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Analytical Matching of Optimal Damping Characteristics Curve for Vehicle Passive Suspensions

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ABSTRACT

To facilitate the damping matching of dampers for vehicle passive suspensions, this paper proposes an analytical matching method of the optimal piecewise linear damping characteristics curve. Based on the vehicle vibration model, taking the suspension dynamic deflection as the constraint, by the vibration acceleration and the wheel dynamic load, an objective function about the relative damping coefficient was created. Using the integral mid-value law, an analytical formula of the optimal relative damping coefficient was established and the analytical matching method of the piecewise linear damping characteristics curve was presented. With a practical example, the effectiveness of the proposed matching method was validated by road test. The results showed that the method presented is workable, which has important reference value for fast damping matching of vehicle passive suspensions.

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1. INTRODUCTION

The suspension system is an essential part of vehicles. Its performance influences ride comfort and driving safety of vehicles [1-3]. At present, the passive suspension system is widely used in vehicles and its relative damping coefficient has an important guidevline which is significance for the design and the selection of dampers [4-6].

At present, many scholars have done a lot of research works on the relative damping coefficient and the damping matching method of passive suspensions. For vehicles fitted with the passive suspension system, the relative damping coefficient of the system effects riding comfort [7, 8]. Therefore, its determination is the key problem that plagues the damping matching of vehicle suspensions. Usually, the relative damping coefficient is selected by trial and error in its practical range of 0.2~0.4 [9, 10]. Although this method is feasible, because of its high design cost and long cycle, it is difficult to meet the requirements of the rapid development of suspension systems in modern vehicle industry. Yang et al. [11] gave the design range of the relative damping coefficient of passive suspensions for in-wheel motor vehicles, which has a certain guiding significance for the initial selection of passive suspensions. Zhao et al. [12] studied the optimal damping of truck cushion systems. Smith et al. [13] investigated the application of the least relative damping coefficient. Xing et al. [14] optimized the suspension damping based on ADAMS software and ant colony algorithm. These studies have important contributions to the damping matching of passive suspensions. However, it is not convenient for engineers to master the complex modelling methods and optimization algorithms. Thus, there is an urgent need to solve the analytical calculation of the relative damping coefficient and to create a fast matching method of damper damping characteristics. Therefore, it can provides effective guidance for suspension design and damper selection.

To avoid repeated trials in the process of damping matching, this paper proposes an analytical matching method of the optimal piecewise linear damping characteristics curve. With a practical example, through road test, the effectiveness of the presented method was validated.

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2. ANALYTICAL CALCULATION OF VEHICLE VIBRATION

The vehicle dynamic model is shown in Figure 1. Where, m_2 represents the vehicle body mass; m_1 represents the tire system mass; K is the suspension stiffness coefficient; C is the suspension damping coefficient; K_t is the tire vertical stiffness coefficient; q is the road excitation; z_1 and z_2 are the displacements of m_1 and m_2, respectively.

Based on the vehicle vibration model, its vibration equation can be expressed as follows:

$$\begin{cases} m_2\ddot{z}_2 + C(\dot{z}_2 - \dot{z}_1) + K(z_2 - z_1) = 0 \\ m_1\ddot{z}_1 + C(\dot{z}_1 - \dot{z}_2) + K(z_1 - z_2) + K_t(z_1 - q) = 0 \end{cases} \quad (1)$$

Let $r_k = K_t / K$, $r_m = m_2 / m_1$, $\omega_0 = \sqrt{K / m_2}$, $\xi = C / (2\sqrt{m_2 K})$.

Where, r_m, r_k, and ξ are the dimensionless ratios related to vehicle parameters; ω_0 is the natural frequency of the vehicle body; ξ is called the relative damping coefficient of the suspension system.

According to Equation (1), the mean square acceleration $\sigma^2_{\ddot{z}_2}$ can be expressed as follows [15]:

$$\sigma^2_{\ddot{z}_2} = \pi^2 G_q(n_0) n_0^2 v \frac{\omega_0^3(1 + r_m + 4r_m r_k \xi^2)}{2\xi r_m} \quad (2)$$

where, $n_0 = 0.1\text{m}^{-1}$, v is the vehicle speed, $G_q(n_0)$ the road roughness coefficient.

The mean square WDL (wheel dynamic load) $\sigma^2_{F_d}$ can be expressed as follows [15]:

$$\sigma^2_{F_d} = K_t^2 \pi^2 G_q(n_0) n_0^2 v [r_m r_k (r_m r_k - 2 - 2r_m) + (1 + r_m)^3 + 4r_m r_k \xi^2 (1 + r_m)^2] / (2r_m^3 r_k^2 \omega_0 \xi) \quad (3)$$

The mean square SDD (suspension dynamic deflection) $\sigma^2_{f_d}$ can be expressed as follows [15]:

$$\sigma^2_{f_d} = \pi^2 G_q(n_0) n_0^2 v \frac{1 + r_m}{2\xi \omega_0 r_m} \quad (4)$$

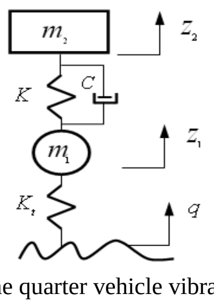

Figure 1. The quarter vehicle vibration model

3. ANALYTICAL MATCHING METHOD OF DAMPER DAMPING

3. 1. Analytic Formula of Optimal Relative Damping Coefficient

To get the optimal relative damping coefficient for passive suspensions, create the objective function J as:

$$J(\xi) = \alpha \left(\frac{\sigma_{\ddot{z}_2}}{g} \right)^2 + (1 - \alpha) \left[\frac{\sigma_{F_d}}{(m_1 + m_2)g} \right]^2 \quad (5)$$

where, α is a weighted factor and $\alpha \in [0,1]$.

The constraint is as follows:

$$\sigma_{f_d} \le [f_d] / 3 \quad (6)$$

where, $[f_d]$ represents the limit travel of the SDD f_d.

Substituting Equations (2) and (3) into Equation (5), obtain the following:

$$J(\xi) = \frac{\pi^2 G_q(n_0) n_0^2 v \omega_0^3}{2r_m g^2} \left(\frac{a}{\xi} + b\xi \right) \quad (7)$$

where, $b = 4r_m r_k$,

$a = \alpha + \alpha r_m + \frac{(1 - \alpha)[r_m r_k (r_m r_k - 2 - 2r_m) + (1 + r_m)^3]}{(1 + r_m)^2}$.

Under the constraint of Equation (6), the relative damping coefficient ξ^* minimizing J can be obtained as

$$\xi^* = \sqrt{\frac{a}{b}} = \frac{1}{2}\sqrt{\frac{1 + r_m}{r_m r_k} + \frac{(1 - \alpha)(r_m r_k - 2 - 2r_m)}{(1 + r_m)^2}} \quad (8)$$

According to $\frac{\partial \sigma^2_{\ddot{z}_2}}{\partial \xi} = 0$, obtain $\xi = \frac{1}{2}\sqrt{\frac{1 + r_m}{r_m r_k}}$ and denote it as ξ_{oc} [15]:

$$\xi_{oc} = \frac{1}{2}\sqrt{\frac{1 + r_m}{r_m r_k}} \quad (9)$$

According to $\frac{\partial \sigma^2_{F_d}}{\partial \xi} = 0$, obtain $\xi = \frac{1}{2}\sqrt{\frac{1 + r_m}{r_m r_k} + \frac{r_m r_k - 2 - 2r_m}{(1 + r_m)^2}}$ and denote it as ξ_{os} [15]:

$$\xi_{os} = \frac{1}{2}\sqrt{\frac{1 + r_m}{r_m r_k} + \frac{r_m r_k - 2 - 2r_m}{(1 + r_m)^2}} \quad (10)$$

Based on Equation (8), it can be seen that when α=0, ξ^*=ξ_{os} and when α=1, ξ^*=ξ_{oc}. For example, r_m=10 and r_k=9 for a car. The curve of the relative damping coefficient ξ^* along with the weighted factor α is obtained, as shown in Figure 2.

Figure 2 shows that when α=0, ξ^*=ξ_{os}=0.41 and when α =1, ξ^*=ξ_{oc}=0.18. For controllable suspension systems, ξ^* can change from 0.18 to 0.41 along with α from 1 to 0. However, for passive suspensions, ξ^* is basically a constant. A compromise between ξ_{oc} and ξ_{os} must be made to select the optimal relative damping coefficient ξ_{op} for passive suspensions.

Mathematically, the optimal mean value is often determined by the method of the integral mean value theorem. Based on the integral mean value law, obtain the following:

$$\xi_{op} = \frac{1}{1-0}\int_0^1 \xi^* \mathrm{d}\alpha \tag{11}$$

Substituting Equation (10) into Equation (11), obtain the following:

$$\int_0^1 \xi^* \mathrm{d}\alpha = \int_0^1 \frac{1}{2}\sqrt{\frac{1+r_m}{r_m r_k} + \frac{(1-\alpha)(r_m r_k - 2 - 2r_m)}{(1+r_m)^2}}\mathrm{d}\alpha \tag{12}$$

Based on Equations (9) and (10), by variable substitution, obtain the following:

$$\int_0^1 \xi^* \mathrm{d}\alpha = \frac{2(\xi_{os}^2 + \xi_{os}\xi_{oc} + \xi_{oc}^2)}{3(\xi_{os} + \xi_{oc})} \tag{13}$$

According to Equations (11) and (13), the optimal relative damping coefficient ξ_{op} can be obtained as:

$$\xi_{op} = \frac{2(\xi_{os}^2 + \xi_{os}\xi_{oc} + \xi_{oc}^2)}{3(\xi_{os} + \xi_{oc})} \tag{14}$$

According to the vehicle parameters m_2, m_1, K, and K_t, the mass ratio $r_m = m_2/m_1$ and the stiffness ratio $r_k = K_t/K$ can be obtained. Then, ξ_{oc} and ξ_{os} can be calculated, respectively. Thirdly, according to (14), ξ_{op} can be achieved. According to the analysis above, it can seen that ξ_{op} is related to r_m and r_k, and has nothing to do with other parameters. The surface of the optimal relative damping coefficient ξ_{op} versus the mass ratio r_m and the stiffness ratio r_k is shown in Figure 3.

Figure 3 illustrates that the optimal relative damping coefficient ξ_{op} decreases with the increase of the mass ratio r_m and ξ_{op} increases with the increase of the stiffness ratio r_k.

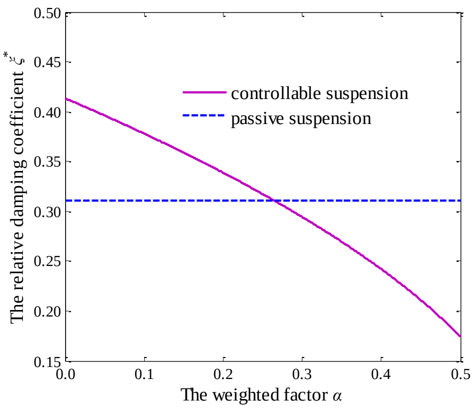

Figure 2. The curve of the relative damping coefficient ξ^* versus the weighted factor α

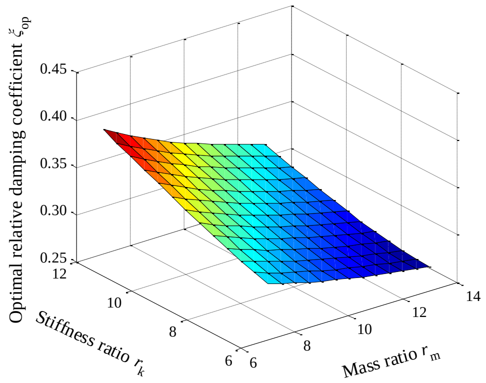

Figure 3. The surface of the optimal relative damping coefficient ξ_{op} versus the mass ratio r_m and the stiffness ratio r_k

When r_m belongs to [7, 13] and r_k belongs to [6, 12], the maximum value of ξ_{op} is 0.382, the minimum value of ξ_{op} is 0.275. Thus, the range of ξ_{op} is [0.275, 0.382].

3. 2. Matching of Optimal Piecewise Linear Damping Characteristics Curve For suspension dampers, the curve of the damping force F_c versus the relative motion velocity V is nonlinear. For the convenience of the damper design, the curve is often simplified as the piecewise linear damping characteristics curve, as shown in Figure 4.

In Figure 4, V_{k1} and V_{k1y} are the initial opening valve velocity of the rebound stroke and the compression stroke, respectively; F_{k1} and F_{k1y} are the initial opening valve damping force of the rebound stroke and the compression stroke, respectively; V_{k2} and V_{k2y} are the second opening valve velocity of the rebound stroke and the compression stroke, respectively; F_{k2} and F_{k2y} are the second opening valve damping force of the rebound stroke and the compression stroke, respectively.

According to the vehicle parameters m_2, K, and $\xi = \xi_{op}$ calculated by equation (14)，the damping coefficient C_{k1} for $V \in [0, V_{k1}]$ can be determined as

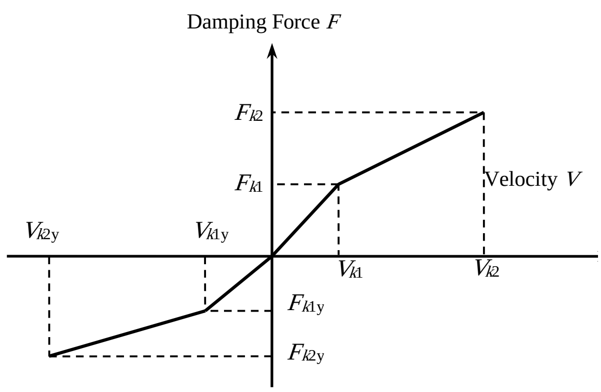

Figure 4. The curve of the damping force F_c versus the velocity V

$$C_{k1} = \frac{2\xi_{op}\sqrt{Km_2}}{i^2\cos^2\theta} \quad (15)$$

where, i is the lever ratio for the damper installation, θ is the damper installation angle.

Based on C_{k1} and V_{k1}, the damping force F_{k1} for $V=V_{k1}$ can be determined as:

$$F_{k1} = C_{k1}V_{k1} \quad (16)$$

The slope k_1 of the curve for $V \in [0, V_{k1}]$ equals C_{k1}. Thus, the slope k_2 of the curve for $V \in [V_{k1}, V_{k2}]$ can be determined as:

$$k_2 = \frac{k_1}{\eta} = \frac{C_{k1}}{\eta} \quad (17)$$

where, η is the smoothness-to-safety ratio.

On the basis of V_{k1}, V_{k2}, F_{k1}, and k_2, the damping force for $V=V_{k2}$ can be determined as:

$$F_{k2} = F_{k1} + k_2(V_{k2} - V_{k1}) \quad (18)$$

The damping force F_{k1y} for $V=V_{k1y}$ and F_{k2y} for $V=V_{k2y}$ can be respectively determined as:

$$F_{k1y} = \beta C_{k1}V_{k1} \quad (19)$$

$$F_{k2y} = \beta[F_{k1} + k_2(V_{k2} - V_{k1})] \quad (20)$$

where, β is the damper bi-directional ratio.

4. CASE STUDY AND TEST VERIFICATION

For example, the vehicle parameters of a mini-truck for are as follows: m_2=350 kg, m_1=35 kg, K=19897 N/m for the front suspensions, K_t=179073 N/m, η=1.5, θ=10°, i=0.9; V_{k1}=0.3 m/s, V_{k2}=1.0 m/s, V_{k1y}=-0.15 m/s, V_{k2y}=-1.0 m/s, β=1/3. In order to achieve the optimal damping effect, it is necessary to match the optimal damping damper for the front suspensions. In the study, the original dampers of the rear suspensions remained unchanged.

4. 1. Example of Damper Damping Matching

According to the parameters of the mini-truck, using equations (9) and (10), the calculated ξ_{oc} and ξ_{os} equal 0.175 and 0.414, respectively. Using equation (14), the calculated ξ_{op}=0.31. Based on ξ_{op}=0.31 and the parameters above, using the damping matching method in Section 3.2, the optimal piecewise linear damping characteristics curve of the damper was determined. For comparative analysis, the piecewise linear damping characteristics curves for ξ=0.22, 0.25, 0.28, 0.34 were also designed, respectively.

The dampers for ξ=0.22, 0.25, 0.28, 0.34, and 0.31 were produced based on their piecewise linear damping characteristics curves. The damping characteristics of the dampers were tested. For the damper with ξ_{op}=0.31, the comparison of the tested curve and the designed curve is depicted in Figure 5 and the comparison of the damping forces is shown in Table 1.

From Figure 5, it can be seen that the tested curve is very close to the designed curve. Table 1 shows that the damping force values of the tested and the designed at different velocities are coincident. The test results show that the produced damper with ξ_{op}=0.31 meet the design requirements of the characteristic curves. Other produced dampers also meet their design requirements. Due to the similarity of the curves, it's not described here.

4. 2. Road Test Verification

To verify the effectiveness of the proposed matching method of the damper damping, the road tests were conducted through changing different dampers for ξ=0.22, 0.25, 0.28, 0.34, and 0.31, respectively. The used test equipment is LMS Test Lab. To measure the vehicle acceleration response, an accelerometer was mounted on the cab floor of the test vehicle.

According to the pulse test requirement and method of the standard GB/T 4970-2009, pulse tests were carried out. The measured maximum values of the floor vibration acceleration $\ddot{z}_{max}$ are shown in Table 2. In addition, the random vibration tests were carried out on the highway through changing different dampers for ξ=0.22, 0.25, 0.28, 0.34, and 0.31, respectively.

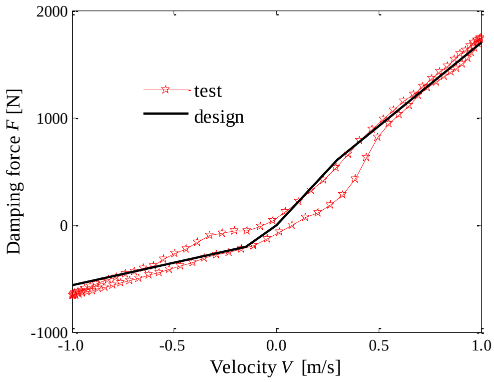

Figure 5. Comparison of the damper damping characteristics curves

TABLE 1. Comparison of damping force values

Damping force F / (N)	Velocity V / (m·s^{-1})				
	-1.0	**-0.15**	**0**	**0.3**	**1.0**
Tested value	-621.5	-181.4	0	556.2	1796.3
Designed value	-567.3	-203.0	0	608.9	1701.9
Relative deviation (%)	-8.72	11.9	0	9.48	-5.26

The floor weighted RMS acceleration σ_w calculated from the measured data are shown in Table 3.

Table 2 shows that when the relative damping coefficient ξ equals ξ_{op}=0.31, the maximum values of the floor vibration acceleration are mostly less than the other four groups ξ=0.22, 0.25, 0.28, and 0.34. The results clearly state that the damper for ξ=0.31 has the better damping effect. Although the maximum values are relatively larger when the speed v equals 40km/h and v=70km/h, they are quite close. Moreover, the relative deviation is only 1.55% between the condition ξ=0.31 and the condition ξ=0.34 for v=40km/h. The relative deviation is only 0.75% between the condition ξ=0.31 and the condition ξ=0.28 for v=70km/h.

From Table 3, it can be seen that when the relative damping coefficient ξ equals ξ_{op}=0.31, the floor weighted RMS acceleration σ_w are mostly not larger than the other four groups ξ=0.22, 0.25, 0.28, and 0.34. Moreover, the driver felt that the ride comfort of the vehicle equipped with the damper for ξ=0.31 is better than the others. The results show that under the highway condition, the damper for ξ=0.31 is beneficial to improve ride comfort.

TABLE 2. Results of pulse tests

Speed v /(km/h)	**Maximum acceleration $\ddot{z}_{max}$ / (m/s^2)**				
	ξ=0.22	**ξ=0.25**	**ξ=0.28**	**ξ=0.31**	**ξ=0.34**
30	14.59	14.16	14.11	12.92	13.99
40	11.26	11.09	10.68	10.45	10.29
50	10.25	10.28	10.12	9.69	9.97
60	9.98	9.54	9.16	8.98	10.23
70	10.35	9.87	9.39	9.46	10.12
80	11.62	11.46	10.72	10.56	10.98
90	11.06	11.19	11.28	10.39	10.56
100	11.97	12.63	12.94	11.57	12.24

TABLE 3. Results of random vibration tests

Speed v /(km/h)	**The floor weighted RMS acceleration σ_w / (m/s^2)**				
	ξ=0.22	**ξ=0.25**	**ξ=0.28**	**ξ=0.31**	**ξ=0.34**
30	0.29	0.25	0.27	0.26	0.28
40	0.38	0.35	0.36	0.33	0.37
50	0.37	0.37	0.35	0.34	0.39
60	0.46	0.39	0.41	0.40	0.43
70	0.54	0.52	0.50	0.51	0.54
80	0.67	0.59	0.56	0.56	0.69
90	0.69	0.63	0.61	0.62	0.68
100	0.65	0.62	0.63	0.60	0.71

5. CONCLUSIONS

To facilitate the damping matching of dampers for vehicles, this paper proposes an analytical matching method of the optimal piecewise linear damping characteristics curve and validates its effectiveness. The main conclusions are as follows:

(1) The optimal relative damping coefficient ξ_{op} can be calculated by a simple analytical formula. The formula is only related to the mass ratio $r_m=m_2/m_1$ and the stiffness ratio $r_k=K_t/K$.

(2) Based on the optimal relative damping coefficient ξ_{op}, the damper optimal damping characteristics curve can be determined by the proposed method of the optimal piecewise linear damping characteristics curve.

(3) The test results show that the ride comfort of the vehicle equipped with the damper for ξ=0.31 is better than ξ=0.22, 0.25, 0.28, and 0.34. It can be seen that the proposed method is effective.

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چکیده

برای تسهیل تطبیق لغزش دمپر برای تعلیق های غیرفعال خودرو، این مقاله یک روش تطبیق تحلیلی برای منحنی ویژگی های منحنی خنک کننده خطی به صورت تحلیل آنالیز بهینه ارائه می دهد. بر اساس مدل ارتعاش خودرو، با توجه به شتاب ارتعاش و بار دینامیکی چرخش پویایی تعلیق به عنوان محدودیت، یک تابع هدف در مورد ضریب آرام سازی نسبی ایجاد شد. با استفاده از قانون ثانویه مقادیر انتگرال، یک فرمول تحلیلی از ضریب آرام سازی نسبی بهینه تهیه شد و روش تطبیق تحلیلی منحنی ویژگی های خمشی خطی بسته بندی شده ارائه شد. با یک مثال عملی، اثربخشی روش تطبیق پیشنهادی با آزمون جاده تأیید شد. نتایج نشان داد که روش ارائه شده عملی است که ارزش مرجع قابل توجهی را برای تطبیق سرعت مکانیسم تعلیق های غیر فعال خودرو دارد.

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