IJE TRANSACTIONS C: Aspects Vol. 31, No. 12, (December 2018) 2059-2067

International Journal of Engineering

Journal Homepage: www.ije.ir

Mathematical Formulation and Solving of Green Closed-loop Supply Chain Planning Problem with Production, Distribution and Transportation Reliability

M. B. Fakhrzad*, P. Talebzadeh, F. Goodarzian

Faculty of Industrial Engineering, Yazd University, Yazd, Iran

PAPER INFO

Paper history:
Received 30 January 2018
revised form 27 October 2018
Accepted 06 November 2018

Keywords:
Green Closed-loop Supply Chain
Uncertainty
Reliability
NSGA-II Algorithm

ABSTRACT

In this paper, developed a new multi-product, multi-period, and multi-level closed-loop green supply chain planning model under uncertain conditions. The formulated model consists of five objective functions, which minimize the cost of the supply chain, minimize the CO_2 emission of transportation vehicles, maximize the reliability of manufacturing and distribution centers, maximize the reliability of the transportation system, and maximize the level of service provided. Therefore, the problem of model formulated as a multi-objective mixed integer nonlinear programming. Also, since the proposed model is complex and NP-hard in large size, therefore, for the investigation of the results, we have used a Non-Dominated Sorting Genetic II Algorithm (NSGA-II). In addition, the small of size results of the problem achieved by GAMS software. Therefore, we try to solve these problems by analyzing and comparing them with the help of these algorithm. For this purpose, various size has been considered.

doi: *10.5829/ije.2018.31.12c.10*

1. INTRODUCTION

In the field of supply chain management (SCM), the Forward Supply Chain (FSC) refers to a series of activities that transform raw materials into one or several end products [1-3]. Since the independent design of FSC and RSC may result in sub-optimal solutions, supply chain planners are recommended to design forward and reverse logistics in an integrated fashion. Such integration can be made with a horizontal or vertical configuration [1, 4]. In this regard, Zeballos et al. [5] proposed a method for the design and planning of multi-product multi-period CLSC with 10-layer network structure (five forward and five reverse) with uncertainty in the amount of raw materials and demand. In a study by Dai and Zhang [6], they presented a model for the design and planning of multi-product multi-period multi-level CLSC under uncertain conditions. Soleimani et al. [7] and Garg et al. [8] attempted to improve the process of CLSC optimization with the aid of mathematical programming. Pasandideh et al. [9] proposed a probabilistic multi-objective model for the planning of sustainable supply chain network under uncertainty conditions according to economic, environmental and social objectives. Kalaitzidou et al. [10] developed a new mixed integer linear programming model for the planning of multi-period single-product capacitated CLSC with the goal of maximizing the expected profit. Cardoso et al. [11] utilized an integer linear programming model to incorporate the financial risks into the design and planning of CLSCs with uncertain demand for end products. Garg et al. [8]. studied a CLSC network with four levels in the forward chain and five levels in the reverse chain. In another work by Pasandideh et al. [9] and Ning et al. [12] developed the bi-objective optimization of a multi-product multi-period supply chain consisting of factories, distribution centers with non-deterministic services, and customers. Khatami et al. [4] proposed a model for the simultaneous planning of multi-product FSC and RSC networks with uncertainties in demand

*Corresponding author Email: *mfakhrzad@yazd.ac.ir* (M. B. fakhrzad)

Please cite this article as: M. B. fakhrzad, P. Talebzadeh, F. Goodarzian, Mathematical Formulation and Solving of Green Closed-loop Supply Chain Planning Problem with Production, Distribution and Transportation Reliability, International Journal of Engineering (IJE), IJE TRANSACTIONS C: Aspects Vol. 31, No. 12, (December 2018) 2059-2067

and returned products over multiple periods and levels. Ramazani et al. [13]are designing a supply chain network that includes supply centers, production centers, distribution centers, customers, warehouse centers, recruitment centers and recycling centers. Jabbarzadeh et al. [14] a robust randomized optimization model has been designed to design a closed-loop supply chain network that works flexibly during disruptions. Jerbia et al. [15] developed closed-loop supply chain network with various recycling options. Initially, the definite problem is considered as a mixed integer linear program (MILP). Kim et al. [16] designed a three-level closed loop supply chain, including suppliers of raw materials, manufacturers, and customers. Ghahremani et al. [17] proposed a location-allocation facilities model for multi-product, multi-period, multi-level under the lack, uncertainty. In addition, extensive work conduted on multi-product, uncertainty and purchase of raw materials of the CLSC network [18-20].

In this paper, we consider a multi-objective, multi-product, and multi-period GCLSC. In addition, none of the above reviewed articles have simultaneously considered the maximization of the reliability of centers, the minimization of CO_2 emission, and the maximization of customer satisfaction. All articles have been reviewed maximizing reliability of the centers, minimization of released gas production, customer satisfaction and the minimization of cost is not considered simultaneously. The paper mathematically formulates the problem as an MILP model. The problem is solved by commercial software of "GAMS", for the small sized problems to optimality. Moreover, the paper proposes a NSGA-II entitled Non-Dominated Sorting Genetic Algorithm II while solving the large-sized problems.

2. PROBLEM DESCRIPTION

The overall problem can be stated as follows:
1. Throughout the planning horizon, the demand for products is uncertain. 2. The price of raw materials and the costs of procurement, manufacturing, transportation, facility construction, and CO_2 emission are uncertain. 3. The inventory of raw materials and products at the start and end of each period is zero. 4. The shortage is allowed, but failure to meet the demand results in incurring a penalty cost. Nevertheless, the total demand should be met by the end of last period. 5. To ensure the proper supply of materials, each supplier is assumed to have a reliability index. 6. Facilities that are less vulnerable to disruptions increase the efficiency and thus the reliability of the supply chain. 7. There are several types of transport vehicles with known capacities. 8. Transporting goods between facilities results in the generation of CO_2 emission. 9. The model is multilevel, multi-product and multi-period. 10. The locations of customers are fixed and known. 11. The maximum number of facilities that can be constructed and their capacity are limited. 12. All returned products must be collected.

3. MODELLING AND FORMULATING THE PROBLEM

The paper considers a five level green closed-loop supply chain network with multiple suppliers, multiple plants, multiple DCs and multiple customers. The required indices and parameters are presented in Tables 1 and 2. The decision variables of the model are given in Table 3.

TABLE 1. Indices used in the mathematical model

Notations	Descriptions		
		M	Set of raw materials (m = 1,2,…,M)
I	Set of potential sites for suppliers (i = 1, 2,…, I)	J	Set of potential sites for manufacturing/remanufacturing centers (j = 1, 2,..., J)
K	Set of potential sites for distribution/collection centers (k = 1,2,…,K)	C	Set of customer locations (c = 1,2,…,C)
R	Set of products (r = 1,2,…,R)	F	Set of potential sites for disposal centers (f = 1,2,…,F)
T	Set of periods (t = 1,2,…,T)	L	Type of transportation system (l = 1,2,…,T)

TABLE 2. Parameters used in the mathematical model

pcr_{mit}	Cost of procuring raw material *m* from supplier *i* in period *t*	${ftr^l}_{kjt}$	Fixed cost of transportation system *l* for a transport from distribution/collection center *k* to manufacturing/remanufacturing center *j* in period *t*
rp_{mit}	Price of raw material *m* supplied by supplier *i* in period *t*	${vtr^l}_{rkjt}$	Cost of transporting recoverable product *r* from distribution/collection center *k* to manufacturing/remanufacturing center *j* using transportation system *l* in period *t*
$\widetilde{pc_{rjt}}$	Cost of procuring raw material *m* from supplier *i* in period *t*	${ftf^l}_{kft}$	Fixed cost of transportation system *l* for a transport from distribution/collection center *k* to disposal center *f* in period *t*

rp_{mit}	Price of raw material *m* supplied by supplier *i* in period *t*	vtf^{l}_{rkft}	Cost of transporting unrecoverable product *r* from distribution/collection center *k* to disposal center *f* using transportation system *l* in period *t*
pc_{rjt}	Per unit cost of manufacturing product *r* in manufacturing/remanufacturing center *j* in period *t*	hc_{rkt}	Distribution and inventory holding cost of product *r* at distribution/collection center *k* in period *t*
fts^{l}_{ijt}	Fixed cost of transportation system *l* for a transport from supplier *i* to manufacturing/remanufacturing center *j* in period *t*	shc_{rct}	Per unit cost of shortage of product *r* for customer *c* in period *t*
vts^{l}_{ijmt}	Cost of transporting raw material m from supplier *i* to manufacturing/remanufacturing center *j* using transportation system *l* in period *t*	dc_{rft}	Cost of disposal of product *r* at disposal center *f* in period *t*
ftp^{l}_{jkt}	Fixed cost of transportation system *l* for a transport from manufacturing/remanufacturing center *j* to distribution/collection center *k* in period *t*	F_{jt}	Fixed cost of establishing manufacturing/remanufacturing center *j* in period *t*
vtp^{l}_{rjkt}	Cost of transporting product *r* from manufacturing/remanufacturing center *j* to distribution/collection center *k* using transportation system *l* in period *t*	F_{kt}	Fixed cost of establishing distribution/collection center *k* in period *t*
ftd^{l}_{kct}	Fixed cost of transportation system *l* for a transport from distribution/collection center *k* to customer *c* in period *t*	F_{ft}	Fixed cost of establishing disposal center *f* in period *t*
vtd^{l}_{rkct}	Cost of transporting product *r* from distribution/collection center *k* to customer *c* using transportation system *l* in period *t*	F_{it}	Fixed cost of selecting supplier *i* in period *t*
ftc^{l}_{ckt}	Fixed cost of transportation system *l* for a transport from customer *c* to distribution/collection center *k* in period *t*	$\theta1_{ijmt}$	Quantity of CO_2 released per unit of raw material *m* transported from supplier *i* to manufacturing/remanufacturing center j in period *t*
vtc^{l}_{rckt}	Cost of transporting returned product *r* from customer c to distribution/collection center *k* using transportation system *l* in period *t*	$\theta2_{rjkt}$	Quantity of CO_2 released per unit of product *r* transported manufacturing/remanufacturing center *j* to distribution/collection center *k* in period *t*
$\theta3_{rkct}$	Quantity of CO_2 released per unit of product *r* transported from distribution/collection center *k* to customer *c* in period *t*	rak^{l}_{jk}	Reliability of transport from manufacturing/remanufacturing center *j* to distribution/collection center *k* using transportation system *l*
$\theta4_{rckt}$	Quantity of CO_2 released per unit of returned product *r* transported from customer *c* to distribution/collection center *k* in period *t*	rac^{l}_{kc}	Reliability of transport from distribution/collection center *k* to customer *c* using transportation system *l*
$\theta5_{rkft}$	Quantity of CO_2 released per unit of returned product *r* transported from distribution/collection center *k* to disposal center f in period *t*	raq^{l}_{ck}	Reliability of transport from customer *c* to distribution/collection center *k* using transportation system *l*
$\theta6_{rkjt}$	Quantity of CO_2 released per unit of recoverable product *r* transported from distribution/collection center *k* to manufacturing/remanufacturing center *j* in period *t*	rap^{l}_{kj}	Reliability of transport from distribution/collection center *k* to manufacturing/remanufacturing center *j* using transportation system *l*
R1	Reliability index of manufacturing/remanufacturing center *j*	rao^{l}_{kf}	Reliability of transport from distribution/collection center *k* to disposal center *f* using transportation system *l*
R2	Reliability index of distribution/collection center *k*	γ	Weight of importance of forward-looking responsiveness
RI_{mit}	Reliability index of supplier *i* for raw material m period *t*	ur_{rmt}	Rate of use of raw material m for manufacturing product *r* in period *t*
rai^{l}_{ij}	Reliability of transport from supplier *i* to manufacturing/remanufacturing center *j* using transportation system *l*	dem_{rct}	Demand for product *r* in period *t*
RT_r	Rate of return of product *r*	$capp_{jt}$	Storage capacity of manufacturing center *j* in period *t*
RR_r	Rate of recoverability of product *r*	$capr_{jt}$	Storage capacity of remanufacturing center *j* in period *t*
v_r	Unit volume of product *r*	MJ_j	Maximum number of manufacturing/remanufacturing centers *j* that can be established
$capd_{kt}$	Maximum storage capacity of distribution/collection center *k* in period *t*	MK_k	Maximum number of distribution/collection centers *k* that can be established
v_m	Unit volume of raw material *m*	MF_f	Maximum number of disposal centers *f* that can be established
$capl_{lt}$	Potential capacity of Type *l* transport vehicle in period *t*	MI_i	Maximum number of suppliers *i* that can be selected
$capI_{it}$	Storage capacity of supplier *i* in period *t*	$capf_{ft}$	Capacity of disposal center *f* in period *t*

TABLE 3. Decision variables used in the mathematical model

W_{it}	Equals 1 if supplier *i* is selected, 0 otherwise	TRS^{l}_{ijmt}	Quantity of raw material *m* transported from supplier *i* to manufacturing/remanufacturing center *j* using transportation system *l* in period *t*
$T1_{rjk}$	Time required for transporting each unit of product *r* from manufacturing center *j* to distribution center *k*	SP_{rjt}	Quantity of product *r* manufactured in manufacturing/remanufacturing *j* in period *t*
$T2_{rkc}$	Time required for transporting each unit of product *r* from distribution center *k* to customer *c*	TPP^{l}_{rjkt}	Quantity of product *r* transported from manufacturing/remanufacturing center *j* to distribution/collection center *k* using transportation system *l* in period *t*

$T3_{rck}$	Time required for transporting each unit of product *r* from customer *c* to collection center *k*	TPr^{l}_{rkjt}	Quantity of product *r* transported from distribution/collection center *k* to manufacturing/remanufacturing centre *j* using transportation system *l* in period *t*
$T4_{rkj}$	Time required for transporting each unit of product *r* from collection center *k* to remanufacturing center *j*	SH_{rct}	Quantity of unsatisfied demand of customer c for product r at the end of period t
SR_{mit}	Quantity of raw material *m* procured from supplier *i* in period *t*	TPD^{l}_{rkct}	Quantity of product *r* transported from distribution/ collection center *k* to customer *c* using transportation system *l* in period *t*
TPf^{l}_{rkft}	Quantity of product *r* transported from distribution/collection center *k* to disposal center *f* using transportation system *l* in period *t*	ND^{l}_{kct}	The number of trips made from distribution/collection center *k* to customer *c* using transportation system *l* in period *t*
ID_{rkt}	Inventory of product *r* at distribution/collection center *k* at the end of period *t*	NC^{l}_{ckt}	The number of trips made from customer *c* to distribution/collection center *k* using transportation system *l* in period *t*
NS^{l}_{ijt}	The number of trips made from supplier *i* to manufacturing/remanufacturing center *j* using transportation system *l* in period *t*	NN^{l}_{kjt}	The number of trips made from distribution/collection center *k* to manufacturing/remanufacturing center *j* using transportation system *l* in period *t*
NP^{l}_{jkt}	The number of trips made from manufacturing/remanufacturing center *j* to distribution/collection center *k* using transportation system *l* in period *t*	NFK^{l}_{kft}	The number of trips made from distribution/collection center *k* to disposal center *f* using transportation system *l* in period *t*
X_{jt}	Equals 1 if manufacturing/remanufacturing center *j* is established in period *t*, 0 otherwise	KC_{kc}	Equals 1 if distribution/collection center *k* is supplying customer *c*, 0 otherwise
U_{kt}	Equals 1 if distribution/collection center *k* is established in period *t*, 0 otherwise	C_{ck}	Equals 1 if customer *c* is allocated to distribution/collection center *k*, 0 otherwise
Z_{ft}	Equals 1 if disposal center *f* is established in period *t*, 0 otherwise		

$$\begin{aligned}Min\ Z_1 = &\sum_{\forall i}\sum_{\forall t}\sum_{\forall m}(pcr_{mit}\,.SR_{mit}) + \sum_{\forall l}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t}\sum_{\forall m}(rp_{mit}\,.TRS^{l}_{ijmt}) + \sum_{\forall r}\sum_{\forall j}\sum_{\forall t}(pc_{rjt}\,.SP_{rjt}) + \sum_{\forall l}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t}({fts^{l}}_{ijt}\,.NS^{l}_{ijt}) +\\ &\sum_{\forall l}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t}\sum_{\forall m}({vts^{l}}_{ijmt}\,.TRS^{l}_{ijmt}) + \sum_{\forall l}\sum_{\forall j}\sum_{\forall k}\sum_{\forall t}\left({ftp^{l}}_{jkt}\,.NP^{l}_{jkt}\right) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall j}\sum_{\forall k}\sum_{\forall t}\left({vtp^{l}}_{rjkt}\,.TPP^{l}_{rjkt}\right) +\\ &\sum_{\forall l}\sum_{\forall k}\sum_{\forall c}\sum_{\forall t}({ftd^{l}}_{kct}\,.ND^{l}_{kct}) + + \sum_{\forall l}\sum_{\forall k}\sum_{\forall c}\sum_{\forall t}({ftd^{l}}_{kct}\,.ND^{l}_{kct}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall c}\sum_{\forall t}(vtd^{l}_{rkct}.TPD^{l}_{rkct}) +\\ &\sum_{\forall l}\sum_{\forall c}\sum_{\forall k}\sum_{\forall t}({ftc^{l}}_{ckt}\,.NC^{l}_{ckt}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall c}\sum_{\forall t}({vtc^{l}}_{rckt}\,.TPC^{l}_{rckt}) +\\ &\sum_{\forall l}\sum_{\forall j}\sum_{\forall k}\sum_{\forall t}\left({ftr^{l}}_{kjt}\,.NN^{l}_{kjt}\right) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall j}\sum_{\forall t}({vtr^{l}}_{rkjt}\,.TPr^{l}_{rkjt}) +\\ &\sum_{\forall l}\sum_{\forall f}\sum_{\forall k}\sum_{\forall t}\left({ftf^{l}}_{kft}\,.NF^{l}_{kft}\right) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall f}\sum_{\forall t}\left({vtf^{l}}_{rkft}\,.TPf^{l}_{rkft}\right) + \sum_{\forall r}\sum_{\forall k}\sum_{\forall t}(hc_{rkt}\,.ID_{rkt}) + \sum_{\forall r}\sum_{\forall c}\sum_{\forall t}(shc_{rct}\,.SH_{rct}) +\\ &\sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall f}\sum_{\forall t}(dc_{rft}\,.TPf^{l}_{rkft}) + \sum_{\forall j}\sum_{\forall t}(X_{jt}\,.F_{jt}) + \sum_{\forall k}\sum_{\forall t}(U_{kt}\,.F_{kt}) + \sum_{\forall f}\sum_{\forall t}(Z_{ft}\,.F_{ft}) + \sum_{\forall i}\sum_{\forall t}(W_{it}\,.F_{i,t})\end{aligned} \tag{1}$$

$$\begin{aligned}Min\ Z_2 = &\sum_{\forall l}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t}\sum_{\forall m}(\theta 1_{\mathrm{ijmt}}.TRS^{l}_{ijmt}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall j}\sum_{\forall k}\sum_{\forall t}\theta 2_{\mathrm{rjkt}}\,.TPP^{l}_{rjkt}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall c}\sum_{\forall t}(\theta 3_{\mathrm{rkct}}\,.TPD^{l}_{rkct}) +\\ &\sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall j}\sum_{\forall t}(\,\theta 6_{rkjt}\,.TPr^{l}_{rkjt}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k}\sum_{\forall f}\sum_{\forall t}\left(\theta 5_{\mathrm{rkft}}\,.TPf^{l}_{rkft}\right)\end{aligned} \tag{2}$$

$$Max\ Z_3 = \sum_{l}\sum_{r}\sum_{t}\frac{\sum_{m}\sum_{j}\frac{\sum_{i} RI_{mit}\,.TRS^{l}_{ijmt}}{v_{\mathrm{m}}}}{\sum_{c} dem_{rct}} + \sum_{\forall t}\sum_{\forall j}(\mathrm{R1}\,.X_{jt}) + \sum_{\forall t}\sum_{\forall k}(\mathrm{R2}\,.U_{kt}) \tag{3}$$

$$\begin{aligned}Max\ Z_4 = &\sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t}(rai^{l}_{ij}\,.TRS^{l}_{ijmt}) + \sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t} rak^{l}_{jk}.TPP^{l}_{rjkt} + \sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t} rac^{l}_{kc}.TPD^{l}_{rkct} +\\ &\sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t} racw^{l}_{ck}.TPC^{l}_{rckt} + \sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t} rap^{l}_{kj}.TPr^{l}_{rkjt} + \sum_{\forall l}\sum_{\forall r}\sum_{\forall i}\sum_{\forall j}\sum_{\forall t} rao^{l}_{kf}.TPf^{l}_{rkft}\end{aligned} \tag{4}$$

$$\begin{aligned}Min\ Z_5 = &\gamma\left(\left(\sum_{\forall \mathrm{j}}\sum_{\forall r}\sum_{\forall k}\sum_{\forall t} T1_{rjk} \times TPP^{l}_{rjkt}\right) + \left(\sum_{\forall \mathrm{c}}\sum_{\forall r}\sum_{\forall k}\sum_{\forall t} T2_{rkc} \times TPD^{l}_{rkct}\right)\right) + (1-\gamma) * \left(\left(\sum_{\forall \mathrm{c}}\sum_{\forall r}\sum_{\forall k}\sum_{\forall t} T2_{rck} \times TPC^{l}_{rckt}\right) +\right.\\ &\left.\left(\sum_{\forall \mathrm{j}}\sum_{\forall r}\sum_{\forall k}\sum_{\forall t} T4_{rkj} \times TPr^{l}_{rkjt}\right)\right)\end{aligned} \tag{5}$$

s.t.

$$\sum_{\forall l}\sum_{\forall j} TRS^{l}_{ijmt} = SR_{mit} \qquad \forall\ i,t \tag{6}$$

$$\sum_{\forall l}\sum_{\forall k} TPP^{l}_{rjkt} = SP_{rjt} \qquad \forall\ r,j,t \tag{7}$$

$$\sum_{\forall l}\sum_{\forall r}\sum_{\forall k} TPP^{l}_{rjkt}\ .ur_{rmt} = \sum_{\forall l}\sum_{\forall i} TRS^{l}_{ijmt} + \sum_{\forall l}\sum_{\forall r}\sum_{\forall k} TPr^{l}_{rkjt}\,.ur_{rmt} \qquad \forall\ j,t,m \tag{8}$$

$$SH_{rct} - SH_{rc(t-1)} + \sum_{\forall l}\sum_{\forall k}(TPD^{l}_{rkct}) = dem_{rct} \qquad \forall\ r,c,t>1 \tag{9}$$

$$\sum_{\forall l}\sum_{\forall k} TPC^{l}_{rckt} = \left(dem_{rc(t-1)} - SH_{rc(t-1)}\right) \times RT_{\mathrm{r}} \qquad \forall\ r,c,t > 1 \tag{10}$$

$$\sum_{\forall l}\sum_{\forall j} TPr^{l}_{rkjt} = RR_r . \sum_{\forall l}\sum_{\forall c} TPC^{l}_{rckt} \ \forall \ r,k,t \quad (11)$$

$$\sum_{\forall l}\sum_{\forall f} TPf^{l}_{rkft} = (1 - RR_r) . \sum_{\forall l}\sum_{\forall c} TPC^{l}_{rckt} \quad \forall \ r,k,t \quad (12)$$

$$\sum_{\forall l}\sum_{\forall j} TPP^{l}_{rjkt} - ID_{rkt} + ID_{rk(t-1)} = \sum_{\forall l}\sum_{\forall c} TPD^{l}_{rkct} \quad \forall \ r,k,t > 1 \quad (13)$$

$$\sum_{\forall r} ID_{rkt} . v_r \le capd_{kt} \quad \forall \ k,t \quad (14)$$

$$v_m . TRS^{l}_{ijmt} \le capl_{lt} . NS^{l}_{ijt} \quad \forall \ l,j,i,t,m \quad (15)$$

$$\sum_{\forall r}(v_r . TPP^{l}_{rjkt}) \le capl_{lt} . NP^{l}_{jkt} \quad \forall \ l,j,k,t \quad (16)$$

$$\sum_{\forall r}(v_r . TPD^{l}_{rkct}) \le capl_{lt} . ND^{l}_{kct} \quad \forall \ l,k,c,t \quad (17)$$

$$\sum_{\forall r}(v_r . TPC^{l}_{rckt}) \le capl_{lt} . NC^{l}_{ckt} \quad \forall \ l,c,k,t \quad (18)$$

$$\sum_{\forall r}(v_r . TPr^{l}_{rkjt}) \le capl_{lt} . NN^{l}_{kjt} \quad \forall \ l,k,j,t \quad (19)$$

$$\sum_{\forall r}(v_r . TPf^{l}_{rkft}) \le capl_{lt} . NRK^{l}_{kft} \quad \forall \ l,k,f,t \quad (20)$$

$$\sum_{\forall l}\sum_{\forall j} TRS^{l}_{ijmt} \le capI_{it} \quad \forall \ i,t,m \quad (21)$$

$$\sum_{\forall l}\sum_{\forall f} TPf^{l}_{rkft} \le capf_{ft} . Z_{ft} \quad \forall \ r,k,t \quad (22)$$

$$\sum_{\forall l}\sum_{\forall k} TPP^{l}_{rjkt} \le capp_{jt} . X_{jt} \quad \forall \ r,j,t \quad (23)$$

$$\sum_{\forall l}\sum_{\forall j} TPP^{l}_{rjkt} \le capd_{kt} . U_{kt} \quad \forall \ r,k,t \quad (24)$$

$$\sum_{\forall l}\sum_{\forall c} TPD^{l}_{rkct} \le capd_{kt} . U_{kt} \quad \forall \ r,k,t \quad (25)$$

$$\sum_{\forall l}\sum_{\forall j} TPr^{l}_{rkjt} + \sum_{\forall l}\sum_{\forall f} TPf^{l}_{rkft} \le capd_{kt} . Z_{ft} \quad \forall \ r,k,t \quad (26)$$

$$\sum_{\forall l}\sum_{\forall c} TPC^{l}_{rckt} \le capdr_{kt} . Z_{ft} \quad \forall \ r,k,t \quad (27)$$

$$\sum_{\forall l}\sum_{\forall k} TPr^{l}_{rkjt} \le capr_{jt} . X_{jt} \quad \forall \ r,j,t \quad (28)$$

$$\sum_{\forall l}\sum_{\forall i} TRS^{l}_{ijmt} \le \mathrm{capI}_{it} . \mathrm{W}_{it} \quad \forall \ \mathrm{m,j,t} \quad (29)$$

$$\sum_{\forall j} X_{jt} \le MJ_j \quad \forall \ \mathrm{t} \quad (30)$$

$$\sum_{\forall k} U_{kt} \le MK_k \quad \forall \ \mathrm{t} \quad (31)$$

$$\sum_{\forall f} Z_{ft} \le MF_f \quad \forall \ \mathrm{t} \quad (32)$$

$$\sum_{\forall i} W_{it} \le MI_i \quad \forall \ \mathrm{t} \quad (33)$$

$$TPD^{l}_{rkct} = \mathrm{KC}_{kc} * dem_{rct} \quad \forall \ r,c,k,t \quad (34)$$

$$TPC^{l}_{rckt} \le C_{ck} . \mathrm{capd}_{kt} \quad \forall \ r,c,k,t \quad (35)$$

$$X_{jt}, U_{kt}, Z_{ft} \in \{0,1\}, SR_{mit}, SP_{rjt}, ID_{rkt}, TPf^{l}_{rkft}, RPf^{l}_{rk'f't} TRS^{l}_{ijt}, RRS^{l}_{ij't}, TPK^{l}_{rkjt} \geqq 0$$
$$, RPK^{l}_{rk'j't}, TPC^{l}_{rckt}, RPC^{l}_{rck't}, RPD^{l}_{rk'ct}, RPP^{l}_{rj'k't} TPD^{l}_{rkct}, SH_{rct}, TPP^{l}_{rjkt} \geqq 0 \quad (36)$$
$$NS^{l}_{ijt}, NP^{l}_{jkt}, ND^{l}_{kct}, NC^{l}_{ckt}, RNC^{l}_{ck't}, RNN^{l}_{ckt},, NN^{l}_{ckt}, NRK^{l}_{ckt}, RNS^{l}_{ij't}, RNP^{l}_{j'k't}, RND^{l}_{k'ct} \geqq 0$$

Objective function (1) of the proposed model minimizes the sum of operating and production cost and the cost of transportation of products and raw materials. Objective function (2) minimizes the CO_2 emission generated by the movement of transport vehicles between centers. Objective function (3) maximizes the reliability of demand satisfaction based on the reliability values defined for suppliers, and also maximizes the reliability of manufacturing and distribution centers. Objective function (4) maximizes the reliability of the transportation system. Objective function (5) minimizes the total time required for the products manufactured in the factory to pass through the distribution centers (via the forward supply chain) and reach the customers and also minimizes the total time required for the returned products to be transported back (through the inverse supply chain) and reach the factory. This function attempts to shorten the time it takes to serve customers in order to improve the level of service. Constraint (6) of the proposed model states that all raw materials supplied by a supplier should be transferred to the factories within the same period. Constraint (7) guarantees that all products manufactured in each factory are distributed to distribution centers within the same period. Constraint (8) urges that for each raw material, in each period, the outflow of products from a factory should be equal to the total inflow of inputs received from all suppliers and collection centers. Constraints (9) and (10) maintain the balance of shortage accrued from the past period. Constraints (11) and (12) compute the flows of returned, remanufactured and disposed products. Constraint (13) maintains the balance of inventory held in distribution centers. Constraint (14) controls the inventory remained at the distribution center by the end of each period. Constraints (15) to (20) guarantee that the capacity of the used transportation system is respected. Constraint (21) ensures that the sum of all outflows from a supplier to all manufacturing centers does not exceed its capacity. Constraints (22) to (29) ensure that the flow is restricted to the paths between the points where a facility is established or selected and also that the total flows in any facility does not exceed its capacity. Constraints (30) to (33) control the number of manufacturing/ remanufacturing centers, distribution/ collecting centers, and disposal centers. Constraint (34) states that the total quantity of products distributed via the distribution centers must be equal to total demand. Constraint (35) ensures that a product will be retrieved from a customer to be delivered to a distribution center only if that customer is assigned to that distribution center. Constraints (36) identify the types of variables.

4. SOLUTION METHODOLOGY

In this paper, NSGA-II algorithm has been proposed to handle this problem. This algorithm is solved some problems with previous algorithms [21]. In order to numerically evaluate the proposed NSGA-II for performance, we consider 5 sets of small sized and 5 sets of large sized instances Table 4. We generate 5 instances for each problem size. Table 5 shows the data range of each parameter. We implement the algorithms in MATLAB V. 7.10.0.499 and run on a Pentium IV 2.8 GHz processor with 6 GB memory. In the section, the considered problem is NP-hard and exact method can only solve small sized instances [22, 23]. All five kinds of small sized instances can be solved by GAMS, 24. 1. 3, but with increasing the number of each index of tenth small size instances, we cannot obtain any feasible solutions with use of exact methods.

We use relative percentage deviation (RPD) in order to compare the performance of NSGA-II algorithm and B&B approach [20].

TABLE 4. The sets different instances size

Instances size		**i**	**j**	**k**	**c**	**f**	**r**	**t**	**l**	**m**
small	1	1	2	2	2	2	1	2	2	2
	2	1	2	2	1	2	3	2	2	2
	3	1	2	3	2	2	1	3	2	2
	4	1	2	1	1	2	2	3	1	2
	5	3	3	2	2	3	3	3	2	2
Large	1	4	3	5	4	3	3	4	2	2
	2	4	5	5	3	3	3	4	2	2
	3	4	3	1	3	2	4	4	4	2
	4	5	3	3	2	3	4	5	4	2
	5	5	4	2	2	3	4	5	5	3

To calculate RPD for minimization objective, Equation (37) is used.

$$RPD = \frac{Alg - Min}{Min} \times 100 \qquad (37)$$

where Alg and Min are the solution of the algorithm and the optimum solution if the problem is solved in single objective manner for a given instance. To calculate RPD for maximization objective, Equation (38) is used.

$$RPD = \frac{Max - Alg}{Max} \times 100 \qquad (38)$$

where Alg and Max are the solution of the algorithm and the optimum solution if the problem is solved in single objective manner for a given instance. Table 6 show the average RPD of algorithms for each objective (Z_1, Z_2, Z_3, Z_4 and Z_5 mean objectives functions). The MILP mode only solves small sized instances.

TABLE 5. Data range of parameter

parameter	**value**	**parameter**	**value**	**parameter**	**value**	**parameter**	**Value**
pcr_{mit}	$uniform(100{,}1000)$	$vtf^{l}{}_{rkft}$	$uniform(50{,}100)$	$R1_j$	$uniform(0{,}0.5)$	v_r	$uniform(30{,}50)$
rp_{mit}	$uniform(100{,}500)$	hc_{rkt}	$uniform(20{,}60)$	$R2_k$	$uniform(0{,}0.5)$	$capd_{kt}$	$uniform(10{,}100)$
pc_{rjt}	$uniform(100{,}500)$	shc_{rct}	$uniform(40{,}60)$	RI_{mit}	$uniform(0{,}0.5)$	v_m	$uniform(30{,}50)$
$fts^{l}{}_{ijt}$	$uniform(50{,}100)$	dc_{rft}	$uniform(10{,}50)$	rai^{l}_{ij}	$uniform(0{,}0.5)$	$capl_{lt}$	$uniform(30{,}50)$
$vts^{l}{}_{ijmt}$	$uniform(50{,}100)$	F_{jt}	$uniform(100{,}150)$	rak^{l}_{jk}	$uniform(0{,}0.5)$	$capI_{it}$	$uniform(10{,}100)$
$ftp^{l}{}_{jkt}$	$uniform(50{,}100)$	F_{kt}	$uniform(100{,}150)$	rac^{l}_{kc}	$uniform(0{,}0.5)$	$capf_{ft}$	$uniform(30{,}50)$
$vtp^{l}{}_{rjkt}$	$uniform(50{,}100)$	F_{ft}	$uniform(100{,}150)$	raq^{l}_{ck}	$uniform(0{,}0.5)$	$capp_{jt}$	$uniform(10{,}100)$
$ftd^{l}{}_{kct}$	$uniform(50{,}100)$	$F_{i,t}$	$uniform(100{,}150)$	rap^{l}_{kj}	$uniform(0{,}0.5)$	$capr_{jt}$	$uniform(10{,}100)$
vtd^{l}_{rkct}	$uniform(50{,}100)$	$\theta1_{ijmt}$	$uniform(100{,}300)$	rao^{l}_{kf}	$uniform(0{,}0.5)$	MJ_j	$uniform(10{,}100)$
$ftc^{l}{}_{ckt}$	$uniform(50{,}100)$	$\theta2_{rjkt}$	$uniform(100{,}300)$	γ	0.5	MK_k	$uniform(10{,}100)$
$vtc^{l}{}_{rckt}$	$uniform(50{,}100)$	$\theta3_{rkct}$	$uniform(100{,}300)$	ur_{rmt}	$uniform(0{,}0.5)$	MF_f	$uniform(10{,}100)$
$ftr^{l}{}_{kjt}$	$uniform(50{,}100)$	$\theta4_{rckt}$	$uniform(100{,}300)$	dem_{rct}	$uniform(1{,}20)$	MI_i	$uniform(10{,}100)$
$vtr^{l}{}_{rkjt}$	$uniform(50{,}100)$	$\theta5_{rkft}$	$uniform(100{,}300)$	RT_r	$uniform(0{,}0.5)$		
$ftf^{l}{}_{kft}$	$uniform(50{,}100)$	$\theta6_{rkjt}$	$uniform(100{,}300)$	RR_r	$uniform(0{,}0.5)$		

TABLE 6. The average RPD of NSGA-II on different small and large problems

Small sized instances	Obj. function	MILP model	GAMS CPU Time	NSGA-II	CPU Time	Large sized instances	NSGA-II	CPU Time
	Z_1	16.743	0.032	38.227	0.056	**1**	43.766	0.154
	Z_2	14.233	0.057	28.672	2.566		32.077	3.776
1	Z_3	28.566	1.02	49.672	10.302		54.233	15.699
	Z_4	31.866	1.56	54.828	14.677		59.455	19.344
	Z_5	9.623	1.15	17.226	3.665		19.344	6.788
	Z_1	14.566	0.78	41.178	0.12	**2**	46.098	0.567
	Z_2	15.704	1.24	34.584	2.988		37.233	4.843
2	Z_3	32.242	2.25	54.577	12.432		61.788	21.233
	Z_4	34.233	2.58	59.014	18.603		69.908	26.244
	Z_5	11.012	1.36	22.005	4.013		25.544	9.122
	Z_1	18.856	0.88	46.066	0.463	**3**	52.322	1.122
	Z_2	17.906	1.45	39.322	3.456		45.607	7.688
3	Z_3	35.654	1.78	61.226	16.788		68.322	29.433
	Z_4	37.432	2.75	66.889	25.522		73.356	34.245
	Z_5	12.089	1.56	24.676	7.632		27.355	13.821
	Z_1	22.678	1.05	51.889	0.789	**4**	58.677	1.789
	Z_2	20.789	2.05	45.032	5.344		50.422	10.443
4	Z_3	41.789	2.89	68.454	19.533		77.344	35.566
	Z_4	43.889	3.02	72.655	31.998		81.599	41.332
	Z_5	15.322	1.68	26.877	10.455		29.843	16.455
	Z_1	27.665	1.24	57.655	1.257	**5**	63.221	2.021
	Z_2	23.543	2.35	49.543	8.433		56.411	15.588
5	Z_3	47.886	3.23	75.677	25.667		81.566	43.433
	Z_4	51.233	3.42	78.533	39.522		89.311	51.78
	Z_5	18.677	1.77	28.033	14.599		30.133	19.623

5. CONCLUSIONS

In this paper, we formulated the problem of multi-product multi-period multi-level closed-loop green supply chain planning under uncertainty. The problem was formulated as a multi-objective mixed integer nonlinear programming model. To solve the small instances of the problem under uncertainty conditions, the robust counterpart of the model was derived. For more realistic large problems, a multi-objective NSGA-II was developed. Given the small difference between the optimal solutions obtained using the GAMS software and the near-optimal solutions obtained from NSGA-II, the metaheuristic algorithm can be considered a reliable method for solving the proposed formulation, and is particularly preferable for larger problems. Future studies are recommended to test and compare the performance of other efficient multi-objective algorithms such as ant colony optimization algorithm, tabu search, simulated annealing, etc. for this problem. The incorporation of other objectives such as quality, net present value, and emission due to manufacturing, remanufacturing and disposal activities into the formulation may also contribute to the development of the literature.

6. REFERENCES

1. Hatefi, S. and Jolai, F., "Robust and reliable forward–reverse logistics network design under demand uncertainty and facility disruptions", ***Applied Mathematical Modelling***, Vol. 38, No. 9-10, (2014), 2630-2647.

2. Ramazankhani, M.-E., Mostafaeipour, A., Hosseininasab, H. and Fakhrzad, M.-B., "Feasibility of geothermal power assisted hydrogen production in iran", ***International Journal of Hydrogen Energy***, Vol. 41, No. 41, (2016), 18351-18369.

3. Pasandideh, S.H.R., Niaki, S.T.A. and Asadi, K., "Bi-objective optimization of a multi-product multi-period three-echelon supply chain problem under uncertain environments: Nsga-ii and nrga", ***Information Sciences***, Vol. 292, (2015), 57-74.

4. Khatami, M., Mahootchi, M. and Farahani, R.Z., "Benders' decomposition for concurrent redesign of forward and closed-loop supply chain network with demand and return uncertainties", ***Transportation Research Part E: Logistics and Transportation Review***, Vol. 79, (2015), 1-21.

5. Zeballos, L.J., Méndez, C.A., Barbosa-Povoa, A.P. and Novais, A.Q., "Multi-period design and planning of closed-loop supply chains with uncertain supply and demand", ***Computers & Chemical Engineering***, Vol. 66, (2014), 151-164.

6. Dai, Z. and Zheng, X., "Design of close-loop supply chain network under uncertainty using hybrid genetic algorithm: A fuzzy and chance-constrained programming model", ***Computers & Industrial Engineering***, Vol. 88, (2015), 444-457.

7. Soleimani, H., Govindan, K., Saghafi, H. and Jafari, H., "Fuzzy multi-objective sustainable and green closed-loop supply chain network design", ***Computers & Industrial Engineering***, Vol. 109, (2017), 191-203.

8. Garg, K., Kannan, D., Diabat, A. and Jha, P., "A multi-criteria optimization approach to manage environmental issues in closed loop supply chain network design", ***Journal of Cleaner Production***, Vol. 100, (2015), 297-314.

9. Pasandideh, S.H.R., Niaki, S.T.A. and Asadi, K., "Optimizing a bi-objective multi-product multi-period three echelon supply chain network with warehouse reliability", ***Expert Systems with Applications***, Vol. 42, No. 5, (2015), 2615-2623.

10. Kalaitzidou, M.A., Longinidis, P. and Georgiadis, M.C., "Optimal design of closed-loop supply chain networks with multifunctional nodes", ***Computers & Chemical Engineering***, Vol. 80, (2015), 73-91.

11. Cardoso, S.R., Barbosa-Póvoa, A.P. and Relvas, S., "Integrating financial risk measures into the design and planning of closed-loop supply chains", ***Computers & Chemical Engineering***, Vol. 85, (2016), 105-123.

12. Ning Ma, M. S., Ashayeri, P. d. i. J., and Sotirov, A. P. R., "Robust optimization of supply chain downsizing problem under bankruptcy", Tilburg University, (2010),

13. Ramezani, M., Kimiagari, A.M., Karimi, B. and Hejazi, T.H., "Closed-loop supply chain network design under a fuzzy environment", ***Knowledge-Based Systems***, Vol. 59, (2014), 108-120.

14. Jabbarzadeh, A., Haughton, M. and Khosrojerdi, A., "Closed-loop supply chain network design under disruption risks: A robust approach with real world application", ***Computers & Industrial Engineering***, Vol. 116, (2018), 178-191.

15. Jerbia, R., Boujelben, M.K., Sehli, M.A. and Jemai, Z., "A stochastic closed-loop supply chain network design problem with multiple recovery options", ***Computers & Industrial Engineering***, Vol. 118, (2018), 23-32.

16. Kim, J., Do Chung, B., Kang, Y. and Jeong, B., "Robust optimization model for closed-loop supply chain planning under reverse logistics flow and demand uncertainty", ***Journal of Cleaner Production***, Vol. 196, (2018), 1314-1328, doi: org/10.1016/j.jclepro.2018.06.157.

17. Ghahremani-Nahr, J., Kian, R. and Sabet, E., "A robust fuzzy mathematical programming model for the closed-loop supply chain network design and a whale optimization solution algorithm", ***Expert Systems with Applications***, Vol. 116, (2019), 454-471.

18. Ben-Tal, A., El Ghaoui, L. and Nemirovski, A., "Robust optimization, Princeton University Press, Vol. 28, (2009).

19. Srinivas, N. and Deb, K., "Muiltiobjective optimization using nondominated sorting in genetic algorithms", ***Evolutionary Computation***, Vol. 2, No. 3, (1994), 221-248.

20. Naderi, B., Zandieh, M., Balagh, A.K.G. and Roshanaei, V., "An improved simulated annealing for hybrid flowshops with sequence-dependent setup and transportation times to minimize total completion time and total tardiness", ***Expert Systems with Applications***, Vol. 36, No. 6, (2009), 9625-9633.

21. Fakhrzad, M., Sadeghieh, A. and Emami, L., "A new multi-objective job shop scheduling with setup times using a hybrid genetic algorithm", ***International Journal of Engineering-Transactions B: Applications,*** Vol. 26, No. 2, 207-218

22. Fakhrzad, M. and Heydari, M., "A heuristic algorithm for hybrid flow-shop production scheduling to minimize the sum of the earliness andf tardiness costs", ***Journal of the Chinese Institute of Industrial Engineers***, Vol. 25, No. 2, (2008), 105-115.

23. Hosseini-Nasab, H., Fereidouni, S., Ghomi, S.M.T.F. and Fakhrzad, M.B., "Classification of facility layout problems: A review study", ***The International Journal of Advanced Manufacturing Technology***, Vol. 94, No. 1-4, (2018), 957-977.

Mathematical Formulation and Solving of Green Closed-loop Supply Chain Planning Problem with Production, Distribution and Transportation Reliability

M. B. fakhrzad, P. Talebzadeh, F. Goodarzian

Faculty of Industrial Engineering, Yazd University, Yazd, Iran

PAPER INFO

Paper history:
Received 30 January 2018
revised form 27 October 2018
Accepted 06 November 2018

Keywords:
Green Closed-loop Supply Chain
Uncertainty
Reliability
NSGA-II Algorithm

چکیده

در این مقاله یک شبکه زنجیره تأمین حلقه-بسته سبز چندمحصولی، چنددوره‌ای، چندسطحی تحت عدم قطعیت مورد بررسی واقع شده است. مدل ارائه شده شامل پنج تابع هدف: کمینه‌سازی هزینه‌های شبکه زنجیره‌تامین، کمینه‌سازی انتشار گازهای خروجی وسیله نقلیه در بین مراکز، حداکثرسازی قابلیت اطمینان مراکز تولید، توزیع و حمل‌ونقل، حداکثرسازی نرخ قابلیت اطمینان سیستم حمل‌ونقل و در نهایت حداکثرسازی سطح سرویس‌دهی به مشتریان می‌باشد. سطوح زنجیره‌تامین این مدل شامل مراکز تأمین‌کننده، مراکز تولید/ احیا، مراکز توزیع/ جمع‌آوری، مراکز مشتریان و مراکز دفع می‌باشد. برای حل مساله از روش استوارسازی فازی و روش بهینه‌سازی چند هدفه ژنتیک با مرتب سازی نا مغلوب NSGA-II استفاده شده است. می‌توان نتیجه گرفت الگوریتم NSGA-II یک روش مطمئن برای حل مدل پیشنهادی در اندازه بزرگ مساله می‌باشد.

doi*: 10.5829/ije.2018.31.12c.10*