add the solution (Obj 1, Obj 2) to the Pareto optimal solutions and go to step 5.

Step 5: Set the value of ε to (Obj 2-1) and go to step 2.

Problem 1 in ε-constraint MILP model of BOMMHCP is computationally intractable such that even small-size examples cannot be solved optimally in a reasonable runtime using a commercial solver. Accordingly, we proposed a valid inequality to accelerate the solving process of the problem. Since BOMMHCP is a single allocation model and each non-hub node can be connected to at most one hub, then at most one path would be existed to transfer the flow between each O/D pair. Therefore, valid inequality can be stated as follows:

$$\sum_{k=1}^{N}\sum_{l=1}^{N} y_{ijklt} \le 1 \qquad i,j=1,...,N, t=1,...,T \tag{17}$$

By increasing the number of network nodes and periods, *Problem 1* will be intractable due to the existence of many binary variables and constraints and addition of valid inequality (17) does not help either to solve the problem in a reasonable time. As a result, to accelerate the solution process, the Benders decomposition algorithm will be developed in the following section.

3. 2. Benders Decomposition (BD) BD is a classical solution approach that was initially introduced in 1962 by Benders to solve the NP-hard Mixed Integer Programming (MIP) problems. In this algorithm, the MIP model is decomposed into two smaller problems: the Master Problem (MP) and the Sub-problem (SP). The MP only contains integer variables and the SP includes continuous variables of the original problem. The MP and SP are iteratively solved to produce lower bound (LB) and upper bound (UB) for optimal objective value. Then, by the convergence of LB and UB, the optimal solution is found. To illustrate the Benders decomposition algorithm, consider the following MIP model [19]:

$$\begin{aligned} &P: \min C^T x + f^T y \\ &\quad Ax + By \ge b \\ &\quad Dy \ge d \\ &\quad y \in Z^+ \\ &\quad x \ge 0 \end{aligned} \tag{18}$$

Or equivalently,

$$\begin{aligned} &MP: \min f^T y + \eta(y) \\ &\quad Dy \ge d \\ &\quad y \in Z^+ \end{aligned} \tag{19}$$

where,

$$\begin{aligned} &SP:\ \eta(y) = \min C^T x \\ &\quad Ax \ge b - By \\ &\quad x \ge 0 \end{aligned} \tag{20}$$

By fixing y to the feasible values ($\bar{y}$), the dual of sub-problem (DSP) is shown as follows:

$$\begin{aligned} &DSP:\ \max\ (b - B\bar{y})^T u \\ &\quad A^T u \le c \\ &\quad u \ge 0 \end{aligned} \tag{21}$$

By obtaining the optimal values of u ($\bar{u}$) the relaxed master problem is shown as follows:

$$\begin{aligned} &RMP:\ \min\ Z \\ &\quad Z \ge (b - By)^T \bar{u} + f^T y \quad \forall \bar{u} \in P\Omega \subseteq \Omega \\ &\quad (b - By)^T \bar{u} \le 0 \qquad \forall \bar{u} \in R\Omega \subseteq \Omega \\ &\quad Dy \ge d \\ &\quad y \in Z^+ \end{aligned} \tag{22}$$

where $P\Omega$ is all of the extreme point sets and $R\Omega$ is all of the extreme ray sets of polyhedron Ω. Also, Ω is defined by the constraint sets of DSP model (). The classical BD algorithm is operated as follows for the minimization problem [19]:

$\bar{y}$:= initial feasible integer solution
$LB := -\infty$ and $UB := \infty$
while $UB - LB > \delta$ (δ is the user-defined optimality gap) **do**
solve the DSP ()
if the DSP () **Unbounded then**
Get unbounded ray $\bar{u}$ and Add the cut $(b - By)^T \bar{u} \le 0$ to the RMP ()
else
Get the extreme point $\bar{u}$ and Add the cut $Z \ge (b - By)^T \bar{u} + f^T y$ to the RMP ()
$UB := \min\left(UB, (b - B\bar{y})^T \bar{u} + f^T \bar{y}\right)$
end if
solve the RMP ()
$\bar{Z} := \min\{Z \mid cuts, y \in Z^+\}$ *and* $LB := \bar{Z}$
end while

In the following, we develop BD for the ε-constraint model of BOMMHCP.

3. 2. 1. The Sub-Problem The SP for the fixed integer variable $x = \bar{x}$, is created as follows:

$$Min \quad -\sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} W_{ijt} y_{ijklt} \tag{23}$$

s.t.

$$2y_{ijklt} \le \bar{x}_{ikt} + \bar{x}_{jlt} \qquad i \ne j, k, l = 1,...,N, t = 1,...,T \tag{24}$$

$$(D_{ik} + \alpha D_{kl} + D_{lj}) \times y_{ijklt} \le \beta \qquad i \ne j, k, l = 1,...,N, t = 1,...,T \tag{25}$$

$$\sum_{k=1}^{N}\sum_{l=1}^{N} y_{ijklt} \le 1 \qquad i, j = 1,...,N, t = 1,...,T \tag{26}$$

$$y_{ijklt} \in \{0,1\} \qquad i, j, k, l = 1,...,N, t = 1,...,T \tag{27}$$

BD uses the results of the dual SP solutions to create cutting constraints in each iteration. SP has binary variables and its dual representation is not possible. So, firstly, the structure of SP model is changed in order to transform it into a model with continuous variables. So:

$$Min \quad -\sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} W_{ijt} \times A_{ijklt} \times y_{ijklt} \tag{28}$$

s.t.

$$y_{ijklt} \le \bar{x}_{kkt} \qquad i \ne j, k, l = 1,...,N, t = 1,...,T \tag{29}$$

$$y_{ijklt} \le \bar{x}_{llt} \qquad i \ne j, k, l = 1,...,N, t = 1,...,T \tag{30}$$

$$\sum_{k=1}^{N}\sum_{l=1}^{N} y_{ijklt} \le 1 \qquad i, j = 1,...,N, t = 1,...,T \tag{31}$$

$$0 \le y_{ijklt} \le 1 \qquad i, j, k, l = 1,...,N, t = 1,...,T \tag{32}$$

The parameter A_{ijklt} is used to specify the feasible paths with respect to the coverage radius, as follows:

$$A_{ijklt} = \begin{cases} 1 \; if \;\; (D_{ik} + \alpha D_{kl} + D_{lj}) \times y_{ijklt} \le \beta \\ 0 \; else \end{cases}$$

3. 2. 2. The Dual of Sub-Problem Given the dual variables e_{ijklt} for constraints set (29), f_{ijklt} for constraints set (30) and g_{ijt} for constraints (31), the dual of SP (DSP) is created as follows:

$$\begin{aligned} Max \; & \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} \bar{x}_{kkt} \times e_{ijklt} \\ & + \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} \bar{x}_{llt} \times f_{ijklt} \\ & + \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{t=1}^{T} g_{ijt} \end{aligned} \tag{3}$$

s.t.

$$e_{ijklt} + f_{ijklt} + g_{ijt} \le -W_{ijt} A_{ijklt} \qquad i \ne j, k, l = 1,...,N, t = 1,...,T \tag{4}$$

$$e_{ijklt}, f_{ijklt}, g_{ijt} \le 0 \qquad i, j, k, l = 1,...,N, t = 1,...,T \tag{5}$$

Constraints set (4) are related to the primal variable y_{ijklt}.

3. 2. 3. The Relaxed Master Problem (RMP) SP always has a feasible and finite optimal solution. Therefore, by dual theory, DSP has an optimal finite solution $(\bar{e}_{ijklt}, \bar{f}_{ijklt}, \bar{g}_{ijt})$. Thus, by the principle of weak duality, the optimality Benders cut is obtained as constraint (37). As a result, the RMP is created as follows:

$$Min \quad Z \tag{36}$$

s.t.

$$\begin{aligned} Z \ge & \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} x_{kkt} \times \bar{e}_{ijklt} \\ & + \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}\sum_{t=1}^{T} x_{llt} \times \bar{f}_{ijklt} \\ & + \sum_{i=1}^{N}\sum_{j=1,\neq i}^{N}\sum_{t=1}^{T} \bar{g}_{ijt} \end{aligned} \tag{37}$$

$$\sum_{k=1}^{N}\sum_{t=1}^{T} OP_{kt} r_{kt} + \sum_{k=1}^{N}\sum_{t=2}^{T} CL_{kt} s_{kt} \le \varepsilon \tag{38}$$

$$\sum_{k=1}^{N} x_{ikt} \le 1 \qquad i=1,...,N, t=1,...,T \tag{39}$$

$$\sum_{k=1}^{N} x_{kkt} = P_t \qquad t=1,...,T \tag{40}$$

$$x_{ikt} \le x_{kkt} \qquad i,k=1,...,N, t=1,...,T \tag{41}$$

$$\sum_{i=1, i\ne k}^{N} x_{ikt} \ge x_{kkt} \qquad k=1,...,N, t=1,...,T \tag{42}$$

$$r_{k1} \le x_{kk1} \qquad k=1,...,N \tag{43}$$

$$r_{kt} - s_{kt} \le x_{kkt} - x_{kk(t-1)} \qquad k=1,...,N, t=2,...,T \tag{44}$$

$$x_{ikt}, r_{kt}, s_{kt} \in \{0,1\} \qquad i,j,k,l=1,...,N, t=1,...,T \tag{45}$$

3. 2. 4. Accelerating the Proposed BD In some cases, direct use of classic BD may not result in a significant reduction in solution runtime. Some of the main reasons for the slow convergence rate of the classical BD are: (1) solving the excessive numbers of RMPs and SPs, and (2) the low quality of the cuts created in each iteration. Therefore, various methods and techniques have been developed to increase the convergence speed of the BD [20].

In early runs of BD, it was observed that there was a low convergence rate at the lower bound of the objective function (RMP problem). Therefore, to increase the efficiency and speed of BD, the cutting constraints set (46)-(47) are added to RMP problem. These constraints limit the maximum flow through the network.

$$h_{ijt} \le \sum_{k=1}^{N} A_{ijklt} \times x_{ikt} - x_{jlt} + 1 \qquad i,j,l=1,...,N, t=1,...,T \tag{46}$$

$$Z \ge -\sum_{i=1}^{N}\sum_{j=1, \ne i}^{N}\sum_{t=1}^{T} W_{ijt} h_{ijt} \tag{47}$$

4. COMPUTATIONAL RESULTS

Experiments were conducted to assess the performance of the valid inequality and proposed Benders decomposition approach. In the following, we will explain the data generation approach and analyze the obtained results.

4. 1. Data Generation The well known standard benchmark that is refers to data obtained from the Civil Aeronautics Board of the United States of America (CAB dataset [21]), is used for evaluating BOMMHCP model. Since the CAB dataset is not provided for multi-period problems. Therefore, according to Alumur et al. [22], flows in the CAB dataset are considered for the first period only. Also, for following periods, it is obtained from the multiplication of the flows of the previous period by a random number from the uniform interval (0.9, 1.2). The cost of establishing and closing hubs has been generated according to Gelareh et al. [23]. Moreover, the cost of establishing the hub for the first period is randomly generated from the interval (500,700). Likewise, for following periods, it is obtained from the multiplication of the establishment costs of the previous period by a random number from the uniform interval (1, 1.05). Additionally, the cost of closing the hub for the first period is randomly generated from the interval (200,300) and for following periods, it is obtained from the multiplication of the closing costs of the previous period by a random number from the uniform interval (1, 1.05). Different values for discount factor and coverage radius are considered according to the article by Silva and Cunha [24]. The various values of the parameters are shown in

TABLE .

4. 2. Results Analysis The GAMS software (version 24.9.1) and CPLEX solvers (version 12.7.1) were used for solving ε-constraint MILP model of BOMMHCP and the proposed BD algorithm. The software runs on a 4 GHz Intel processor (Intel Core i7-6700k) and 32GB of RAM. A time limit of 7200 and 3600 seconds was considered for the implementation of ε-constraint MILP model with valid inequality and BD algorithm, respectively. In continue, we will explain the results from various aspects.

TABLE 1. Parameter Values for BOMMHCP

Parameter	Values
N	*10,15,20,25*
P	*2,3,4,5*
W_{ijt}	*CAB dataset* $t=1$ $W_{ij(t-1)} \times U[0.9,1.2]$ $t>1$
OP_{kt}	$U[500,700]$ $t=1$ $OP_{k(t-1)} \times U[1.05,1.1]$ $t>1$
CL_{kt}	$U[200,300]$ $t=2$ $CL_{k(t-1)} \times U[1.05,1.1]$ $t>2$

4. 2. 1. Performance of The Valid Inequality Different problems with 10 and 15 nodes were solved to illustrate the efficiency of valid inequality and the results are presented in Table 2. The stars (*) indicate that the optimal solution has been achieved. As it is evident, the optimal solution is obtained by applying the valid inequality (17) for all sample issues, while without considering the inequality, only the optimal solution to the problems with ten nodes is obtained. Also, the results confirm the high performance of a valid inequality in terms of runtime and show a significant decrease.

4. 2. 2. Evaluation of Speeding Up The BD In this paper, we proposed a modification to speeding up the classical BD algorithm. The computational results are presented in Figures 1 and 2. Several experiments have been conducted to verify the effectiveness of correction. As an example, the N10P2T3α0.2R1425 problem (problem with ten nodes, two hubs, three periods, α=0.2 and the coverage radius of 1425) is solved by the classical BD and improved BD. The results show that the classical BD converges in iteration 39 (Figure 1**Figure**), while, the improved BD converges in two iterations (Figure 2).

TABLE 2. Results of adding Valid Inequality

Number of Nodes	Number of Hubs	Discount Factor	Coverage Radius	Epsilon (Budget Constraint)	With Valid Inequality		Without Valid Inequality	
					Flow Objective	Runtime (seconds)	Flow Objective	Runtime (seconds)
10	2	0.2	1425	1599	**3,174,972***	3.2	**3,174,972***	77.6
10	3	0.2	1117	2433	**3,139,640***	3.0	**3,139,640***	193
10	4	0.2	811	3273	**2,794,350***	4.7	**2,794,350***	185.2
10	5	0.2	736	4134	**2,733,823***	7.9	**2,733,823***	147.3
10	2	0.6	1671	1599	**3,136,301***	4.0	**3,136,301***	73.9
10	3	0.6	1387	2433	**3,091,015***	2.6	**3,091,015***	179.7
10	4	0.6	1148	3273	**3,021,212***	1.7	**3,021,212***	281.4
10	5	0.6	1079	4134	**2,896,000***	1.9	**2,896,000***	138.7
15	2	0.2	2004	1564.5	**7,470,656***	275	7,407,953	4634.4
15	3	0.2	1638	2349	**7,341,687***	183	7,142,206	7203.2
15	4	0.2	1324	3138	**7,051,643***	278	6,912,159	7203.4
15	5	0.2	1149	3964.5	**7,075,740***	115	6,706,032	7203.9
15	2	0.6	2103	1564.5	**7,179,215***	340	7,133,948	7203.3
15	3	0.6	1844	2349	**7,073,270***	513	6,933,837	7203.5
15	4	0.6	1756	3138	**7,146,254***	125	6,589,729	7203.2
15	5	0.6	1560	3964.5	**6,871,335***	134	6,430,333	7203.8

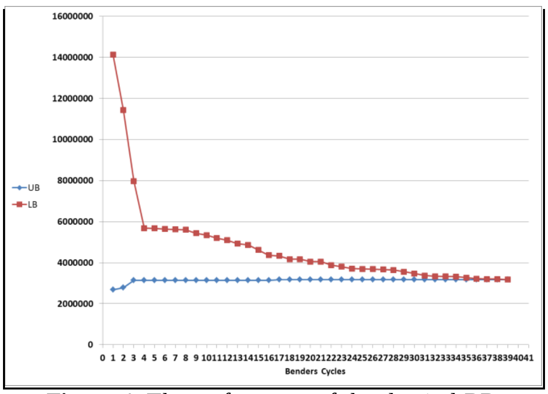

Figure 1. The performance of the classical BD

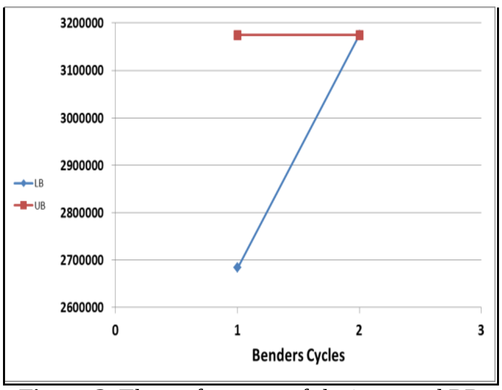

Figure 2. The performance of the improved BD

4. 2. 3. Verifying and Validating the BD Algorithm Several examples were solved to confirm the efficacy of the improved BD algorithm. The obtained results are presented in Table 3. The stars (*) indicate that the optimal solution has been achieved.

The results demonstrated that the performance of the ε-constraint MILP models with and without valid inequality and the Improved BD algorithm are similar and they reach to the optimal solution for all instances with ten nodes in less than 3600 seconds. However, in all instances with 15 nodes, only ε-constraint MILP models with valid inequality and the improved BD algorithm performed similarly, while ε-constraint MILP models without valid inequality was not capable of solving the samples at the proposed runtime limitation and made even worse objective values. In addition, the results obtained from instances with 20 and 25 nodes demonstrated that the proposed BD algorithm performed better than the others and it had less runtime, and more appropriate solutions were obtained. The comparison of solution times and objective values are shown in Figures 3 and 4, respectively.

TABLE 1. Results of improved BD

Problem	Nodes	Hubs	Discount Factor	Without Valid Inequality		With Valid Inequality		Improved BD	
				obj. value	runtime	obj. value	Runtime (seconds)	obj. value	Runtime (seconds)
1	10	2	0.2	**3,177,612***	246.3	**3,177,612***	3.191	**3,177,612***	3.15
2	10	3	0.2	**3,132,909***	607.36	**3,132,909***	3.07	**3,132,909***	3.019
3	10	4	0.2	**3,203,761***	22.94	**3,203,761***	2.2	**3,203,761***	2.93
4	10	5	0.2	**3,156,751***	86.4	**3,156,751***	2.72	**3,156,751***	2.88
5	10	2	0.6	**3,130,815***	707.95	**3,130,815***	3.18	**3,130,815***	3.05
6	10	3	0.6	**3,143,466***	636.5	**3,143,466***	2.52	**3,143,466***	3.11
7	10	4	0.6	**3,148,697***	53.87	**3,148,697***	2.09	**3,148,697***	3.02
8	10	5	0.6	**3,163,102***	107.2	**3,163,102***	2.14	**3,163,102***	3.01
9	15	2	0.2	5,712,456	3604	**7,470,656***	314.7	**7,470,656***	14.6
10	15	3	0.2	7,305,615	3603.7	**7,404,374***	1762.9	**7,404,374***	36.7
11	15	4	0.2	6,755,050	3603.6	**7,521,565***	163.3	**7,521,565***	10.9
12	15	5	0.2	7,190,805	3603.7	**7,394,625***	48.69	**7,394,625***	7.6
13	15	2	0.6	6,495,672	3603.7	**7,446,354***	59.3	**7,446,354***	11.6
14	15	3	0.6	7,016,642	3603.3	**7,255,354***	322.7	**7,255,354***	30.2
15	15	4	0.6	6,969,136	3603.8	**7,443,425***	20.1	**7,443,425***	6.6
16	15	5	0.6	6,998,628	3603.5	**7,458,756***	10.6	**7,458,756***	6.5
17	20	2	0.2	17,685,355	3608.7	**18,210,653***	1184.9	**18,210,653***	212.8
18	20	3	0.2	13,936,717	3609.3	17,815,161	3608.5	**17,947,235***	268.1
19	20	4	0.2	13,854,186	3609.2	18,153,547	3609.2	**18,176,821***	19.6
20	20	5	0.2	13,326,320	3608.4	18,101,487	3609.1	**18,114,561***	249.3
21	20	2	0.6	15,110,702	3608.6	**18,153,692***	1027.6	**18,153,692***	90.1
22	20	3	0.6	14,717,260	3608.4	17,881,134	3608.8	**17,937,160***	230.7
23	20	4	0.6	17,070,718	3609.6	**18,197,128***	333.7	**18,197,128***	46.4
24	20	5	0.6	14,471,042	3608.9	**18,128,884***	138.1	**18,128,884***	17.2
25	25	2	0.2	20,714,822	3620.2	22,402,229	3619.9	**26,896,201***	921.4
26	25	3	0.2	19,383,882	3625.3	20,206,324	3619.8	**26,876,295***	1578.9
27	25	4	0.2	19,064,977	3619.5	19,333,815	3619.2	**26,952,730***	1246.2
28	25	5	0.2	18,061,867	3618.6	21,150,505	3619.5	**26,927,569***	736.2
29	25	2	0.6	0	3618.5	21,498,335	3628.2	**26,714,270***	658.6
30	25	3	0.6	20,363,454	3640.1	22,702,714	3619.9	**26,923,193***	724.9
31	25	4	0.6	20,675,391	3621.6	22,331,966	3619.7	**26,783,695***	863.3
32	25	5	0.6	20,133,147	3619.6	20,538,530	3619.9	**26,834,055***	1169.2

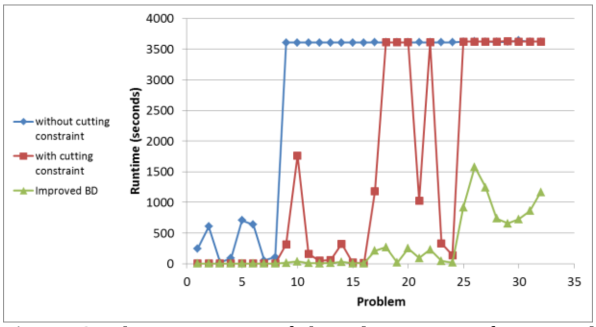

Figure 3. The comparison of the solution time of improved BD versus **ε**-constraint MILP model

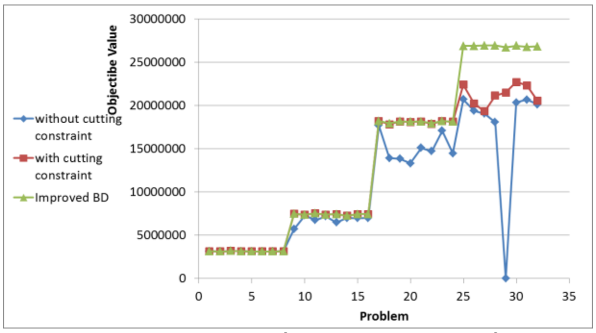

Figure 4. The comparison of the objective value of improved BD versus **ε**-constraint MILP model

4. 2. 4. Pareto Optimal Solutions Two sample problems have been solved to illustrate Pareto's optimal solutions in ***ε***-constraint method, including the problem with ten nodes, four hubs, three time periods and discount factor of 0.6 (N10P4T3α0.6) and the problem with 15 nodes, four hubs, three time periods and discount factor of 0.6 (N15P4T3α0.6). Results are shown in Table 4, Figures 5 and 6.

TABLE 4. Pareto optimal solutions for problems with 10 and 15 nodes

Solution Numbers	N10P4T3α0.6		N15P4T3α0.6	
	Flow Obj.	Cost Obj.	Flow Obj.	Cost Obj.
1	3,021,212	2402	7,146,254	2430
2	3,017,047	2398	7,124,188	2396
3	2,977,080	2389	7,096,680	2290
4	2,951,872	2338	7,083,419	2288
5	2,892,576	2330	7,074,650	2260
6	2,862,641	2319	7,015,516	2225
7	2,858,476	2315	6,884,141	2204
8	2,834,144	2256	6,721,777	2179
9	2,793,301	2255	6,702,545	2151
10	2,788,025	2241	6,550,317	2126
11	2,743,380	2196	6,543,344	2120
12	2,712,631	2182		

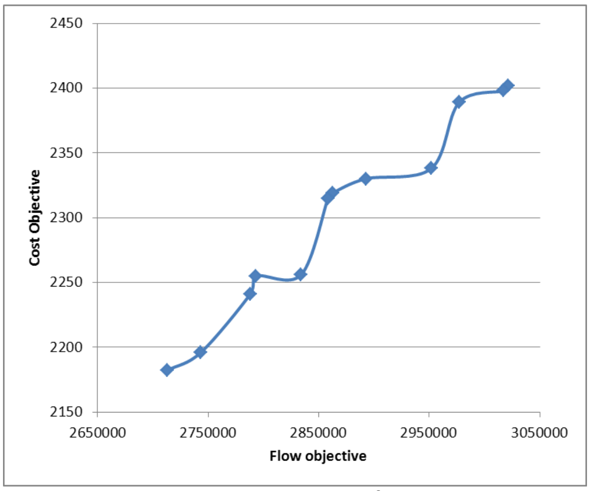

Figure 5. Optimal Pareto Front for N10P4T3α0.6

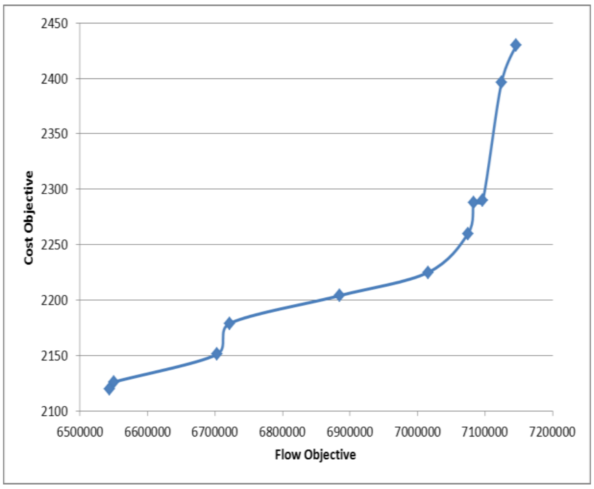

Figure 1. Optimal Pareto Front for N15P4T3α0.6

As an example, for a network with 15 nodes, three hubs and three periods (N15P3T3α0.6), the designed hub network obtained in the first period (Figure 7), in the second period (Figure 8) and the third period (Figure 9) are shown. According to Figures, in the first period, hubs are located in nodes 9 (Detroit), 12 (Los Angeles) and 13 (Memphis), and in the second and third periods hubs are located in nodes 6 (Cleveland), 12 (Los Angeles) and 13 (Memphis). Although the locations of the hubs are the same in second and third periods, the allocation of nodes to hubs is different.

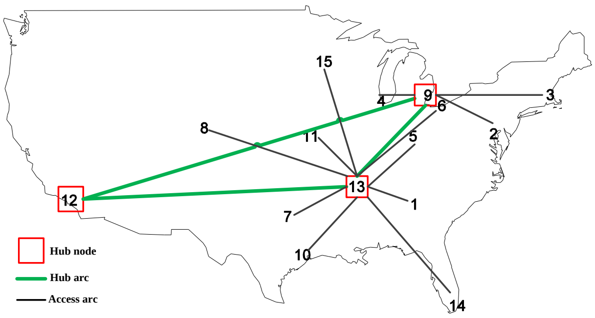

Figure 7. Network structure for N15P3T3 in the first period

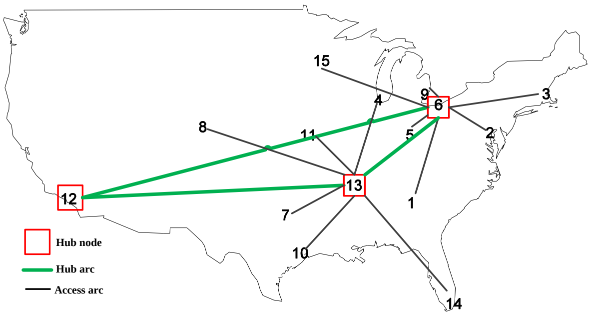

Figure 8. Network structure for N15P3T3 in the Second period

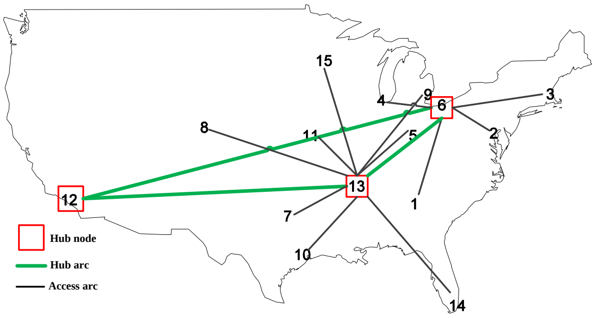

Figure 9. Network structure for N15P3T3 in the third period

5. CONCLUSIONS

In this paper, by considering the periodic changes in O/D flows and the costs of establishing and closing hubs, a mathematical model was proposed for a bi-objective multi-period maximal hub covering problem. Furthermore, the ε-constraint method was used to solve the proposed model. Given that the single-objective problem found in the ε-constraint method is computationally intractable, we added a new valid inequality and also developed the BD algorithm to accelerate the solution process. Results showed that the proposed BD has a high performance in terms of runtime and optimal attainment.

This work covers the application of HLPs in periodic variation of parameters. However, there are still issues for development. One of them includes taking into account stochastic parameters (such as O/D flows, traveling time and costs) along with periodic parameters to increase the efficiency of the BOMMHCP model when the decision parameters are changing. Meanwhile, it is useful to extend the proposed model by considering other constraints of the real world, including the amount of available budget and capacity constraint in the network.

6. REFERENCES

1. Contreras, I., Hub location problems, in Location science, G. Laporte, S. Nickel, and F. Saldanha da Gama, Editors. 2015, Springer International Publishing: Cham. 311-344.

2. Campbell, J.F., "Integer programming formulations of discrete hub location problems", ***European Journal of Operational Research***, Vol. 72, No. 2, (1994), 387-405.

3. Karimi, H. and Bashiri, M., "Hub covering location problems with different coverage types", ***Scientia Iranica***, Vol. 18, No. 6, (2011), 1571-1578.

4. Hwang, Y.H. and Lee, Y.H., "Uncapacitated single allocation p-hub maximal covering problem", ***Computers & Industrial Engineering***, Vol. 63, No. 2, (2012), 382-389.

5. Jabalameli, M.S., Barzinpour, F., Saboury, A. and Ghaffari-Nasab, N., "A simulated annealing-based heuristic for the single allocation maximal covering hub location problem", ***International Journal of Metaheuristics***, Vol. 2, No. 1, (2012), 15-37.

6. Ebrahimi-zade, A., Sadegheih, A. and Lotfi, M.M., "A modified nsga-ii solution for a new multi-objective hub maximal covering problem under uncertain shipments", ***Journal of Industrial Engineering International***, Vol. 10, No. 4, (2014), 185-197.

7. Pasandideh, S.H.R., Niaki, S.T.A. and Sheikhi, M., "A bi-objective hub maximal covering location problem considering time-dependent reliability and the second type of coverage", ***International Journal of Management Science and Engineering Management***, Vol. 11, No. 4, (2015), 195-202.

8. Bashiri, M. and Rezanezhad, M., "A reliable multi-objective p-hub covering location problem considering of hubs capabilities", ***International Journal of Engineering-Transactions B: Applications***, Vol. 28, No. 5, (2015), 717-729.

9. Karimi, H., Bashiri, M. and Nickel, S., "Capacitated single allocation p-hub covering problem in multi-modal network using tabu search", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 29, No. 6, (2016), 797-808.

10. Ebrahimi-zade, A., Sadegheih, A. and Lotfi, M.M., "Fuzzy multi-objective linear programming for a stochastic hub maximal covering problem with uncertain shipments", ***International Journal of Industrial and Systems Engineering***, Vol. 23, No. 4, (2016), 482-499.

11. Janković, O. and Stanimirović, Z., "A general variable neighborhood search for solving the uncapacitated r-allocation p-hub maximal covering problem", ***Electronic Notes in Discrete Mathematics***, Vol. 58, (2017), 23-30.

12. Janković, O., Mišković, S., Stanimirović, Z. and Todosijević, R., "Novel formulations and vns-based heuristics for single and multiple allocation p-hub maximal covering problems", ***Annals of Operations Research***, Vol. 259, No. 1-2, (2017), 191-216.

13. Madani, S.R., Shahandeh Nookabadi, A. and Hejazi, S.R., "A bi-objective, reliable single allocation p-hub maximal covering location problem: Mathematical formulation and solution approach", ***Journal of Air Transport Management***, Vol. 68, (2018), 118-136.
14. Gelareh, S., "Hub location models in public transportation planning", Kaiserslautern University of Technology, Ph.D, Thesis (2008),
15. Ebrahimi-zade, A., Hosseini-Nasab, H., zare-mehrjerdi, Y. and Zahmatkesh, A., "Multi-period hub set covering problems with flexible radius: A modified genetic solution", ***Applied Mathematical Modelling***, Vol. 40, No. 4, (2016), 2968-2982.
16. Campbell, J.F. and O'Kelly, M.E., "Twenty-five years of hub location research", ***Transportation Science***, Vol. 46, No. 2, (2012), 153-169.
17. Balaman, Ş.Y., Matopoulos, A., Wright, D.G. and Scott, J., "Integrated optimization of sustainable supply chains and transportation networks for multi technology bio-based production: A decision support system based on fuzzy ε-constraint method", ***Journal of Cleaner Production***, Vol. 172, (2018), 2594-2617.
18. Yu, H. and Solvang, W.D., "An improved multi-objective programming with augmented ε-constraint method for hazardous waste location-routing problems", ***International Journal of Environmental Research and Public Health***, Vol. 13, No. 6, (2016), DOI: 10.3390/ijerph13060548.
19. Emami, S., Moslehi, G. and Sabbagh, M., "A benders decomposition approach for order acceptance and scheduling problem: A robust optimization approach", ***Computational and Applied Mathematics***, Vol. 36, No. 4, (2017), 1471-1515.
20. Saharidis , K.D., Minoux , M. and Ierapetritou , G., "Accelerating benders method using covering cut bundle generation", ***International Transactions in Operational Research***, Vol. 17, No. 2, (2010), 221-237.
21. O'Kelly, M.E., "A quadratic integer program for the location of interacting hub facilities", ***European Journal of Operational Research***, Vol. 32, No. 3, (1987), 393-404.
22. Alumur, S.A., Nickel, S., Saldanha-da-Gama, F. and Seçerdin, Y., "Multi-period hub network design problems with modular capacities", ***Annals of Operations Research***, Vol. 246, No. 1, (2016), 289-312.
23. Gelareh, S., Neamatian Monemi, R. and Nickel, S., "Multi-period hub location problems in transportation", ***Transportation Research Part E: Logistics and Transportation Review***, Vol. 75, (2015), 67-94.
24. Silva, M.R. and Cunha, C.B., "A tabu search heuristic for the uncapacitated single allocation p-hub maximal covering problem", ***European Journal of Operational Research***, Vol. 262, No. 3, (2017), 954-965.

Mathematical Model for Bi-objective Maximal Hub Covering Problem with Periodic Variations of Parameters

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چکیده

مسأله حداکثر پوشش هاب یکی از مسائل چالش برانگیز در تحقیق عملیات و برنامه‌ریزی حمل و نقل بوده که هدف آن یافتن مکان بهینه‌ی مجموعه‌ای از هاب‌ها برای رسیدن به حداکثر جریان در یک شبکه است. از آنجائی‌که در دنیای واقعی پارامترهای اصلی مسأله مانند جریان بین مبدأ و مقصد، هزینه‌ها و زمان سفر به طور دوره‌ای تغییر می‌کنند، در مواجهه با آنها می‌بایست تمهیداتی به‌کار گرفته شوند. در این مقاله، برای مقابله با تغییرات دوره‌ای پارامترها، یک مدل ریاضی دو هدفه برای مساله حداکثر پوشش هاب پویا با تخصیص تکی ارائه شده است. برای دسترسی به جواب‌های ناچیره از رویکرد محدودیت اپسیلون استفاده شده است. با توجه به اینکه مسئله‌ی تک هدفه ایجاد شده در روش محدودیت اپسیلون از نظر محاسباتی پیچیده است، الگوریتم تجزیه‌ی بندرز با اضافه نمودن نامعادلات معتبر برای سرعت بخشیدن به فرآیند حل توسعه داده شده است. در نهایت، روش پیشنهادی با استفاده از مجموعه داده CAB اجرا شده و نتایج به‌دست آمده نشان‌دهنده کارایی الگوریتم پیشنهادی از نظر بهینگی و زمان اجرا می‌باشد.

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